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More general families of exact solitary wave solutions of the nonlinear Schrödinger equation with their applications in nonlinear optics

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Abstract. In this article we analytically studied the complex nonlinear Schrödinger equation with Kerr law nonlinearity using the auxiliary equation mapping method, as a result, we found a series of more general and new families of exact solutions, which are more powerful in the development of soliton dynamics, quantum plasma, adiabatic parameter dynamics, biomedical problems, fluid dynamics, industrial studies, nonlinear optics and many other fields. The calculations demonstrate that this method is more reliable, straightforward and effective to analytically study other nonlinear complicated physical problems modeled by complex nonlinear partial differential equations arising in mathematical physics, hydrodynamics, fluid mechanics, mathematical biology, plasma physics, engineering disciplines, chemistry and many other natural sciences. We have also expressed our solutions graphically with the help of Mathematica 10.4 to physically understand the behavior of different shapes of solutions including kink-type, anti-kink-type, half-bright and dark solitons.

1 Introduction and problem formulation

It is well known that a large variety of nonlinear wave problems arising in physics, chemistry, biomedical problems, fluid dynamics and in many other natural sciences are governed by nonlinear partial differential equations (NLPDEs) [1–42]. The analytical study of nonlinear partial differential equations is one of the most fascinating and exciting areas of research for many researchers in recent years. The development of new mathematical techniques to find out a more compact and general form of exact solutions is one of the most important tasks to understand the complete dynamical process modeled by complex nonlinear partial differential equations from the past few decades. Extracting exact solutions of nonlinear partial differential equations is also important to check the stability of numerical solutions as well as to develop a wide range of new mathematical solvers to simplify the routine calculation. In recent time, an abundance of new more powerful and effective methods have been developed with the help of different computer softwares like Mathematica, Maple and Matlab, such as the Kudryashov method [1, 2], the truncated expansion method [3, 4], the Bäcklund transform method [5, 6], the inverse scattering method [7], the extended Fan sub-equation method [8], the homogeneous balance method [9], the Jacobi elliptic function method [10], the tanh-function method [11], and many more in several theoretical works about solitons and their applications [12–19].

It is well known that the propagation of ultrashort pulses in fibers is governed by the higher-order nonlinear Schrödinger equation (HONLSE). Nowadays the propagation of ultrashort pulses plays an important role to fulfill the increased demand of high rate data transmission in optical communication systems. To study the propagation of ultrashort pulses is practically important in various areas of research; these are studied in plasma physics, nuclear physics, mathematical physics, nonlinear optics, and many other physical sciences [20,21]. So, to extract optical solitons and understand the dynamics of ultrashort pulses completely has received a great attention of researchers in recent time. The present work has been motivated to study analytically the cubic nonlinear Schrödinger equation with Kerr

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law nonlinearity, which is obtained from the HONLSE. The higher order nonlinear Schrödinger equation (HONLSE) in dimensionless form is given as [22]

$$i\phi_t - \frac{\beta_2}{2}\phi_{xx} + \gamma_1\phi|\phi|^2 = i\frac{\beta_3}{6}\phi_{xxx} + \frac{\beta_4}{24}\phi_{xxxx} - \gamma_2\phi|\phi|^4 + i\alpha_1(\phi|\phi|^2)_x + i\alpha_2\phi(|\phi|^2)_x. \tag{1}$$

Here $\phi(x,t)$ is a complex wave function, where β_i, $i = 2,3,4$ are dispersion coefficients, β_2 is the group velocity dispersion (GVD), β_3 is the third-order dispersion (TOD), and β_4 is the fourth-order dispersion (FOD), respectively, while γ_1 is the coefficient of cubic nonlinearity, γ_2 is the coefficient of quintic nonlinearity, α_1, α_2 are related to self-steeping (SS) and self-frequency shift coefficients. The standard nonlinear Schrödinger equation, given as

$$i\phi_t - \frac{\beta_2}{2}\phi_{xx} + \gamma_1\phi|\phi|^2 = 0, \tag{2}$$

is obtained by setting $\beta_3 = \beta_4 = \gamma_2 = \alpha_1 = \alpha_2 = 0$. The exact solutions of (1) by considering $\beta_4 = \gamma_2 = 0$ has been studied, called as the perturbed nonlinear Schrödinger equation using the extended Fan sub-equation method given as

$$i\phi_t - \frac{\beta_2}{2}\phi_{xx} + \gamma_1\phi|\phi|^2 = i\frac{\beta_3}{6}\phi_{xxx} + i\alpha_1(\phi|\phi|^2)_x + i\alpha_2\phi(|\phi|^2)_x. \tag{3}$$

Many mathematicians and physicists have studied the HONSLE extensively with some special choices of parameters to find out exact solutions using different methods [15–20], like [26] studied with $\beta_3 = \alpha_2 = 0$ [24], with $\alpha_2 = 0$, and [25] studied with $\beta_3 = \alpha_1 = \alpha_2 = 0$. Our focus is to study (3) by setting $\beta_3 = \alpha_1 = \alpha_2 = 0$ with Kerr law nonlinearity given as [43]

$$i\phi_t - \phi_{xx} + 2\phi|\phi|^2 - 2\sigma_0^2\phi = 0. \tag{4}$$

The main outline of the paper is as follow. In sect. 1 a brief introduction of the model is given. In sect. 2 we applied the auxiliary equation mapping method, the detailed description of the method is given in ref. [21], on the complex nonlinear Schrödinger equation with Kerr law nonlinearity. In sect. 3 a graphical representation and discussion of the solutions is given. In sect. 4 the concluding remarks are given.

