

Meson spectral function and screening masses in magnetized quark gluon plasma

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Abstract. We calculate the spectral function in the pseudoscalar (scalar) channel in the high-temperature phase of QCD in the presence of a background magnetic field. The spatial and temporal screening masses are determined from the long-distance behavior of the corresponding correlation functions.

1 Introduction

Under extreme conditions of temperature and baryon density, the strongly interacting matter of quantum chromodynamics (QCD) liberates a large number of degrees of freedom indicating a phase transition to a deconfined, quasi-ideal state known as the quark gluon plasma (QGP) [1]. There is an overall consensus that heavy-ion collision experiments have shown us glimpses of such a state of matter. It has been known for some time that the ultra-relativistic motion of charged particles create an intense magnetic field in the early stage of non-central heavy-ion collision. The energy scale of the magnetic field thus generated is comparable with the characteristic scale of QCD, for example, $\mathcal{B} \sim m_\pi^2/e$ at RHIC and could be as high as $\mathcal{B} \sim 10m_\pi^2/e$ at LHC [2–6]. Here m_π is pion mass in vacuum and e is the charge of the proton. An external magnetic field modifies the QCD vacuum and entails a rich spectrum of phenomena - chiral magnetic effect (CME) [2, 7–9], chiral vortical effect (CVE) [10, 11], magnetic catalysis [12–14], inverse magnetic catalysis [15, 16], modification of the phase diagram [15, 17, 18] and so on. There is a tremendous amount of activities, both theoretical and experimental, going on to understand the properties of QCD matter under an external magnetic field [19]. Apart from QCD, the effects induced by an external magnetic field are important in astrophysics [20], cosmology [21], physics beyond the standard model [22], or condensed matter physics [23].

Hadronic correlation functions are useful objects to understand the intricate dynamics of QCD [24–26]. By studying the correlation functions of colorless currents at large separation one can, in principle, determine the hadronic spectrum. At zero temperature properties of the hadronic spectra are determined by nonperturbative fluctuations of quark and gluon fields. Average characteristics of these fluctuations, called condensates, are fairly known at zero temperature. Although QCD as a confining theory is yet to be fully understood, it is a remarkable fact that the knowledge about a few condensates allows to estimate the masses of hadrons quite successfully. Temperature, density or external field modify the condensates and hence the hadronic masses and decay widths. In fact, first-principle calculation of lattice QCD established that the modification of the spectrum of QCD at high temperature is linked with the melting of quark and gluon condensates. For temperature (T) much higher than the pseudo critical temperature (T_c) of QCD, mesons of course do not exist in the medium. The large distance asymptotic of the correlators are now determined by the normal modes of excitations in the plasma.

Thus spectral functions of hadronic correlators are of topical interest. These objects encode complete information of in-medium hadron properties, transport coefficients and electromagnetic emissivity from the hot and dense plasma. Mesonic spectral functions at finite temperature have been calculated in the literature using analytic methods [27–31] or numerical simulations of lattice QCD [32–35]. Hadronic correlators in a background magnetic field have also been studied in different settings, see [36–47] for the latest developments in the field.

The purport of the present paper is to discuss the modification of mesonic spectral densities in the high-temperature deconfined phase of QCD. We work to $\mathcal{O}(\alpha_s^0)$ in the strong-coupling constant albeit the effect of magnetic field, by construction, is included to all orders. Neglect of QCD radiative corrections provides a clean benchmark to understand

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the effect of a magnetic field on the propagation of mesons and it serves to define an appropriate starting point for a refined analysis with higher-order QCD effects systematically embedded.

For brevity, we shall consider only neutral pseudo scalar (scalar) mesons of chiral quarks in this paper. We also assume that mesons are composed of only one kind of quark flavor which will be either u or d . These meson states constructed from the connected diagrams are not physical states *per se*. Nonetheless, they appear in QCD inequalities [48] and can be constructed in lattice QCD [49, 50]. Let us note that physical meson states in a background electromagnetic field are nontrivial admixture of spin eigenstates and for $N_f > 1$ one should also include the contribution from disconnected diagrams, see for example [49, 50]. However, these subtleties show up in higher-order calculation and lie outside the scope of the present paper.

Analytic studies in a background magnetic field have rarely been pushed beyond one loop and even at one-loop order the calculations are arranged for some special configurations of fields most of the time. If the magnetic field dominates other scales in the problem, then it makes sense to place the charged particles in the lowest Landau level (LLL) because states at higher Landau levels are too heavy to be excited. The virtue of the LLL approximation is that it allows complete separation of motion along the direction parallel to the magnetic field and gyromagnetic motion in the transverse space. Furthermore, it allows much simpler tensorial structure of n point functions and easy Gaussian integrations over transverse momenta of the virtual particles. In general, long-distance properties are sensitive to the LLL. However, barring special observable like chiral magnetic current or spin polarization where only the LLL contribute, restriction to the LLL brings in uncertainty in the calculation. Effect of higher Landau levels are accommodated in the loop calculation either by choosing special direction of propagation with respect to the external magnetic field ($\vec{p} \parallel$ or $\perp \vec{B}$) or through a partial resummation of arbitrary Landau levels [51] in the strong-field limit. Another kind of resummation of Landau levels is applicable when the magnetic field is weaker than pertinent mass scales in the problem. Here the resummation is equivalent to the expansion of the propagator in powers of the magnetic field [52] which is mostly useful to calculate the high frequency tail of the massive correlators [53, 54]. Such expansion coincides with the operator product expansion which has been widely used in the context of QCD sum rule calculations in nonperturbative background of color electromagnetic fields. For massless particle, the exercise of OPE needs certain care. The point is that the propagator becomes increasingly sensitive to the infrared as one moves to higher order in the expansion which is translated in the infrared sensitivity of the correlator. To save the whole scheme from doom, one needs to absorb long-distance divergences in the definition of condensates leaving short-distance contributions in coefficient functions. The procedure is known for QCD [55–57] and we have explicitly checked in the case of electromagnetic correlator that it works in the presence of a background $U(1)$ field too. We do not discuss this type of calculation here which in a sense is redundant when the complete result is known. The interested reader may see [58] for a case in this point.

2 Formalism

For definiteness, we assume a spatio-temporally constant magnetic field along the z -direction. The hadronic current is given by $J_h(\tau, \vec{x}) = \bar{q}(\tau, \vec{x}) \Gamma_h q(\tau, \vec{x})$. Here, $\Gamma_h = 1, \gamma_5$ for scalar (S), pseudo-scalar (PS), respectively.

2.1 Correlation function

Since the magnetic field breaks the isotropy of space, the in-medium correlation functions depend on longitudinal (z) and transverse (x_\perp) coordinates separately,

$$\begin{aligned} \chi_h(\tau, x_\perp, z) &= \left\langle J_h(\tau, x_\perp, z) J_h^\dagger(0, 0_\perp, 0) \right\rangle_\beta, \\ &= \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3 p}{(2\pi)^3} e^{-i(\omega_n \tau - \vec{p} \cdot \vec{x})} \chi_h(\omega_n, p_\perp, p_z). \end{aligned} \quad (1)$$

The spectral density $\sigma_h(\omega, p_\perp, p_z)$, upto possible subtractions, is defined as

$$\begin{aligned} \chi_h(\omega_n, p_\perp, p_z) &= \int_{-\infty}^{+\infty} du \frac{\sigma_h(u, p_\perp, p_z)}{u - i\omega_n}, \\ \Rightarrow \sigma_h(\omega, p_\perp, p_z) &= \frac{1}{\pi} \Im \chi_h(i\omega_n = \omega + i\epsilon, p_\perp, p_z). \end{aligned} \quad (2)$$

Correlation functions of interest can be expressed in terms of the spectral function.

