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New classes of generalized anisotropic polytropes pertaining radiation density

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Abstract. In this article, we consider the generalized polytropic equation of state with anisotropic matter distribution in isotropic coordinates. The static spherically symmetric configuration is considered for the development of mathematical models of compact objects incorporating the radiation factor. We have examined 12 different stars with the developed models. Analysis of the models shows that they are well-behaved and physically viable.

1 Introduction

In general relativity, polytropes refer to the solution of the Lane-Emden equation. They are greatly responsible for the formation and demonstration of the cosmic structure of compact objects and the internal structure of relativistic stars. By using the laws of thermodynamics, Chandrasekhar [1] developed the theory of polytropes in the Newtonian framework. Tooper [2] established the fundamental polytropic structure for spherically symmetric compressible fluids. He also derived the equation of equilibrium for polytropes. Kovetz [3] corrected some irregularities in Chandrasekhar's theory of rotating polytropes. Komatsu *et al.* [4] presented a numerical technique to develop the structure of rapidly rotating and uniformly rotating polytropes for general relativistic stars. Polytropes can be used to describe compact objects with stable gravitational fields very effectively. The polytropic equation of state, in which the pressure is defined as a function of density, Lane [5] was the first to use them in the modeling of the stellar structures of relativistic objects. Later, Herrera *et al.* [6] discussed conformally flat spherically symmetric fluid distributions with the help of the polytropic equation of state.

In order to study the formation of stars, the discussion of the anionotropy factor is of considerable significance. The anisotropic fluid is used to illustrate the behavior of pressure components of compact objects. Anisotropic spherically symmetric fluids are widely studied in general relativity. Bower and Liang [7] analyzed different aspects of the local anisotropic fluid distribution with the help of the generalized hydrostatic equilibrium equation for relativistic objects. Cosenza *et al.* [8] constructed a general structure for solving Einstein's equations by considering anisotropic fluid inner distribution of compact objects. Herrera and Santos [9] investigated the significance of local anisotropy in a self-gravitating system, considering Newtonian and general relativistic domains. Herrera *et al.* [10] conducted a detailed study on the evolution of spherically symmetric self-gravitating dissipative fluid utilizing anisotropic stresses. Herrera and Barreto [11] established a general polytropic model with anisotropic pressure for Newtonian stars. Reddy *et al.* [12] applied the perturbation technique to examine the effects of anisotropic pressure and heat dissipation on gravitational collapse of compact objects. Azam *et al.* [13–15] investigated the stability of charged self-gravitating compact objects using the local density perturbation by utilizing linear and quadratic equations of state. Maurya *et al.* [16] discussed exact solutions of Einstein-Maxwell field equations with the anisotropic configuration of spherically symmetric relativistic stars.

In the evolution of stars, it has been observed that a large amount of radiations are emitted, through neutrinos and quantized energy (photons); these radiations per unit volume are termed as radiation density. Diffusion and free-streaming approximation are the two categories of radiating density. The diffusion limit can only exist when the

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particles' mean free path is less then the original length of the object. Here a heat flow type vector defines dissipation. The free-streaming approximation is related to the discharge of null fluid. Herrera and Santos [10,17] discussed the shear-free condition for self-gravitating relativistic stars using both the diffusion and the free streaming dissipation. Kutschera [18] showed that gravitational waves are monopoles for astrophysical sources. Prisco *et al.* [19] physically analyzed the gravitational collapse of charged dissipative spherically symmetric compact objects. Dissipation is considered with both free-streaming and diffusion approximations. According to Herrera *et al.* [20] with a non-dissipative fluid, if the system involves zero expansion condition, then the energy density should be inhomogeneous. Herrera *et al.* [21–24] then discussed the structure, properties, stability and discovered some features defining the irregularities in the matter distribution of spherical stars which are self-gravitating. For such objects with different matter configurations, Sharif with his coworkers [25–27] unfolded new aspects which are used to describe inhomogeneous radiation density distribution.

The generalized polytropic equation of state is widely used in the modeling of relativistic stars. Chavanis [28,29], by merging the linear equation of state ($p_r = \beta_0\rho_0$) and the polytropic equation of state ($p_r = \beta_1\rho^{1+\frac{1}{n}}$), constructed the generalized polytropic equation of state, $p_r = \beta_0\rho + \beta_1\rho^{1+\frac{1}{n}}$, where ρ is the density, p_r denotes the radial pressure, and n is the polytropic index. Freitas and Goncalves [30] analyzed the evolution of this universe at constant density using the generalized polytropic equation of state and considered different aspects of the universe. Azam *et al.* [31,32] developed general structures of charge compact stars which are spherical and cylindrical in nature under conformally flat condition having anisotropic matter distributions.

In mathematical modeling, stability analysis of compact objects is significantly important. Bondi [33] established hydrostatic equilibrium equations for the analysis of the stability of compact objects. In the past, different scholars have considered Schwarzschild coordinates as well as isotropic coordinates for the development of their models. To discuss the stability of stellar structures, the range $1 < n < 5$ for polytropic index is used. Pandey *et al.* [34], while illustrating the structure and properties of compact objects, presented the range $\frac{1}{2} < n < 3$. Herrera *et al.* [35] introduced the cracking technique, using local density perturbation for the stability of system. Different values of n were considered by Takisa and Maharaj [36] to acquire exact solutions for the Einstein Maxwell field equations. To address the instability problem, Azam *et al.* [37–39] used the local density perturbation technique by evaluating the cracking points for numerous compact stars in spherical and cylindrical symmetry. Ngubelanga and Maharaj [40] used different values of the polytropic index for generating new classes of polytropic models, along with obtaining masses of several stars. Mardan *et al.* [41] investigated the gravitational behavior of compact objects by new classes of polytropic models for $n = 1, \frac{1}{2}, \frac{1}{3}$, which are physically well behaved.

A pulsar is a stellar object which emits radiations. They are also termed as highly magnetized neutron stars (stars composed of highly dense matter). After the discovery of pulsars in 1968 by Hewish *et al.* [42], many discoveries were made, related to the physical properties of pulsars. Demorest *et al.* [43] predicted the mass of PSR J1614-2230 by using the Shapiro Delay at National Radio Astronomy Observatory. The most stable and brightest millisecond pulsar PSR J0437-4715 was discovered by Johnston *et al.* [44] in the Parkes southern radio pulsar survey. Recently, Ozel *et al.* [45] predicted its mass and radii. A rapidly spinning neutron star which emits X-rays in a strong magnetic field is known as an X-ray binary pulsar. Rawls *et al.* [46] redefined the mass of six eclipsing X-ray binary pulsars, Vela X-1, 4U 1538-52, SMC X-1, LMC X-4, Cen X-3, and Her X-1, with the help of the numerical Roche lobe geometry. These are the high-mass X-ray binaries. The low-mass X-ray binaries 4U 1608-52, EXO 1785-248, Sax J1808.4-3658, 4U 1820-30, obtained from spectroscopic phenomena, are discussed in [45,47].