2 NLSE with Kerr law nonlinearity

Here our focus is to apply the auxiliary equation mapping method [21] on the complex nonlinear Schrödinger equation (CNLSE) with Kerr law nonlinearity [21]

$$i\phi_t - \phi_{xx} + 2\phi|\phi|^2 - 2\sigma_0^2\phi = 0. \tag{5}$$

Here $\phi(x,t)$ describes the complex wave function with σ_0 as a constant and x, t represents the partial derivatives. The perturbed NLSE has been studied by many researchers [32–37] but our focus is to extract the exact traveling wave solutions of (5) with Kerr law nonlinearity by the implementation of the auxiliary equation mapping method in a unified manner. It is important to mention here that by the choice of $\sigma_0 = 0$, eq. (5) reduces to NLSE with non-Kerr law nonlinearity given as

$$i\phi_t - \phi_{xx} + 2\phi|\phi|^2 = 0. \tag{6}$$

To convert eq. (5) into NLODE consider the following wave transformation:

$$\phi(x,t) = W(\xi)e^{i(\gamma x + \alpha t)}, \quad \xi = \kappa(x + 2\gamma t), \tag{7}$$

then eq. (5) becomes in the following form:

$$-\kappa^2\left(\frac{\partial^2 W(\xi)}{\partial \xi^2}\right) + 2W^3(\xi) + (\gamma^2 - 2\sigma_0^2 - \alpha)W(\xi) = 0, \tag{8}$$

where γ, α and κ are arbitrary constants to be determined later. Balancing W^3 and W'' in eq. (8), we obtain $m = 1$, by the auxiliary equation mapping method eq. (8) admits the general solution in the form of a series as

$$W(\xi) = \sum_{j=0}^{m} a_j F^j(\xi) + \sum_{j=-1}^{-m} b_{-j} F^j(\xi) + \sum_{j=2}^{m} c_j F^{j-2}(\xi)F'(\xi) + \sum_{j=-1}^{-m} d_{-j} F^j(\xi)F'(\xi), \tag{9}$$

where the a_j, b_j, c_j, d_j are constants to be determined later, and $F(\xi)$ satisfies the following auxiliary ordinary differential equation with its derivatives:

$$F'^2 = \left(\frac{\mathrm{d}F}{\mathrm{d}\xi}\right)^2 = pF^2(\xi) + qF^3(\xi) + rF^4(\xi), \tag{10}$$

$$F''(\xi) = pF(\xi) + \frac{3}{2}qF^2(\xi) + 2rF^3(\xi), \tag{11}$$

$$F'''(\xi) = (p + 3qF(\xi) + 6rF^2(\xi))F'(\xi). \tag{12}$$

For $m = 1$, the general solution of eq. (8) has the following form:

$$W(\xi) = a_0 + a_1 F(\xi) + \frac{b_1}{F(\xi)} + d_1 \frac{F'(\xi)}{F(\xi)}. \tag{13}$$

Substituting eq. (13) with the help of auxiliary ordinary differential equation (10) into (8), collecting all coefficients of $F'^k(\xi)F^j(\xi)$ ($k = 0, 1$, $j = 0, 1, 2, 3, \ldots, n$) and setting them equal to zero, we obtain a system of algebraic equations, by solving this system with the help of Maple or Mathematica softwares, different sets of values of constants a_j, b_j, c_j, d_j and frequency are obtained, by substituting them in eq. (13) different more general and new families of exact solutions of eq. (5) are obtained as mentioned below.

– *Family 1:*

$$a_0 = \pm\sqrt{\frac{1}{2}(\alpha - \gamma^2) + \sigma_0^2}, \qquad b_1 = d_1 = 0, \qquad p = \frac{2(\alpha - \gamma^2 + 2\sigma_0^2)}{\kappa^2}$$

$$a_1 = \pm\frac{q\kappa^2}{2\sqrt{2}\sqrt{\alpha - \gamma^2 + 2\sigma_0^2}}, \qquad r = \frac{q^2\kappa^2}{8(\alpha - \gamma^2 + 2\sigma_0^2)}. \tag{14}$$