– Temporal meson correlation function:

$$\begin{aligned}\chi_h^\tau(\tau, p_\perp, p_z) &= \int d^3x e^{-i\vec{p}\cdot\vec{x}} \chi_h(\tau, x_\perp, z), \\ &= \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} e^{-i\omega_n \tau} \chi_h(\omega_n, p_\perp, p_z), \\ &= \int_0^\infty d\omega \sigma_h(\omega, p_\perp, p_z) K(\omega, \tau),\end{aligned}\tag{3}$$

where, $K(\omega, \tau) = \frac{\cosh \omega(\tau - \frac{\beta}{2})}{\sinh \frac{\beta\omega}{2}}$.

– Longitudinal correlation function:

$$\begin{aligned}\chi_h^z(z) &= \int_0^\beta d\tau \int d^2x_\perp \chi_h(\tau, x_\perp, z), \\ &= \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} e^{ip_z z} \int_0^\infty d\omega \frac{\sigma_h(\omega, p_\perp = 0, p_z)}{\omega}.\end{aligned}\tag{4}$$

– Transverse plane correlation function

$$\begin{aligned}\chi_h^{xy}(\vec{x}_\perp) &= \int_0^\beta d\tau \int dz \chi_h(\tau, x_\perp, z), \\ &= \int \frac{d^2p_\perp}{(2\pi)^2} e^{i\vec{p}_\perp \cdot \vec{x}_\perp} \int_0^\infty d\omega \frac{\sigma_h(\omega, p_\perp, p_z = 0)}{\omega}.\end{aligned}\tag{5}$$

If the spectrum of the theory is characterized by simple poles, then the corresponding Fourier transforms will feature exponential fall off at long distance. The inverse of the characteristic range of the correlation function is called the screening mass. In the chiral limit, temporal and spatial screening masses are equal in free theory and given by $m_{scr} = 2\pi T$ which has simple interpretation as arising due to independent propagation of two quarks. If we neglect QCD effects, equality of the screening masses still hold for longitudinal and temporal directions even when a magnetic field is present. In the lowest order of perturbation theory, the screening masses are in fact independent of magnetic field and given by the free theory value. This is a consequence of the fact that the long-distance properties of the correlator are determined by the LLL which is independent of \mathcal{B} . It will be shown later that the transverse-plane correlator shows a Gaussian fall-off at large distance with a mass scale $\sim \sqrt{|q_f \mathcal{B}|}$, where q_f is the charge of the flavor f . This is not a screening behavior per se, but reminiscent of the magnetic confinement in the transverse plane with a characteristic scale $r_\perp \sim 1/\sqrt{|q_f \mathcal{B}|}$.

2.2 Fermion propagator

The exact charged fermion propagator in a homogeneous external field can be written as

$$\widetilde{S}_f(x, x') = e^{\lambda(x, x')} \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-x')} S_f(k).\tag{6}$$

Here $S_f(k)$ is the translation and gauge-invariant part of the fermion propagator in a background potential $A_\mu^{ext}(x)$. The holonomy factor $\lambda(x, x')$ breaks gauge and translation invariance. The explicit form of λ is not important here, it drops out in a gauge-invariant calculation.

S_f can be decomposed as sum over the discrete Landau levels [59, 60],

$$iS_f(k) = ie^{-\rho} \sum_{n=0}^{\infty} (-1)^n D_n(k_\parallel, k_\perp) \Delta_f(k_\parallel, \epsilon_n),\tag{7a}$$

$$D_n(k_\parallel, k_\perp) = 2 \left(k_\parallel + m \right) \left(\mathcal{P}^- L_n(2\rho) - \mathcal{P}^+ L_{n-1}(2\rho) \right) - 4k_\perp L_{n-1}^1(2\rho),\tag{7b}$$

$$\Delta_f(k_\parallel, \epsilon_n) = \left(k_\parallel^2 - (\epsilon_n)^2 \right)^{-1} = \left(k_\parallel^2 - m_f^2 - 2n|q_f \mathcal{B}| \right)^{-1}.\tag{7c}$$

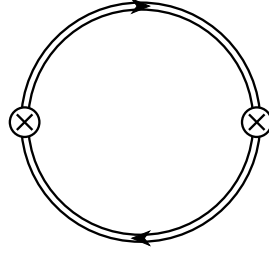


Fig. 1. Magnetically confined but otherwise free meson correlator. Double lines represent the exact fermion propagator in a magnetic field.

Our notation here is as follows: the four-vectors are decomposed into components parallel and perpendicular to magnetic field, $a^\mu = a_\parallel^\mu + a_\perp^\mu$, where $a_\parallel^\mu = (a^0, 0, 0, a^3)$ and $a_\perp^\mu = (0, a^1, a^2, 0)$. The metric tensor is written as $g^{\mu\nu} = g_\parallel^{\mu\nu} + g_\perp^{\mu\nu}$, where $g_\parallel^{\mu\nu} = \text{diag}(1, 0, 0, -1)$ and $g_\perp^{\mu\nu} = \text{diag}(0, -1, -1, 0)$. The scalar product naturally splits as $a \cdot b = (a \cdot b)_\parallel + (a \cdot b)_\perp$ where $(a \cdot b)_\parallel = a^0 b^0 - a^3 b^3$ and $(a \cdot b)_\perp = -(a^1 b^1 + a^2 b^2)$. Let us also note that q_f and m_f are the charge and mass of the fermion, respectively. We have taken $\rho = \mathbf{k}_\perp^2 / |q_f \mathcal{B}|$. $P^\pm = \frac{1}{2}(1 \pm i\gamma^1 \gamma^2 \text{sgn}(|q_f \mathcal{B}|))$ are spin projection operators along the magnetic-field direction. L_n^α are associated Laguerre polynomials. By definition, $L_n = 0$, if $n < 0$.

3 Spectral function

The correlation function is given by the convolution of fermion propagators (fig. 1). Using the propagator (7) the pseudoscalar correlator can be recast in the form

$$\begin{aligned} \chi_{ps}(\omega, p_\perp, p_z) &= -4N_c \sum_{rs} \int_{k_\parallel} \int \frac{d^2 k_\perp}{(2\pi)^2} (-1)^{r+s} e^{-(\rho_k + \rho_q)} \Delta_f(k_\parallel, \epsilon_r) \Delta_f(q_\parallel, \epsilon_s) \\ &\quad \times [\mathcal{F}_{r,s} \{L_r(2\rho_k) L_s(2\rho_q) + L_{r-1}(2\rho_k) L_{s-1}(2\rho_q)\} - 16(k \cdot q)_\perp L_{r-1}^1(2\rho_k) L_{s-1}^1(2\rho_q)] + \mathcal{C}, \\ \text{with } \mathcal{F}_{r,s} &= (s_\parallel - 2|q_f \mathcal{B}|r - 2|q_f \mathcal{B}|s). \end{aligned} \quad (8)$$

\mathcal{C} represents terms without discontinuities. Here $s_\parallel = \omega^2 - p_z^2$ and the sum integral stands for

$$\int \frac{d^2 k_\parallel}{(2\pi)^2} \rightarrow iT \sum_{k_0} \int \frac{dk_z}{2\pi} = i \int_{k_\parallel}. \quad (9)$$

The frequency sum in (8) is most conveniently done using the mixed representation of the propagator

$$\Delta_f(\tau) = T \sum_{k_0} e^{-k_0 \tau} \Delta_f(k_\parallel, \epsilon_r), \quad (10)$$

where

$$\Delta_f(\tau) = -\frac{1}{2\omega_r(k)} \left[\left(1 - n_f(\omega_r(k)) e^{-\omega_r(k)\tau}\right) - n_f(\omega_r(k)) e^{\omega_r(k)\tau} \right]. \quad (11)$$

