Eur. Phys. J. Plus (2018) **133**: 298
DOI 10.1140/epjp/i2018-12101-4

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

Markovian thermal evolution of entanglement and decoherence of GHZ state

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Received: 24 November 2017 / Revised: 6 June 2018
Published online: 2 August 2018

Abstract. The thermal evolution of decoherence and entanglement of an open quantum system consisting of three uncoupled oscillators is investigated using the Lindblad equation. We consider T and GHZ states as the initial states of the system and a bosonic bath in a thermal equilibrium state as the environment. Using the PPT criterion and the degree of purity, the dependence of the entanglement and decoherence of the system on the parameters of the environment (temperature, dissipation coefficient) and the initial state (noise, squeezing) is investigated. It is observed that the relaxation rate to entanglement sudden death (RRESD) is an increasing function of temperature, dissipation coefficient and noise, while it is a decreasing function of squeezing. In addition, it is observed that decoherence occurs sooner with increase of all the involved parameters. Moreover, by comparing T and GHZ states as initial states of the system, it is observed that the entanglement of the system with GHZ initial states can survive longer.

1 Introduction

Entanglement, a marvelous feature of quantum mechanics, is an essential ingredient of quantum information theory and quantum technologies [1]. Hence, it is important to produce and preserve entangled states. Physical systems known as open quantum systems are always interacting with their surroundings. This interaction often causes a waste of quantum correlations which is termed as decoherence. Decoherence and attenuation of quantum correlations are fundamental difficulties in the implementation of quantum information processing. Thus, it is valuable to study the dynamics of quantum correlations in open quantum systems.

Since the evolution of open quantum systems is irreversible, the unitary transformation and also the Liouville equation are not suitable to describe them. There are different methods to calculate the environment influence on the dynamics of the system that lead to various master equations [2,3]. In certain cases, the dynamics of open quantum system can be described by the Lindblad equation obtained under the Born-Markov approximation; this approximation is valid as long as the interaction between the system and the environment is weak (Born approximation) and the correlation time of environment is less than the relaxation time of the system (Markov approximation) [4].

Recent studies have shown that Gaussian states play an important role in the continuous variable quantum information theory, due to the simplicity in their production, control and mathematical description [5,6]. Hence, much attention has been paid to the investigation of quantum correlations and decoherence of these states in the framework of open systems [7–13]. The entanglement evolution of two- and three-mode Gaussian states in various open quantum systems has been studied with regard to the Markovian approximation in several reports [14–17]. These reports demonstrated that increasing the temperature of the environment causes the entanglement sudden death to take place sooner and the entanglement can survive the longest at zero temperature. Moreover, the study of the evolution of entanglement of multimode Gaussian states in noisy channels shows that the more the temperature of the channels increases, the sooner the entanglement sudden death occurs [18]. However, entanglement can be induced into the system when the Penning-trap Hamiltonian and its coherent states are considered as the system and initial

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states, respectively [19]. In addition, investigation of non-Markovian evolution of quantum correlations of two-mode Gaussian states in noisy channels has shown that quantum correlations decrease over the time and may vanish and revive again [12,20].

Recently, experimental methods have been proposed for the production of three-mode Gaussian states known as the optimal resources for quantum communications [21]. These states can be divided into three classes: fully symmetric states (for example GHZ and T states), bisymmetric states (for example, basset hound states) and fully asymmetric states. The entanglement evolution of basset hound states was investigated in ref. [22]. Fully symmetric three-mode states are optimally useful in quantum communications due to their special features of sharing entanglement between the modes. Therefore, this paper is devoted to fully symmetric states, namely GHZ and T states. These states are invariant under the exchange of any two modes. GHZ states have maximal genuine tripartite entanglement, and also have maximal bipartite entanglement in any reduced state. In addition, T states have maximal genuine tripartite entanglement but all their two-mode reduced states are separable [21,23]. These states can be used as shared sources in CV teleportation [24], quantum secret sharing [25] and dense coding [26]. A recent study by O'Gorman and Campbell revealed that magic states, including T states, can be used in some block protocols to reduce the global output error [27]. Moreover, they provided some devices with a new approach to distillate these states [27]. Furthermore, it has been shown that GHZ state can be realized by usual technology and used as an efficient quantum source for reliable quantum communications [28–32]. Also, utilization of maximally entangled GHZ states as a quantum channel guarantees success in quantum information protocols such as CV teleportation network [24,33,34].

The goal of this study is to investigate the evolution of entanglement and decoherence of a three-mode Gaussian system in a thermal bath. To this end, we consider a system consisting of three non-interacting oscillators coupled to an equilibrium thermal bath. Also, two fully symmetric three-mode states (GHZ and T states) are considered as initial states. Considering the Born-Markov approximation and using the Lindblad equation, we obtain the evolution dynamics of the system. Then, the effect of the environmental and initial-state parameters on the evolution of entanglement and decoherence is investigated. In addition, the dynamics of entanglement of the two initial states, GHZ and T states, is compared.

This paper is organized as follows: in sect. 2, we explain three-mode Gaussian states. In sect. 3 we study the PPT criterion. Sections 4 and 5, are dedicated to description of GHZ states, T states and open quantum systems, respectively. In sect. 6 we review Lindblad dynamics. Then, we investigate the evolution of entanglement and decoherence of open quantum system in sect. 7 and 8. Finally, the conclusion is given in sect. 9.

