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Quasi–four-body treatment of charge transfer in the collision of protons with atomic helium: II. Second-order non-Thomas mechanisms and the cross sections

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Abstract. A fully quantum mechanical four-body treatment of charge transfer collisions between energetic protons and atomic helium is developed here. The Pauli exclusion principle is applied to both the wave function of the initial and final states as well as the operators involved in the interaction. Prior to the collision, the helium atom is assumed as a two-body system composed of the nucleus, He^{2+}, and an electron cloud composed of two electrons. Nonetheless, four particles are assumed in the final state. As the double interactions contribute extensively in single charge transfer collisions, the Faddeev-Lovelace-Watson scattering formalism describes it best physically. The treatment of the charge transfer cross section, under this quasi–four-body treatment within the FWL formalism, showed that other mechanisms leading to an effect similar to the Thomas one occur at the same scattering angle. Here, we study the two-body interactions which are not classically described but which lead to an effect similar to the Thomas mechanism and finally we calculate the total singlet and triplet amplitudes as well as the angular distributions of the charge transfer cross sections. As the incoming projectiles are assumed to be plane waves, the present results are calculated for high energies; specifically a projectile energy of 7.42 MeV was assumed as this is where experimental results are available in the literature for comparison. Finally, when possible we compare the present results with the other available theoretical data.

1 Introduction

In a single charge transfer process, one electron from an atomic species is attached to the second interacting particle, as reported in the literature theoretically [1–14] and experimentally [15–23]. These processes should be studied under a many-body model, except for the interaction of a fully charged ion and atomic hydrogen which could be studied under a three-body formalism [24–27]. The same is also true in the investigation of interactions leading to ionization [28–30] or discrete excitation [31,32]. For other interactions, where two interacting atomic objects carry two electrons, a four-body treatment is physically more sensible. The Born approximation [33,34], the Faddeev-Watson-Lovelace (FWL) formalism [25,35–37] and the Continuous Distorted Wave (CDW) method [38] are examples of methods implemented frequently in order to study single charge transfer under a three-particle formalism, where some other assumptions are made for the treatment to be physically feasible. The study of single charge transfer, based on a four-body interaction has been extended under the CDW [26,39,40], Distorted Wave Born (DWB) [41–43], B1B, B2B [4,44,45] and FWL [46–51] approximations or formalisms as well.

Following up from our two recent papers [52,53], the single charge transfer in proton-helium collisions is considered here in order to introduce a detailed four-body treatment of the process. We will achieve this by implementing the FWL formalism, which is a quantum mechanical treatment and defines the different terms in the interaction properly. In this paper we will specifically treat the second-order terms, *i.e.* nuclear-nuclear terms, and we will calculate the scattering partial amplitudes and differential cross sections.

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The charge transfer interaction between the proton, as a bare ion, and atomic helium is shown as

$$P + (T + e_1 + e_2) \rightarrow (P + e_P) + (T + e_T), \tag{1}$$

where P and $(T+e_1+e_2)$ stand for the proton as the projectile and helium as the target, respectively. Note that we have defined the two electrons after collision as e_P and e_T to take the indistinguishability of these electrons into account. Previously we implemented the FWL formalism, for the first time, to investigate the first-order terms of the prior FWL equation form [52], as well as the second-order terms leading to a Thomas peak under FWL formalism [53]. We will denote the latter as paper I. Here we intend to discuss the other terms, specifically the nuclear-nuclear interaction terms, and to find the angular distribution of the cross sections. We will not repeat the work discussed in paper I and the first order paper [52].

In order to properly calculate the transition amplitude for single charge transfer in proton-helium collisions, one needs a form of the transition matrix derived by Nutt and Schwinger [54,55], in integral form, which can be implemented either as its exact off-shell Coulomb T-matrix [56–58] or Chen's [59] approximate off-shell Coulomb T-matrix or via Haar resolution analysis which is implemented to calculate the transition amplitudes [60,61]. The three off-shell Coulomb T-matrix forms are used here, and the results are too close to be distinguished. Therefore, we are confident that our choice does not contribute to possible errors.