The main theme for this work is to establish mathematical models with anisotropic matter distribution by considering isotropic coordinates. The models are constructed by using generalized polytropic equation of state. In sect. 2, we discuss the Einstein field equation with anisotropic fluid in isotropic coordinates and consider the generalized polytropic equation of state. In sect. 3, the quadratic form of the gravitational potential is used for the purpose of integration, which leads us to new classes of polytropes. Section 4 is about formulating models for the generalized polytropic equation of state by applying different polytropic indices. In sect. 5, we discuss the properties of the solution by polytropic index $n = 1$, and the analysis of the developed model is presented graphically.

2 Einstein field equations

To generate a model for generalized polytropic equation of state, a general static spherically symmetric line element in isotropic coordinates $(x^a) = (t, r, \theta, \phi)$ is considered as

$$\mathrm{d}s^2 = -A^2(r)\mathrm{d}t^2 + B^2(r)[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)], \tag{1}$$

with $A(r)$ and $B(r)$ being the metric quantities, also known as gravitational potentials. Considering the anisotropic fluid, we have the energy momentum tensor of the form [35]

$$T_{\alpha\beta} = (\rho + p_\perp)v_\alpha v_\beta + p_\perp g_{\alpha\beta} + (p_r - p_\perp)\chi_\alpha\chi_\beta + q_\alpha v_\beta + v_\alpha q_\beta + \epsilon\ell_\alpha\ell_\beta. \tag{2}$$

The quantities used in the above equation are defined as

$$v^\alpha = A^{-1}\delta_0^\alpha, \qquad q^\alpha = qB^{-1}\delta_1^\alpha, \qquad l^\alpha = A^{-1}\delta_0^\alpha + B^{-1}\delta_1^\alpha, \qquad \chi^\alpha = B^{-1}\delta_1^\alpha. \tag{3}$$

In eq. (2), ρ is the energy density, $p_\perp$ is the tangential pressure, p_r is the radial pressure, ϵ is the radiation density, q^α is the heat flux, v^α is the four-velocity of the fluid, χ^α is the unit four-vector along the radial direction, ℓ^α is the radial null four-vector, q is a function of r, and $q^\alpha = q\chi^\alpha$. The Einstein field equations, from eq. (1) to eq. (2), take the form

$$8\pi(\rho+\epsilon) = -\frac{1}{B^2}\left[2\frac{B''}{B} - \frac{B'}{B}\left(\frac{B'}{B} - \frac{4}{r}\right)\right], \tag{4}$$

$$8\pi(p_r+\epsilon) = 2\frac{A'}{A}\left(\frac{B'}{B^3} + \frac{1}{r}\frac{1}{B^2}\right) + \frac{B'}{B^3}\left(\frac{B'}{B} + \frac{2}{r}\right), \tag{5}$$

$$8\pi p_\perp = 2\frac{1}{B^2}\left[\frac{B''}{B} - \frac{B'}{B}\left(\frac{B'}{B} - \frac{1}{r}\right)\right] + \frac{1}{B^2}\left(\frac{A''}{A} + \frac{1}{r}\frac{A'}{A}\right). \tag{6}$$

It is worthwhile to mention here that matter configuration is anisotropic together with the contribution of heat flux and radiation density. However the onset of field equations clearly shows that heat flux has no or negligible impact on static gravitational source.

In the above system of eqs. (4)–(6) we have six variables (ρ, p_r, $p_\perp$, ϵ, $A(r)$, $B(r)$) with three independent equations. For convenience, we use the following transformation [41]:

$$x \equiv r^2, \qquad M \equiv B^{-1}, \qquad F \equiv MA. \tag{7}$$

By using eq. (7), the Einstein field equations (4)–(6) transform to the following set of equation:

$$8\pi(\rho+\epsilon) = 4[2xMM_{xx} - 3(xM_x - M)M_x], \tag{8}$$

$$8\pi(p_r+\epsilon) = 4M(M-2xM_x)\frac{F_x}{F} - 4(2M-3xM_x)M_x, \tag{9}$$

$$8\pi p_\perp = 4xM^2\frac{F_{xx}}{F} + 4M(M-2xM_x)\frac{F_x}{F} - 4(2M-3xM_x)M_x - 8xMM_{xx}. \tag{10}$$

The subscript x represents derivative w.r.t. the x-coordinate. Here, the Newton gravitational constant and speed of light are assumed to be unity. Now, we consider the generalized polytropic equation of state [31]

$$p_r = \beta_0\rho + \beta_1\rho^{(1+\frac{1}{n})}, \tag{11}$$

with polytropic index n. Now using eq. (11), in the system eq. (8)–(10) we get

$$8\pi\rho = 4[2xMM_{xx} - 3(xM_x - M)M_x] - 8\pi\epsilon, \tag{12}$$

$$p_r = \beta_0\rho + \beta_1\rho^{1+\frac{1}{n}}, \tag{13}$$

$$\begin{aligned}\varDelta = {} & 4xM^2\frac{F_{xx}}{F} + 4M(M-2xM_x)\frac{F_x}{F} - 4(2M-3xM_x)M_x \\ & - 8xMM_{xx} - 4\beta_0(2xMM_{xx} - 3(xM_{xx} - M)M_x - 2\pi\epsilon) \\ & - 8\pi\beta_1\left(\frac{2xMM_{xx} - 3(xM_x - M)M_x - 2\pi\epsilon}{2\pi}\right)^{1+\frac{1}{n}},\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{F_x}{F} = {} & \frac{\beta_0}{M(M-2xM_x)}(2xMM_{xx} - 3(xM_x - M)M_x - 2\pi\epsilon) \\ & + \frac{2\pi\beta_1}{M(M-2xM_x)}\left(\frac{2xMM_{xx} - 3(xM_x - M)M_x - 2\pi\epsilon}{2\pi}\right)^{1+\frac{1}{n}} \\ & + \frac{2\pi\epsilon}{M(M-2xM_x)} + \frac{(2M-3xM_x)M_x}{M(M-2xM_x)},\end{aligned} \tag{15}$$

where $\varDelta = 8\pi(p_\perp - p_r)$ is the measure of anisotropy, which is attractive in nature if $\varDelta < 0$, repulsive if $\varDelta > 0$, and vanishes if $\varDelta = 0$. Equations (12)–(15) are nonlinear for M and F. We write the gravitational mass function as [36]

$$m(x) = 2\pi\int_0^x \frac{1}{\sqrt{z}}\big[z\rho(z)\big]\mathrm{d}z, \tag{16}$$

where z is an integration variable.