Here $\omega_r(k) = \sqrt{k_z^2 + \epsilon_r^2}$ and n_f is Fermi-Dirac distribution function. Let us define

$$\mathcal{T}_{r,s}^{\alpha,\beta,\gamma} = (-1)^{r+s} \int \frac{d^2 k_\perp}{(2\pi)^2} e^{-(\rho_k + \rho_q)} L_r^\alpha(2\rho_k) L_s^\beta(2\rho_q) (\vec{k}_\perp \cdot \vec{q}_\perp)^\gamma, \quad (12a)$$

$$\mathcal{I}_{r,s} = - \int_{k_\parallel} \Delta_f(k_\parallel, \epsilon_r) \Delta_f(q_\parallel, \epsilon_s). \quad (12b)$$

Then the spectral function can be written as

$$\sigma_{ps}(\omega, \vec{p}_\perp, p_z) = -4N_c \sum_{rs} \left[\mathcal{F}_{rs} \left(\mathcal{T}_{r,s}^{0,0,0} + \mathcal{T}_{r-1,s-1}^{0,0,0} \right) + 16\mathcal{T}_{r-1,s-1}^{111} \right] \Im \left(\frac{\mathcal{I}_{r,s}}{\pi} \right). \quad (13)$$

Similarly the spectral function in the scalar channel can be written as

$$\sigma_s(\omega, \vec{p}_\perp, p_z) = -4N_c \sum_{rs} \left[\mathcal{G}_{rs} \left(\mathcal{T}_{r,s}^{0,0,0} + \mathcal{T}_{r-1,s-1}^{0,0,0} \right) + 16\mathcal{T}_{r-1,s-1}^{111} \right] \Im \left(\frac{\mathcal{I}_{r,s}}{\pi} \right), \quad (14)$$

where $\mathcal{G}_{rs} = (s_\parallel - 4m^2 - 2|q_f \mathcal{B}|r - 2|q_f \mathcal{B}|s)$.

Expressions for $\Im(\mathcal{I}_{r,s})$ and \mathcal{T} integrals are worked out in appendices. Let us note that $\mathcal{I}_{r,s}$ are property of the longitudinal (\parallel) space, whereas \mathcal{T} integrals belong to the transverse (\perp) space. This near complete factorization makes the interpretation of the spectral function clear. The correlator in (8) can be thought of as superposition of mesonic states with quark-antiquark pair in (r, s) Landau levels. Each such mesonic state has its own spectral density $\sigma_{r,s}$. $\sigma_{r,s}$ consist of annihilation contribution and scattering contribution which is typical of a thermal medium. What is different in the magnetized plasma is the dimensional reduction. Since the discontinuity of the correlator is determined by $\mathcal{I}_{r,s}$, the structure of the spectral function is essentially that of the two-dimensional field theory in the $(0, 3)$ -plane. The gyromagnetic motion of the charged particles in the transverse plane does not lead to any new cut in the energy plane and its effect on the cut structure is subtle. Let us note that the quantized momentum of the charged particles in the transverse plane acts like a mass term for motion in the longitudinal direction. Thus the background magnetic field, in a sense, acts just like a medium and it shifts the location of cuts in the energy plane by endowing quarks an effective mass $\sim \sqrt{q_f B}$.

$\mathcal{F}_{r,s}$ and \mathcal{T} integrals can be explained in the same way as the corresponding expressions in the free theory [61]. The magnetic field modifies the scattering amplitudes and these modifications are contained in the \mathcal{T} integrals. $\mathcal{F}_{r,s}$ can be obtained from the free theory by dropping all references to transverse dynamics ($s \rightarrow s_{\parallel}$) and augmenting the quark mass by the quantized transverse momentum ($m^2 \rightarrow m^2 + 2l|q_f B|$).

We note that the spectral function of pseudoscalar and scalar channels is degenerate in the chiral limit $\sigma_{ps} = \sigma_s$ although fermions of the theory became “massive”. Thus the magnetically generated mass does not lead to chiral-symmetry breaking, at least it is not captured in the lowest order of perturbation theory.

The structure of the spectral function is much simplified in two special circumstances - 1) when momentum of the meson is aligned with the magnetic field or 2) when the quark-antiquark pair occupies the LLL. Let us pause for a while to discuss these two special cases before we disseminate the results.

3.1 Spectral function for $\vec{p}_{\perp} = 0$

Let us set $\vec{p}_{\perp} = 0$ in (13). \mathcal{T} integrals now reduce to normalization integrals (see (B.4)),

$$\mathcal{T}_{r,s}^{0,0,0} = \frac{|q_f B|}{8\pi} \delta_{r,s}, \quad \mathcal{T}_{r,s}^{1,1,1} = \frac{|q_f B|^2}{16\pi} \delta_{r,s}. \tag{15}$$

Substituting (15) in (8), we get

$$\chi^{ps}(\omega, p_z) = -N_c \frac{|q_f B|}{2\pi} \sum_r (2 - \delta_{r,0}) \mathcal{F}_{rr} \mathcal{I}_{r,r} - N_c \frac{4|q_f B|^2}{\pi} \sum_r r \mathcal{I}_{r,r}. \tag{16}$$

The magnetic-field-dependent contribution in the first term from \mathcal{F}_{rr} cancels a similar contribution in the second term. The spectral function now follows as

$$\chi^{ps}(\omega, p_z) = -N_c \frac{|q_f B|}{2\pi} s_{\parallel} \sum_r (2 - \delta_{r,0}) \mathcal{I}_{r,r}. \tag{17}$$

Using (A.2), the spectral function follows as

$$\begin{aligned} \sigma_{ps}(\omega, p_z) &= N_c \frac{|q_f B|}{8\pi^2} \sum_r (2 - \delta_{r,0}) \theta(s_{\parallel} - 4\epsilon_r^2) \frac{1}{\sqrt{1 - \frac{4\epsilon_r^2}{s_{\parallel}}}} (1 - n_f(\omega_r^+) - n_f(\omega_r^-)) \\ &+ N_c \frac{|q_f B|}{8\pi^2} \sum_r (2 - \delta_{r,0}) \theta(-s_{\parallel}) \frac{1}{\sqrt{1 - \frac{4\epsilon_r^2}{s_{\parallel}}}} (n_f(\tilde{\omega}_r^+) - n_f(-\tilde{\omega}_r^-)), \end{aligned}$$

where $\omega_r^{\pm} = \frac{\omega}{2} \pm \frac{p_z}{2} \sqrt{1 - \frac{4\epsilon_r^2}{s_{\parallel}}}$, $\tilde{\omega}_r^{\pm} = \frac{\omega}{2} \pm \frac{|p_z|}{2} \sqrt{1 - \frac{4\epsilon_r^2}{s_{\parallel}}}$. (18)

In the limit of very weak field, the difference in energy between adjacent Landau levels becomes very small and r in this case can be taken as a continuous variable. Replacing summation over r by an integration and using the following identities:

$$n_f(x) = \sum_{r=1}^{\infty} (-1)^{r+1} e^{-rx}, \tag{19}$$

$$\log(1 + e^{-x}) = \sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r} e^{-rx}, \tag{20}$$