Then, substituting these values into eq. (13) and with the mentioned solutions of eq. (10) in ref. [21] using the auxiliary equation mapping method, the following solutions of eq. (5) are obtained in this family:

$$q_1 = \left(\frac{2(\alpha - \gamma^2 + 2\sigma_0^2)}{2\sqrt{2(\alpha - \gamma^2 + 2\sigma_0^2)}} - \frac{p\kappa^2}{2\sqrt{2(\alpha - \gamma^2 + 2\sigma_0^2)}}\left(1 + s\tanh\left(\frac{\sqrt{p}}{2}(\kappa(x + 2\gamma t)) + \xi_0\right)\right)\right)e^{i(\gamma x + \alpha t)}, \tag{15}$$

$$q_2 = \left(\frac{4(\alpha - \gamma^2 + 2\sigma_0^2)}{4\sqrt{2(\alpha - \gamma^2 + 2\sigma_0^2)}} + \frac{q\kappa^2\sqrt{\frac{p}{r}}}{4\sqrt{2(\alpha - \gamma^2 + 2\sigma_0^2)}}\left(1 + \frac{s\sinh(\sqrt{p}(\kappa(x + 2\gamma t)))}{\rho + \cosh(\sqrt{p}(\kappa(x + 2\gamma t)))}\right)\right)e^{i(\gamma x + \alpha t)}, \tag{16}$$

$$q_3 = \left(\frac{2(\alpha - \gamma^2 + 2\sigma_0^2)}{2\sqrt{2(\alpha - \gamma^2 + 2\sigma_0^2)}} - \frac{p\kappa^2}{2\sqrt{2(\alpha - \gamma^2 + 2\sigma_0^2)}}\left(1 + \frac{s(\rho\sqrt{1 + \sigma^2} + \cosh(\sqrt{p}(\kappa(x + 2\gamma t))))}{\sigma + \sinh(\sqrt{p}(\kappa(x + 2\gamma t)))}\right)\right)e^{i(\gamma x + \alpha t)}. \tag{17}$$

– *Family 2:*

$$a_0 = \pm\frac{\sqrt{\alpha - \gamma^2 + 2\sigma_0^2}}{2\sqrt{2}}, \qquad b_1 = a_1 = 0, \qquad p = \frac{-\alpha + \gamma^2 - 2\sigma_0^2}{\kappa^2}, \qquad q = r = 0, \qquad d_1 = -\frac{i\,\kappa}{2\sqrt{2}}. \tag{18}$$

Then, substituting these values into eq. (13) and with the mentioned solutions of eq. (10) in ref. [21] using the auxiliary equation mapping method, the following solutions of eq. (5) are obtained in this family:

$$q_4 = \left(\frac{2\sqrt{\alpha - \gamma^2 + 2\sigma_0^2}}{4\sqrt{2}} - \frac{i\,\sqrt{p}\kappa\, s\,\mathrm{sech}(\frac{\sqrt{p}}{2}(\kappa(x + 2\gamma t)) + \xi_0)^2}{4\sqrt{2}(1 + s\tanh(\frac{\sqrt{p}}{2}(\kappa(x + 2\gamma t)) + \xi_0))}\right)e^{i(\gamma x + \alpha t)} \quad p < 0, \tag{19}$$

$$q_5 = \left(\frac{\sqrt{\alpha - \gamma^2 + 2\sigma_0^2}}{2\sqrt{2}} - \frac{i\kappa\sqrt{p}s(1+\rho\cosh(\sqrt{p}(\kappa(x+2\gamma t))))}{2\sqrt{2}(\rho+\cosh(\sqrt{p}(\kappa(x+2\gamma t))))(\rho+\cosh(\sqrt{p}(\kappa(x+2\gamma t)))+s\sinh(\sqrt{p}(\kappa(x+2\gamma t))))} \right) e^{i(\gamma x+\alpha t)} \quad p<0, \tag{20}$$

$$q_6 = \left(\frac{\sqrt{\alpha - \gamma^2 + 2\sigma_0^2}}{2\sqrt{2}} + \frac{i\,\kappa\sqrt{p}s(1+\rho\sqrt{1+\sigma^2}\cosh(\sqrt{p}(\kappa(x+2\gamma t)))-\sigma\sinh(\sqrt{p}(\kappa(x+2\gamma t))))}{2\sqrt{2}(\sigma+\sinh(\sqrt{p}(\kappa(x+2\gamma t))))(\sigma+s\,\rho\sqrt{1+\sigma^2}+s\cosh(\sqrt{p}(\kappa(x+2\gamma t)))+\sinh(\sqrt{p}(\kappa(x+2\gamma t))))} \right) \times e^{i(\gamma x+\alpha t)} \quad p<0. \tag{21}$$

– *Family 3:*

$$a_1 = \frac{\sqrt{r}\kappa}{2}, \qquad d_1 = \frac{\kappa}{2}, \qquad b_1 = a_0 = 0, \qquad p = \frac{2(\alpha-\gamma^2+2\sigma_0^2)}{\kappa^2}, \qquad q=r=0. \tag{22}$$

Then, substituting these values into eq. (13) and with the mentioned solutions of eq. (10) in ref. [21] using the auxiliary equation mapping method, the following solutions of eq. (5) are obtained in this family:

$$q_7 = \left(-\frac{p\sqrt{r}\kappa(1+s\tanh(\frac{\sqrt{p}(\kappa(x+2\gamma t))}{2}+\xi_0))}{2q} + \frac{\sqrt{p}\kappa\, s\,\mathrm{sech}(\frac{\sqrt{p}(\kappa(x+2\gamma t))}{2}+\xi_0)^2}{4(1+s\tanh(\frac{\sqrt{p}(\kappa(x+2\gamma t))}{2}+\xi_0))} \right) e^{i(\gamma x+\alpha t)}, \tag{23}$$