3 Integration

The system of eqs. (12)–(15) has five unknown variables $(\rho, \Delta, \epsilon, M, F)$ with four equations so, to make the system viable, we need another equation. Thus the gravitational potential M is introduced as

$$M = kx^2 + gx + h, \tag{17}$$

which is quadratic in nature, where k, g and h are constants. If we consider M to be linear, then it will result in elimination of variables, so we choose the quadratic form as in [41]. Using the above relation equation (17) in eq. (12), we get energy density as

$$8\pi\rho = -8k^2x^3 + 4kgx^2 + 40khx + 12gh - 8\pi\epsilon, \tag{18}$$

and the radial pressure from eq. (13) as

$$p_r = \beta_0\left[\frac{-8k^2x^3 + 4kgx^2 + 40khx + 12gh - 8\pi\epsilon}{8\pi}\right] + \beta_1\left[\frac{-8k^2x^3 + 4kgx^2 + 40khx + 12gh - 8\pi\epsilon}{8\pi}\right]^{1+\frac{1}{n}}. \tag{19}$$

Hence eq. (15) becomes

$$\begin{aligned}\frac{F_x}{F} = {} & \frac{\beta_0}{(kx^2+gx+h)(-3kx^2-gx+h)}\big(-2k^2x^3 + kgx^2 + 10khx + 3gh - 2\pi\epsilon\big)\\ & + \frac{2\pi\beta_1}{(kx^2+gx+h)(-3kx^2-gx+h)}\left(\frac{-2k^2x^3 + kgx^2 + 10khx + 3gh - 2\pi\epsilon}{2\pi}\right)^{1+\frac{1}{n}}\\ & + \frac{2\pi\epsilon}{(kx^2+gx+h)(-3kx^2-gx+h)} + \frac{(-4kx^2-gx+2h)(2kx+g)}{(kx^2+gx+h)(-3kx^2-gx+h)}. \end{aligned} \tag{20}$$

The above equation cannot be easily integrated due to the presence of the polytropic index n. Hence to get exact models we assign specific different values to n.

4 Polytropic models

In this section, we introduce constant values for the polytropic index n from [41], as $n = 1$, $n = \frac{1}{2}$, $n = \frac{1}{3}$ to integrate eq. (20) and obtain exact polytropic models w.r.t. coordinate r, since it is difficult to integrate F for the polytropic index n.

4.1 Polytropic index n = 1

We take $n = 1$, in eq. (20), and get

$$F(r) = C(kr^4 + gr^2 + h)^I(-3kr^4 - gr^2 + h)^J[Y(r)]^L[Z(r)]^W e^{N(r)}, \tag{21}$$

where C is a constant of integration and we consider

$$\begin{aligned} I = {} & -\frac{1}{8\pi hk(-g^2+4hk)}\big[2\pi k(6g^2h - g^3h - 24h^2k + 4gh^2k + 2\pi g\epsilon) - 4\pi k(-3g^2h + 12h^2k + g\pi\epsilon)\beta_0\\ & -(9g^5h - 72g^3h^2k - 24g^2hk\pi\epsilon + 96h^2k^2\pi\epsilon + 4gk(36h^3k + \pi^2\epsilon^2))\beta_1\big], \end{aligned} \tag{22}$$

$$\begin{aligned} J = {} & \frac{1}{648\pi hk(-g^2+4hk)}\big[54\pi k(2g^2h - 3g^3h - 8h^2k + 6g(2h^2k + \pi\epsilon)) - 108\pi k(-7g^2h + 28h^2k + 3g\pi\epsilon)\beta_0\\ & + (25g^5h + 824g^3h^2k - 1512g^2hk\pi\epsilon + 6048h^2k^2\pi\epsilon + 12gk(-308h^3k + 27\pi^2\epsilon^2))\beta_1\big], \end{aligned} \tag{23}$$

$$Y = \left[\frac{g + \sqrt{-g^2+4hk} + 2kr^2}{-g + \sqrt{-g^2+4hk} - 2kr^2}\right], \tag{24}$$

$$L = -\frac{1}{8\pi hk\sqrt{-g^2+4hk}}\big[2k\pi(2gh+g^2h-2\pi\epsilon)+4k\pi^2\epsilon\beta_0+(9g^4h-72g^2h^2k+144h^3k^2-4k\pi^2\epsilon^2)\beta_1\big], \tag{25}$$

$$Z = \left[\frac{g+\sqrt{-g^2-12hk}+6kr^2}{-g+\sqrt{-g^2-12hk}-6kr^2}\right], \tag{26}$$

$$\begin{aligned} W = {} & \frac{1}{648hk\pi\sqrt{-g^2-12hk}(-g^2+4hk)}\big[-54k\pi(-10g^3h+3g^4h+40h^2gk+72hk\pi\epsilon-6g^2(2h^2k+\pi\epsilon)) \\ & +108k\pi(2g^3h-8gh^2k-3g^2\pi\epsilon+36hk\pi\epsilon)\beta_0+(-25g^6h+484g^4h^2k-432g^3hk\pi\epsilon+1728gh^2k^2\pi\epsilon \\ & -144hk^2(196h^3k+27\pi^2\epsilon^2)+12g^2k(460h^3k+27\pi^2\epsilon^2))\beta_1\big], \end{aligned} \tag{27}$$

$$N = -\frac{r^2(109g^2-42gkr^2+12k(-32h+kr^4))\beta_1}{54\pi}. \tag{28}$$

The degree of anisotropy becomes

$$\begin{aligned} \Delta = {} & \frac{4J(-g-6kr^2)(kr^4+gr^2+h)}{(-3kr^4-gr^2+h)^2}\Bigg[2I(g+2kr^2)(-3kr^4-gr^2+h)+(J-1)(-g-6kr^2) \\ & +r^2(-3kr^4-gr^2+h)(kr^4+gr^2+h)-\frac{-6}{J(-g-6kr^2)}\Bigg]+8r^2I(g+2kr^2)(kr^4+gr^2+h)\left[L\frac{Y'}{Y}+W\frac{Z'}{Z}\right] \\ & \times\left(N'2L\frac{Y'}{Y}+2W\frac{Z'}{Z}\right)+4r^2(kr^4+gr^2+h)\left[L\frac{Y''}{Y}+W\frac{Z''}{Z}\right] \\ & +4r^2(kr^4+gr^2+h)^2\left[L(L-1)\left(\frac{Y'}{Y}\right)^2+W(W-1)\left(\frac{Z'}{Z}\right)^2\right] \\ & +4r^2(-3kr^4-gr^2+h)\left[L\frac{Y'}{Y}+W\frac{Z'}{Z}\right]+4r^2(kr^4+gr^2+h) \\ & \times\Bigg[N''+2LW(kr^4+gr^2+h)\frac{Y'Z'}{YZ}+\Bigg(I(g+2kr^2)+L(kr^4+gr^2+h)\frac{Y'}{Y} \\ & +W(kr^4+gr^2+h)\frac{Z'}{Z}+(kr^4+gr^2+h)N'\Bigg)N'\Bigg]+4I(2kr^2+g) \\ & \times\big[r^2(I-1)(2kr^2+g)+(-3kr^4-gr^2+h)\big]+4(kr^4+gr^2+h)\big[2kr^2I+J(-g-6kr^2)\big] \\ & +4(kr^4+gr^2+h)(-3kr^4-gr^2+h)N'-4(2kr^2+g)(-4kr^4-gr^2+h)-16kr^2(kr^4+gr^2+h) \\ & -4\beta_0\big[4kr^2(kr^4+gr^2+h)-3(2kr^2+g)(kr^4-h)-2\pi\epsilon\big] \\ & -\frac{2\beta_1}{\pi}\big[4kr^2(kr^4+gr^2+h)-3(2kr^2+g)(kr^4-h)-2\pi\epsilon\big]^2. \end{aligned} \tag{29}$$

