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# Study of charged spherical collapse in f($\mathcal{G}$, T) gravity

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**Abstract.** This paper deals with the dynamics of charged spherical perfect fluid collapse in the framework of $f(\mathcal{G},T)$ gravity. We formulate dynamical equations through the Misner-Sharp formalism and investigate the effects of correction terms, fluid parameters as well as electromagnetic field on the collapse rate. We also construct a relationship among Weyl scalar, correction terms, electric field intensity and energy density. For zero electric charge and constant $f(\mathcal{G},T)$, it is found that if the metric is conformally flat then energy density is homogeneous and vice versa. We conclude that the electric charge and positive correction terms behave as anti-gravitational force and hence diminish the collapse rate.

## 1 Introduction

The current cosmic accelerated expansion has been found from different observational evidence including large scale structures, cosmic microwave background radiations and supernova type Ia, etc. This expansion is considered as the result of a cryptical force named as dark energy which possesses large negative pressure. The mysterious nature of dark energy has stimulated many researchers to reveal its salient features. Modified theories of gravity are supposed as the most optimistic and promising approaches to uncover its ambiguous nature. Such theories can be modeled by replacing the corresponding scalar invariants with their generic functions in the geometric part of the Einstein-Hilbert action.

The Lovelock gravity is one of the modified versions of general relativity (GR) in $n$-dimensional space which is found to be equivalent to GR for the case of 4 dimensions [1,2]. It is worth mentioning here that the first Lovelock scalar contains only the Ricci scalar $R$, while the second Lovelock scalar contains the Gauss-Bonnet (GB) invariant which gives Einstein GB gravity in 5 dimensions [3, 4]. The GB invariant is defined as $\mathcal{G} = R_{\alpha\beta\xi\eta}R^{\alpha\beta\xi\eta} + R^2 - 4R_{\alpha\beta}R^{\alpha\beta}$, where $R_{\alpha\beta\xi\eta}$ and $R_{\alpha\beta}$ represent the Riemann and Ricci tensors, respectively which is found to be 4-dimensional topological quantity. Nojiri and Odintsov [5] introduced $f(\mathcal{G})$ gravity which gives interesting features of current cosmic expansion. This theory is free from instability problems like ghost spin-2 instabilities [6] and is consistent with solar system constraints as well as cosmological structure [7–9].

Nojiri and Odintsov [10] introduced the notion of curvature-matter coupling and Harko *et al.* [11] established such coupling in $f(R)$ theory referred as $f(R,T)$ gravity. The curvature-matter couplings can describe the rotation curves of galaxies and different epochs of the universe evolution. These couplings also provide non-conservation of stress energy tensor implying the presence of an extra force; consequently, the path of particles is changed. These coupling models are extremely useful for explaining the current cosmic expansion as well as dark matter and dark energy interactions [12]. Recently, Sharif and Ikram [13] introduced this coupling in the $f(\mathcal{G})$ theory named as the $f(\mathcal{G},T)$ gravity. The same authors [14] also reconstructed several cosmological models including power-law solutions, de Sitter universe and phantom as well as non-phantom eras in this gravity. They also studied the effect of $f(\mathcal{G},T)$ gravity on the stability of Einstein universe with non-conserved background [15]. Recently, Bhatti *et al.* [16] investigated the stability of some stellar solutions in this gravity and inferred that power-law and logarithmic correction terms could give an arena to host stable configurations of stellar models.

Gravitational collapse is one of the key aspects of stellar evolution and is referred as the fundamental mechanism in the formation of new stars. The process of gravitational collapse was first studied by Chandrasekhar [17], who

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concluded that a star remains in its equilibrium position if the inward gravitational force is balanced by outward directed pressure. Oppenheimer and Snyder [18] explored the dynamics of dust collapse and found that a black hole is the eventual result of dust collapse. Misner and Sharp [19] discussed the dynamics of dissipative spherical collapse and concluded that interior energy is transformed into a flux directed in the outward direction. Chan [20] studied the collapse of a radiating star and found that anisotropic pressure of the fluid increases due to shear viscosity. Herrera and Santos [21] investigated spherical collapse by using the Misner-Sharp technique and found that energy disappears in the shape of radiation and heat flow. Herrera [22] studied the dynamics of dissipative collapse through dynamical and transport equations. Herrera *et al.* [23] worked on cylindrical collapse and discussed the effects of anisotropic pressure on the collapsing phenomenon.

The study of spherical collapse with different fluid distributions has also been studied in various modified gravity models. Sharif and Abbas [24] investigated the spherical collapse for dissipative fluids in $f(\mathcal{G})$ gravity. They also discussed the dynamics of shearfree charged radiating spherical systems in this theory [25]. Chakrabarti and Banerjee [26] studied the collapse of spherical stars for the charged dissipative fluids in $f(R)$ gravity. Recently, Sharif and Farooq [27] examined the collapse of charged spherical stars incorporated with perfect fluid in $f(R)$ gravity.

In the phenomenon of collapse, the electric charge plays a role of Coulomb repulsive force which reduces the gravitational force and hence prevents the collapse process. The effects of an electromagnetic field on the dynamics of a stellar structure were first studied by Rosseland [28]. Bekenstein [29] investigated the charged spherical ideal collapse. Di Prisco *et al.* [30] studied spherical collapse coupled with non-adiabatic charged fluid and observed a dissipation procedure through transport and dynamical equations. Sharif and Abbas [31] discussed cylindrical collapse of charged non-adiabatic fluid in $f(\mathcal{G})$ gravity and studied the effects of electromagnetic field, anisotropy and heat flux on the collapse rate. Guha and Benerji [32] studied the dynamics of cylindrical collapse with anisotropic charged fluid and discussed the effects of heat dissipation through dynamical equations. Sharif and his collaborators [33–35] explored the influence of the electromagnetic field on self-gravitating compact objects by investigating instability constraints during the collapse process. Sharif and Yousaf [36] found the effects of electric charge on the evolution of a sphere and concluded that the electromagnetic field decreases the collapse rate. Sharif and Farooq [37] explored the charged cylindrical perfect fluid collapse in $f(R)$ gravity and concluded that correction terms, anisotropic pressure as well as electric charge prevent the collapse process.

In this paper, we analyze the role of correction terms, matter variables and electric charge on the dynamical behavior of a collapsing perfect fluid sphere in the background of $f(\mathcal{G},T)$ gravity. For this purpose, the interior region of a spherical star is matched with exterior charged Viadya spacetime through Darmois conditions [38]. The paper is planned as follows. In sect. 2, we construct $f(\mathcal{G},T)$ equations of motion and formulate junction conditions. Section 3 is devoted to develop the dynamical equations by using the Misner-Sharp formalism. The relationship among Weyl scalar, correction terms, electric charge and energy density is established in sect. 4. In the last section, we provide a summary of the results.