$$q_8 = \left(\frac{\kappa}{4}\left(\frac{2\sqrt{p}s(1+\rho\cosh(\sqrt{p}(\kappa(x+2\gamma t))))}{(\rho+\cosh(\sqrt{p}(\kappa(x+2\gamma t))))(\rho+\cosh(\sqrt{p}(\kappa(x+2\gamma t)))+s\sinh(\sqrt{p}\kappa(x+2\gamma t))} \right) + \sqrt{\frac{p}{r}}\sqrt{r}\left(1+\frac{s\sinh(\sqrt{p}\kappa(x+2\gamma t))}{\rho+\cosh(\sqrt{p}\kappa(x+2\gamma t))}\right)\right) e^{i(\gamma x+\alpha t)} \quad p>0, \quad r>0, \quad \rho>0, \tag{24}$$

$$q_9 = \left(\frac{s\sqrt{p}\kappa(-1-\rho\sqrt{1+\sigma^2}\cosh(\sqrt{p}\kappa(x+2\gamma t))+\sigma\sinh(\sqrt{p}\kappa(x+2\gamma t)))}{2(\sigma+\sinh(\sqrt{p}\kappa(x+2\gamma t)))(\sigma+s\rho\sqrt{1+\sigma^2}+s\cosh(\sqrt{p}\kappa(x+2\gamma t))+\sinh(\sqrt{p}\kappa(x+2\gamma t)))} - \frac{p\kappa\sqrt{r}(\sigma+s\rho\sqrt{1+\sigma^2}+s\cosh(\sqrt{p}\kappa(x+2\gamma t))+\sinh(\sqrt{p}\kappa(x+2\gamma t)))^2}{2q(\sigma+\sinh(\sqrt{p}\kappa(x+2\gamma t)))(\sigma+s\rho\sqrt{1+\sigma^2}+s\cosh(\sqrt{p}\kappa(x+2\gamma t))+\sinh(\sqrt{p}\kappa(x+2\gamma t)))} \right) e^{i(\gamma x+\alpha t)}. \tag{25}$$

– *Family 4:*

$$d_1 = \frac{\kappa}{2}, \qquad b_1 = a_1 = a_0 = 0, \qquad p = \frac{2(\alpha-\gamma^2+2\sigma_0^2)}{\kappa^2}, \qquad q=r=0. \tag{26}$$

Then, substituting these values into eq. (13) and with the mentioned solutions of eq. (10) in ref. [21] using the auxiliary equation mapping method, the following solutions of eq. (5) are obtained in this family:

$$q_{10} = \left(\frac{\sqrt{p}\kappa\, s\,\mathrm{sech}(\frac{\sqrt{p}(\kappa(x+2\gamma t))}{2}+\xi_0)^2}{4(1+s\tanh(\frac{\sqrt{p}(\kappa(x+2\gamma t))}{2}+\xi_0))} \right) e^{i(\gamma x+\alpha t)}, \tag{27}$$

$$q_{11} = \left(\frac{\kappa\sqrt{p}s(1+\rho\cosh(\sqrt{p}(\kappa(x+2\gamma t))))}{2\rho+\cosh(\sqrt{p}(\kappa(x+2\gamma t))))(\rho+\cosh(\sqrt{p}(\kappa(x+2\gamma t)))+s\sinh(\sqrt{p}\kappa(x+2\gamma t)))} \right) e^{i(\gamma x+\alpha t)}, \tag{28}$$

$$q_{12} = \left(\frac{s\sqrt{p}\kappa(-1-\rho\sqrt{1+\sigma^2}\cosh(\sqrt{p}\kappa(x+2\gamma t))+\sigma\sinh(\sqrt{p}\kappa(x+2\gamma t)))}{2(\sigma+\sinh(\sqrt{p}\kappa(x+2\gamma t)))(\sigma+s\rho\sqrt{1+\sigma^2}+s\cosh(\sqrt{p}\kappa(x+2\gamma t))+\sinh(\sqrt{p}\kappa(x+2\gamma t)))} \right) e^{i(\gamma x+\alpha t)}. \tag{29}$$

– *Family 5:*

$$a_0 = \pm\frac{\sqrt{\alpha-\gamma^2+2\sigma_0^2}}{2\sqrt{2}}, \qquad b_1 = a_1 = 0, \qquad d_1 = \pm\frac{i\sqrt{-\alpha+\gamma^2-2\sigma_0^2}}{2\sqrt{2}\sqrt{p}} \qquad q=r=0. \tag{30}$$