The line element in eq. (1), for $n=1$, takes the form

$$\begin{aligned} \mathrm{d}s^2 = {} & -C(kr^4+br^2+h)^{2I}(-3kr^4-br^2+h)^{2(J-1)}\big[Y(r)\big]^{2L}\big[Z(r)\big]^{2W}e^{2N(r)}\mathrm{d}t^2 \\ & +(-3kr^4-br^2+h)^{-2}\big[\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2)\big]. \end{aligned} \tag{30}$$

This solution is categorized as $p_r=\beta_0\rho+\beta_1\rho^2$.

4.2 Polytropic index $n=\frac{1}{2}$

We take $n=\frac{1}{2}$, in eq. (20), and get

$$F(r) = C(kr^4+gr^2+h)^I(-3kr^4-gr^2+h)^J[Y(r)]^L[Z(r)]^W e^{N(r)}, \tag{31}$$

where C is a constant of integration and we consider

$$\begin{aligned} I = {} & \frac{1}{16hk^2(-g^2+4hk)\pi^2}\big[4k^2\pi^2(-6g^2h+g^3h+24h^2k-2g(2h^2k+\pi\epsilon))+8k^2\pi^2(-3g^2h+12h^2k+g\pi\epsilon)\beta_0 \\ & +(-27g^8h+378g^6h^2k-1944g^4h^3k^2+54g^5hk\pi\epsilon-432g^3h^2k^2\pi\epsilon+72g^2hk^2(60h^3k-\pi^2\epsilon^2) \\ & +288h^2k^3(-12h^3k+\pi^2\epsilon^2)+8gk^2\pi\epsilon(108h^3k+\pi^2\epsilon^2))\beta_1\big], \end{aligned} \tag{32}$$

$$
\begin{aligned}
J =& -\frac{1}{34992hk^2(-g^2+4hk)\pi^2}\big[2916k^2\pi^2(2g^2h-3g^3h-8h^2k+6g(2h^2k+\pi\epsilon))-5832k^2\pi^2(-7g^2h+28h^2k+3g\pi\epsilon)\beta_0\\
&+(125g^8h+1150g^6h^2k+41784g^4h^3k^2-4050g^5hk\pi\epsilon-133488g^3h^2k^2\pi\epsilon+1944gk^2\pi\epsilon(308h^3k-9\pi^2\epsilon^2)\\
&+1512g^2hk^2(68h^3k+81\pi^2\epsilon^2)-6048h^2k^3(196h^3k+81\pi^2\epsilon^2))\beta_1\big], \qquad (33)\\
Y =& \left[\frac{g+\sqrt{-g^2+4hk}+2kr^2}{-g+\sqrt{-g^2+4hk}-2kr^2}\right], \qquad (34)\\
L =& -\frac{1}{16\pi^2hk^2\sqrt{-g^2+4hk}}\big[4k^2\pi^2(2gh+g^2h-2\pi\epsilon)+8k^2\pi^3\epsilon\beta_0\\
&+(27g^7h-324g^5h^2k+1296g^3h^3k^2-1728gh^4k^3-(54g^4hk-432g^2h^2k^2+864h^3k^3)\pi\epsilon+8k^2\pi^3\epsilon^3)\beta_1\big], \qquad (35)\\
Z =& \left[\frac{g+\sqrt{-g^2-12hk}+6kr^2}{-g+\sqrt{-g^2-12hk}-6kr^2}\right], \qquad (36)\\
W =& \frac{-1}{34992hk^2\pi^2\sqrt{-g^2-12hk}(-g^2+4hk)}\big[2916k^2\pi^2(-10g^3h+3g^4h\\
&+40h^2gk+72hk\pi\epsilon-6g^2(2h^2k+\pi\epsilon))-5832k^2\pi^2(2g^3h-8gh^2k-3g^2\pi\epsilon\\
&+36hk\pi\epsilon)\beta_0+(125g^9h+1900g^7h^2k-71664g^5h^3k^2-4050g^6hk\pi\epsilon\\
&+78408g^4h^2k^2\pi\epsilon-23328hk^3\pi\epsilon(196h^3k+9\pi^2\epsilon^2)+1944g^2k^2\pi\epsilon(460h^3k\\
&+9\pi^2\epsilon^2)+1728gh^2k^3(1960h^3k+81\pi^2\epsilon^2)-44g^3hk^2(4156h^3k+243\pi^2\epsilon^2))\beta_1\big], \qquad (37)\\
N =& \frac{1}{29160k\pi^2}\big[r^2(-97790g^5+48270g^4kr^2+30g^3k(29737h-1031kr^4)\\
&+135g^2k(-2966hkr^2+151k^2r^6+1308\pi\epsilon)-72gk^2(35100h^2-2995hkr^4\\
&+153k^2r^8+945r^2\pi\epsilon)+540k^2(1544h^2kr^2+6kr^4(k^2r^6+6\pi\epsilon)-3h(47k^2r^6+384\pi\epsilon)))\beta_1\big]. \qquad (38)
\end{aligned}
$$