2 Background and post-transition operator

The four-body FWL equations for the post (+) and prior (−) forms of the transition operator are obtained as

$$U^{\pm}_{\beta\alpha} = (1 - \delta_{\beta\alpha}) \begin{Bmatrix} V_\beta - V_{\beta\alpha} \\ V_\alpha - V_{\beta\alpha} \end{Bmatrix} + \sum_{j \in E(\alpha)} \sum_{i \in E(\alpha)} T_{ji}. \tag{2}$$

Based on the Faddeev three-body treatment of the scattering process [36], and its expansion to four-body interactions [51], we introduced a quasi–four-body treatment for single charge transfer [52,53] where we obtained a post-form of the FWL transition operator as

$$U^{+}_{\beta\alpha} = V_{Pe_T} + V_{Te_P} + V_{e_Pe_T} + T_{Pe_T,PC} + T_{Te_P,PC} + T_{e_Pe_T,PC} + T_{PT,PC} + T_{Pe_T,PT} + T_{Te_P,PT} + T_{e_Pe_T,PT} + T_{PT,PT}, \tag{3}$$

in which C and T stand for the electronic cloud and the target's nucleus, respectively. We will use this post-form of the interaction operator in this paper, in order to calculate the cross sections. As noted above, the indistinguishability of the electrons results in naming them as e_P and e_T after collision. Finally, we find the post-form of the FWL transition operator, in this quasi–four-body treatment for single charge in terms of the two particle interactions, V, and the two particle transition operators, t, as

$$\begin{aligned} U^{+}_{\beta\alpha} = {} & V_{Te_P} + V_{e_Pe_T} + \left(G_0^{-1} + t_{Te_P} + t_{e_Pe_T} + t_{Pe_T}\right) G_0 t_{PT} + V_{Pe_T} + \frac{1}{2}\left(G_0^{-1} + t_{Te_P} + t_{e_Pe_T} + t_{PT}\right) G_0 t_{Pe_T} \\ & + \frac{1}{2}\left(t_{Te_P} + t_{Pe_T} + t_{e_Pe_T} + t_{PT}\right) G_0 t_{Pe_P} \end{aligned} \tag{4}$$

where G_0 stands for the free particle Green's operator. Although the method presented here is applied to proton-helium collisions, eq. (4) can be applied to any interaction between a bare ion and a two-electron atomic system when describing a single charge transfer process.

Applying the Pauli principle to the two electrons of the four-particle interacting system, the final asymptotic singlet (+) or triplet (−) states are given by eqs. (I19) in paper I [53]. The singlet and triplet forms of the final wave function result in singlet and triplet partial amplitudes.

The transition amplitude is defined as

$$A = \langle \psi_f | U^{+}_{\beta\alpha} | \psi_i \rangle, \tag{5}$$

which is written as 14 terms for the final triplet states and 23 terms for the final singlet states. The partial amplitudes for the first-order terms of the prior form [52] and the second-order nuclear (electron) – electron, $T_{Te_P,PC}(T_{e_Pe_T,PC})$, terms were calculated in paper I, eqs. (I-26) and (I-35). Those second-order terms confirmed the Thomas mechanisms. The first and the second-order terms of the post-form of two particle interactions to be discussed here, respectively, are

$$\{V_{Pe_T}, V_{Te_P}, V_{e_Pe_T}\} \tag{6}$$

and

$$\left\{T_{Pe_T,PC}, T_{PT,PC}, T_{Pe_T,PT}, T_{Te_P,PT}, T_{e_Pe_T,PT}, T_{PT,PT}\right\}. \tag{7}$$

Note that any partial amplitude obtained by inserting eq. (4) into eq. (5) contains two sub-amplitudes depending on the final coordinate used to define the wave function. A sub-amplitude either expresses the probability of charge transfer from the initial channel $|i\rangle$ into final channel $|f_1\rangle$, or the second one shows the probability of this transformation into final channel $|f_2\rangle$. To become familiar with Jacobi coordinate see [52]. Therefore, we will calculate the angular distribution of a single charge transfer collision from the ground state of atomic helium into the ground state of the He^+ and the H atom.

It is, also, necessary to define the momentum transfer experienced by the target ion and the projectile during the collision to be $\vec{T}$ and $\vec{J}$, respectively. Additionally the vectors $\vec{V}_i$ and $\vec{V}_f$ stand for the initial and final heavy-particles velocities, respectively. One can conclude that $\vec{V}_i = \vec{V}_f$ for a direct collision, which can therefore be simply set as $\vec{V}$.