## 2 Field equations and junction conditions

In this section, we construct the field equations for a charged spherical system with perfect fluid as well as junction conditions with exterior charged Viadya spacetime. The Einstein-Hilbert action for this gravity is given by [13]

$$S_{f(\mathcal{G},T)} = \frac{1}{2\kappa^2}\int\left[f\left(\mathcal{G},T\right)+R\right]\sqrt{-g}\mathrm{d}^4x + \int\mathcal{L}_m\sqrt{-g}\mathrm{d}^4x, \tag{1}$$

where $\kappa^2$, $\mathcal{L}_m$ and $g$ represent the coupling constant, matter Lagrangian density and determinant of the metric tensor $g_{\alpha\beta}$, respectively. Here, $f(\mathcal{G},T)$ is an arbitrary function of a GB invariant and trace of the stress energy tensor. We consider that the spherical configuration is filled with perfect fluid given by

$$T^{(\mathcal{M})}_{\alpha\beta} = (\rho+p)\mathcal{U}_\alpha\mathcal{U}_\beta + pg_{\alpha\beta},$$

where $\rho$, $p$ and $\mathcal{U}_\alpha$ are the energy density, pressure and four-velocity (with $\mathcal{U}^\alpha\mathcal{U}_\alpha=-1$) of the fluid, respectively. The field equations for the action (1) are

$$\begin{aligned} G_{\alpha\beta} &= 8\pi T_{\alpha\beta} - \left(\Theta_{\alpha\beta}+T_{\alpha\beta}\right)f_T\left(\mathcal{G},T\right) + \frac{1}{2}g_{\alpha\beta}f\left(\mathcal{G},T\right)\\ &+ \left(4R_{\xi\beta}R^\xi_\alpha + 4R^{\xi\eta}R_{\alpha\xi\beta\eta} - 2RR_{\alpha\beta} - 2R_{\beta\xi\eta\delta}R^{\xi\eta\delta}_\alpha\right)f_\mathcal{G}\left(\mathcal{G},T\right)\\ &+ (4R_{\alpha\beta}\nabla^2 + 4g_{\alpha\beta}R^{\xi\eta}\nabla_\xi\nabla_\eta + 2R\nabla_\alpha\nabla_\beta - 2g_{\alpha\beta}R\nabla^2 - 4R^\xi_\alpha\nabla_\beta\nabla_\xi\\ &- 4R^\xi_\beta\nabla_\alpha\nabla_\xi - 4R_{\alpha\xi\beta\eta}\nabla^\xi\nabla^\eta)f_\mathcal{G}(\mathcal{G},T), \end{aligned} \tag{2}$$

where $\nabla^2 = \Box = \nabla_\alpha \nabla^\alpha$ and $G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$ represent the d'Alembert operator and Einstein tensor, respectively. Also, $f_{\mathcal{G}}(\mathcal{G}, T) = \frac{\partial f(\mathcal{G},T)}{\partial \mathcal{G}}$ and $f_T(\mathcal{G}, T) = \frac{\partial f(\mathcal{G},T)}{\partial T}$. It is noteworthy that for $T = 0$, the field equations of this theory are reduced to that of $f(\mathcal{G})$ gravity. Furthermore, when $f(\mathcal{G}, T) = 0$ the field equations of GR are recovered. Rearranging eq. (2), we obtain

$$G_{\alpha\beta} = 8\pi T^{(\mathrm{eff})}_{\alpha\beta} = 8\pi \left( T^{(\mathcal{M})}_{\alpha\beta} + T^{(\mathcal{G}T)}_{\alpha\beta} \right), \tag{3}$$

where $T^{(\mathrm{eff})}_{\alpha\beta}$ is the effective stress energy tensor and $T^{(\mathcal{G}T)}_{\alpha\beta}$ are the correction terms of $f(\mathcal{G}, T)$ gravity given as follows:

$$\begin{aligned} T^{(\mathcal{G}T)}_{\alpha\beta} = \frac{1}{8\pi} \Big[ & (\rho + p)\mathcal{U}_\alpha \mathcal{U}_\beta f_T(\mathcal{G}, T) + \frac{1}{2} g_{\alpha\beta} f(\mathcal{G}, T) \\ & + \left( 4R_{\xi\beta} R^{\xi}_{\alpha} + 4R^{\xi\eta} R_{\alpha\xi\beta\eta} - 2RR_{\alpha\beta} - 2R_{\beta\xi\eta\delta} R^{\xi\eta\delta}_{\alpha} \right) f_{\mathcal{G}}(\mathcal{G}, T) \\ & + (4R_{\alpha\beta}\nabla^2 + 4g_{\alpha\beta} R^{\xi\eta}\nabla_\xi \nabla_\eta + 2R\nabla_\alpha \nabla_\beta - 2g_{\alpha\beta} R\nabla^2 - 4R^{\xi}_{\alpha}\nabla_\beta \nabla_\xi \\ & - 4R^{\xi}_{\beta}\nabla_\alpha \nabla_\xi - 4R_{\alpha\xi\beta\eta}\nabla^\xi \nabla^\eta) f_{\mathcal{G}}(\mathcal{G}, T) \Big]. \end{aligned} \tag{4}$$

The line element for the interior geometry $(\mathcal{U}^-)$ is

$$\mathrm{d}s^2_- = -X^2(t, r)\mathrm{d}t^2 + Y^2(t, r)\mathrm{d}r^2 + Z^2(t, r)(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2), \tag{5}$$

where $Z$ represents the areal radius of a spherical star and $\mathcal{U}^\alpha = \frac{1}{X}\delta^\alpha_0$ defines the comoving velocity of the fluid. Including the effects of the electromagnetic field, eq. (3) becomes

$$G_{\alpha\beta} = 8\pi \left( T^{(\mathcal{M})}_{\alpha\beta} + T^{(\mathcal{G}T)}_{\alpha\beta} + T^{(E)}_{\alpha\beta} \right), \tag{6}$$

where $T^{(E)}_{\alpha\beta}$ is the electromagnetic stress energy tensor defined by

$$T^{(E)}_{\alpha\beta} = \frac{1}{4\pi} \left[ F^{\mu}_{\alpha} F_{\beta\mu} - \frac{1}{4} g_{\alpha\beta} F^{\mu\nu} F_{\mu\nu} \right]. \tag{7}$$