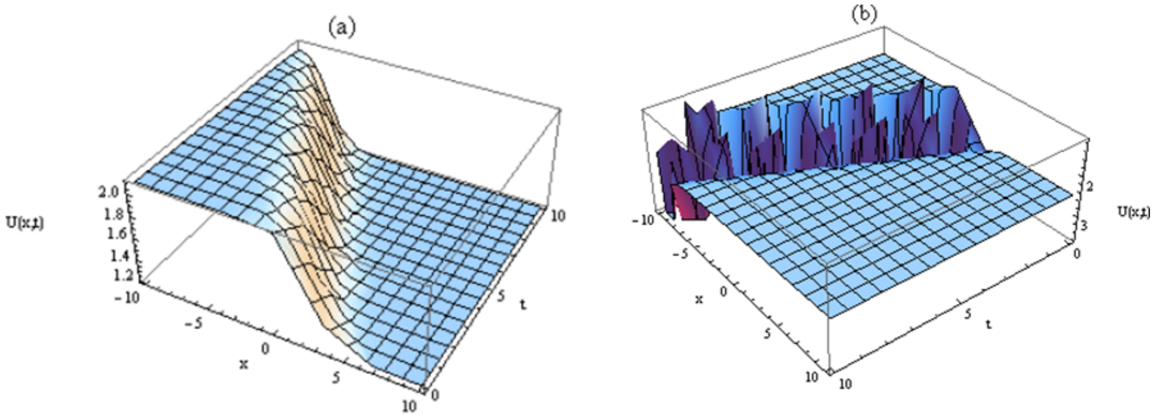

Fig. 1. Graphical representation of solitary wave solutions (15) and (16): (a) three-dimensional half-bright soliton of anti-kink-type with $\xi_0 = -2$, $\kappa = 1$, $\gamma = .5$, $\rho = -1.8$, $s = 4$, and $\alpha = 1.5$; (b) three-dimensional graph of (16) as a dark soliton with $\xi_0 = -2$, $\kappa = 1.1$, $\gamma = .5$, $\rho = -1.8$, $s = 4$, and $\alpha = 1.5$ in intervals $(-10, 10)$, $(0, 10)$.

Then, substituting these values into eq. (13) and with the mentioned solutions of eq. (10) in ref. [21] using the auxiliary equation mapping method, the following solutions of eq. (5) are obtained in this family:

$$q_{13} = \left(\frac{2\sqrt{\alpha - \gamma^2 + 2\sigma_0^2}}{4\sqrt{2}} + \frac{i\,\sqrt{-\alpha + \gamma^2 - 2\sigma_0^2}\, s\, \mathrm{sech}(\frac{\sqrt{p}}{2}(\kappa(x + 2\gamma t)) + \xi_0)^2}{4\sqrt{2}(1 + s\tanh(\frac{\sqrt{p}}{2}(\kappa(x + 2\gamma t)) + \xi_0))} \right) e^{i(\gamma x + \alpha t)} \quad p < 0, \tag{31}$$

$$\begin{aligned} q_{14} = {} & \left(\frac{\sqrt{\alpha - \gamma^2 + 2\sigma_0^2}}{2\sqrt{2}} \right. \\ & \left. + \frac{i\sqrt{-\alpha + \gamma^2 - 2\sigma_0^2}\, s(1 + \rho\cosh(\sqrt{p}(\kappa(x + 2\gamma t))))}{2\sqrt{2}(\rho + \cosh(\sqrt{p}(\kappa(x + 2\gamma t))))(\rho + \cosh(\sqrt{p}(\kappa(x + 2\gamma t))) + s\sinh(\sqrt{p}(\kappa(x + 2\gamma t))))} \right) \\ & \times e^{i(\gamma x + \alpha t)} \quad p < 0, \end{aligned} \tag{32}$$

$$\begin{aligned} q_{15} = {} & \left(\frac{\sqrt{\alpha - \gamma^2 + 2\sigma_0^2}}{2\sqrt{2}} \right. \\ & \left. - \frac{i\,\sqrt{-\alpha + \gamma^2 - 2\sigma_0^2}\, s(1 + \rho\sqrt{1 + \sigma^2}\cosh(\sqrt{p}(\kappa(x + 2\gamma t))) - \sigma\sinh(\sqrt{p}(\kappa(x + 2\gamma t))))}{2\sqrt{2}(\sigma + \sinh(\sqrt{p}(\kappa(x + 2\gamma t))))(\sigma + s\,\rho\sqrt{1 + \sigma^2} + s\cosh(\sqrt{p}(\kappa(x + 2\gamma t))) + \sinh(\sqrt{p}(\kappa(x + 2\gamma t))))} \right) \\ & \times e^{i(\gamma x + \alpha t)} \quad p < 0. \end{aligned} \tag{33}$$

3 Graphical representation of the solutions

In this section we graphically present our new derived families of solutions, including rational functions, hyperbolic functions and trigonometric functions with different shapes to understand the physical description of the NLSE using Mathematica 10.4. See figs. 1–7.

Results and discussion

In this section our focus is to highlight the similarities and differences of our results with solutions already obtained in the literature by applying different methods. We have obtained a collection of more general and new solutions, the key point of this is the structure of our proposed solution (9), which has a different structure with the range of three parameters, by obtaining different sets of values of constants a_j, b_j, c_j, d_j with the help of Mathematica, eq. (10) has solutions of different types including rational, trigonometric, and hyperbolic functions. By this powerful method we have obtained a collection of new families of solutions but still some of our results are similar to others. In the following we made a comparison of our results with some other methods.