The degree of anisotropy becomes

$$
\begin{aligned}
\Delta =& \frac{4J(-g-6kr^2)(kr^4+gr^2+h)}{(-3kr^4-gr^2+h)^2}\Bigg[2I(g+2kr^2)(-3kr^4-gr^2+h)+(J-1)(-g-6kr^2)\\
&+r^2(-3kr^4-gr^2+h)(kr^4+gr^2+h)-\frac{-6}{J(-g-6kr^2)}\Bigg]+8r^2I(g+2kr^2)(kr^4+gr^2+h)\left[L\frac{Y'}{Y}+W\frac{Z'}{Z}\right]\\
&\times\left(N'2L\frac{Y'}{Y}+2W\frac{Z'}{Z}\right)+4r^2(kr^4+gr^2+h)\left[L\frac{Y''}{Y}+W\frac{Z''}{Z}\right]+4r^2(kr^4+gr^2+h)^2\\
&\times\left[L(L-1)\left(\frac{Y'}{Y}\right)^2+W(W-1)\left(\frac{Z'}{Z}\right)^2\right]+4r^2(-3kr^4-gr^2+h)\left[L\frac{Y'}{Y}+W\frac{Z'}{Z}\right]+4r^2(kr^4+gr^2+h)\\
&\times\Bigg[N''+2LW(kr^4+gr^2+h)\frac{Y'Z'}{YZ}+\Bigg(I(g+2kr^2)+L(kr^4+gr^2+h)\frac{Y'}{Y}+W(kr^4+gr^2+h)\frac{Z'}{Z}\\
&+(kr^4+gr^2+h)N'\Bigg)N'\Bigg]+4I(2kr^2+g)\big[r^2(I-1)(2kr^2+g)+(-3kr^4-gr^2+h)\big]+4(kr^4+gr^2+h)\\
&\times\big[2kr^2I+J(-g-6kr^2)\big]+4(kr^4+gr^2+h)(-3kr^4-gr^2+h)N'-4(2kr^2+g)(-4kr^4-gr^2+h)\\
&-16kr^2(kr^4+gr^2+h)-4\beta_0\big[4kr^2(kr^4+gr^2+h)-3(2kr^2+g)(kr^4-h)-2\pi\epsilon\big]\\
&-\frac{2\beta_1}{\pi}\big[4kr^2(kr^4+gr^2+h)-3(2kr^2+g)(kr^4-h)-2\pi\epsilon\big]^3. \qquad (39)
\end{aligned}
$$

The line element in eq. (1) for $n=\frac{1}{2}$ takes the form

$$
\begin{aligned}
\mathrm{d}s^2 =& -C(kr^4+br^2+h)^{2I}(-3kr^4-br^2+h)^{2(J-1)}\big[Y(r)\big]^{2L}\big[Z(r)\big]^{2W}e^{2N(r)}\mathrm{d}t^2\\
&+(-3kr^4-br^2+h)^{-2}\big[\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2)\big]. \qquad (40)
\end{aligned}
$$

This solution is categorized as $p_r=\beta_0\rho+\beta_1\rho^{\frac{3}{2}}$.

4.3 Polytropic index $\mathbf{n = \frac{1}{3}}$

We have taken $n = \frac{1}{3}$, in eq. (20), and get

$$F(r) = C(kr^4 + gr^2 + h)^I(-3kr^4 - gr^2 + h)^J[Y(r)]^L[Z(r)]^W e^{N(r)}, \tag{41}$$

where C is a constant of integration and we consider

$$\begin{aligned} I = &-\frac{1}{32hk^3(-g^2+4hk)\pi^3}\big[8k^3\pi^3(-6g^2h - g^3h - 24h^2k + 2g(2h^2k + \pi\epsilon)) \\ &- 16k^3\pi^3(-3g^2h + 12h^2k + g\pi\epsilon)\beta_0 + (-81g^{11}h - 1539g^9h^2k + 11664g^7h^3k^2 \\ &- 216g^8hk\pi\epsilon + 3024g^6h^2k^2\pi\epsilon - 15552g^4h^3k^3\pi\epsilon + 1728g^3h^2k^3(48h^3k - \pi^2\epsilon^2) \\ &+ 192g^2hk^3\pi\epsilon(180h^3k - \pi^2\epsilon^2) - 216g^5hk^2(204h^3k - \pi^2\epsilon^2) + 768h^2k^4\pi\epsilon(36h^3k \\ &- \pi^2\epsilon^2) + 16gk^3(-3888h^6k^2 + 216h^3k\pi^2\epsilon^2 + \pi^4\epsilon^4))\beta_1\big], \end{aligned} \tag{42}$$

$$\begin{aligned} J = &\frac{1}{1889568hk^3(-g^2+4hk)\pi^3}\big[157464k^3\pi^3(2g^2h - 3g^3h - 8h^2k + 6g(2h^2k \\ &+ \pi\epsilon)) - 314928k^3\pi^3(-7g^2h + 28h^2k + 3g\pi\epsilon)\beta_0 + (625g^{11}h + 9125g^9h^2k \\ &- 92400g^7h^3k^2 - 27000g^8hk\pi\epsilon - 248400g^6h^2k^2\pi\epsilon - 9025344g^4h^3k^3\pi\epsilon \\ &- 326592g^2hk^3\pi\epsilon(68h^3k + 27\pi^2\epsilon^2) + 1306368h^2k^4\pi\epsilon(196h^3k + 27\pi^2\epsilon^2) \\ &+ 216g^5hk^2(18644h^3k + 2025\pi^2\epsilon^2) + 1728g^3h^2k^3(20944h^3k + 8343\pi^2\epsilon^2) \\ &- 1296gk^3(159152h^6k^2 + 49896h^3k\pi^2\epsilon^2 - 729\pi^4\epsilon^4))\beta_1\big], \end{aligned} \tag{43}$$

$$Y = \left[\frac{g + \sqrt{-g^2+4hk} + 2kr^2}{-g + \sqrt{-g^2+4hk} - 2kr^2}\right], \tag{44}$$

$$\begin{aligned} L = &-\frac{1}{32\pi^3hk^3\sqrt{-g^2+4hk}}\big[8k^3\pi^3(2gh + g^2h - 2\pi\epsilon) + 16k^3\pi^4\epsilon\beta_0 + (81g^{10}h \\ &- 1377g^8h^2k + 9072g^6h^3k^2 + \pi\epsilon(-216g^7hk + 2592g^5h^2k^2 - 10368g^3h^3k^3 \\ &+ 13824gh^4k^4) + 1728g^2h^2k^3(24h^3k - \pi^2\epsilon^2) - 216g^4hk^2(132h^3k - \pi^2\epsilon^2) \\ &- 16k^3(1296h^6k^2 - 216h^3k\pi^2\epsilon^2 + \pi^4\epsilon^4))\beta_1\big], \end{aligned} \tag{45}$$

$$Z = \left[\frac{g + \sqrt{-g^2-12hk} + 6kr^2}{-g + \sqrt{-g^2-12hk} - 6kr^2}\right], \tag{46}$$

$$\begin{aligned} W = &\frac{1}{1889568hk^3\pi^3\sqrt{g^2-4hk}(-g^2+4hk)}\big[157464k^3\pi^3(-10g^3h + 3g^4h \\ &+ 40h^2gk + 72hk\pi\epsilon - 6g^2(2h^2k + \pi\epsilon)) - 314928k^3\pi^3(2g^3h - 8gh^2k \\ &- 3g^2\pi\epsilon + 36hk\pi\epsilon)\beta_0 + (625g^{12}h + 12875g^{10}h^2k - 48900g^8h^3k^2 \\ &- 27000g^9hk\pi\epsilon - 410400g^7h^2k^2\pi\epsilon + 15479424g^5h^3k^3\pi\epsilon \\ &- 72g^6hk^2(85972h^3k - 6075\pi^2\epsilon^2) - 373248gh^2k^4\pi\epsilon(1960h^3k + 27\pi^2\epsilon^2) \\ &+ 31104g^3hk^3\pi\epsilon(4156h^3k + 81\pi^2\epsilon^2) - 864g^4h^2k^3(56836h^3k + 9801\pi^2\epsilon^2) \\ &+ 1296g^2k^3(112112h^6k^2 - 74520h^3k\pi^2\epsilon^2 - 729\pi^4\epsilon^4) \\ &+ 15552hk^4(38416h^6k^2 + 31752h^3k\pi^2\epsilon^2 + 729\pi^4\epsilon^4))\beta_1\big], \end{aligned} \tag{47}$$