The amplitude of the direct interaction between the particles, *i.e.*, Coulomb potential, making use of the post-form of the transition operator in charge transfer helium-proton scattering, is summarized as

$$A_e = \langle\psi_f|V_{Pe_2} + V_{Te_1} + V_{e_1e_2}|\psi_i\rangle = \frac{1}{\sqrt{N}}\begin{cases} 2\langle\psi_{f_1}|V_{PC} + V_{e_1e_2}|\psi_i\rangle + \sqrt{N}\langle\psi_f|V_{Te_1}|\psi_i\rangle & \text{Singlet} \\ \langle(1-P_{12})\psi_{f_1}|V_{Pe_2} + V_{Te_1}|\psi_i\rangle & \text{Triplet.} \end{cases} \tag{8}$$

This amplitude resembles the Coulomb interactions between the projectile-electron, nucleus-electron and electron-electron and is similar to the first-order Born approximation. Therefore, we call it the Born amplitude. The Born amplitude could be written in terms of five integrals for which their simplified forms and their momentum conditions are listed as

$$I_{01} = \langle\psi_{f_1}|V_{Pe_1}|\psi_i\rangle = (2\pi)^{3/2}\int \mathrm{d}\vec{k}_{1P}\mathrm{d}\vec{k}_{2T}\phi^*_H\left(\vec{k}_{1P}\right)\phi^*_{He^+}\left(\vec{k}_{2T}\right)V_{Pe_1}\left(\vec{k}_{1P}-\vec{J}\right)\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{9}$$

where $\vec{k}_{1i} = -\vec{T}$ and $\vec{k}_{2T} - \vec{k}_{2i} = 0$;

$$I_{02} = \langle\psi_{f_2}|V_{Pe_1}|\psi_i\rangle = (2\pi)^{3/2}\int \mathrm{d}\vec{k}_{1i}\mathrm{d}\vec{k}_{2i}\phi^*_H\left(\vec{k}_{2P}\right)\phi^*_{He^+}\left(\vec{k}_{1T}\right)V_{Pe_1}\left(\vec{k}_{2i}+\vec{T}\right)\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{10}$$

where $\vec{k}_{1T} = \vec{k}_{1i} + \vec{k}_{2i} + \vec{T}$ and $\vec{k}_{2P} = \vec{k}_{2i} - \vec{V}$;

$$I_{03} = \langle\psi_{f_1}|V_{Te_1}|\psi_i\rangle = (2\pi)^{3/2}\int \mathrm{d}\vec{k}_{1i}\mathrm{d}\vec{k}_{2i}\phi^*_H\left(\vec{k}_{1P}\right)\phi^*_{He^+}\left(\vec{k}_{2T}\right)V_{Te_1}\left(\vec{k}_{1i}+\vec{T}\right)\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{11}$$

where $\vec{k}_{1P} = \vec{J}$ and $\vec{k}_{2T} - \vec{k}_{2i} = 0$;

$$I_{04} = \langle\psi_{f_2}|V_{Te_1}|\psi_i\rangle = (2\pi)^{3/2}\int \mathrm{d}\vec{k}_{1i}\mathrm{d}\vec{k}_{1T}\phi^*_H\left(\vec{k}_{2P}\right)\phi^*_{He^+}\left(\vec{k}_{1T}\right)V_{Te_1}\left(\vec{k}_{1T}-\vec{k}_{1i}\right)\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{12}$$

where $\vec{k}_{2P} = \vec{J}$ and $\vec{k}_{2i} = -\vec{T}$; and

$$I_{05} = \langle\psi_{f_1}|V_{e_1e_2}|\psi_i\rangle = (2\pi)^{3/2}\int \mathrm{d}\vec{k}_{1i}\mathrm{d}\vec{k}_{2i}\phi^*_H\left(\vec{k}_{1P}\right)\phi^*_{He^+}\left(\vec{k}_{2T}\right)V_{Te_1}\left(\vec{k}_{1i}+\vec{T}\right)\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{13}$$

where $\vec{k}_{2T} = \vec{k}_{1i} + \vec{k}_{2i} + \vec{T}$ and $\vec{k}_{1P} = \vec{J}$ are the respective momentum conditions.