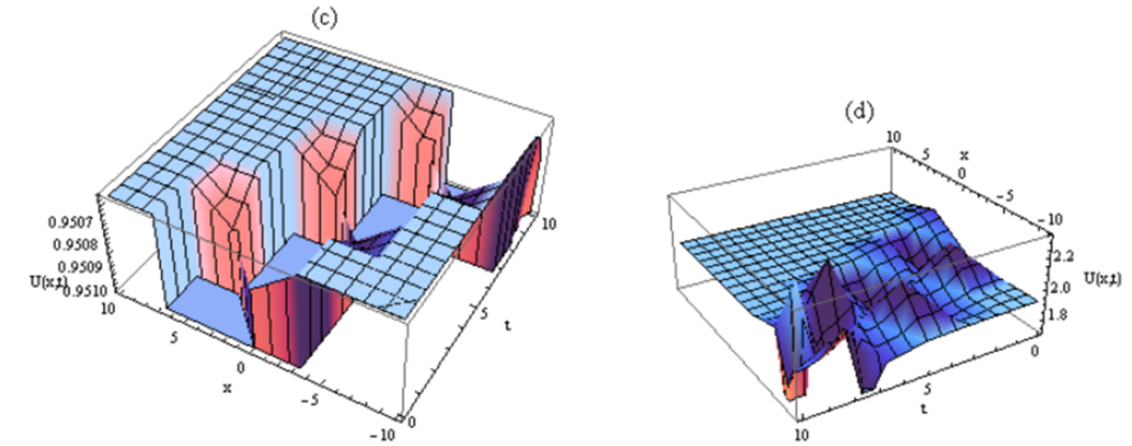

Fig. 2. Graphical representation of solitary wave solutions of (19) and (20): (c) (19) as a dark soliton of different shape with $\xi_0 = -2$, $\kappa = 1$, $\gamma = .5$, $\rho = -1.8$, $s = 4$, and $\alpha = 1.5$; (d) three-dimensional graph of (20) as a half-bright soliton of periodic type with $\xi_0 = 2$, $\kappa = 1$, $\gamma = .5$, $\rho = -1.8$, $s = 4$, and $\alpha = 1.5$ in intervals $(-10, 10)$, $(0, 10)$.

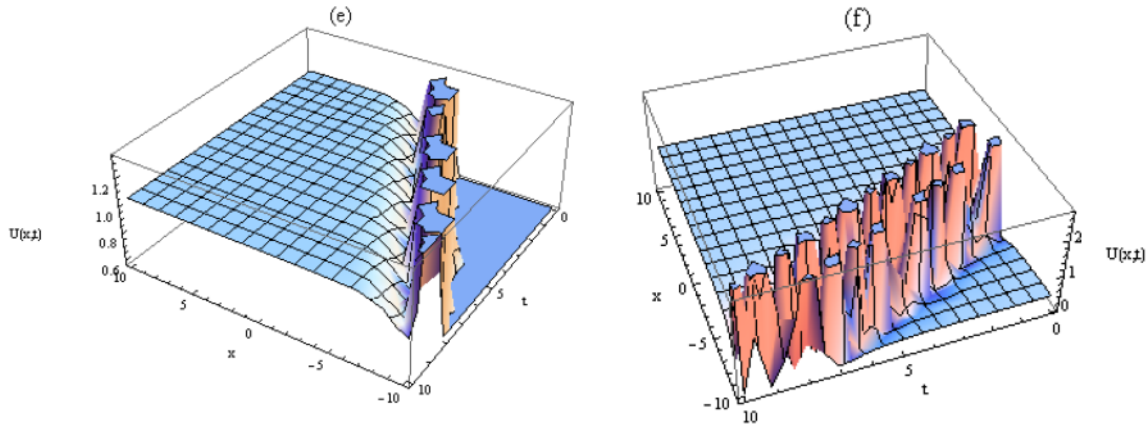

Fig. 3. Graphical representation of solitary wave solutions (24) and (25): (e) three-dimensional graph of (24) as a bright soliton of different shape with $\xi_0 = -1$, $\kappa = 1.1$, $\gamma = .5$, $\rho = 1.8$, $s = 1.8$, and $\alpha = 1$; (f) three-dimensional graph of (25) as a bright soliton of different shape with $\xi_0 = -1$, $\kappa = 1$, $\gamma = .5$, $\rho = -1.8$, $s = 1.8$, and $\alpha = 1.7$ in intervals $(-10, 10)$, $(0, 10)$.