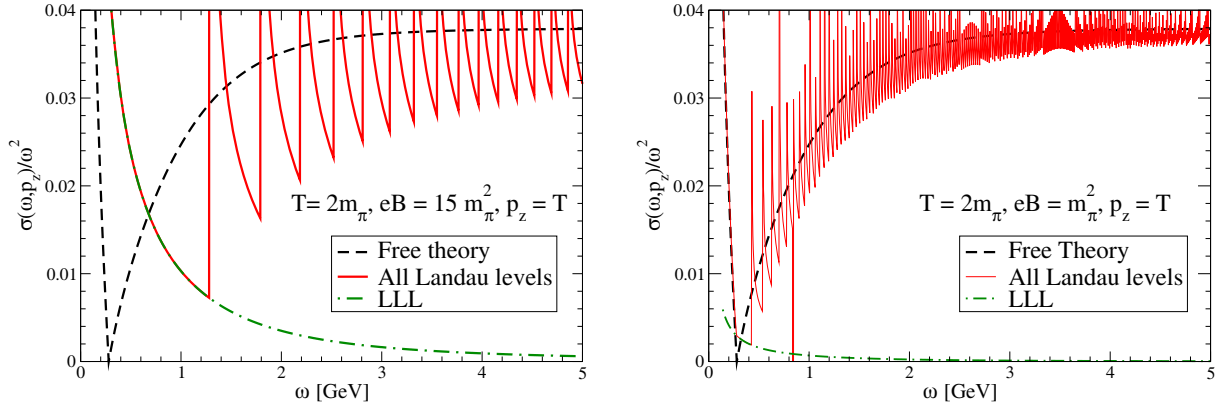


Fig. 2. Spectral function in the pseudoscalar channel for two different values of field strength. The meson state is assumed to be made up of massless u quarks.

it is not difficult to show that in the limit of very weak field, the spectral function is approximated by free field value

$$\sigma_{ps}(\omega, p_z) \Big|_{|q_f \mathcal{B}| \rightarrow 0} \simeq \frac{N_c}{8\pi^2} \frac{T}{|p_z|} \left[\theta(s_{\parallel} - 4m^2) \log \frac{\cosh \frac{\beta \tilde{\omega}_+}{2}}{\cosh \frac{\beta \tilde{\omega}_-}{2}} + \theta(-s_{\parallel}) \log \frac{(1 + e^{-\beta \tilde{\omega}_+})}{(1 + e^{\beta \tilde{\omega}_-})} \right]. \quad (21)$$

3.2 Spectral function in the strong field limit

Suppose all mass scales in the problem are smaller than $|q_f \mathcal{B}|$. We may assume that quarks are occupying the lowest Landau level, which is the lowest energy state. The fermion propagator in LLL is obtained by setting $n = 0$ in (7),

$$iS_f^{\text{LLL}}(k) = 2ie^{-\frac{\mathbf{k}_{\perp}^2}{|q_f \mathcal{B}|}} (\not{k}_{\parallel} + m_f) \mathcal{P}^- \Delta_f(k_{\parallel}). \quad (22)$$

Apart from spin projection operator, $(\not{k}_{\parallel} + m_f) \Delta_f(k_{\parallel})$ is just the free particle propagator in \parallel space. We notice that in the lowest Landau level, the dynamics in \parallel and \perp space have been completely separated at the propagator level. The spectral function can be obtained from (A.2) and (B.11) as

$$|\sigma_s| = |\sigma_{ps}| = \frac{|q_f \mathcal{B}|}{4\pi^2} \frac{e^{-\frac{p_{\perp}^2}{2|q_f \mathcal{B}|}}}{\left(\sqrt{1 - \frac{4m_f^2}{s_{\parallel}}} \right)} (1 - n_F(\omega_+) - n_f(\omega_-)) \theta(s_{\parallel} - 4m_f^2). \quad (23)$$

3.3 Results for spectral function

In fig. 2, we show the spectral function in the limit $p_{\perp} = 0$. The sawtooth nature of the spectral function is due to singularities at particle thresholds. Physically at the threshold point the meson is unstable with respect to the decay into a quark-antiquark pair. The origin of these singularities is dimensional reduction and infinitesimally narrow Landau levels which are artefact of lowest order of perturbation theory. Let us note that the peaks become narrower and their number increases as the intensity of the magnetic field decreases. This is easy to understand. For a given ω , the highest Landau level that contributes to the spectral function is $r_{\text{max}} = \lfloor (s_{\parallel} - 4m^2)/(8|q_f \mathcal{B}|) \rfloor$ which increases with dwindling magnetic field. On the other hand, in the high-frequency tail of the spectral function, the spacing between two successive peaks is given by $\delta\omega \simeq |q_f \mathcal{B}|/4\omega$ ($\omega \gg |q_f \mathcal{B}|$), which decreases when the magnetic field decreases or the frequency increases.

We show in fig. 3 the spectral function for nonzero transverse momentum. The results are similar in nature to those of fig. 2 but more spiky and stay above the corresponding result at $p_{\perp} = 0$. These can be understood as a consequence of the availability of more decay channels at nonzero p_{\perp} .

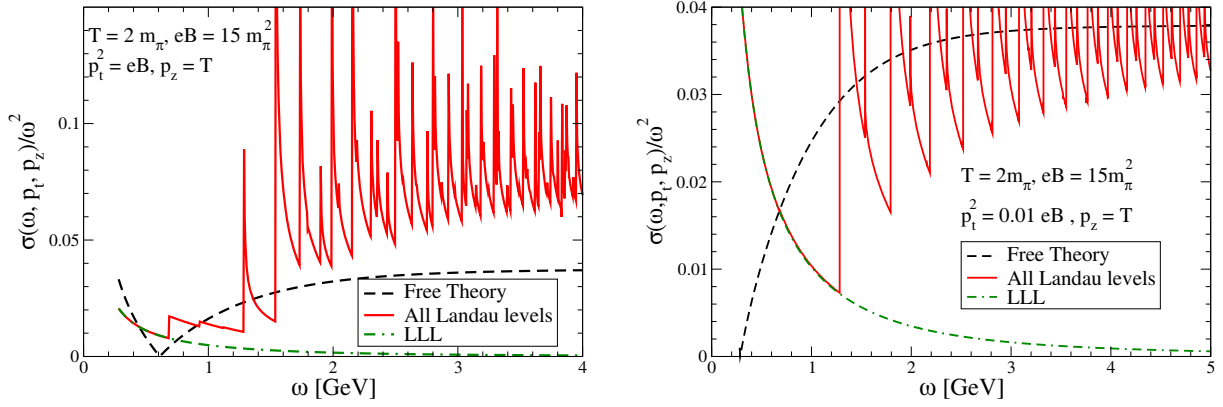


Fig. 3. Same as fig. 2 but for nonzero p_{\perp} (p_{\perp} in the figures has the same meaning as p_{\perp}) and only annihilation contribution is shown in the magnetized case. For $p_{\perp} \rightarrow 0$ we should get back fig. 2. This is shown in the right panel.

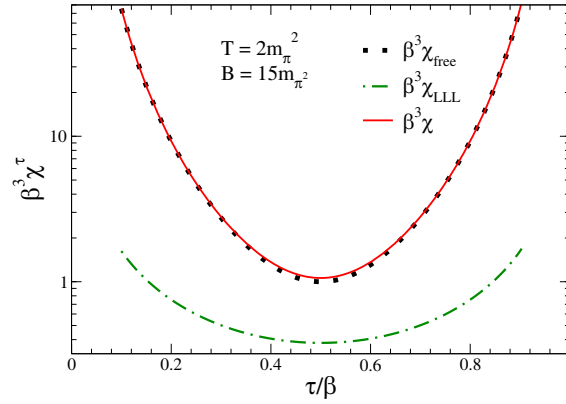


Fig. 4. Pseudoscalar correlator made up of up-quark in the presence of a magnetic field.

4 Screening masses

At zero momenta, the spectral function is obtained from

$$\sigma(\omega) = N_c \sum_l (2 - \delta_{l0}) \frac{|q_f \mathcal{B}|}{4\pi^2} \theta(\omega^2 - 8l|q_f \mathcal{B}|) \frac{\omega}{\sqrt{\omega^2 - 8l|q_f \mathcal{B}|}} \tanh \frac{\beta\omega}{4}. \quad (24)$$

We notice that far away from the threshold, the spectral function for each mode consist of a frequency-independent part together with power-suppressed corrections. This is a consequence of dimensional reduction and is in contrast to the free field limit where the spectral function grows as ω^2 .