3 Charge transfer from $|i\rangle$ to $|f_1\rangle$

The charge transfer from the initial state $|i\rangle$ to the final state $|f_1\rangle$, due to the first-order sub-amplitude, t_{PT}, is simplified as

$$I_{31} = \langle\psi_{f_1}|t_{PT}|\psi_i\rangle = \int \Phi^*_{He^+}\left(\vec{k}_{2T}\right)\Phi^*_H\left(\vec{k}_{1P}\right)t_{PT}\left(\mu_n\vec{V}-\vec{k}_{1i}-\vec{T},\mu_n\vec{V},E_{PT}\right)\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right)\mathrm{d}\vec{k}_{1i}\mathrm{d}\vec{k}_{2i}, \tag{14}$$

where the momentum conditions $\vec{k}_{2T} - \vec{k}_{2i} = 0$ and $\vec{k}_{1i} - \vec{k}_{1P} = \vec{V}$ emerge from the delta functions in simplifying this term. Similar momentum conditions should be present for any term that contributes to the charge transfer.

The other first-order sub-amplitude in this charge transfer process, due to t_{Pe_2}, is

$$I_{32} = \langle\psi_{f_1}|t_{Pe_2}|\psi_i\rangle = \int \mathrm{d}\vec{k}_{1P}\mathrm{d}\vec{k}_{2i}\Phi^*_{He^+}\left(\vec{k}_{2T}\right)\Phi^*_H\left(\vec{k}_{1P}\right)t_{Pe_2}\left(\vec{k}_{1P}+\vec{k}_{2i}+\vec{T},\vec{k}_{2i}-\vec{V};E_{Pe_2}\right)\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{15}$$

where $\vec{k}_{2T} - \vec{k}_{2i} = \vec{k}_{1P} - \vec{J}$ and $\vec{k}_{1i} - \vec{k}_{1P} = \vec{V}$ now emerge as the momentum condition. The scattering energies in integrals (14) and (15) are defined as

$$E_{PT} = \varepsilon_f + \frac{1}{2}\mu_n V^2 - \frac{1}{2}k_{1P}^2 - \frac{1}{2}k_{2T}^2 = \varepsilon_i + \frac{1}{2}\mu_n V^2 - \frac{1}{2}k_{1i}^2 - \frac{1}{2}k_{2i}^2 \cong \frac{1}{2}\mu_n V^2 \tag{16}$$

and

$$E_{Pe_2} \cong \frac{1}{2}V^2 - \vec{k}_{2T}\cdot\vec{V} - \frac{1}{2}k_{1P}^2 + \varepsilon_f = \frac{1}{2}V^2 - \vec{k}_{2i}\cdot\vec{V} - \frac{1}{2}k_{1i}^2 + \varepsilon_i. \tag{17}$$

The scattering energy E_{PT} defined by eq. (16) is present in all the terms containing the two particle interaction PT. The term $\mu_n V^2/2$ in E_{PT} is dominated over the other terms as the heavy-particle reduced mass $\mu_n = M_T M_P/(M_T + M_P)$ is a very large number. The second-order sub-amplitudes, whose interactions result in the charge transfer of the initial state $|i\rangle$ to $|f_1\rangle$ are the following.

a) The term $I_{33} = \langle\psi_{f_1}|t_{Te_1}G_0 t_{PT}|\psi_i\rangle$ is simplified in momentum space as

$$\begin{aligned} I_{33} = (2\pi)^{-3}\int \mathrm{d}\vec{k}_{1P}\mathrm{d}\vec{k}_{1i}\mathrm{d}\vec{k}_{2i}\Phi_{He^+}^*\left(\vec{k}_{2T}\right)\Phi_H^*\left(\vec{k}_{1P}\right)\left\langle\vec{k}_{1P}+\vec{V}|t_{Te_1}|\vec{k}_{1i}\right\rangle G_0(E_{PT}) \\ \times\left\langle\mu_n\vec{V}-\vec{k}_{1P}+\vec{J}|t_{PT}|\mu_n\vec{V}\right\rangle\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right) \end{aligned} \tag{18}$$