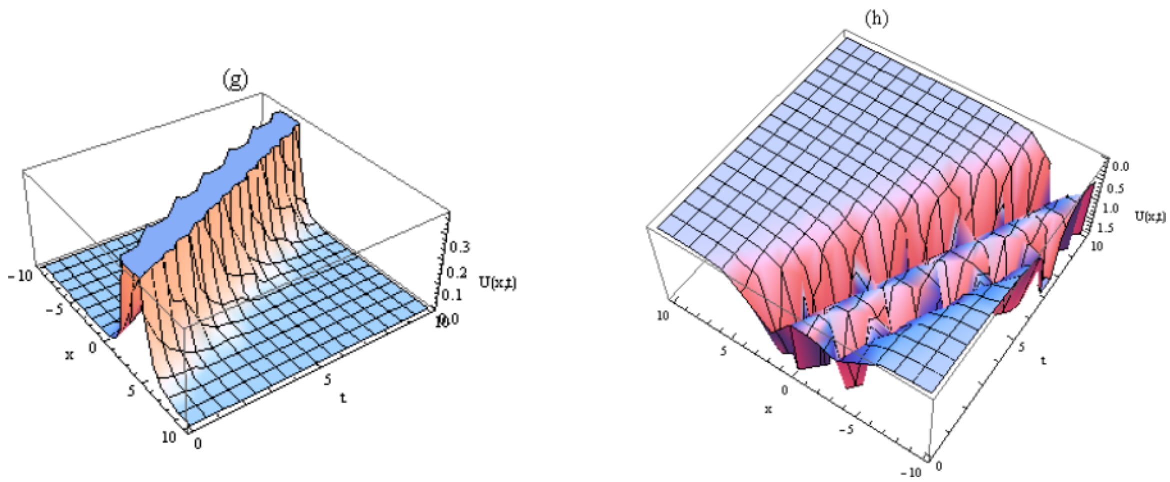

Fig. 4. Graphical representation of solitary wave solutions (27) and (28): (g) three-dimensional graph of (27) as a bright soliton with $\xi_0 = -1$, $\kappa = 1.1$, $\gamma = .5$, $\rho = 1.8$, $s = 1.8$, and $\alpha = 1$; (h) three-dimensional graph of (28) as a half-dark soliton of different shape with $\xi_0 = 3$, $\kappa = 1$, $\gamma = .5$, $\rho = -1.8$, $s = 1.8$, and $\alpha = 1.7$ in intervals $(-10, 10)$, $(0, 10)$.

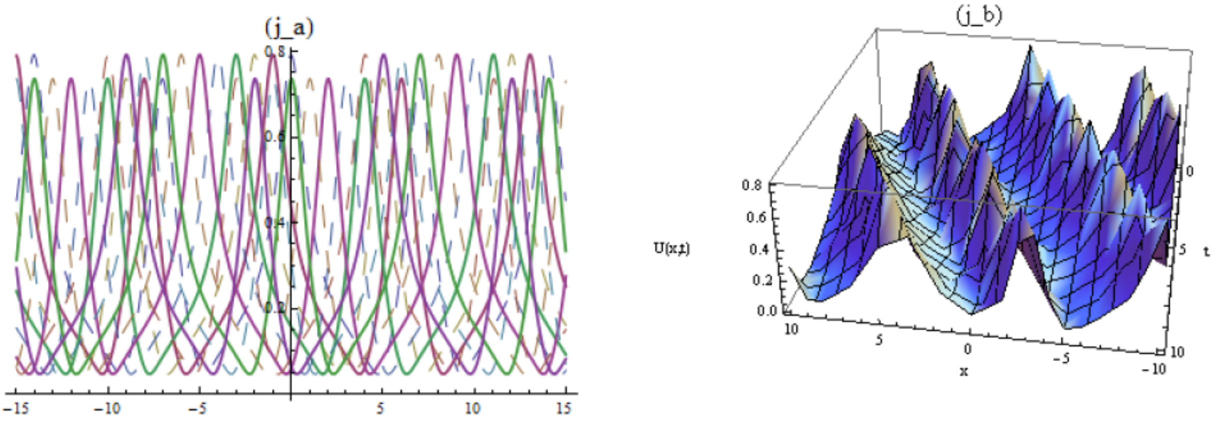

Fig. 5. Graphical representation of the solitary wave solution (29): (j-a) two-dimensional graph of (29) as a periodic soliton with $\xi_0 = 3$, $\kappa = 1.1$, $\gamma = 1.5$, $\rho = 1.8$, $s = 3$, and $\alpha = 1.7$; (j-b) three-dimensional periodic soliton of (29) in intervals $(-10, 10)$, $(0, 10)$.