Now, from (3) and (24), the temporal correlation function can be written as

$$\chi^\tau = \chi_0^\tau + \sum_{l>0} \chi_l^\tau, \quad (25)$$

where χ_0^τ is the correlator with the LLL approximation. χ_0^τ is obtained as

$$\tilde{\chi}_0^\tau = N_c \frac{|q_f \mathcal{B}|}{2\pi^2 T^2} (1 - 2\tilde{\tau}) \frac{\pi}{\sin(2\pi\tilde{\tau})}, \quad (26)$$

where $\tilde{\tau} = \tau/\beta$ and $\tilde{\chi}^\tau = \beta^3 \chi^\tau$.

We show the temporal correlator in the presence of a magnetic field in fig. 4. The effect of the magnetic field on the temporal correlator is marginal even for extreme value of the field achievable in the heavy-ion collision. Let us also note that the contribution of the LLL on the temporal correlator is small and an all-order summation over the Landau levels is necessary. The screening mass in the time direction, on the other hand, is dictated by the LLL. The screening mass is given by $m_{\text{scr}}^\tau = 2\pi T$ which is same as in the free field theory. This behavior is understood as a consequence of the \mathcal{B} -independence of the LLL.

Now the immunity of χ^τ to the exposure of the \mathcal{B} -field can be understood in the following way. Using the Euler-Mclaurin kind of relation between the sum of series and the integral of a function [62], it can be shown that

$$\chi_{free}^\tau - \chi_0^\tau \leq \chi^\tau \leq \chi_{free}^\tau + \chi_0^\tau, \quad (27)$$

where χ_{free} is the correlator in the absence of magnetic field [27],

$$\begin{aligned} \tilde{\chi}_{free}^\tau &= \beta^3 \chi_{free}^\tau = \frac{N_c}{\pi^2} \partial_{\tilde{\tau}^2} \left[(1 - 2\tilde{\tau}) \frac{\pi}{\sin(2\pi\tilde{\tau})} \right] \\ &= 2N_c \frac{\cos(2\pi\tilde{\tau})}{\sin^2(2\pi\tilde{\tau})} + N_c \pi (1 - 2\tilde{\tau}) \frac{(1 + \cos^2(2\pi\tilde{\tau}))}{\sin^3(2\pi\tilde{\tau})}. \end{aligned} \quad (28)$$

We have checked that inequality (27) is satisfied in our case. Thus the change in the spectral function $\delta\chi = \chi - \chi_{free} \simeq \chi_0$, and it is small. Let us note that the derivation of (27) rests on the assumption that the correlator can be written as a sum over Landau levels and that it is a smooth function of the quantum number of orbital motion taken as a continuous variable. These are not overly restrictive assumptions and relations similar to (27) presumably hold in general.

The transverse plane correlator in the LLL can be written from (5) and (23) as

$$\chi^{xy}(\vec{x}_\perp) = N_c \frac{|q_f \mathcal{B}|}{4\pi^2} \int_0^\infty \frac{d\omega}{\omega} \tanh\left(\frac{\beta\omega}{4}\right) \int \frac{d^2 p_\perp}{(2\pi)^2} e^{i\vec{p}_\perp \cdot \vec{x}_\perp} e^{-\frac{p_\perp^2}{2|q_f \mathcal{B}|}}. \quad (29)$$

The integration over transverse coordinates is a Gaussian one. The frequency integration is logarithmically divergent and hence needs regularization. This is done by subtracting the zero temperature divergence from (29). Note that the zero temperature divergence is not identical with vacuum divergence ($\mathcal{B} = 0, T = 0$) in (29) since the quarks are dressed by the magnetic field. We isolate the divergence by placing a cutoff in the frequency integration,

$$\int_0^\infty \frac{d\omega}{\omega} \tanh\frac{\beta\omega}{4} = \log\omega \tanh\frac{\beta\omega}{4} \Big|_0^A - \frac{\beta}{4} \int_0^\infty d\omega \log\omega \operatorname{sech}^2\frac{\beta\omega}{4}. \quad (30)$$

The first term on the right of (30) $\approx \log A$ for $A \gg T$, which is subtracted. The integration in the second term is convergent and hence the upper limit of integration can be set to ∞ . It can be shown that

$$\int_0^\infty dx \log x \operatorname{sech}^2 ax = -\frac{\log\left(\frac{4a}{\pi} e^{\gamma_E}\right)}{a}, \quad a > 0, \quad (31)$$

where γ_E is the Euler-Mascheroni constant. Hence the correlator can be written as

$$\chi^{xy}(\vec{x}_\perp) = N_c \log\left(\frac{\beta e^{\gamma_E}}{\pi}\right) \frac{|q_f \mathcal{B}|^2}{8\pi^3} \exp\left(-\frac{1}{2}|q_f \mathcal{B}| x_\perp^2\right). \quad (32)$$

The correlator decays in the transverse direction with a characteristic range $\sqrt{|q_f \mathcal{B}|}$. But as alluded, this Gaussian falloff is not characteristics of the typical screening behavior but rather a manifestation of magnetic confinement.

Using the following identity:

$$n_f(x) = \frac{1}{2} - 2 \sum_{l=1}^{\infty} \frac{x}{(2l-1)^2 \pi^2 + x^2}, \quad (33)$$

we obtain the longitudinal correlator from (4) and (23) as

$$\chi^z(z) = N_c \frac{|q_f \mathcal{B}|}{4\pi\beta} \sum_{l=1}^{\infty} \exp\left(-2\pi \frac{(2l-1)|z|}{\beta}\right). \quad (34)$$

At large distance the $l = 1$ contribution dominates in (34). Thus the longitudinal screening mass m_{scr}^z is $2\pi T$ which coincides with m_{scr}^τ . Let us note, however, that the asymptotic behavior of the correlator is different from free theory.

5 Outlook

We have derived an analytic expression for the spectral function in the pseudoscalar (scalar) channel in the deconfined and magnetized phase of QCD and found spatial and temporal screening masses. While the results presented herein are nontrivial as no approximation has been made regarding the kinematics or strength of the magnetic field, it would be of interest to include QCD corrections. The strong interaction of the quarks will inevitably mix the Landau levels [63], fuzzi the distinction between \parallel and \perp spaces and smooth out the threshold singularities.

The present analysis can be straightforwardly extended to other channels. We note that the spectral function in the vector channel gives the emissivity of the plasma [36, 64]. The calculation presented herein will be useful in the calculation of a host of transport coefficients [64–66] and susceptibilities [67, 68] in the presence of a homogeneous magnetic field. Also it would be interesting to develop a framework to take into account a general background of inhomogeneous and time-dependent electromagnetic field in the calculation. Work along this direction is in progress and will be reported elsewhere. The interested reader may see [69–71] for a glimpse of closely related problems.