2 Three-mode Gaussian states

A continuous variable system consists of N bosonic modes. Gaussian states are the most important and commonly used continuous variable states. These are called Gaussian due to the characteristic of their wave functions [30,35–37]. In recent years, it has been shown that these states are essential elements in continuous variable quantum information theory [38]. Different types of these states are referred to as vacuum state, squeezed state and thermal state of electromagnetic field. Gaussian states can easily be generated with the use of optical devices in the lab. In addition, these states have been implemented successfully in quantum information processes and communication protocols.

Gaussian states are completely described by the covariance matrix in the phase space with finite dimension as follows:

$$\sigma_{ij} = \frac{1}{2}\langle R_i R_j + R_j R_i\rangle - \langle R_i\rangle\langle R_j\rangle, \tag{1}$$

where

$$R = (x_1, p_1, x_2, p_2, x_3, p_3)^T. \tag{2}$$

In addition, every covariance matrix needs to satisfy Robertson-Schrödinger uncertainty relation to be physically acceptable. In terms of the covariance matrix this condition is given by [39]

$$\sigma + i\Omega \geq 0, \quad \Omega = \oplus_{k=1}^{N} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{3}$$

where, all eigenvalues of $\sigma + i\Omega$ must be non-negative.

In the case of N-mode Gaussian states, covariance matrix is $2N \times 2N$, real, symmetric and blocked. In this paper, we consider the three-mode Gaussian states with the following covariance matrix:

$$\sigma = \begin{pmatrix} \sigma_1 & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12}^T & \sigma_2 & \varepsilon_{23} \\ \varepsilon_{13}^T & \varepsilon_{23}^T & \sigma_3 \end{pmatrix}, \tag{4}$$

where σ_i and ε_{ij} $(i, j = 1, 2, 3)$ are covariance matrices of each mode and the correlation matrices between the modes, respectively. Generally, an appropriate symplectic transformation can be used to convert every covariance matrix to a normal form as follows [40,41]:

$$\nu = \oplus_{k=1}^{n} \operatorname{diag}\{\nu_k, \nu_k\}, \tag{5}$$

where, ν_ks are the positive eigenvalues of the matrix $|i\Omega\sigma|$ termed as sympelectic eigenvalues. These eigenvalues and any function of them are invariant under the sympelectic transformations; for example, determinant of the covariance matrix and purity as functions of ν_ks are given by [41,42]

$$\mathrm{Det}[\sigma] = \prod_{k=1}^{N} \nu_k^2, \qquad \mu = \prod_{k=1}^{N} \frac{1}{\nu_K}\,. \tag{6}$$

Purity determines the degree of mixedness of quantum states; the purity is in the range $1/N \leq \mu \leq 1$ where its upper and lower bounds refer to pure states and completely mixed states, respectively. In the case of continuous variable systems, $N \to \infty$; therefore, the minimum of purity is zero.

All information about Gaussian states can be extracted with the use of symplectic eigenvalues; for example, the uncertainly relation in terms of the smallest symplectic eigenvalue is given by

$$\nu_- \geq 1. \tag{7}$$

3 PPT criterion of three-mode Gaussian states

To determine the entanglement of three-mode Gaussian states, we use the PPT criterion. It is the sufficient and necessary condition of separability of all bipartite $(1+N)$-mode Gaussian states. According to this criterion, a three-mode Gaussian state has entanglement between one- and two-mode parts if and only if the following condition holds [38–42]:

$$\tilde{\sigma} + i\Omega \leq 0, \tag{8}$$

where

$$\tilde{\sigma} = \tau_j \cdot \sigma \cdot \tau_j, \quad j = 1, 2, 3, \tag{9}$$

and τ_j's are defined as

$$\tau_1 = \operatorname{diag}(1, -1, 1, 1, 1, 1), \qquad \tau_2 = \operatorname{diag}(1, 1, 1, -1, 1, 1), \qquad \tau_3 = \operatorname{diag}(1, 1, 1, 1, 1, -1). \tag{10}$$

In addition, eq. (8) can be written in terms of the smallest symplectic eigenvalue $(\tilde{\nu}_-)$ as follows [42]:

$$\tilde{\nu}_- < 1. \tag{11}$$

Moreover, the less the $\tilde{\nu}_-$, the more the entanglement will be.

4 Description of fully symmetric states

The three-mode symmetric Gaussian states which are invariant under exchange of any two modes are fully described by the following covariance matrix:

$$\sigma_s = \begin{pmatrix} \alpha & \varepsilon & \varepsilon \\ \varepsilon^T & \alpha & \varepsilon \\ \varepsilon^T & \varepsilon^T & \alpha \end{pmatrix}. \tag{12}$$

They have been produced successfully using optical devices in laboratory [31,32]. Also, they are utilized as quantum resources for important tasks in quantum information such as quantum teleportation [24,33,34]. Two example of these states are GHZ and T states on which we concentrate in this paper.

Mixed GHZ states are fully symmetric and belong to the family of squeezed thermal states. They are derived from the evolution of pure GHZ states in dissipative channels; it is for this reason, it is said that they shows noise [42]. Pure GHZ states, GHZ states without noise, with finite squeezing can be generated by different methods. For example, they can be produced by mixing three light beams with the use of two beam splitters; one of the beams is squeezed in position and the two other beams are squeezed in momentum [21].