$$N = -\frac{1}{5511240k^2\pi^3}\big[r^2(-3kr^2(4644640g^7 - 3089135g^6kr^2 - 315g^5k(192128h - 7303kr^4) + 21g^4k(1776470hkr^2 - 85761k^2r^6 - 579240\pi\epsilon) + 1260g^3k^2(218351h^2 - 20427hkr^4 + 1113k^2r^8 + 6186r^2\pi\epsilon) - 108g^2k^2(1432270h^2kr^2 + 9090k^3r^{10} + 47565k\pi\epsilon r^4 - 126h(1324k^2r^6 + 7415\pi\epsilon)) + 756gk^2(-688640h^3k + 127890h^2k^2r^4 - 120h(121k^3r^8 + 599k\pi\epsilon r^2) + 27(25k^4r^{12} + 136k^2\pi\epsilon r^6 + 420\pi^2\epsilon^2)) - 432k^3(-582400h^3kr^2 + 63h^2(1477k^2r^6 + 7720\pi\epsilon) - 135h(62k^3r^{10} + 329k\pi\epsilon r^4) + 315(k^4r^{14} + 6k^2\pi\epsilon r^8 + 18\pi^2\epsilon^2r^2))) + 35(796849g^8 - 11165136g^6hk + 56633229g^4h^2k^2 - 2112264g^5k\pi\epsilon + 19269576g^3hk^2\pi\epsilon - 54587520gh^2k^3\pi\epsilon - 216g^2k^2(542176h^3k - 8829\pi^2\epsilon^2) + 20736hk^3(5521h^3k - 324\pi^2\epsilon^2)))\beta_1\big]. \tag{48}$$

The degree of anisotropy becomes

$$\begin{aligned}\varDelta = {}& \frac{4J(-g-6kr^2)(kr^4+gr^2+h)}{(-3kr^4-gr^2+h)^2}\Bigg[2I(g+2kr^2)(-3kr^4-gr^2+h)\\
&+(J-1)(-g-6kr^2)+r^2(-3kr^4-gr^2+h)(kr^4+gr^2+h)\\
&-\frac{-6}{J(-g-6kr^2)}\Bigg]+8r^2I(g+2kr^2)(kr^4+gr^2+h)\left[L\frac{Y'}{Y}+W\frac{Z'}{Z}\right]\\
&\times\left(N'2L\frac{Y'}{Y}+2W\frac{Z'}{Z}\right)+4r^2(kr^4+gr^2+h)\left[L\frac{Y''}{Y}+W\frac{Z''}{Z}\right]\\
&+4r^2(kr^4+gr^2+h)^2\left[L(L-1)\left(\frac{Y'}{Y}\right)^2+W(W-1)\left(\frac{Z'}{Z}\right)^2\right]\\
&+4r^2(-3kr^4-gr^2+h)\left[L\frac{Y'}{Y}+W\frac{Z'}{Z}\right]+4r^2(kr^4+gr^2+h)\\
&\times\Bigg[N''+2LW(kr^4+gr^2+h)\frac{Y'Z'}{YZ}+\Bigg(I(g+2kr^2)+L(kr^4+gr^2+h)\frac{Y'}{Y}\\
&+W(kr^4+gr^2+h)\frac{Z'}{Z}+(kr^4+gr^2+h)N'\Bigg)N'\Bigg]+4I(2kr^2+g)\\
&\times\big[r^2(I-1)(2kr^2+g)+(-3kr^4-gr^2+h)\big]+4(kr^4+gr^2+h)\\
&\times\big[2kr^2I+J(-g-6kr^2)\big]+4(kr^4+gr^2+h)(-3kr^4-gr^2+h)N'\\
&-4(2kr^2+g)(-4kr^4-gr^2+h)-16kr^2(kr^4+gr^2+h)\\
&-4\beta_0\big[4kr^2(kr^4+gr^2+h)-3(2kr^2+g)(kr^4-h)-2\pi\epsilon\big]\\
&-\frac{2\beta_1}{\pi}\big[4kr^2(kr^4+gr^2+h)-3(2kr^2+g)(kr^4-h)-2\pi\epsilon\big]^4.\end{aligned} \tag{49}$$

The line element in eq. (1) for $n=\frac{1}{3}$, takes the form

$$\begin{aligned}\mathrm{d}s^2 = {}& -C(kr^4+br^2+h)^{2I}(-3kr^4-br^2+h)^{2(J-1)}\big[Y(r)\big]^{2L}\big[Z(r)\big]^{2W}e^{2N(r)}\mathrm{d}t^2\\
&+(-3kr^4-br^2+h)^{-2}\big[\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2)\big].\end{aligned} \tag{50}$$

This solution is categorized as $p_r=\beta_0\rho+\beta_1\rho^{\frac{4}{3}}$.

5 Properties of the new solution

The physical properties of the polytropic model $n=1$, w.r.t. the coordinate r, are discussed in this section (as in [41]). In the previous section three exact models were developed for gravitational system, anisotropy and for the line element equation (1); out of all, $n=1$ is relatively less complicated. The radial pressure for the considered value becomes

$$p_r=\beta_0\rho+\beta_1\rho^2, \tag{51}$$

Table 1. Radius (R), mass ($M_\odot$), central radial pressure (p_{r_0}) and central density (ρ_0) with $\beta_1 = 0.931$, $g = -0.0558811$ and $\epsilon = 7.5657 \times 10^{-16}$ of various relativistic stars.