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Appendix A. Imaginary part of one-loop self-energy

The integration over k_z can easily be done with the help of the well known relation

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|\frac{\partial f(x)}{\partial x}|_{x_i}}. \quad (\text{A.1})$$

x_i is simple zero of $f(x)$, $f(x_i) = 0$. The imaginary part of $\mathcal{I}_{r,s}$ is given by

$$\begin{aligned} \Im \left(\frac{\mathcal{I}_{r,s}}{\pi} \right) &= -\frac{1}{8\pi} \int \frac{dk_z}{\omega_r(k)\omega_s(q)} \left[(1 - n_f(\omega_r(k)) - n_f(\omega_s(q))) \left\{ \delta(\omega - \omega_r(k) - \omega_s(q)) - \delta(\omega + \omega_r(k) + \omega_s(q)) \right\} \right. \\ &\quad \left. + (n_f(\omega_r(k)) - n_f(\omega_s(q))) \left\{ \delta(\omega - \omega_r(k) + \omega_s(q)) - \delta(\omega + \omega_r(k) - \omega_s(q)) \right\} \right], \end{aligned} \quad (\text{A.2})$$

where, $\omega_r(k) = \sqrt{k_z^2 + \epsilon_r^2}$ and $\omega_s(q) = \sqrt{q_z^2 + \epsilon_s^2}$. ϵ_r is transverse mass in the r -th Landau level, $\epsilon_r^2 = m^2 + 2r|q_f \mathcal{B}|$. Let us introduce, $\chi_{r,s} = \frac{1}{2}(1 + \frac{\epsilon_r^2 - \epsilon_s^2}{s_{\parallel}})$, $\lambda_{rs} = \lambda_{sr} = \frac{1}{2}A^{1/2}(1, \frac{\epsilon_r^2}{s_{\parallel}}, \frac{\epsilon_s^2}{s_{\parallel}})$ where, $A(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ is the triangle function. We also take $\omega_{rs}^{\pm} = \chi_{rs}\omega \pm \lambda_{rs}p_z$, $\tilde{\omega}_{rs}^{\pm} = \chi_{rs}\omega \pm \lambda_{rs}|p_z|$. The imaginary part can be written as

$$\begin{aligned} \Im \left(\frac{\mathcal{I}_{r,s}}{\pi} \right) &= -\theta \left(s_{\parallel} - (\epsilon_r + \epsilon_s)^2 \right) \frac{1}{8\pi s_{\parallel} \lambda_{rs}} \left(2 - n_f(\omega_{rs}^+) - n_f(\omega_{rs}^-) - n_f(\omega_{sr}^+) - n_f(\omega_{sr}^-) \right) \\ &\quad + \left(\theta(s_{\parallel}) - \theta \left(s_{\parallel} - (\epsilon_r - \epsilon_s)^2 \right) \right) \frac{1}{4\pi s_{\parallel} \lambda_{rs}} \left(2 - n_f(\omega_{rs}^+) - n_f(\omega_{rs}^-) - n_f(\omega_{sr}^+) - n_f(\omega_{sr}^-) \right) \\ &\quad - \theta^*(\epsilon_r - \epsilon_s) \theta(-s_{\parallel}) \frac{1}{8\pi s_{\parallel} \lambda_{rs}} \left(n_f(\tilde{\omega}_{rs}^+) - n_f(-\tilde{\omega}_{sr}^-) \right) \\ &\quad - \theta^*(\epsilon_s - \epsilon_r) \theta(-s_{\parallel}) \frac{1}{8\pi s_{\parallel} \lambda_{rs}} \left(n_f(\tilde{\omega}_{sr}^+) - n_f(-\tilde{\omega}_{rs}^-) \right) \\ &\quad - \delta_{\epsilon_r, \epsilon_s} \theta(-s_{\parallel}) \frac{1}{4\pi s_{\parallel} \lambda_{rs}} \left(n_f(\tilde{\omega}_{rr}^+) - n_f(-\tilde{\omega}_{rr}^-) \right), \end{aligned} \quad (\text{A.3})$$

where $\theta^*(x)$ is Heaviside theta function with the condition that $\theta^*(0) = 0$.

Appendix B. Evaluation of \mathcal{T} integrals

The \mathcal{T} integrals in the main text have following structure:

$$\mathcal{T}_{m,n}^{\alpha,\beta,\gamma} = (-1)^{m+n} \int \frac{d^2 k_{\perp}}{(2\pi)^2} e^{-(\rho_k + \rho_q)} L_m^{\alpha}(2\rho_k) L_n^{\beta}(2\rho_q) (\vec{k} \cdot \vec{q})^{\gamma}. \quad (\text{B.1})$$

Let us scale momentum variables as $x = \frac{2k^2}{|q_f B|}$ and $\xi = \frac{2p^2}{|q_f B|}$ in (B.1). We can write

$$\mathcal{T}_{m,n}^{\alpha,\beta,\gamma} = (-1)^{m+n} \frac{|q_f B|}{16\pi^2} \left(\frac{|q_f B|}{2} \right)^\gamma e^{-\frac{\xi}{2}} \chi_{m,n}^{\alpha,\beta,\gamma}, \tag{B.2}$$

where

$$\chi_{m,n}^{\alpha,\beta,\gamma} = \int_0^\infty dx \int_0^{2\pi} d\phi e^{-(x-\sqrt{x\xi} \cos \phi)} L_m^\alpha(x) L_n^\beta(x + \xi - 2\sqrt{x\xi} \cos \phi) \left(x - \sqrt{x\xi} \cos \phi \right)^\gamma. \tag{B.3}$$

Equation (B.3) is defined for arbitrary positive values of α, β or γ . For our purpose, we need to evaluate a small subset of this where $\alpha = \beta = \gamma \in \{0, 1\}$. When $\xi = 0$, $\chi_{m,n}^\alpha = \chi_{m,n}^{\alpha,\alpha,\alpha}$ express the orthogonality relation for Laguerre polynomials,

$$\chi_{m,n}^{\alpha,\alpha,\alpha} = 2\pi \frac{\Gamma(\alpha + n + 1)}{\Gamma(n + 1)} \delta_{m,n}. \tag{B.4}$$

For nonzero value of ξ deterministic numerical integrator perform fairly well to evaluate diagonal elements χ_{mm}^α for moderate value of ξ . For extreme values of ξ or when $|m - n|$ is large, the success of cubature routines are uncertain due to the oscillatory nature of the integrand in (B.3). We can, however, circumvent this problem by expanding χ in a polynomial basis. Since χ is an analytic function of ξ , such an expansion is always possible and it provides a fast, stable and accurate method for the numerical evaluation of (B.3). In addition to this, analytic expressions for χ may prove to be useful to test the accuracy of the results obtained from approximate forms of propagators.

We assume a Laguerre-Fourier expansion of χ ,

$$\chi(\xi) = \sum_{l=0}^\infty c_l^\delta L_l^\delta(\xi), \tag{B.5}$$

and our task boils down to finding out the coefficients c_l^δ which we will do in a heuristic way.

Let us first consider χ_{mn}^0 . The strategy here is to exponentiate the angular dependence in one of the Laguerre polynomial in (B.3) using a nice addition formula due to Bateman [72],

$$e^{\sqrt{x\xi} e^{i\phi}} L_n \left(x + \xi - 2\sqrt{x\xi} \cos \phi \right) = \sum_{k=0}^\infty \left(\sqrt{x\xi} e^{i\phi} \right)^{k-n} \frac{n!}{k!} L_n^{k-n}(x) L_n^{k-n}(\xi). \tag{B.6}$$

We multiply both sides of (B.6) by $e^{-i\sqrt{x\xi} \sin \phi}$ and substitute it in (B.3). The innermost angular integration can be performed using Sommerfeld’s representation of the Bessel function,

$$J_\nu(u) = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{iu \sin \phi - i\nu\phi}, \quad \nu \text{ is an integer.} \tag{B.7}$$

The x integration then can be done with the following identity [73]:

$$\int_0^\infty dx e^{-x} (\sqrt{x})^{\gamma+\lambda} J_{\gamma+\lambda}(b\sqrt{x}) L_m^\gamma(x) L_n^\lambda(x) = (-1)^{m+n} \left(\frac{b}{2} \right)^{\gamma+\lambda} e^{-\frac{b^2}{4}} L_m^{\gamma+m-n} \left(\frac{b^2}{4} \right) L_n^{\lambda+n-m} \left(\frac{b^2}{4} \right). \tag{B.8}$$