The Maxwell field equations are

$$F_{[\alpha\beta;\nu]} = 0, \qquad F^{\alpha\beta}_{;\beta} = 4\pi J^\alpha, \tag{8}$$

where $F_{\mu\nu} = \phi_{\nu,\mu} - \phi_{\mu,\nu}$ is the Maxwell field tensor, while the terms $\phi_\mu$ and $J^\alpha$ correspond to the four-current and four-potential, respectively. For comoving coordinates, the charge is at rest so, the magnetic field is zero. Consequently, the four-current and four-potential become

$$J^\alpha = \mu_0 V^\alpha, \qquad \phi_\alpha = \phi \delta^0_\alpha,$$

here $\mu_0 = \mu_0(t, r)$ is the charge density and $\phi = \phi(t, r)$ is the scalar field potential. For the interior spacetime, the charge conservation law yields

$$q(r) = 4\pi \int_0^r \mu_0 Y Z^2 \mathrm{d}r, \tag{9}$$

where $q(r)$ is the total amount of electric charge in the interior region of the spherical star. The electric field intensity for spherically symmetric metric can be given as follows:

$$E = \frac{q(r)}{4\pi Z^2}.$$

The Maxwell field equations become

$$\phi'' - \phi' \left( \frac{X'}{X} + \frac{Y'}{Y} - 2\frac{Z'}{Z} \right) = 4\pi \mu_0 X Y^2, \tag{10}$$

$$\dot{\phi}' - \phi' \left( \frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} - 2\frac{\dot{Z}}{Z} \right) = 0. \tag{11}$$

In the above equations, prime and dot represent differentiation with respect to radial and temporal coordinates, respectively. The corresponding field equations are

$$8\pi\left(T_{00}^{(\mathcal{M})}+T_{00}^{(\mathcal{GT})}+T_{00}^{(E)}\right)=8\pi\left(\rho+\frac{T_{00}^{(\mathcal{GT})}}{X^2}+2\pi E^2\right)X^2$$
$$=\frac{X^2}{Z^2}\left(-\frac{Z'^2}{Y^2}+2\frac{Y'Z'Z}{Y^3}-2\frac{Z''Z}{Y^2}\right)+\left(\frac{X^2}{Z^2}+\frac{\dot{Z}^2}{Z^2}+\frac{2\dot{Y}\dot{Z}}{YZ}\right), \tag{12}$$

$$8\pi\left(T_{11}^{(\mathcal{M})}+T_{11}^{(\mathcal{GT})}+T_{11}^{(E)}\right)=8\pi\left(P+\frac{T_{11}^{(\mathcal{GT})}}{Y^2}-2\pi E^2\right)Y^2$$
$$=\frac{Y^2}{Z^2}\left(2\frac{\dot{X}\dot{Z}Z}{X^3}-\frac{\dot{Z}^2}{X^2}-2\frac{\ddot{Z}Z}{X^2}\right)+\left(-\frac{Y^2}{Z^2}+2\frac{X'Z'}{XZ}+\frac{Z'^2}{Z^2}\right), \tag{13}$$

$$8\pi\left(T_{22}^{(\mathcal{M})}+T_{22}^{(\mathcal{GT})}+T_{22}^{(E)}\right)=8\pi\left(P+\frac{T_{22}^{(\mathcal{GT})}}{Z^2}+2\pi E^2\right)Z^2$$
$$=\frac{Z^2}{Y^2}\left(\frac{X'Z'}{XZ}+\frac{X''}{X}-\frac{Z'Y'}{ZY}+\frac{Z''}{Z}-\frac{X'Y'}{XY}\right)+\frac{Z^2}{X^2}\left(\frac{\dot{X}\dot{Y}}{XY}-\frac{\ddot{Y}}{Y}-\frac{\dot{Y}\dot{Z}}{YZ}-\frac{\ddot{Z}}{Z}+\frac{\dot{X}\dot{Z}}{XZ}\right), \tag{14}$$

$$8\pi\left(T_{33}^{(\mathcal{M})}+T_{33}^{(\mathcal{GT})}+T_{33}^{(E)}\right)=8\pi\left(P+\frac{T_{33}^{(\mathcal{GT})}}{Z^2\sin^2\theta}+2\pi E^2\right)Z^2\sin^2\theta$$
$$=\frac{Z^2}{Y^2}\left(\frac{X'Z'}{XZ}+\frac{X''}{X}-\frac{Z'Y'}{ZY}+\frac{Z''}{Z}-\frac{X'Y'}{XY}\right)+\frac{Z^2}{X^2}\left(\frac{\dot{X}\dot{Y}}{XY}-\frac{\ddot{Y}}{Y}-\frac{\dot{Y}\dot{Z}}{YZ}-\frac{\ddot{Z}}{Z}+\frac{\dot{X}\dot{Z}}{XZ}\right), \tag{15}$$

$$8\pi\left(T_{01}^{(\mathcal{M})}+T_{01}^{(\mathcal{GT})}+T_{01}^{(E)}\right)=8\pi T_{01}^{(\mathcal{GT})}$$
$$=\left(2\frac{X'\dot{Z}}{XZ}+2\frac{Z'\dot{Y}}{ZY}-2\frac{\dot{Z}'}{Z}\right), \tag{16}$$

where $T_{00}^{(\mathcal{GT})}$, $T_{01}^{(\mathcal{GT})}$, $T_{11}^{(\mathcal{GT})}$, $T_{22}^{(\mathcal{GT})}$ and $T_{33}^{(\mathcal{GT})}$ are the correction terms given in appendix A. The mass function for the interior geometry is defined as [39]

$$m(t,r)=\frac{Z}{2}-\frac{Z}{2}\left(g^{\alpha\beta}\partial_\alpha Z\partial_\beta Z\right).$$

Inclusion of electromagnetic field yields

$$m(t,r)=\frac{Z}{2}+\frac{Z\dot{Z}^2}{2X^2}-\frac{ZZ'^2}{2Y^2}+\frac{q^2}{2Z}. \tag{17}$$

For the exterior geometry ($\mathcal{U}^+$), we consider the charged Vaidya spacetime

$$\mathrm{d}s_+^2=-\left(1+\frac{Q^2}{R^2}-\frac{2M}{R}\right)\mathrm{d}\nu^2-2\,\mathrm{d}\nu\,\mathrm{d}R+R^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right), \tag{18}$$

where $Q$ and $M$ determine the charge and mass of the exterior geometry, respectively. We use the Darmois junction conditions for smooth matching of both regions. These conditions lead to [38]

$$M=m \quad\Longleftrightarrow\quad Q=q, \tag{19}$$

$$\frac{Q^2}{2Z^3}+4\pi Z\left[\frac{T_{01}^{(\mathcal{GT})}}{XY}+\left(p+\frac{T_{11}^{(\mathcal{GT})}}{Y^2}+2\pi E^2\right)\right]=0. \tag{20}$$

These equations give the necessary and sufficient conditions for the smooth matching of both spacetimes. Equation (19) shows that if masses of interior and exterior metrics are equal then their corresponding charges are the same and vice versa. Equation (20) provides the relationship between the correction terms of the interior spacetime and the electromagnetic field.