where $\vec{k}_{2T} - \vec{k}_{2i} = 0$ is the momentum condition which is expected as the non-interacting electron does not experience a momentum transfer. This term has no classical counterpart and contributes to the charge transfer not only in quasi–four-body FWL formalism but also in three-body FWL formalism. The free Green operator is simplified as

$$\begin{aligned} G_0^+(E_{PT}) &= \left(E_{PT} - \left|\mu_n\vec{V} - \vec{k}_{1P} + \vec{J}\right|^2 \Big/ 2\mu_n + i\eta\right)^{-1} \\ &= \left(E_{Te_1} - \left|\vec{k}_{1i}\right|^2 \Big/ 2 + i\eta\right)^{-1} = G_0^+(E_{Te_1}) \quad \eta\to 0, \end{aligned} \tag{19}$$

where the scattering energy E_{PT} is defined by eq. (16) and the scattering energy E_{Te_1} is defined as

$$E_{Te_1} = \frac{1}{2}V^2 + \varepsilon_f + \vec{k}_{1P}\cdot\vec{V} - \frac{1}{2}k_{2T}^2. \tag{20}$$

One should note that the free Green's function derived throughout this manuscript has two distinct but equal forms.

b) The term $I_{34} = \langle\psi_{f_1}|t_{Pe_2}G_0 t_{PT}|\psi_i\rangle$ is simplified as

$$\begin{aligned} I_{34} = (2\pi)^{-3}\int \mathrm{d}\vec{k}_{2T}\mathrm{d}\vec{k}_{1i}\mathrm{d}\vec{k}_{2i}\Phi_{He^+}^*\left(\vec{k}_{2T}\right)\Phi_H^*\left(\vec{k}_{1P}\right)\left\langle\vec{k}_{2T}-\vec{V}|t_{Pe_2}|\vec{k}_{2i}-\vec{V}\right\rangle G_0(E_{PT}) \\ \times\left\langle\mu_n\vec{V}+\vec{k}_{2T}-\vec{T}|t_{PT}|\mu_n\vec{V}\right\rangle\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \end{aligned} \tag{21}$$

where the momentum condition is now $\vec{k}_{1i} - \vec{k}_{1P} = \vec{V}$, This conditions clarifies that the momentum transfer to the first electron is as large as the projectile velocity in magnitude; *i.e.* it is the momentum of an electron moving at the same velocity as the projectile. The scattering energy E_{Pe_2} is defined by the first equality in eq. (17). Also,

$$\begin{aligned} G_0(E_{PT}) &= \left(E_{PT} - \left|\mu_n\vec{V} + \vec{k}_{2T} - \vec{T}\right|^2 \Big/ 2\mu_n + i\eta\right)^{-1} \\ &\equiv \left(E_{Pe_2} - \left|\vec{k}_{2i} - \vec{V}\right|^2 \Big/ 2 + i\eta\right)^{-1} = G_0(E_{Pe_2}); \quad \eta\to 0 \end{aligned} \tag{22}$$

is the free Green's function. The form of the Green's function, $G_0(E_{Pe_2})$, on the second side of eq. (22), will be often repeated and, therefore, we omit noting it for the future cases. This term contributes to charge transfer due to the Pauli exclusion principle.

c) The term $I_{35} = \langle\psi_{f_1}|t_{e_1e_2}G_0t_{PT}|\psi_i\rangle$ can be written as

$$I_{35} = (2\pi)^{-3}\int \mathrm{d}\vec{k}_{1P}\mathrm{d}\vec{k}_{1i}\mathrm{d}\vec{k}_{2i}\Phi^*_H\left(\vec{k}_{1P}\right)\Phi^*_{He^+}\left(\vec{k}_{2T}\right)\left\langle\frac{1}{2}\left(\vec{k}_{2T}-\vec{k}_{1P}-\vec{V}\right)|t_{e_1e_2}|\frac{1}{2}\left(\vec{k}_{2i}-\vec{k}_{1i}\right)\right\rangle G_0(E_{PT})$$
$$\times\left\langle\mu_n\vec{V}-\vec{k}_{1P}+\vec{J}|t_{PT}|\mu_n\vec{V}\right\rangle\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{23}$$

where $(\vec{k}_{1i}+\vec{k}_{2i})-(\vec{k}_{1P}+\vec{k}_{2T})=\vec{V}$ is the momentum condition for this double interaction to lead to charge transfer. This shows that the two-electron momentum transfer equals to the speed of the projectile. The scattering energy E_{ee} is defined as

$$E_{ee} = \frac{1}{2}V^2+\vec{k}_{1P}\cdot\vec{V}-\frac{1}{4}\left(\vec{k}_{1i}+\vec{k}_{2i}\right)^2+\varepsilon_f \tag{24}$$

and the Green's function in eq. (23) is similar to that of the sub-amplitude I_{33} which is defined in eq. (19). Not considering the electronic cloud, this term disappears.