So we have

$$\chi_{m,n}^0 = (-1)^{m+n} 2\pi \Gamma(n + 1) \left(\frac{\xi}{2} \right)^{-n} e^{-\frac{\xi}{4}} L_n^{m-n} \left(\frac{\xi}{4} \right) \sum_{k=0}^n \frac{1}{k!} \left(\frac{\xi}{2} \right)^k L_n^{k-n}(\xi) L_m^{m-n} \left(\frac{\xi}{4} \right). \tag{B.9}$$

We can further simplify (B.9) by using the following identity [74]:

$$\sum_{k=0}^\infty L_m^{(k+\alpha)}(x) L_n^{(k+\beta)}(y) \frac{z^k}{k!} = e^z \sum_{k=0}^{\min(m,n)} L_{m-k}^{(k+\alpha)}(x-z) L_{n-k}^{(k+\beta)}(y-z) \frac{z^k}{k!}, \tag{B.10}$$

together with the fact that $L_\nu^{-\nu}(x) = (-x)^\nu / \nu!$. The upshot is an amazingly simple expression for $\chi_{m,n}^0$,

$$\chi_{m,n}^0 = (-1)^m \frac{2\pi e^{\frac{\xi}{4}}}{\Gamma(m + 1)} U \left(-m, -m + n + 1, \frac{\xi}{4} \right) L_n^{m-n} \left(\frac{\xi}{4} \right), \tag{B.11}$$

where $U(a, b, z)$ is Tricomi’s confluent hypergeometric function.

The expression for $\chi_{m,n}^1$ is somewhat complicated. To evaluate it, we start with the following generalization of (B.6) [75],

$$e^{\sqrt{x\xi}e^{i\phi}} L_n^\beta(x + \xi - 2\sqrt{x\xi} \cos \phi) = \sum_{k=0}^{\infty} \sum_{s=0}^n C_{nks}^\beta (\sqrt{x\xi})^{k+s} L_{n-s}^{\beta+k+s}(x) L_{n-s}^{\beta+k+s}(\xi) e^{i\phi(k-s)}, \quad (\text{B.12})$$

where

$$C_{nks}^\beta = \frac{(\beta + k + s)\Gamma(\beta + s)\Gamma(\beta + k)\Gamma(n - s + 1)}{\Gamma(\beta)\Gamma(s + 1)\Gamma(k + 1)\Gamma(\beta + n + k + 1)}, \quad \beta \neq 0. \quad (\text{B.13})$$

As in the earlier case, (B.12) allows to perform the angular integration. The x integration is likewise recast in the form of a Hankel transform, a convenient closed-form expression for which can be derived as

$$\begin{aligned} \mathcal{H}_{m,n}^{\mu,\nu,\alpha,\beta} &= \int_0^\infty dx e^{-x} x^{\mu+\frac{\nu}{2}} L_m^\alpha(x) L_n^\beta(x) J_\nu(\sqrt{x\xi}) \\ &= (-1)^{m+n} \Gamma(\mu + 1) \left(\frac{\sqrt{\xi}}{2}\right)^\nu e^{-\frac{\xi}{4}} \sum_{r=0}^{\mu} \sum_{l=0}^{\gamma} \frac{(-1)^r}{r!} \binom{\mu + \nu}{\mu - r} \binom{\gamma}{l} \\ &\quad \times L_n^{\alpha+m+l-n} \left(\frac{\xi}{4}\right) L_{m+l-\gamma}^{\beta+n+\gamma-l-m} \left(\frac{\xi}{4}\right), \end{aligned} \quad (\text{B.14})$$

where $\gamma = \mu + \nu - \alpha - \beta + r$ and $\mu + \nu > -1$. For $\gamma < 0$, the upper limit of l summation in (B.14) has to be replaced by ∞ . For $\mu = 0, \nu = \alpha + \beta$ (B.14) reduces to (B.8).

After a short algebra $\chi_{m,n}^1$ can be expressed as

$$\begin{aligned} \chi_{m,n}^1 &= {}^a\chi_{m,n}^1 - {}^b\chi_{m,n}^1, \\ {}^a\chi_{m,n}^1 &= 2\pi \sum_{k=0}^{\infty} \sum_{s=0}^n C_{nks}^\beta \left(\sqrt{\xi}^{k+s}\right) \mathcal{H}_{m,n-s}^{s+1,k-s,1,k+s+1}, \\ {}^b\chi_{m,n}^1 &= 2\pi \sum_{k=0}^{\infty} \sum_{s=0}^n C_{nks}^\beta \left(\sqrt{\xi}^{k+s}\right) (k-s) \mathcal{H}_{m,n-s}^{s,k-s,1,k+s+1}. \end{aligned} \quad (\text{B.15})$$

Appendix C. On the gauge dependence of mesonic correlation functions

Let us recall that the Schwinger representation of a fermion propagator in background electromagnetic field is given by

$$\widetilde{S}_f(x, x') = e^{\lambda_f(x, x')} \times \mathcal{FT} [S_f(k)].$$

The holonomy factor is given by

$$\lambda_f(x, x') = iq_f \int_{x'}^x d\xi_\mu \left[A^\mu(\xi) + \frac{1}{2} F^{\mu\nu}(\xi - x')_\nu \right]. \quad (\text{C.1})$$

q_f is the charge of the fermion and $F^{\mu\nu}$ is assumed constant. A general correlator has the form

$$\chi_{AB}(x, x') \sim \widetilde{S}_f(x, x') \Gamma_A \widetilde{S}_{f'}(x', x) \Gamma_B \quad (\text{C.2})$$

We note that the holonomy factors from the propagators do not cancel in the correlator if $f \neq f'$. Thus charged-meson correlators are gauge variant and break translation invariance. This problem is not specific to QED but have been discussed in QCD [76, 77].

Now Green's function are gauge-dependent objects and one has to identify and isolate the gauge-independent information from the gauge-dependent objects. We note that the gauge-dependent phase factor in Schwinger propagators does not alter the the pole positions. The poles of the exact meson propagator are also gauge invariant as the propagator and the polarization function share the same holonomy factor. Hence the spectrum of collective excitations is gauge invariant. The screening masses are likewise gauge independent. We note that the spectral function for charged mesons correspond to the gauge- and translation-invariant part of the correlator. For the two-vertex fermion loop, the translation-invariant part can be isolated by gauging away the holonomy factor by a suitable transformation [78].