## 3 Dynamical equations

Here, we discuss the dynamical behavior of a spherical star with the influence of electric charge and correction terms. The dynamical equations obtained by the contraction of Bianchi identity are given as

$$\left[T^{(\mathcal{M})\alpha\beta}+T^{(\mathcal{GT})\alpha\beta}+T^{(E)\alpha\beta}\right]_{;\beta}\mathcal{X}_\alpha=0,$$
$$\left[T^{(\mathcal{M})\alpha\beta}+T^{(\mathcal{GT})\alpha\beta}+T^{(E)\alpha\beta}\right]_{;\beta}\mathcal{U}_\alpha=0,$$

where $\mathcal{X}^\alpha=\frac{1}{Y}\delta^\alpha_1=(0,y^{-1},0,0)$ is a four-vector. After simplification, it follows that:

$$\frac{X'}{Y^2X}\left[\rho+p+\frac{T^{(\mathcal{GT})}_{00}}{X^2}+\frac{T^{(\mathcal{GT})}_{11}}{Y^2}\right]+\frac{1}{Y^2}\left[\frac{T^{(\mathcal{GT})}_{11}}{Y^2}\right]'-\frac{1}{YX}\left[\frac{T^{(\mathcal{GT})}_{01}}{YX}\right]^{\cdot}+\frac{p'}{Y^2}-\frac{2T^{(\mathcal{GT})}_{01}}{Y^2X^2}\left[\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right]$$
$$+\frac{2Z'}{ZY^2}\left[\frac{T^{(\mathcal{GT})}_{11}}{Y^2}-\frac{T^{(\mathcal{GT})}_{22}}{Z^2}\right]-\frac{4\pi E}{ZY^2}[E'Z+2Z'E]=0, \tag{21}$$

$$\frac{\dot{Y}}{YX^2}\left[\rho+p+\frac{T^{(\mathcal{GT})}_{00}}{X^2}+\frac{T^{(\mathcal{GT})}_{11}}{Y^2}\right]+\frac{\dot{\rho}}{X^2}+\left[\frac{T^{(\mathcal{GT})}_{00}}{X^4}\right]^{\cdot}+\frac{2\dot{X}}{X^3}\left[\frac{T^{(\mathcal{GT})}_{00}}{X^2}\right]-\frac{1}{YX}\left[\frac{T^{(\mathcal{GT})}_{01}}{YX}\right]'-\frac{2T^{(\mathcal{GT})}_{01}}{Y^2X^2}\left[\frac{X'}{X}+\frac{Z'}{Z}\right]$$
$$+\frac{2\dot{Z}}{ZX^2}\left[\rho+p+\frac{T^{(\mathcal{GT})}_{00}}{X^2}+\frac{T^{(\mathcal{GT})}_{22}}{Z^2}\right]=0. \tag{22}$$

These equations are used to study the change in evolution of a stellar structure. To discuss the properties of a dynamical system, let us define the proper time derivative $D_t$ and radial derivative $D_r$ as [39]

$$D_t=\mathcal{U}^\alpha\frac{\partial}{\partial x^\alpha}=\frac{1}{X}\frac{\partial}{\partial t},\qquad D_r=\frac{1}{Z'}\frac{\partial}{\partial r}. \tag{23}$$

In the collapsing procedure, the proper time derivative of areal radius defines the corresponding velocity of fluid particles, *i.e.*,

$$\mathcal{V}=D_t(Z)=\frac{\dot{Z}}{X}<0. \tag{24}$$

Using eqs. (17) and (24), we obtain

$$\frac{Z'}{Y}=\left[1-\frac{2m}{Z}+\frac{q^2}{Z^2}+\mathcal{V}^2\right]^{\frac{1}{2}}=\varepsilon. \tag{25}$$

The proper time variation of $m$ is given by

$$D_t(m)=-4\pi Z^2\left[\left(p+\frac{T^{(\mathcal{GT})}_{11}}{Y^2}-2\pi E^2\right)\mathcal{V}-\frac{T^{(\mathcal{GT})}_{01}}{YX}\varepsilon\right]+\frac{\mathcal{V}}{2}\frac{q^2}{Z^2}. \tag{26}$$

This expression describes how internal energy of a star varies with the passage of time. On the right-hand side, the factor $(p+\frac{T^{(\mathcal{GT})}_{11}}{Y^2}-2\pi E^2)\mathcal{V}$ yields the effective electric field intensity and pressure on the collapsing system. The velocity of fluid particles and pre-factor $(-4\pi)$ indicate that this term is positive and the inequality $(p+\frac{T^{(\mathcal{GT})}_{11}}{Y^2})>2\pi E^2$ exists. This implies that the effective outward directed pressure is larger than the effects of charge. The second entity $(\frac{T^{(\mathcal{GT})}_{01}}{YX}\varepsilon)$ manifests the additional effects of correction terms as compared to GR while the remaining factor $(\frac{q^2}{Z^2})$ indicates the Coulomb repulsive effect which decreases the internal energy of star.

The proper radial variation of $m$ yields

$$D_r(m)=4\pi Z^2\left[\left(\rho+\frac{T^{(\mathcal{GT})}_{00}}{X^2}+2\pi E^2\right)-\frac{T^{(\mathcal{GT})}_{01}}{YX}\frac{\mathcal{V}}{\varepsilon}\right]+\frac{q}{Z}D_r(q)-\frac{q^2}{2Z^2}. \tag{27}$$

This demonstrates the change of total energy between the adjacent spherical surfaces of a star. On the right-hand side, the factor $(\rho+\frac{T^{(\mathcal{GT})}_{00}}{X^2}+2\pi E^2)$ depicts how the effective electric field intensity and energy density affect the collapsing process. The effective energy density plays a role of work done which enhances the energy of the system. The second entity $(\frac{T_{01}^{(\mathcal{G}T)}}{YX}\frac{\mathcal{V}}{\varepsilon})$ manifests the additional effects of $f(\mathcal{G},T)$ terms as compared to GR. The next term $(\frac{q}{Z}D_r(q))$ describes the effect of charge. The remaining factor $(\frac{q^2}{2Z^2})$ indicates that the energy of neighboring surfaces of a spherical star decreases due to the repulsive nature of the Coulomb force. Adopting eqs. (23) and (24), we obtain the acceleration for collapsing matter as

$$D_t(\mathcal{V}) = -4\pi Z\left[\left(p+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}-2\pi E^2\right)\right]-\frac{m}{Z^2}+\frac{q^2}{2Z^3}+\frac{\varepsilon}{Y}\frac{X'}{X}. \tag{28}$$

Using eq. (21), we have

$$\frac{X'}{X} = \left(\rho+p+\frac{T_{00}^{(\mathcal{G}T)}}{X^2}+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}\right)^{-1}\left[\frac{Y}{X}\left(\frac{T_{01}^{(\mathcal{G}T)}}{YX}\right)^{\cdot}+\frac{2T_{01}^{(\mathcal{G}T)}}{X^2}\left(\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right)\right.$$
$$\left.-\left(p+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}\right)'-\frac{2Z'}{Z}\left(\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}-\frac{T_{22}^{(\mathcal{G}T)}}{Z^2}\right)+\frac{4\pi E}{Z}[E'Z+2Z'E]\right].$$