d) The second-order term $I_{36} = \langle\psi_{f_1}|t_{PT}G_0t_{Pe_1}|\psi_i\rangle$ in momentum space reads as

$$I_{36} = (2\pi)^{-3}\int \mathrm{d}\vec{k}_{1P}\mathrm{d}\vec{k}_{1i}\mathrm{d}\vec{k}_{2i}\Phi^*_{He^+}\left(\vec{k}_{2T}\right)\Phi^*_H\left(\vec{k}_{1P}\right)\left\langle\mu_n\vec{V}|t_{PT}|\mu_n\vec{V}+\vec{k}_{1i}+\vec{T}\right\rangle G_0(E_{Pe_1})$$
$$\times\left\langle\vec{k}_{1P}|t_{Pe_1}|\vec{k}_{1i}-\vec{V}\right\rangle\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{25}$$

in which the momentum condition similar to I_{33} holds. Both the I_{33} and I_{36} interactions are present in a three-body FWL treatment of the charge transfer interaction. The free Green's function is defined as

$$G_0(E_{PT}) = \left(E_{PT}-\left|\mu_n\vec{V}+\vec{k}_{1i}+\vec{T}\right|^2\Big/2\mu_n+i\eta\right)^{-1}$$
$$\equiv\left(E_{Pe_1}-\left|\vec{k}_{1P}\right|^2\Big/2+i\eta\right)^{-1}=G_0(E_{Pe_1})\quad\eta\to0, \tag{26}$$

while the scattering energy, E_{Pe_1}, is

$$E_{Pe_1} = \varepsilon_i+\frac{1}{2}V^2-\vec{k}_{1i}\cdot\vec{V}-\frac{1}{2}k^2_{2i}. \tag{27}$$

e) The term $I_{37} = \langle\psi_{f_1}|t_{Pe_2}G_0t_{Pe_1}|\psi_i\rangle$, which has no counterpart in a three-body treatment, is

$$I_{37} = (2\pi)^{-3}\int \mathrm{d}\vec{k}_{1P}\mathrm{d}\vec{k}_{2T}\mathrm{d}\vec{k}_{2i}\Phi^*_{He^+}\left(\vec{k}_{2T}\right)\Phi^*_H\left(\vec{k}_{1P}\right)t_{Pe_2}\left(\vec{k}_{2T}-\vec{V},\vec{k}_{2i}-\vec{V};E_{Pe_2}\right)G_0(E_{Pe_1})t_{Pe_1}$$
$$\times\left(\vec{k}_{1P},\vec{k}_{2T}-\vec{k}_{2i}+\vec{J};E_{Pe_1}\right)\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{28}$$

in which the momentum condition, for the charge transfer to take place under this double interaction, is $\vec{k}_{2T}-\vec{k}_{2i}=\vec{k}_{1i}+\vec{T}$. The scattering energies E_{Pe_1} and E_{Pe_2} of sub-amplitude I_{37} are defined in eqs. (27) and (17), respectively. Also the free Green's function inserted in eq. (28) is in the form of the second side of eqs. (26) or (22).

4 Charge transfer from $|i\rangle$ to $|f_2\rangle$

The only first-order sub-amplitude in this group, which defines charge transfer from the initial state $|i\rangle$ to the final state $|f_2\rangle$, is simplified as

$$I_{41} = \langle f_2|t_{Pe_2}|i\rangle = \int \mathrm{d}\vec{k}_{2P}\mathrm{d}\vec{k}_{1i}\Phi^*_{He^+}\left(\vec{k}_{1i},\vec{T}\right)\Phi^*_H\left(\vec{k}_{1P}\right)t_{Pe_2}\left(\vec{k}_{2P},\vec{J};E_{Pe_2}\right)\phi_{He}\left(\vec{k}_{1i},\vec{k}_{2i}\right), \tag{29}$$

where $\vec{k}_{2i}=-\vec{T}$ and $\vec{k}_{1T}\cong\vec{k}_{1i}$ are its momentum conditions and the scattering energy, E_{Pe_2}, is defined by eq. (17).