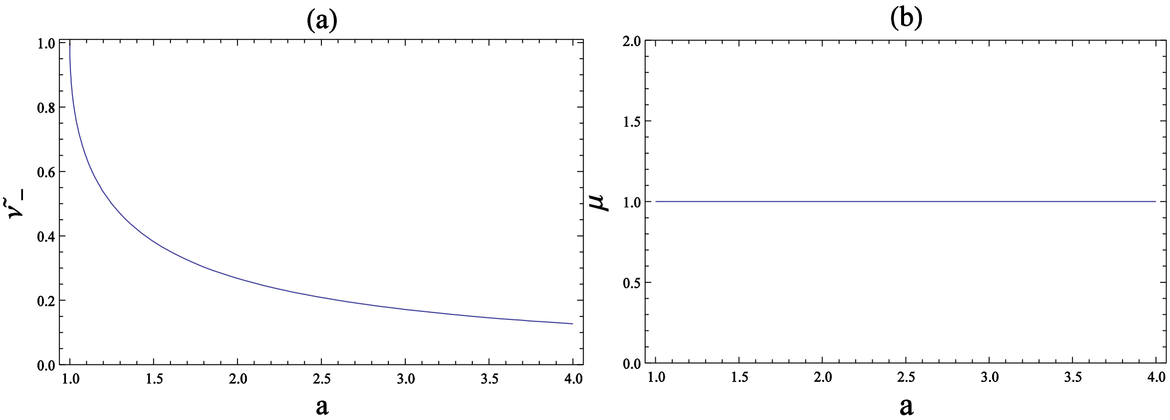

Fig. 1. (a) Entanglement and (b) purity of GHZ state *versus* squeezing parameter for $n = 1$.

These states are described by 6×6 covariance matrices in the form eq. (12) with the following blocks:

$$\begin{aligned} &\alpha = a1_2, \qquad \varepsilon = \text{diag}\{e_+, e_-\},\\ &e_\pm = \frac{a^2 - n^2 \pm \sqrt{(a^2-n^2)(9a^2-n^2)}}{4a}, \end{aligned} \tag{13}$$

where a and n are the squeezed and noise parameters, respectively. Here $n \geq 1$, and if $n = 1$, the lowest noise obtained is equivalent to the three-mode squeezed vacuum state.

In addition, the uncertainly condition for this particular state is expressed as follows [21]:

$$a \geq n. \tag{14}$$

In this study, we consider the initial state of the system of the class of noisy GHZ states, which are inseparable with respect to all bipartitions. The relation between the squeezing and noise parameter in these states is given by

$$s > \frac{\sqrt{9n^4 - 2n^2 + 9 + 3(n^2-1)\sqrt{9n^4 + 14n^2 + 9}}}{4n}. \tag{15}$$

T states are fully symmetric mixed Gaussian states characterized by minimum partial uncertainty; in other words, their smallest eigenvalue is equal to 1. Similar to GHZ states, T states can be generated by mixing three light beams using two beam splitters, one of which is squeezed in momentum and two others are thermal states [21]. Similar to GHZ states, T states are described by covariance matrix in eq. (12) by the following blocks:

$$\begin{aligned} &\alpha = a1_2, \qquad \varepsilon = \text{diag}\{e_+, e_-\},\\ &e_+ = \frac{a^2 - 5 + \sqrt{9a^2(a^2-2)+25}}{4a},\\ &e_- = \frac{5 - 9a^2 + \sqrt{9a^2(a^2-2)+25}}{12a}, \end{aligned} \tag{16}$$

where a is the squeezing parameter. In addition, if $a = 1$, T states are fully separable, and if $a > 1$ are fully inseparable.

Now, we investigate the entanglement and purity of the GHZ and T states *vs.* their squeezing parameter using eqs. (11) and (6). In figs. 1(a) and (b), the entanglement and purity of GHZ states are plotted as a function of squeezing parameter, respectively. It is observed that the entanglement of GHZ states is an increasing function of squeezing but its purity is independent of it.

In figs. 2(a) and (b), the entanglement and purity of T states are plotted as a function of squeezing parameter, respectively. It is observed that the entanglement of T states is an increasing function of squeezing while its purity is a decreasing one.

In fig. 3, the entanglement of GHZ and T states are compared *versus* the squeezing parameter. It is observed that with the increase of squeezing parameter, the entanglement increases more for GHZ states in comparison with T states.