Inserting the value of $\frac{X'}{X}$ in eq. (28), we obtain

$$f_{\text{newtn}} = -f_{\text{grav}}+f_{\text{hyd}}+f_{\text{ds}}, \tag{29}$$

where

$$f_{\text{newtn}} = \left(\rho+p+\frac{T_{00}^{(\mathcal{G}T)}}{X^2}+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}\right)D_t(\mathcal{V}),$$
$$f_{\text{grav}} = \left(\rho+p+\frac{T_{00}^{(\mathcal{G}T)}}{X^2}+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}\right)\left[4\pi Z\left(p+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}-2\pi E^2\right)+\frac{m}{Z^2}-\frac{q^2}{2Z^3}\right],$$
$$f_{\text{hyd}} = -\varepsilon^2\left[D_r\left(p+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}\right)-\frac{q}{4\pi Z^4}D_r(q)+\frac{2}{Z}\left(\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}-\frac{T_{22}^{(\mathcal{G}T)}}{Z^2}\right)\right],$$
$$f_{\text{ds}} = \varepsilon\left[D_t\left(\frac{T_{01}^{(\mathcal{G}T)}}{YX}\right)+\frac{2}{YX}T_{01}^{(\mathcal{G}T)}\left(\frac{\dot{Y}}{YX}+\frac{\mathcal{V}}{Z}\right)\right].$$

Equation (29) demonstrates the effects of different forces on the collapsing phenomenon including Newtonian ($f_{\text{newtn}}$), hydrodynamical ($f_{\text{hyd}}$), gravitational ($f_{\text{grav}}$) forces. It can be observed that all these forces are affected by $f(\mathcal{G},T)$ gravity together with an additional force ($f_{\text{ds}}$). In $f_{\text{newtn}}$, the factors $(\rho+p+\frac{T_{00}^{(\mathcal{G}T)}}{X^2}+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2})$ and $D_t(\mathcal{V})$ describe the inertial mass density and acceleration of collapsing matter, respectively. In $f_{\text{grav}}$, the terms $(\rho+p+\frac{T_{00}^{(\mathcal{G}T)}}{X^2}+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2})$ and $[4\pi Z(p+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}-2\pi E^2)+\frac{m}{Z^2}-\frac{Q^2}{2Z^3}]$ depict the gravitational mass density and acceleration of a collapsing object, respectively. The inertial and gravitational mass density is the same implying that the equivalence principle holds, these masses are not affected by the electric charge.

We analyze how correction terms affect the collapse rate in the existence of realistic fluid $(\rho+p>0)$ with the help of $(\rho+p+\frac{T_{00}^{(\mathcal{G}T)}}{X^2}+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2})$.

- If the inertial mass density is positive then $(\rho+p+\frac{T_{00}^{(\mathcal{G}T)}}{X^2}+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2})$ will be larger than GR. Due to the minus sign, the action of gravitational force may change and depicts the action of the anti-gravity force which slows down the collapse rate.
- If the inertial mass density is negative then the factor $(\rho+p+\frac{T_{00}^{(\mathcal{G}T)}}{X^2}+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2})$ becomes negative, hence the above factor becomes positive and enhances the collapse rate.

The factor $4\pi Z(p+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2}-2\pi E^2)$ manifests the effect of pressure as well as electric field intensity and represents the relativistic part while the second term $(\frac{m}{Z^2})$ describes the effect of mass function on the collapsing system and defines the Newtonian part in the gravitational force. The last factor manifests the Coulomb repulsive force which opposes the collapse rate. In $f_{\text{hyd}}$, the factor $D_r(p+\frac{T_{11}^{(\mathcal{G}T)}}{Y^2})$ prevents the collapse rate due to the negative gradient of the effective pressure and the work done will be in the outward direction. The other terms represent the effect of correction terms and charge. The factor $f_{\text{ds}}$ gives the effect of correction terms on the collapsing system.

## 4 Relation between Weyl scalar and energy density

In this section, we find a relationship between the Weyl scalar, energy density, correction terms and electric charge. The Weyl scalar $\mathcal{C}^2 = \mathcal{C}^{\alpha\beta\mu\nu}\mathcal{C}_{\alpha\beta\mu\nu}$ in terms of Ricci scalar, Kretchmann scalar $\mathcal{R} = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ and Ricci tensor is defined as [30]

$$\mathcal{C}^2 = \frac{1}{3}R^2 + \mathcal{R} - 2R^{\alpha\beta}R_{\alpha\beta}. \tag{30}$$

The expressions of Ricci and Riemann tensors for interior spacetime are given in appendix A. The Kretschmann scalar, after simplification, is given as

$$\begin{aligned}\mathcal{R} &= \frac{48}{Z^6}(m - \frac{q^2}{2Z})^2 - \frac{16}{Z^3}(m - \frac{q^2}{2Z})\left(\frac{G_{00}}{X^2} - \frac{G_{11}}{Y^2} + \frac{G_{22}}{Z^2}\right)\\ &+ 3\left[\left(\frac{G_{00}}{X^2}\right)^2\left(\frac{G_{11}}{Y^2}\right)^2\right] + 4\frac{G_{22}}{Z^2}\left(\frac{G_{00}}{X^2} - \frac{G_{11}}{Y^2}\right)\\ &+ 4\left[\left(\frac{G_{22}}{Z^2}\right)^2 - \left(\frac{G_{01}}{YX}\right)^2\right] - 2\frac{G_{00}G_{11}}{Y^2X^2}.\end{aligned} \tag{31}$$

Inserting the values of $R$, $\mathcal{R}$ and $R_{\alpha\beta}$ in eq. (30), it follows that:

$$\frac{\mathcal{C}Z^3}{\sqrt{48}} = m - \frac{4\pi Z^3}{3}\left(\rho + \frac{T_{00}^{(\mathcal{G}T)}}{X^2} - \frac{T_{11}^{(\mathcal{G}T)}}{Y^2} + \frac{T_{22}^{(\mathcal{G}T)}}{Z^2} + 6\pi E^2\right) - \frac{q^2}{2Z}. \tag{32}$$

Using eqs. (23) and (26), the proper time derivative of the above equation yields

$$\begin{aligned}&D_t\left(\frac{\mathcal{C}Z^3}{\sqrt{48}}\right) + \frac{4\pi Z^3}{3}D_t\left[\left(\rho + \frac{T_{00}^{(\mathcal{G}T)}}{X^2} - \frac{T_{11}^{(\mathcal{G}T)}}{Y^2} + \frac{T_{22}^{(\mathcal{G}T)}}{Z^2} + 6\pi E^2\right)\right]\\ &+ 4\pi Z^2\left[\mathcal{V}\left(\rho + p + \frac{T_{00}^{(\mathcal{G}T)}}{X^2} + \frac{T_{22}^{(\mathcal{G}T)}}{Z^2} + 4\pi E^2\right) - \varepsilon\frac{T_{01}^{(\mathcal{G}T)}}{YX}\right] = 0.\end{aligned}$$