The second-order sub-amplitudes, leading to charge transfer from the initial channel $|i\rangle$ to the final channel $|f_2\rangle$, are the following.

a) The term $I_{42} = \langle \psi_{f_2} | t_{Te_1} G t_{PT} | \psi_i \rangle$ is simplified in momentum space as

$$I_{42} = (2\pi)^{-3} \int \mathrm{d}\vec{k}_{1T} \mathrm{d}\vec{k}_{1i} \mathrm{d}\vec{k}_{2i} \Phi^*_{He^+} \left(\vec{k}_{1T}\right) \Phi^*_H \left(\vec{k}_{2P}\right) \left\langle \vec{k}_{1T} | t_{Te_1} | \vec{k}_{1i} \right\rangle G_0(E_{PT}) \times \left\langle \mu_n \vec{V} - \vec{k}_{2i} - \vec{T} | t_{PT} | \mu_n \vec{V} \right\rangle \phi_{He} \left(\vec{k}_{1i}, \vec{k}_{2i}\right), \tag{30}$$

where the same momentum condition as I_{34} holds by replacing the second electron for the first one. The scattering energy E_{Te_1} in I_{42}, defined as

$$E_{Te_1} = \varepsilon_f - k^2_{2P}/2 \tag{31}$$

differs with the term in I_{34}. The Green's function is obtained as

$$G_0^+(E_{PT}) = \left(E_{PT} - \left| \mu_n \vec{V} - \vec{k}_{2i} - \vec{T} \right|^2 \Big/ 2\mu_n + i\eta \right)^{-1} \equiv \left(E_{Te_1} - k^2_{1i}/2 + i\eta \right)^{-1} = G_0^+(E_{Te_1}) \quad \eta \to 0, \tag{32}$$

where its second form is exactly similar to the second equality in eq. (19). Hence, the first equality in eqs. (19) and (32) are also equal.

b) The next sub-amplitude, $I_{43} = \langle \psi_{f_2} | t_{Pe_2} G_0 t_{PT} | \psi_i \rangle$, is now simplified as

$$I_{43} = (2\pi)^{-3} \int \mathrm{d}\vec{k}_{2P} \mathrm{d}\vec{k}_{1T} \mathrm{d}\vec{k}_{2i} \Phi^*_{He^+} \left(\vec{k}_{1T}\right) \Phi^*_H \left(\vec{k}_{2P}\right) \left\langle \vec{k}_{2P} | t_{Pe_2} | \vec{k}_{2i} - \vec{V} \right\rangle G_0(E_{PT}) \times \left\langle \mu_n \vec{V} - \vec{k}_{2i} - \vec{T} | t_{PT} | \mu_n \vec{V} \right\rangle \phi_{He} \left(\vec{k}_{1i}, \vec{k}_{2i}\right), \tag{33}$$

where $\vec{k}_{1i} = \vec{k}_{1T}$ is the momentum condition, which determines the momentum of the passive electron due to the interaction. The scattering energy, E_{Pe_2}, is now defined as

$$E_{Pe_2} = \varepsilon_f - k^2_{1T}/2. \tag{34}$$

In order to obtain the integral (33), we had to make use of the Green's functions $G_0(E_{PT})$ and $G_0(E_{Pe_2})$ as defined by eqs. (32) and (22), respectively.

c) The sub-amplitude $I_{44} = \langle \psi_{f_2} | t_{PT} G_0 t_{Pe_1} | \psi_i \rangle$ is also simplified as

$$I_{44} = (2\pi)^{-3} \int \mathrm{d}\vec{k}_{2P} \mathrm{d}\vec{k}_{1T} \mathrm{d}\vec{k}_{1i} \Phi^*_{He^+} \left(\vec{k}_{1T}\right) \Phi^*_H \left(\vec{k}_{2P}\right) \left\langle \mu_n \vec{V} | t_{PT} | \mu_n \vec{V} + \vec{k}_{1i} - \vec{J} \right\rangle G_0(E_{Pe_1}) \times \left\langle \vec{k}_{1T} - \vec{V} | t_{Pe_1} | \vec{k}_{1i} - \vec{V} \right\rangle \phi_{He} \left(\vec{k}_{1i}, \vec{k}_{2i}\right). \tag{35}$$