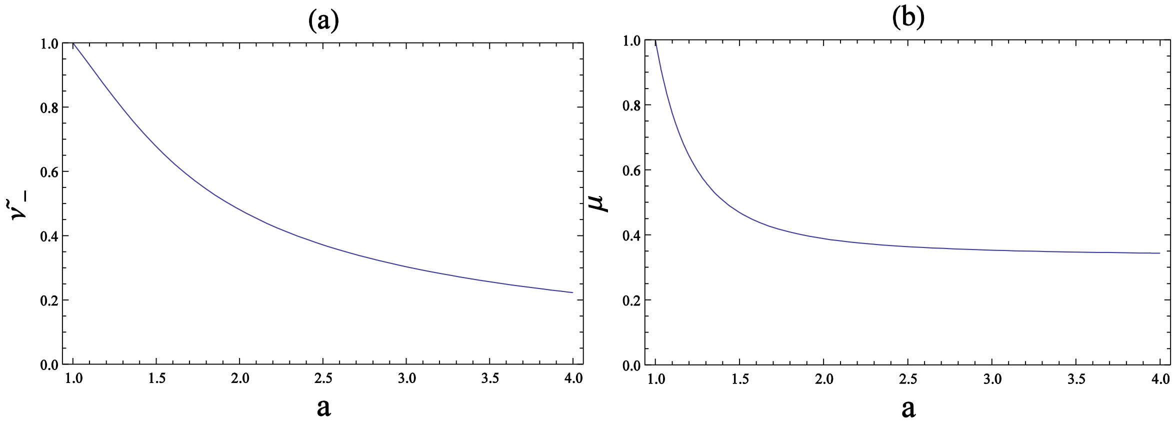

Fig. 2. (a) Entanglement and (b) purity of T states *versus* squeezing parameter.

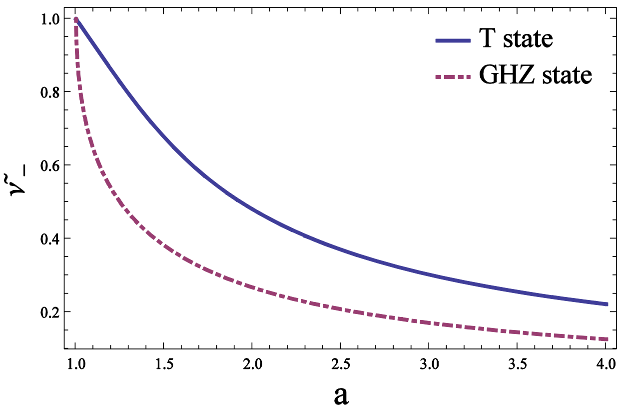

Fig. 3. Entanglement of T and GHZ states *versus* squeezing parameter represented by solid and dot-dashed lines, respectively.

5 Modeling of open quantum system

We consider a Gaussian system consisting of three non-interacting identical oscillators that can describe the modes of bosonic fields, such as electromagnetic fields [2]. We suppose that three oscillators are coupled to a common bosonic bath. Also, bosonic bath contains numerous degrees of freedom and it is described by an ensemble of harmonic oscillators. This model of the system and environment can be described by the following Hamiltonian [2]:

$$H = \sum_{i=1}^{3} \hbar\omega a_i^\dagger a_i + \sum_j \hbar\omega_j b_j^\dagger b_j + \sum_k S_k \otimes B_k, \tag{17}$$

where first and second term are Hamiltonians of the system and the bath, respectively. Also, the third term is the system-bath interaction Hamiltonian. In addition, $a_i^\dagger$ and a_i are creation and annihilation operators of the system, $b_j^\dagger$ and b_j are the same operators of the environment, and S_k and B_k are the general operators acting on the system and the bath, respectively.

6 Lindblad dynamics

Assuming the Born-Markov approximation, the evolution of the density matrix of the system (ρ_s) is obtained with the use of the Lindblad equation as follows [4,43,44]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_s(t) = -i[H,\rho(t)] + \frac{1}{2}\sum_l \left([V_l\rho(t), V_l^\dagger] + [V_l, \rho(t)V_l^\dagger]\right), \tag{18}$$

where the first and second terms after equal sign describe the unitary and non-unitary evolution of the density matrix, respectively. Also, V_ls are Lindblad operators, the super-operators applied on operators in the Liouvil space.

Moreover, these operators are non-Hermitian and describe the dissipation and decoherence due to interaction between the system and the environment. Lindblad operators are given by

$$
\begin{aligned}
V_l^\dagger &= \sum_j^N (\alpha_j^{l*} x_j + \beta_j^{l*} p_j), \quad l = 1, 2, \ldots, 2N, \\
V_l &= \sum_j^N (\alpha_j^l x_j + \beta_j^l p_j), \quad \alpha, \beta \in \mathbb{C},
\end{aligned}
\tag{19}
$$

where α_j^l and β_j^l are the complex coefficients, and x_j and p_j are the position and momentum operators of the system, respectively. Equations (18) and (19) can be used to obtain the evolution of covariance matrix as follows:

$$
\frac{\mathrm{d}\sigma(t)}{\mathrm{d}t} = Y\sigma(t) + \sigma(t)Y^T(t) + 2D, \tag{20}
$$

where matrix Y is given by

$$
Y = \begin{pmatrix}
-\lambda & 1 & 0 & 0 & 0 & 0 \\
-\omega^2 & -\lambda & 0 & 0 & 0 & 0 \\
0 & 0 & -\lambda & 1 & 0 & 0 \\
0 & 0 & -\omega^2 & -\lambda & 0 & 0 \\
0 & 0 & 0 & 0 & -\lambda & 1 \\
0 & 0 & 0 & 0 & -\omega^2 & -\lambda
\end{pmatrix}. \tag{21}
$$