The proper radial derivative of eq. (32) with (27) give

$$D_r\left(\frac{\mathcal{C}Z^3}{\sqrt{48}}\right) + \frac{4\pi Z^3}{3}D_r\left[\left(\rho + \frac{T_{00}^{(\mathcal{G}T)}}{X^2} - \frac{T_{11}^{(\mathcal{G}T)}}{Y^2} + \frac{T_{22}^{(\mathcal{G}T)}}{Z^2} + 6\pi E^2\right)\right] - 4\pi Z^2\left[\left(\frac{T_{11}^{(\mathcal{G}T)}}{Y^2} - \frac{T_{22}^{(\mathcal{G}T)}}{Z^2} - 4\pi E^2\right) - \frac{\mathcal{V}}{\varepsilon}\frac{T_{01}^{(\mathcal{G}T)}}{YX}\right] = 0. \tag{33}$$

If we neglect the effect of electric charge ($Q = 0$), eq. (33) yields

$$D_r\left(\frac{\mathcal{C}Z^3}{\sqrt{48}}\right) + \frac{4\pi Z^3}{3}D_r\left[\left(\rho + \frac{T_{00}^{(\mathcal{G}T)}}{X^2} - \frac{T_{11}^{(\mathcal{G}T)}}{Y^2} + \frac{T_{22}^{(\mathcal{G}T)}}{Z^2}\right)\right] - 4\pi Z^2\left[\left(\frac{T_{11}^{(\mathcal{G}T)}}{Y^2} - \frac{T_{22}^{(\mathcal{G}T)}}{Z^2}\right) - \frac{\mathcal{V}}{\varepsilon}\frac{T_{01}^{(\mathcal{G}T)}}{YX}\right] = 0. \tag{34}$$

Di Prisco *et al.* [30] discussed the collapse of a spherical star for an anisotropic fluid and concluded that homogeneity of energy density implies conformal flatness and vice versa. In order to analyze the validity of this result in the $f(\mathcal{G}, T)$ theory, we assume a particular form of a generic function. There are two possibilities, *i.e.*, the minimal or non-minimal curvature-matter coupling. The assumption of non-minimal coupling $f(\mathcal{G}, T) = f_1(\mathcal{G}) + f_2(\mathcal{G})f_3(T)$ leads to the complicated form of dynamical eq. (22) and we cannot deduce any result from this. So, for convenience we consider the second choice, *i.e.*, $f(\mathcal{G}, T) = f_1(\mathcal{G}) + f_2(T)$. It is worth mentioning here that $f_2(T) = 0$ leads to the correction terms of $f(\mathcal{G})$ gravity while the choice $f_1(\mathcal{G}) = 0 = f_2(T)$ yields the results of GR. Sharif and Ikram [10,11] analyzed the viability of this model by examining the stability of the Einstein universe and some reconstructed cosmological models. The first as well as the generalized second law of thermodynamics are also investigated corresponding to this model [40]. Here, we take $f_1(\mathcal{G})$ and $f_2(T)$ as constants so that $f(\mathcal{G}, T)$ becomes constant. For a constant value of $f(\mathcal{G}, T)$, eq. (34) becomes

$$D_r\left(\frac{\mathcal{C}Z^3}{\sqrt{48}}\right) + \frac{4\pi Z^3}{3}D_r(\rho + f_0) = 0, \tag{35}$$

where $f_0 = -\frac{f_1(\mathcal{G})+f_2(T)}{16\pi}$. This indicates that $\mathcal{C} = 0$ provides $D_r\rho = 0$ and vice versa. Thus the system would experience a homogeneous distribution of energy density if and only if it is conformally flat.

## 5 Concluding remarks

This work deals with the dynamics of charged spherical collapse with perfect fluid distribution in $f(\mathcal{G},T)$ gravity. To examine how the total energy of the system varies with respect to temporal and radial coordinates, we have developed dynamical equations of the spherical region by using the Misner-Sharp formalism and discussed the role of electromagnetic field, correction terms and matter variables on the collapse rate. A relation among the energy density, Weyl scalar, electric charge and correction terms is also established. A direct relationship between energy density and Weyl scalar has been developed with the assumption of zero electric charge and constant $f(\mathcal{G},T)$.

The importance of charge distribution has been discussed in the dynamics of gravitational collapse. In particular, it is noteworthy that the active gravitational mass is not affected by electric charge. The existence of electric charge is significant in the dynamics of collapse which behaves as a Coulomb repulsive force and resists the collapsing process. The results are summarized as follows.

- The matching conditions show that masses of both spacetimes are equal when their corresponding charges are the same and vice versa.
- The change in total energy with the passage of time depicts that the internal energy of spherical star decreases due to the repulsive nature of the Coulomb force and pressure in the outward direction.
- The variation of total energy with respect to radial coordinates manifests that the energy of the system decreases due to the Coulomb repulsive force and increases because of the effective energy density.
- The correction terms strongly affect the inertial mass density. The positive values of inertial mass density resist the collapse rate due to anti-gravitational effects while the negative values enhance the collapse rate.
- The electromagnetic field prevents the collapsing procedure due to the Coulomb repulsive force.
- The rate of collapse becomes slow in the case of a negative gradient pressure. The hydrodynamical force manifests the stable state for constant $f(\mathcal{G},T)$ and prevents the more expanding and collapsing processes.
- In order to obtain the conformal flatness of the spacetime, we have assumed zero electric charge. Consequently, we have found that the energy density of the fluid is homogeneous if and only if the metric is conformally flat and this statement is true only for constant $f(\mathcal{G},T)$.