References

1. Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo, *Nature* **443**, 675 (2006).
2. D.E. Kharzeev, L.D. McLerran, H.J. Warringa, *Nucl. Phys. A* **803**, 227 (2008).
3. V. Skokov, A.Yu. Illarionov, V. Toneev, *Int. J. Mod. Phys. A* **24**, 5925 (2009).
4. A. Bzdak, V. Skokov, *Phys. Lett. B* **710**, 171 (2012).
5. V. Voronyuk, V.D. Toneev, W. Cassing, E.L. Bratkovskaya, V.P. Konchakovski, S.A. Voloshin, *Phys. Rev. C* **83**, 054911 (2011).
6. W.-T. Deng, X.-G. Huang, *Phys. Rev. C* **85**, 044907 (2012).
7. D. Kharzeev, *Phys. Lett. B* **633**, 260 (2006).
8. P.V. Buividovich, M.N. Chernodub, E.V. Luschevskaya, M.I. Polikarpov, *Phys. Rev. D* **80**, 054503 (2009).
9. K. Fukushima, D.E. Kharzeev, H.J. Warringa, *Phys. Rev. Lett.* **104**, 212001 (2010).
10. D. Kharzeev, A. Zhitnitsky, *Nucl. Phys. A* **797**, 67 (2007).
11. D.T. Son, P. Surowka, *Phys. Rev. Lett.* **103**, 191601 (2009).
12. S.P. Klevansky, R.H. Lemmer, *Phys. Rev. D* **39**, 3478 (1989).
13. V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, *Phys. Lett. B* **349**, 477 (1995).
14. I.A. Shushpanov, A.V. Smilga, *Phys. Lett. B* **402**, 351 (1997).
15. G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, A. Schafer, *Phys. Rev. D* **86**, 071502 (2012).
16. R.L.S. Farias, K.P. Gomes, G.I. Krein, M.B. Pinto, *Phys. Rev. C* **90**, 025203 (2014).
17. G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg, A. Schafer, K.K. Szabo, *JHEP* **02**, 044 (2012).
18. E.S. Fraga, L.F. Palhares, *Phys. Rev. D* **86**, 016008 (2012).
19. D. Kharzeev *et al.* (Editor), *Strongly Interacting Matter in Magnetic Fields* (Springer, 2013).
20. A.K. Harding, D. Lai, *Rep. Prog. Phys.* **69**, 2631 (2006).
21. T. Vachaspati, *Phys. Lett. B* **265**, 258 (1991).
22. H. Gies, J. Jaeckel, A. Ringwald, *Phys. Rev. Lett.* **97**, 140402 (2006).
23. V.A. Miransky, I.A. Shovkovy, *Phys. Rep.* **576**, 1 (2015).
24. E.V. Shuryak, *Rev. Mod. Phys.* **65**, 1 (1993).
25. C.E. Detar, J.B. Kogut, *Phys. Rev. Lett.* **59**, 399 (1987).
26. C.E. Detar, J.B. Kogut, *Phys. Rev. D* **36**, 2828 (1987).
27. W. Florkowski, B.L. Friman, *Z. Phys. A* **347**, 271 (1994).
28. F. Karsch, M.G. Mustafa, M.H. Thoma, *Phys. Lett. B* **497**, 249 (2001) hep-ph/0007093.
29. W.M. Alberico, A. Beraudo, P. Czerski, A. Molinari, *Nucl. Phys. A* **775**, 188 (2006).
30. M. Vepsalainen, *JHEP* **03**, 022 (2007).
31. Y. Burnier, M. Laine, *JHEP* **11**, 086 (2012).
32. F. Karsch, E. Laermann, P. Petreczky, S. Stickan, *Phys. Rev. D* **68**, 014504 (2003).
33. G. Aarts, J.M. Martinez Resco, *Nucl. Phys. B* **726**, 93 (2005).
34. B.B. Brandt, A. Francis, H.B. Meyer, H. Wittig, *JHEP* **03**, 100 (2013).
35. M. Cheng *et al.*, *Eur. Phys. J. C* **71**, 1564 (2011).
36. N. Sadooghi, F. Taghinavaz, *Ann. Phys.* **376**, 218 (2017).
37. S.S. Avancini, R.L.S. Farias, M. Benghi Pinto, W.R. Tavares, V.S. Timteo, *Phys. Lett. B* **767**, 247 (2017).
38. P.V. Buividovich, M.I. Polikarpov, *Phys. Rev. D* **83**, 094508 (2011).
39. P.V. Buividovich, M.N. Chernodub, D.E. Kharzeev, T. Kalaydzhyan, E.V. Luschevskaya, M.I. Polikarpov, *Phys. Rev. Lett.* **105**, 132001 (2010).
40. S. Ghosh, A. Mukherjee, M. Mandal, S. Sarkar, P. Roy, *Phys. Rev. D* **96**, 116020 (2017).
41. A. Bandyopadhyay, S. Mallik, *Phys. Rev. D* **95**, 074019 (2017).
42. A. Bandyopadhyay, C.A. Islam, M.G. Mustafa, *Phys. Rev. D* **94**, 114034 (2016).
43. S. Ghosh, A. Mukherjee, M. Mandal, S. Sarkar, P. Roy, *Phys. Rev. D* **94**, 094043 (2016).
44. A. Mukherjee, S. Ghosh, M. Mandal, P. Roy, S. Sarkar, *Phys. Rev. D* **96**, 016024 (2017).
45. Z. Wang, P. Zhuang, *Phys. Rev. D* **97**, 034026 (2018).
46. M. Coppola, D. Gmez Dumm, N.N. Scoccola, *Phys. Lett. B* **782**, 155 (2018).
47. H. Liu, X. Wang, L. Yu, M. Huang, *Phys. Rev. D* **97**, 076008 (2018).
48. S. Nussinov, M.A. Lampert, *Phys. Rep.* **362**, 193 (2002).
49. Y. Hidaka, A. Yamamoto, *Phys. Rev. D* **87**, 094502 (2013).
50. G.S. Bali, B.B. Brandt, G. Endrödi, B. Gläbke, *Phys. Rev. D* **97**, 034505 (2018).
51. A. Kuznetsov, N. Mikheev, *Electroweak Processes in External Electromagnetic Fields* (Springer, 2003).
52. T.-K. Chyi, C.-W. Hwang, W.F. Kao, G.-L. Lin, K.-W. Ng, J.-J. Tseng, *Phys. Rev. D* **62**, 105014 (2000).
53. C.S. Machado, S.I. Finazzo, R.D. Matheus, J. Noronha, *Phys. Rev. D* **89**, 074027 (2014).
54. S. Cho, K. Hattori, S.H. Lee, K. Morita, S. Ozaki, *Phys. Rev. D* **91**, 045025 (2015).
55. D.J. Broadhurst, S.C. Generalis, *Phys. Lett. B* **142**, 75 (1984).
56. S. Zschocke, T. Hilger, B. Kampfer, *Eur. Phys. J. A* **47**, 151 (2011).
57. A.G. Grozin, *Int. J. Mod. Phys. A* **10**, 3497 (1995).
58. A. Patkos, N. Sakai, *Nucl. Phys. B* **168**, 521 (1980).
59. A. Chodos, K. Everding, D.A. Owen, *Phys. Rev. D* **42**, 2881 (1990).

60. V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, Nucl. Phys. B **462**, 249 (1996).
61. H.A. Weldon, Phys. Rev. D **28**, 2007 (1983).
62. G.B. Arfken, H.J. Weber, *Mathematical Methods for Physicists*, 6th edn. (Elsevier, 2005).
63. F. Bruckmann, G. Endrodi, M. Giordano, S.D. Katz, T.G. Kovacs, F. Pittler, J. Wellenhofer, Phys. Rev. D **96**, 074506 (2017).
64. K. Fukushima, Y. Hidaka, Phys. Rev. Lett. **120**, 162301 (2018).
65. X.-G. Huang, A. Sedrakian, D.H. Rischke, Ann. Phys. **326**, 3075 (2011).
66. K. Fukushima, K. Hattori, H.-U. Yee, Y. Yin, Phys. Rev. D **93**, 074028 (2016).
67. P. Chakraborty, M.G. Mustafa, M.H. Thoma, Eur. Phys. J. C **23**, 591 (2002).
68. P. Chakraborty, M.G. Mustafa, M.H. Thoma, Phys. Rev. D **67**, 114004 (2003).
69. H. Gies, L. Roessler, Phys. Rev. D **84**, 065035 (2011).
70. H. Gies, F. Karbstein, N. Seegert, New J. Phys. **17**, 043060 (2015).
71. F. Karbstein, H. Gies, M. Reuter, M. Zepf, Phys. Rev. D **92**, 071301 (2015).
72. H. Bateman, *Partial Differential Equations of Mathematical Physics* (Cambridge University Press, Cambridge, 1932).
73. K.S. Kölbig, H. Scherb, J. Comput. Appl. Math. **71**, 357 (1996).
74. A. Erdélyi, Compos. Math. **7**, 340 (1940).
75. T. Koornwinder, SIAM J. Math. Anal. **8**, 535 (1977).
76. V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Fortsch. Phys. **32**, 585 (1984).
77. S.N. Nikolaev, A.V. Radyushkin, Phys. Lett. B **110**, 476 (1982) **116**, 469(E) (1982).
78. C. Cronstrom, Phys. Lett. B **90**, 267 (1980).