In addition, with the assumption of Gibbs state as the final state of the system, D in eq. (20), the diffusion coefficient matrix, is obtained as follows [45]:

$$
D = \begin{pmatrix}
\frac{\lambda}{2\omega}\coth\left(\frac{\omega}{2T}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\lambda\omega}{2}\coth\left(\frac{\omega}{2T}\right) & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\lambda}{2\omega}\coth\left(\frac{\omega}{2T}\right) & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda\omega}{2}\coth\left(\frac{\omega}{2T}\right) & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\lambda}{2\omega}\coth\left(\frac{\omega}{2T}\right) & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\lambda\omega}{2}\coth\left(\frac{\omega}{2T}\right)
\end{pmatrix}. \tag{22}
$$

Also, T is the temperature of the environment; λ is the dissipation coefficient due to the interaction between the environment and the system and it is defined as follows [46]:

$$
\lambda_{ij} = -\operatorname{Im}(\langle \alpha_i, \beta_j \rangle); \quad i, j = 1, 2, 3. \tag{23}
$$

Finally, the evolution of the covariance matrix is obtained by solving eq. (20), as follows:

$$
\sigma(t) = e^{Yt}[\sigma(0) - \sigma(\infty)](e^{Yt})^T + \sigma(\infty), \tag{24}
$$

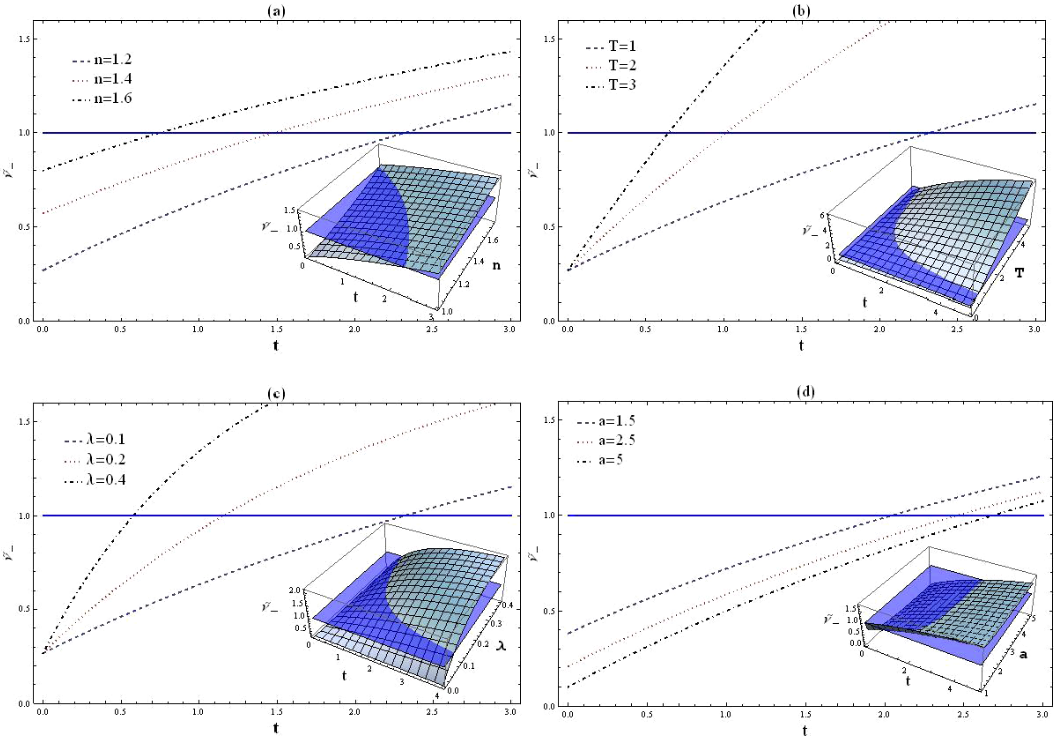

Fig. 4. Entanglement evolution of GHZ states: (a) Two-dimensional plot *versus* time for $n = 1.2$, 1.4, 1.6 and three-dimensional plot *versus* noise and time for $\lambda = 0.1$, $T = 1$, $a = 2$. (b) Two-dimensional plot *versus* time for $T = 1$, 2, 3 and three-dimensional plot *versus* temperature and time for $\lambda = 0.1$, $n = 1$, $a = 2$. (c) Two-dimensional plot *versus* time for $\lambda = 0.1$, 0.2, 0.4 and three-dimensional plot *versus* dissipation coefficient and time for $T = 1$, $a = 2$, $n = 1$. (d) Two-dimensional plot *versus* time for $a = 1.5$, 2.5, 5 and three-dimensional plot *versus* squeezing and time for $T = 1$, $n = 1$, $\lambda = 0.1$. Also, we have taken $m = k = \hbar = \omega = 1$.

where $\sigma(\infty)$ is the covariance matrix of the final state of the system obtained with the use of eqs. (20)–(22); $\sigma(0)$ is the initial state of the system, and Y should satisfy the following condition [48]:

$$\lim_{t\to\infty} e^{Yt} = 0. \tag{25}$$

7 Decoherence and evolution of entanglement

Real quantum systems are never completely isolated from their surroundings. When a quantum system interacts with the environment, it may be entangled with a number of degrees of freedom of the environment. This entanglement eliminates the interference effects caused by quantum superpositions and, as a result, pure quantum states are converted into mixed states. This phenomenon occurs due to the transfer of information of the system to the environment, and is called decoherence [2,47–55]. Decoherence of the system can be investigated by examining its purity. In addition, entanglement disappears under the influence of interaction with the environment after a short time; this phenomenon is also called sudden death of entanglement. In the next section, we investigate the evolution of entanglement and decoherence with the use of the PPT criterion and purity, respectively.