## Appendix A.

The Ricci and Riemann tensors for the interior spacetime (5) are

$$\begin{aligned}
R_{00} &= X^2\left[\frac{G_{00}}{2X^2}+\frac{G_{11}}{2Y^2}+\frac{G_{22}}{Z^2}\right],\\
R_{11} &= Y^2\left[\frac{G_{00}}{2X^2}+\frac{G_{11}}{2Y^2}-\frac{G_{22}}{Z^2}\right],\\
R_{22} &= Z^2\left[\frac{G_{00}}{2X^2}-\frac{G_{11}}{2Y^2}\right],\\
R_{01} &= G_{01},\\
R_{33} &= \sin^2\theta R_{22},\\
R_{0101} &= (X^2Y^2)\left[\frac{G_{00}}{2X^2}-\frac{G_{11}}{2Y^2}+\frac{G_{22}}{Z^2}-\frac{2}{Z^3}\left(m-\frac{q^2}{2Z}\right)\right],\\
R_{0202} &= (X^2Z^2)\left[\frac{G_{11}}{2Y^2}+\frac{1}{Z^3}\left(m-\frac{q^2}{2Z}\right)\right],\\
R_{1212} &= (Y^2Z^2)\left[\frac{G_{00}}{2X^2}-\frac{1}{Z^3}\left(m-\frac{q^2}{2Z}\right)\right],\\
R_{2323} &= 2Z\left(m-\frac{q^2}{2Z}\right)\sin^2\theta, \qquad R_{0212}=\frac{Z}{2}^2G_{01},\\
R_{0303} &= \sin^2\theta R_{0202}, \qquad R_{0313}=\sin^2\theta R_{0212},\\
R_{1313} &= \sin^2\theta R_{1212}.
\end{aligned}$$