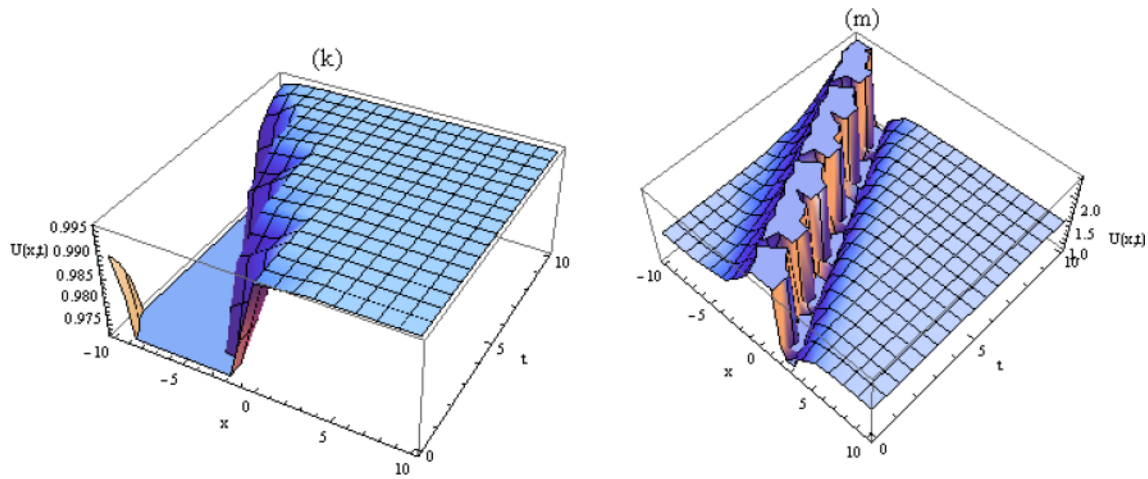

Fig. 6. Graphical representation of solitary wave solutions (31) and (32): (k) three-dimensional graph of (31) as a half-bright soliton with $\xi_0 = 3$, $\kappa = 1$, $\gamma = .5$, $\rho = 1.8$, $s = .3$, and $\alpha = 1.7$; (m) three-dimensional graph of (32) as a bright soliton of different shape with $\xi_0 = 3$, $\kappa = 1$, $\gamma = .5$, $\rho = 3$, $s = -3$, and $\alpha = 1.7$ in intervals $(-10, 10)$, $(0, 10)$.

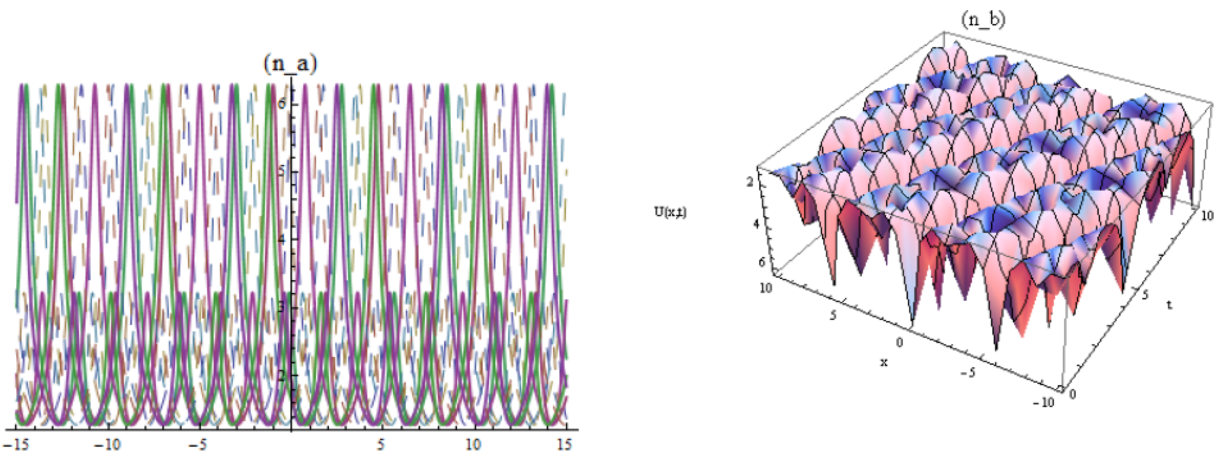

Fig. 7. Graphical representation of the solitary wave solution (33): (n-a) two-dimensional periodic soliton (33) with $\xi_0 = 3$, $\kappa = 1.1$, $\gamma = .5$, $\rho = 3$, $s = 3$, and $\alpha = 1.7$; (n-b) three-dimensional graph of periodic soliton of (33) in intervals $(-10, 10)$, $(0, 10)$.

Similarities with the results obtained by the modified extended direct algebraic method by considering $\lambda = 1$ and $\sigma_0^2 = \gamma$:

- Our solution (19) is approximately the same with the mentioned solution u_{13} in Case 1 of [43].
- Our solution (15) has similarity with the solution u_{21} mentioned in Case 2 of [43].

Results obtained by the extended Fan sub-equation method:

- Exact solitary wave solutions with $\rho = \sigma_0$ obtained by the extended Fan sub-equation method in [43] are different from our obtained results.

In the latter, it is important to note that the solutions obtained in [23], are for the perturbed NLSE while the solutions obtained in the present work are for the NLSE which can be easily obtained by setting the coefficients of dispersion terms and Raman scattering to zero, then the perturbed NLSE becomes NLSE with non-Kerr law nonlinearity which is same as for $\sigma_0 = 0$ in eq. (5). From the above comparison we can conclude that except some solutions (19) and (15) our other obtained solutions are new and have not been formulated before, which shows that our method is more helpful, effective, straightforward and reliable to analytically study other nonlinear complex models.

4 Conclusion

In this paper we analytically investigated the complex nonlinear Schrödinger equation with Kerr law nonlinearity to construct its more general and new solitary wave solutions of different types including rational, trigonometric, and hyperbolic functions. We applied the auxiliary equation mapping method to find a rich variety of new solutions for the range of three parameters; calculations demonstrate that the method is more reliable, straightforward, and effective to analytically study other nonlinear complicated physical problems modeled by complex nonlinear partial differential equations arising in mathematical physics, hydrodynamics, fluid mechanics, mathematical biology, plasma physics, engineering disciplines, chemistry and many other natural sciences. We also have graphically expressed our obtained solutions with the aid of Mathematica 10.4, these combined solutions are more helpful in future in the development of soliton dynamics, adiabatic parameter dynamics, biomedical problems, fluid dynamics, industrial studies and many other fields, more importantly to make a comparison with numerical solutions in the development of numerical techniques as well as to develop a wide range of new mathematical softwares, and to understand the physical interpretation of complicated nonlinear wave problems.

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