8 Results and discussion

In this section, we study the evolution of entanglement and decoherence of GHZ and T states in an open quantum system described in sect. 4. Also, we compare the evolution of entanglement for two initial states (GHZ and T sates).

Using eqs. (20)–(22) and (24) and assuming GHZ and T states as initial states (eqs. (13) and (16)), we obtain time-dependent covariance matrix of the system and its partial transpose with respect to the first mode. Then, using eqs. (6) and (11), we calculate purity and entanglement of the system, respectively. With regard to these calculations, the effect of the environment and initial states on the entanglement and decoherence has been shown in figs. 4 and 5.

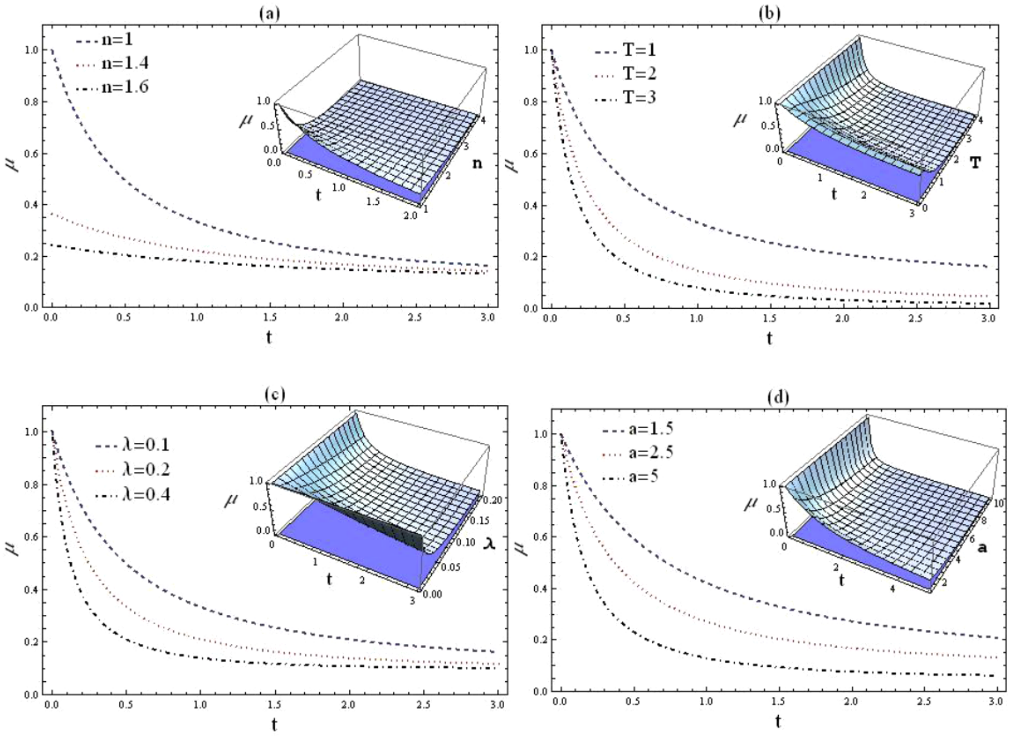

Fig. 5. Evolution of the purity of GHZ states: (a) Two-dimensional plot *versus* time for $n = 1$, 1.4, 1.6 and three-dimensional plot *versus* noise and time for $\lambda = 0.1$, $T = 1$, $a = 2$; (b) Two-dimensional plot *versus* time for $T = 1, 2, 3$ and three-dimensional plot *versus* temperature and time for $\lambda = 0.1$, $n = 1$, $a = 2$. (c) Two-dimensional plot *versus* time for $\lambda = 0.1$, 0.2 and 0.4 and three-dimensional plot *versus* dissipation coefficient and time for $T = 1$, $a = 2$, $n = 1$. (d) Two-dimensional plot *versus* time for $a = 1.5$, 2.5, 5 and three-dimensional plot *versus* squeezing and time for $T = 1$, $n = 1$, $\lambda = 0.1$. Also, we have taken $m = k = \hbar = \omega = 1$.

In figs. 4(a)–(d), we show the evolution of the entanglement of the GHZ state. We have displayed the smallest eigenvalue ($\tilde{\nu}_-$) as a function of the noise, temperature, dissipation coefficient and squeezing, respectively, in two- and three-dimensional plots. With regard to the PPT criterion, the system is entangled or separable if the grid surface of ($\tilde{\nu}_-$) is below or above the transparent horizontal plane (line ($\tilde{\nu}_- = 1$)), respectively. In addition, the intersection with the horizontal plane (line ($\tilde{\nu}_- = 1$)) indicates the entanglement sudden death. Moreover, it is observed that entanglement decreases monotonically with time. Also, RRESD is an increasing function of noise, temperature and dissipation coefficient, while it is a decreasing function of the squeezing parameter. Therefore, RRESD can be controlled by choosing a proper initial state and environmental parameters.