The expressions for the correction terms are

$$
\begin{aligned}
T_{00}^{(\mathcal{G}T)} &= \frac{X^2}{8\pi}\Bigg[f_T(\rho+p)-\frac{1}{2}f+\frac{1}{Y^3X^2Z^2}\Bigg(4Z'^2\dot{Y}\dot{f}_{\mathcal{G}}-4\dot{Z}^2Y'f'_{\mathcal{G}}+8\dot{Z}\dot{Y}Z'f'_{\mathcal{G}}+8Y'Z'\ddot{Z}f_{\mathcal{G}}-4Z'^2\ddot{Y}f_{\mathcal{G}}-8Y'Z'\dot{Z}\dot{f}_{\mathcal{G}}\\
&\quad+\frac{8Z''X'Z'}{Y}f_{\mathcal{G}}+\frac{8\dot{Y}^2Z'^2}{Y}f_{\mathcal{G}}-\frac{8\dot{Z}\dot{Y}X'Z'}{X}f_{\mathcal{G}}+\frac{8Y'Z'\dot{X}\dot{Z}}{X}f_{\mathcal{G}}+\frac{4\dot{Z}^2X'Y'}{X}f_{\mathcal{G}}+\frac{4Z'^2\dot{X}\dot{Y}}{X}f_{\mathcal{G}}\Bigg)\\
&\quad-\frac{1}{Y^2X^2Z^2}\left(8\dot{Z}'^2f_{\mathcal{G}}+8Z''\dot{Z}\dot{f}_{\mathcal{G}}-8Z''\ddot{Z}f_{\mathcal{G}}+\frac{8Z''\dot{X}\dot{Z}}{X}f_{\mathcal{G}}-\frac{4\dot{Z}^2\ddot{Y}}{X}f_{\mathcal{G}}-\frac{16\dot{Z}'\dot{Y}Z'}{Y}f_{\mathcal{G}}-\frac{16\dot{Z}'\dot{Z}X'}{X}f_{\mathcal{G}}\right)\\
&\quad+\frac{1}{YX^2Z^2}\left(4\ddot{Y}f_{\mathcal{G}}-4\dot{Y}\dot{f}_{\mathcal{G}}-\frac{4\dot{X}\dot{Y}}{X}f_{\mathcal{G}}+\frac{4\dot{Z}^2\ddot{Y}}{X^2}f_{\mathcal{G}}+\frac{8\dot{Z}^2X'^2}{YX^2}f_{\mathcal{G}}+\frac{8\dot{Z}\dot{Y}\ddot{Z}}{X^2}f_{\mathcal{G}}-\frac{12\dot{Z}^2\dot{X}\dot{Y}}{X^3}f_{\mathcal{G}}-\frac{12\dot{Z}^2\dot{Y}}{X^2}\dot{f}_{\mathcal{G}}\right)\\
&\quad-\frac{1}{Y^2Z^2}\left(4XX''f_{\mathcal{G}}+\frac{4Y'}{Y}f'_{\mathcal{G}}+\frac{8Z'Z''}{Y^2}f_{\mathcal{G}}-\frac{4Z'^2X''}{Y^2X}f_{\mathcal{G}}-\frac{4X'Y'}{YX}f_{\mathcal{G}}\right)\Bigg],\\
T_{11}^{(\mathcal{G}T)} &= \frac{Y^2}{8\pi}\Bigg[\frac{1}{2}f+\frac{1}{Y^3X^2Z^2}\Bigg(4Z'^2\ddot{Y}f_{\mathcal{G}}-8\ddot{Z}Y'Z'f_{\mathcal{G}}+16\dot{Z}'\dot{Y}Z'f_{\mathcal{G}}-\frac{4\dot{Z}^2X'Y'}{X}f_{\mathcal{G}}-\frac{4Z'^2\dot{X}\dot{Y}}{X}f_{\mathcal{G}}-\frac{8X'Z'\dot{Z}\dot{Y}}{X}f_{\mathcal{G}}\\
&\quad+\frac{8\dot{X}\dot{Z}Y'Z'}{X}f_{\mathcal{G}}\Bigg)+\frac{1}{Y^2X^2Z^2}\Bigg(8\ddot{Z}Z''f_{\mathcal{G}}-8\dot{Z}'^2f_{\mathcal{G}}-8\ddot{Z}Z'f'_{\mathcal{G}}-4Z'^2\ddot{f}_{\mathcal{G}}+\frac{4\dot{Z}^2X''}{X}f_{\mathcal{G}}+\frac{4Z'^2\dot{X}}{X}\dot{f}_{\mathcal{G}}-\frac{4\dot{Z}^2X'}{X}\\
&\quad\times f_{\mathcal{G}}+\frac{8\dot{X}\dot{Z}Z'}{X}f'_{\mathcal{G}}-\frac{8X'Z'\dot{Z}}{X}\dot{f}_{\mathcal{G}}-\frac{8\dot{Z}^2X'^2}{X^2}f_{\mathcal{G}}-\frac{8\dot{Y}^2Z'^2}{Y^2}f_{\mathcal{G}}\Bigg)\\
&\quad+\frac{1}{Z^2X^4}\Bigg(4\dot{Z}^2\ddot{f}_{\mathcal{G}}-4X\dot{X}\dot{f}_{\mathcal{G}}+4X^2\ddot{f}_{\mathcal{G}}-\frac{4\dot{Z}\ddot{Y}}{Y}f_{\mathcal{G}}-\frac{8\dot{Z}\ddot{Z}\dot{Y}}{Y}f_{\mathcal{G}}+\frac{12\dot{Z}^2\dot{X}\dot{Y}}{XY}f_{\mathcal{G}}\\
&\quad-\frac{4\ddot{Y}X^2}{Y}f_{\mathcal{G}}-\frac{8\dot{X}\dot{Z}Z''X}{Y^2}f_{\mathcal{G}}+\frac{4\dot{X}\dot{Y}X}{Y}f_{\mathcal{G}}-\frac{8\dot{Y}^2Z'^2}{Z^2}f_{\mathcal{G}}-\frac{12\dot{Z}^2\dot{X}}{X}\dot{f}_{\mathcal{G}}\Bigg)\\
&\quad+\frac{1}{Y^4XZ^2}\left(12Z'^2X'f'_{\mathcal{G}}-8X'Z'Z''f_{\mathcal{G}}-4X'Y'Yf_{\mathcal{G}}-4Z'^2X''f_{\mathcal{G}}+4X''Y^2f_{\mathcal{G}}+\frac{12Z'^2X'Y'}{Y}f_{\mathcal{G}}\right)\Bigg],\\
T_{22}^{(\mathcal{G}T)} &= \frac{Z^2}{8\pi}\Bigg[\frac{1}{2}f+\frac{1}{Y^3X^3Z^2}(4\ddot{Y}Z'^2f_{\mathcal{G}}+4\dot{X}\dot{Y}Y^2f_{\mathcal{G}}+4X''\dot{Z}^2Yf_{\mathcal{G}}+4Z''\dot{X}Z\dot{f}_{\mathcal{G}}+4\dot{X}\dot{Z}Z'Zf'_{\mathcal{G}}\\
&\quad+4\dot{Y}Z'X'Z\dot{f}_{\mathcal{G}}+4Y'Z'Z\ddot{f}_{\mathcal{G}}+4X'Y'\dot{Z}Z\dot{f}_{\mathcal{G}}+4\ddot{Z}Y'Zf'_{\mathcal{G}}+4\dot{Z}\dot{Y}X'Zf'_{\mathcal{G}}-4X'Y'X^2f_{\mathcal{G}}\\
&\quad-4\dot{X}\dot{Z}Y'Zf'_{\mathcal{G}}-4\dot{X}\dot{Y}Z'^2f_{\mathcal{G}}-4X'Y'\dot{Z}^2f_{\mathcal{G}}+8\dot{X}\dot{Z}Y'Z'f_{\mathcal{G}}-8\dot{Z}\dot{Y}X'Z'f_{\mathcal{G}}-8\dot{Z}'X'YZ\dot{f}_{\mathcal{G}}-8\dot{Z}'\dot{Y}XZf'_{\mathcal{G}}\\
&\quad-8\ddot{Z}Y'Z'Xf_{\mathcal{G}})+\frac{1}{Y^2X^2Z^2}\Bigg(8\ddot{Z}Z''f_{\mathcal{G}}-8\dot{Z}'^2f_{\mathcal{G}}+4X''Xf_{\mathcal{G}}-4\ddot{Y}Yf_{\mathcal{G}}-f_{\mathcal{G}}\\
&\quad\times\frac{8\dot{Y}^2Z'^2}{Y}+\frac{16\dot{Z}'\dot{Z}X'}{X}f_{\mathcal{G}}\Bigg)+\frac{1}{X^4YZ}(4Z''X'f'_{\mathcal{G}}+4\ddot{Y}\dot{Z}\dot{f}_{\mathcal{G}}+4X''Z'f'_{\mathcal{G}}+4\\
&\quad\times\dot{Z}\dot{Y}\ddot{f}_{\mathcal{G}}+4\ddot{Z}\dot{Y}\dot{f}_{\mathcal{G}}+\frac{8\dot{Z}X'^2}{Y}\dot{f}_{\mathcal{G}}-\frac{4X''Z'^2X^3}{Y^3Z}f_{\mathcal{G}}-\frac{12\dot{Z}\dot{X}\dot{Y}}{X}\dot{f}_{\mathcal{G}}-\frac{8\ddot{Z}\dot{Z}\dot{Y}}{Z}f_{\mathcal{G}}-\frac{8X'Z'Z''X^3}{Y^3Z}f_{\mathcal{G}}\Bigg],\\
T_{01}^{(\mathcal{G}T)} &= \frac{1}{8\pi}\Bigg[\frac{1}{ZX^3}\Bigg(8\dot{Z}X'\ddot{f}_{\mathcal{G}}+8\dot{Z}'\dot{X}\dot{f}_{\mathcal{G}}-8\ddot{Z}X'\dot{f}_{\mathcal{G}}-8\dot{Z}'X\ddot{f}_{\mathcal{G}}+\frac{8\dot{X}\dot{Z}\dot{Y}}{Y}f'_{\mathcal{G}}-f'_{\mathcal{G}}\\
&\quad\times\frac{8\ddot{Z}\dot{Y}X}{Y}-\frac{12\dot{Z}^2X'}{Z}\dot{f}_{\mathcal{G}}+\frac{8\dot{Z}'\dot{Z}X}{Z}\dot{f}_{\mathcal{G}}-\frac{8\dot{Y}Z'\dot{X}}{Y}\dot{f}_{\mathcal{G}}\Bigg)\\
&\quad+\frac{1}{Y^2X^2Z}\Bigg(8X'^2Z'\dot{f}_{\mathcal{G}}+8\dot{Z}\dot{Y}^2f'_{\mathcal{G}}-8\dot{Y}^2Z'\dot{f}_{\mathcal{G}}-8\dot{Z}X'^2f'_{\mathcal{G}}+8\dot{Y}Z'Y\ddot{f}_{\mathcal{G}}+8\dot{Z}'\dot{Y}Y\dot{f}_{\mathcal{G}}+8\dot{Z}'X'Xf'_{\mathcal{G}}\\
&\quad-8Z''X'X\dot{f}_{\mathcal{G}}+\frac{4Z'^2X'X}{Z}\dot{f}_{\mathcal{G}}-\frac{4\dot{Z}^2\dot{Y}Y}{Z}f'_{\mathcal{G}}-\frac{8\dot{Y}Z'\dot{Z}Y}{Z}\dot{f}_{\mathcal{G}}+\frac{8\dot{Z}X'Z'X}{Z}f'_{\mathcal{G}}-\frac{8\dot{Z}X'Y'X}{Y}f'_{\mathcal{G}}\Bigg)\\
&\quad+\frac{1}{Z^2}\left(\frac{12Z'^2\dot{Y}}{Y^3}f'_{\mathcal{G}}-\frac{4X'}{X}\dot{f}_{\mathcal{G}}-\frac{4\dot{Y}}{Y}f'_{\mathcal{G}}-\frac{8Z''\dot{Y}Z}{Y^3}f'_{\mathcal{G}}+f'_{\mathcal{G}}\frac{8\dot{Z}'Y'Z}{Y^3}-\frac{8\dot{Z}'Z'}{Y^2}f'_{\mathcal{G}}+\frac{8Y'Z'X'Z}{XY^3}\dot{f}_{\mathcal{G}}\right)\Bigg],\\
T_{33}^{(\mathcal{G},T)} &= \sin^2\theta T_{22}^{(\mathcal{G},T)}.
\end{aligned}
$$

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