Name of the star	$M_\odot$	R	$k(\times 10^{-13})$	h	$\beta_0(\times 10^{-4})$	$\rho_0(\times 10^{-4})$	$p_{r_0}(\times 10^{-14})$
PSR J1614-2230	1.97	10.927	42.19	−0.013514	−3.3356	3.6057	18.251
Vela X-1	1.77	10.608	12.51	−0.013274	−3.2972	3.5416	4.7435
4U 1608-52	1.74	10.558	11.01	−0.013259	−3.2935	3.5376	4.3504
4U 1820-30	1.58	10.271	10.88	−0.013057	−3.2433	3.4837	3.5008
Cen X-3	1.49	10.098	10.76	−0.012987	−3.2260	3.4651	3.2831
PSR J0437-4715	1.4	10.51	10.34	−0.010822	−2.6882	2.8874	3.0692
EXO 1785-248	1.3	9.701	10.03	−0.012748	−3.1666	3.3401	2.5624
LMC X-4	1.29	9.678	9.19	−0.012734	−3.1631	3.3975	2.3232
SMC X-1	1.04	9.070	9.19	−0.012475	−3.0988	3.3284	1.7605
SAX J1808.4-3658	0.90	8.676	8.09	−0.012344	−3.0662	3.2935	1.2867
4U 1538-52	0.87	8.558	7.91	−0.012421	−3.0854	3.3140	1.1996
Her X-1	0.85	8.524	7.75	−0.012281	−3.0506	3.2767	1.1436

the energy density

$$8\pi\rho = -8k^2r^6 + 4kgr^4 + 40khr^2 + 12gh - 8\pi\epsilon, \tag{52}$$

the radial pressure

$$p_r = \beta_0\left[\frac{-2k^2r^6 + kgr^4 + 10khr^2 + 3gh - 2\pi\epsilon}{2\pi}\right] + \beta_1\left[\frac{-2k^2r^6 + kgr^4 + 10khr^2 + 3gh - 2\pi\epsilon}{2\pi}\right]^2, \tag{53}$$

and the gravitational mass takes the form

$$m(r) = \left(-\frac{4}{9}k^2r^9 + \frac{2}{7}kgr^7 + 4khr^5 + 2ghr^3 - \frac{4}{3}\pi\epsilon r^3\right). \tag{54}$$

All the physical quantities discussed above are well behaved. The radial pressure (p_r) and energy density (ρ) remains finite at the center for $r = 0$, as

$$\rho = \left(\frac{3gh}{2\pi} - \epsilon\right), \tag{55}$$

$$p_r = \left[\beta_0\left(\frac{3gh}{2\pi} - \epsilon\right) + \beta_1\left(\frac{3gh}{2\pi} - \epsilon\right)^2\right]. \tag{56}$$

Thus the speed of sound will be

$$\upsilon^2 = \frac{\mathrm{d}p_r}{\mathrm{d}\rho}. \tag{57}$$

For viability, we have the limitation $\upsilon \leq 1$, and at the boundary, radial pressure has to be zero, thus following relation is obtained:

$$\beta_0 = -\beta_1\left[\frac{-2k^2r^6 + kgr^4 + 10khr^2 + 3gh - 2\pi\epsilon}{2\pi}\right]. \tag{58}$$

For all model parameters, the numerical values of twelve various stars are given in table 1. Their masses and the radii from [45,47], have been generated by varying parameters β_0, k and h, which are also mentioned in table 1. The adequate values for ρ_0 and p_{r_0} are also given in table 1. For Pulsar PSR J0437-4715, we regained mass $1.4M_\odot$ and

Table 2. Radial pressure (p_r), energy density (ρ) and mass ($M_\odot$) of the realistic star PSR J0437-4715 from its center core to end limitation.

R	$\rho(\times 10^{-4})$	$p_r(\times 10^{-14})$	$M_\odot$	$\frac{\mathrm{d}p_r}{\mathrm{d}\rho}(\times 10^{-10})$
0.0	2.887445943257	3.06922	0.000000	1.06295
0.75	2.887445943128	3.06887	0.000510	1.06283
1.5	2.887445942391	3.06689	0.004083	1.06215
2.25	2.887445939999	3.06046	0.013776	1.05992
3.0	2.887445934206	3.04489	0.032656	1.05453
3.75	2.887445922567	3.01360	0.063781	1.04369
4.5	2.887445901941	2.95815	0.110215	1.02449
5.25	2.887445868486	2.86822	0.175017	0.99334
6.0	2.887445817664	2.73160	0.261250	0.94602
6.75	2.887445744236	2.53421	0.371975	0.87766
7.5	2.887445642268	2.26010	0.510254	0.78273
8.25	2.887445505124	1.89143	0.679148	0.65505
9.0	2.887445325473	1.40849	0.881719	0.48779
9.75	2.887445095283	0.78968	1.121030	0.27349
10.51	2.887444805826	0.01156	1.400140	0.0040049

radius 10.51 from [45], by choosing $\beta_1 = 0.931$, $g = -0.558811$ and $\epsilon = 7.5657 \times 10^{-16}$ [48]. Table 2 lists the values from the center of the star to its boundary, for ρ, p_r, $M_\odot$ and $\frac{\mathrm{d}p_r}{\mathrm{d}\rho}$. Graphical representation demonstrate that speed of sound, radial pressure and density are gradually decreasing functions of r, while mass, anisotropy and the tangential pressure are gradually increasing function of the radius. All the quantities in the graphical analysis are acceptable.

6 Discussion and conclusion

In this paper, we developed the mathematical models of relativistic objects filled with anisotropic fluid along with the contribution of heat flux and radiation density by making use of generalized polytropic equation of state. The process of gravitational collapse is highly dissipative, because of the emission of massless particles, namely neutrinos and photons (quantized energy), which are responsible for carrying away the binding energy to a black hole or a neutron star. These massless energy particles form a radiative field. In fact, these radiations with respect to volume are termed as radiation density, which is of two kinds, diffusion and free streaming approximation. In diffusion approximation, dissipation is interpreted by the energy flux. The energy flux of radiation is proportional to the temperature gradient; this statement is valid because the mean free path of the particles, which propagates the energy in the interior region of stellar objects, is very small in comparison to the real length of the object [49,50]. However, when the mean free path of the particles becomes large enough they would be referred to as free streaming approximation then to diffusion approximation [51]. Therefore, we have considered the radiative transport in terms of diffusion and free streaming approximation to cover the wide range. This certain selection will make the hydrostatic timescale very small as compared to other stellar timescales in the evolution of a star. The gravitational waves generated by any astrophysical object with high pressure constrains are monopole gravitational radiations [18]. So, the monopole radiation density $\epsilon = 7.5657 \times 10^{-16}$ is used [48]. These monopole waves satisfy Einstein's equations, considering a regular stress-energy tensor.

One important point of discussion is that in matter configuration, we include heat flux as a source of dissipation due to which gravitational radiation arises. However, onset of field equations show that components of heat flux are vanishing, while radiation density can clearly be seen in eqs. (4)–(6). Also, the developed models are well behaved and physically viable, this gives rise to the argument that their might be some other factors that serve as source of radiation other than the heat energy.