In figs. 5(a)–(d), we have displayed the purity of the GHZ state as a function of the noise, temperature, dissipation coefficient and squeezing, respectively, in two- and three-dimensional plots. It is observed that the purity decreases monotonically with time and finally vanishes; therefore, pure state converts to completely mixed state, and coherency is entirely eliminated. In addition, purity is a decreasing function of noise, squeezing, temperature and dissipation coefficient.

In the following, we intend to apply the same approach used in the study of the GHZ state to investigate the entanglement and purity of the T state. In figs. 6(a)–(c), we have displayed the smallest eigenvalue ($\tilde{\nu}_-$) as a function of the squeezing, temperature and dissipation coefficient, respectively, in two- and three-dimensional plots. It is observed that similar to the GHZ state, in general, entanglement decreases monotonically over the time. In addition, RRESD is a decreasing function of the squeezing parameter whereas it is an increasing function of temperature and dissipation coefficient.

In figs. 7(a)–(c), we have displayed the purity of the T state as a function of the squeezing, temperature and dissipation coefficient, respectively, in two- and three-dimensional plots. It is observed that the purity of the T state decreases monotonically with time and finally vanishes. Moreover, purity is a decreasing function of squeezing, temperature and dissipation coefficient.

We compare the evolution of the entanglement of GHZ and T states in fig. 8; the smallest symplectic eigenvalue *versus* time is plotted for these initial states. It is observed that the entanglement sudden death occurs sooner if the T state is the initial one.

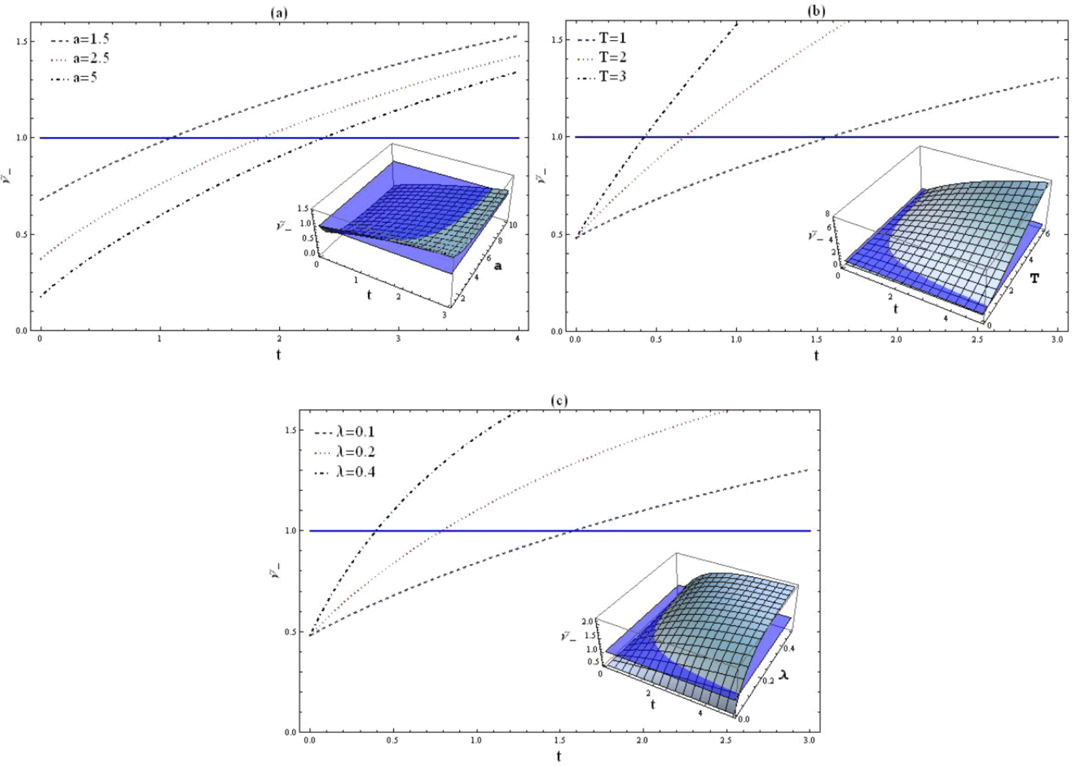

Fig. 6. Entanglement evolution of T states: (a) Two-dimensional plot *versus* time for $a = 1.5$, 2.5, 5 and three-dimensional plot *versus* squeezing and time for $\lambda = 0.1$, $T = 1$. (b) Two-dimensional plot *versus* time for $T = 1$, 2, 3 and three-dimensional plot *versus* temperature and time for $\lambda = 0.1$, $a = 2$. (c) Two-dimensional plot *versus* time for $\lambda = 0.1$, 0.2, 0.4 and three-dimensional plot *versus* dissipation coefficient and time for $T = 1$, $a = 2$. Also, we have taken $m = k = \hbar = \omega = 1$.

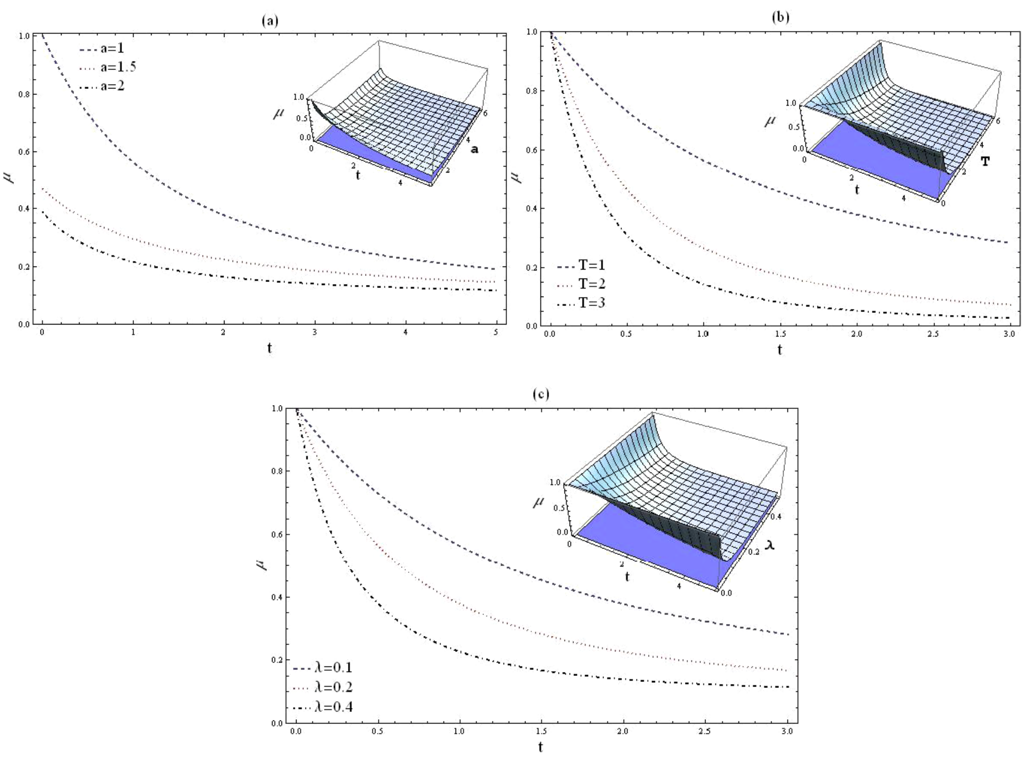

Fig. 7. Evolution of purity of T states: (a) Two-dimensional plot *versus* time for $a = 1$, 1.5, 2 and three-dimensional plot *versus* squeezing and time for $\lambda = 0.1$, $T = 1$. (b) Two-dimensional plot *versus* time for $T = 1$, 2, 3 and three-dimensional plot *versus* temperature and time for $\lambda = 0.1$, $a = 1$. (c) Two-dimensional plot *versus* time for $\lambda = 0.1$, 0.2, 0.4 and three-dimensional plot *versus* dissipation coefficient and time for $T = 1$, $a = 1$. Also, we have taken $m = k = \hbar = \omega = 1$.

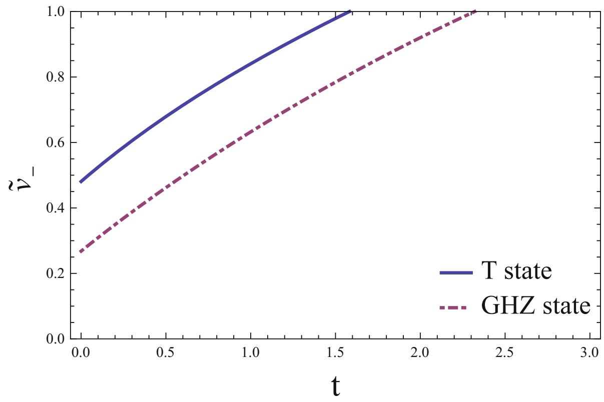

Fig. 8. Entanglement evolution of GHZ and T states for $T = 1$, $a = 2$, $\lambda = 0.1$ and $n = 1$.

9 Conclusion

In this work, we have studied the entanglement and decoherence of a system consisting of three uncoupled symmetric oscillators in a thermal bath. The GHZ states and T states have been considered as the initial states and the three-mode Gibbs state has been considered as the final state of the system. Assuming Born-Markov approximation and using Lindblad equation, we have obtained the evolution of the system. Then, the dependence of the entanglement and decoherence on the initial state and environmental parameters has been investigated with the use of the PPT criterion and purity, respectively. We found that although the entanglements of the GHZ and T states are similarly increasing functions of the squeezing parameter, the entanglement of the GHZ state increases more than that of the T state. However, the purity of T states is a decreasing function of squeezing while that of GHZ states is an independent function of squeezing despite the purity time evolution dependence on squeezing. If GHZ is considered as the initial state of the system, the purity decreases monotonically over time and the initial pure state becomes a mixed state with the increase of all the involved parameters. In addition, entanglement decreases monotonically over time and disappears completely in a short time. Moreover, we have found that RRESD is an increasing function of noise, temperature and dissipation coefficient but it is a decreasing function of the squeezing parameter. Furthermore, if the T state is considered as the initial state of the system, the same results can be obtained. Finally, the comparison of the entanglement evolution of the system with T and GHZ as initial states showed that the entanglement survives longer for the latter. Considering the above findings, we conclude that the RRESD is controllable by manipulation of the environmental and initial state parameters.

The authors very much appreciate the anonymous referees for their nice fruitful comments that were a great help to improve the paper. They would also like to thank Shahid Chamran University of Ahvaz for the grant No. 96/3/02/16670.

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