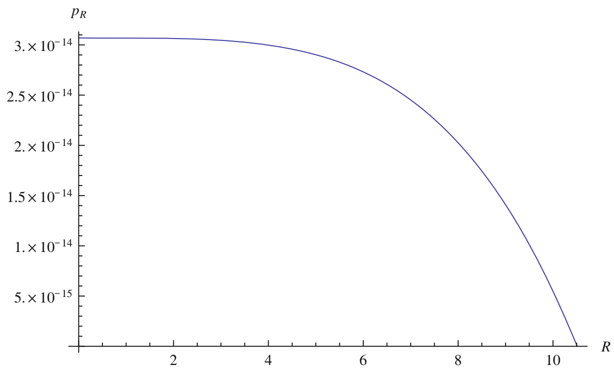

Fig. 1. Radial pressure.

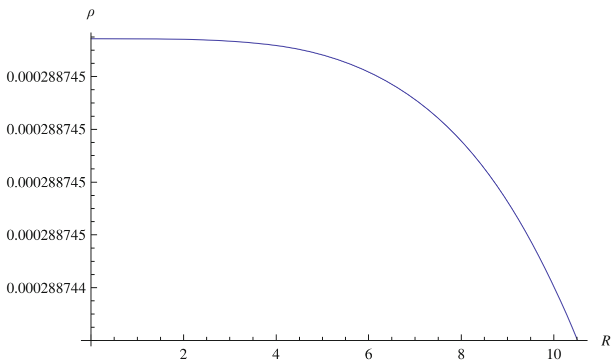

Fig. 2. Density.

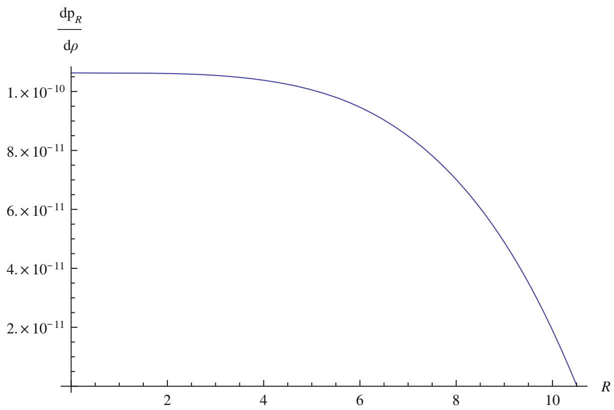

Fig. 3. Speed of sound.

It is important to mention that no naked singularity was observed in the developed models with various parametric values mentioned in table 1. Physically reasonable masses of twelve different astronomical objects has been obtained from the developed models. The masses of six eclipsing X-ray binary pulsars, Vela X-1, Cen X-3, 4U 1538-52, LMC X-4, SMC X-1, Her X-1, four low-mass X-ray binaries, 4U 1608-52, 4U 1820-30, EXO 1785-248, SAX J1808.4-3658 and two pulsars PSR J1614-2230 and PSR J0437-4715, have been regained with recent predicted radii [45–47].

To analyze the effect of gravity on the matter quantities, we have considered, in particular, pulsar PSR J0437-4715, with mass of $1.4M_{\odot}$ and radius $R = 10.51$. Table 2 contains the variation of ρ, p_r, $M_{\odot}$ and $\frac{\mathrm{d}p_r}{\mathrm{d}\rho}$ of pulsar PSR J0437-4715 from the center of the star to its boundaries. To study the variation physically, all the quantities in table 2 are illustrated graphically. Radial pressure, density and pressure gradient are gradually decreasing functions of the radius, as is illustrated in figs. 1, 2, 3, respectively. The radial pressure vanishes at the boundary of the compact object. The central radial pressure and the central density ought to be finite. Taking into account the center of the

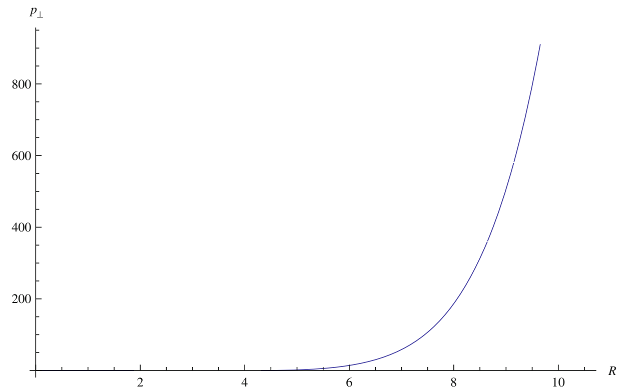

Fig. 4. Tangential pressure.

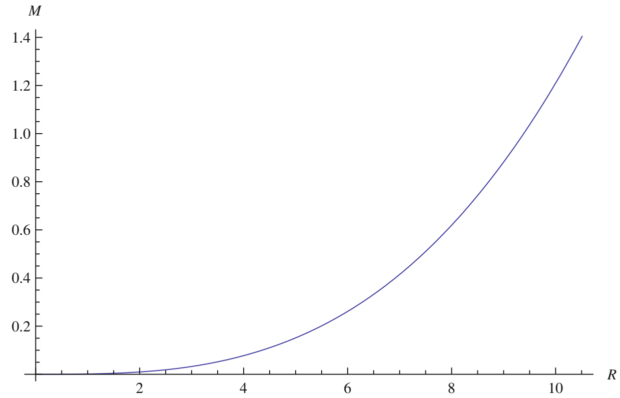

Fig. 5. Mass function.

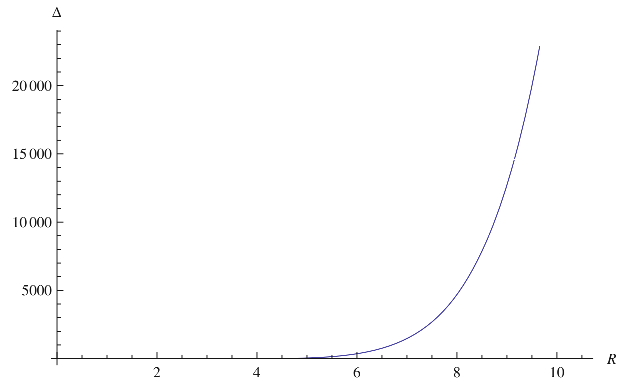

Fig. 6. Degree of anisotropy.

pulsar, the tangential pressure becomes zero and, at the boundary, it attains its maximum value. Figure 4 shows that the tangential pressure is an increasing function of the radius. Similarly, figs. 5 and 6 show that mass and degree of anisotropy are also an increasing function of the radius.

Stability is an essential factor for the acceptance of any model. Numerous techniques have been used in the literature for the stability analysis of stellar objects, where speed of sound (pressure gradient) was considered as a criterion to check the stability of models, as in [41]. The speed of sound remains positive inside the star, as is shown in fig. 3, and thus the casuality condition $\upsilon \leq 1$ is satisfied. The degree of anisotropy is selected in a way that the pressure gradient must be less than zero, setting $r = 0$, at the center, Δ will become zero, which is the basic requirement for stability. In the interior of any star, radial pressure, tangential pressure, energy density, degree of anisotropy and the metric quantities remain non-singular, finite and positive. Hence the models are well behaved and physically viable.

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