Note that the momentum condition for I_{44} is similar to that for I_{42}. The scattering energy E_{Pe_1} used here is the same as the one defined by eq. (27). The Green's function $G_0(E_{Pe_1})$ in eq. (35) and its equivalent form is defined as

$$G_0(E_{PT}) = \left(E_{PT} - \left| \mu_n \vec{V} + \vec{k}_{1i} - \vec{J} \right|^2 \Big/ 2\mu_n \right)^{-1} \equiv \left(E_{Pe_1} - \left| \vec{k}_{1T} - \vec{V} \right|^2 \Big/ 2 \right)^{-1} = G_0(E_{Pe_1}) \quad \eta \to 0. \tag{36}$$

d) The final double interaction term in a four-body treatment of charge transfer from helium to incoming protons, $I_{45} = \langle f_2 | t_{Pe_2} G_0 t_{Pe_1} | i \rangle$, is written as

$$I_{45} = (2\pi)^{-3} \int \mathrm{d}\vec{k}_{2P} \mathrm{d}\vec{k}_{1T} \mathrm{d}\vec{k}_{2i} \Phi^*_{He^+} \left(\vec{k}_{1T}\right) \Phi^*_H \left(\vec{k}_{2P}\right) t_{Pe_2} \left(\vec{k}_{2P}, \vec{k}_{2i} - \vec{V}; E_{Pe_2}\right) G_0(E_{Pe_1}) t_{Pe_1} \times \left(\vec{k}_{1T} - \vec{V}, \vec{k}_{1T} - \vec{k}_{2i} + \vec{J}; E_{Pe_1}\right) \phi_{He} \left(\vec{k}_{1i}, \vec{k}_{2i}\right), \tag{37}$$

in momentum space where the momentum condition so derived is $\vec{k}_{1T} - \vec{k}_{1i} = \vec{k}_{2i} + \vec{T}$. The quantities E_{Pe_1}, E_{Pe_2}, $G_0(E_{Pe_2})$ and $G_0(E_{Pe_1})$ are defined by eqs. (27), (34), (22) and (36), respectively.

5 Singlet and triplet amplitudes

Due to the existence of a singlet or the triplet state of the final state wave function, any sub-amplitude morphs into two different values. In this section, we find the interaction terms for the final singlet and triplet wave forms.

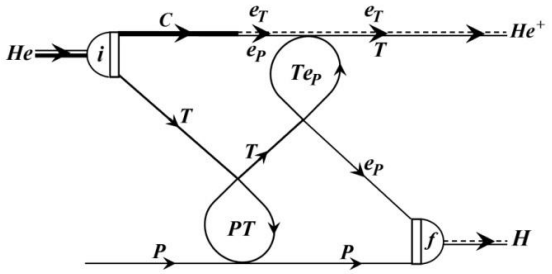

Fig. 1. The graphical description of interaction $T_{Te_P,PT}$.

First-order potential interactions

The first-order potential amplitude, which results from the post-form of the FWL transition operator, can be presented both as the singlet and the triplet forms of the wave function as:

$$A_e = \langle \psi_f | V_{Pe_2} + V_{Te_1} + V_{e_1e_2} | \psi_i \rangle = \frac{1}{\sqrt{N}} \begin{cases} I_{01} + I_{02} + I_{03} + I_{04} + 2I_{05} & \text{Singlet}, \\ -I_{01} + I_{02} + I_{03} - I_{04} & \text{Triplet}. \end{cases} \tag{38}$$

Inter-particle term, A_n

The first inter-particle term $T_{PT,PT}$ appears in all scattering channels such as excitation, ionization and charge transfer, where the incoming projectile shifts the position and/or the momentum of the target's (helium) nucleus and, therefore, changes the symmetry of electronic cloud. Hence the atom loses its "equilibrium" and causes an electron to be emitted, excited or transferred to the projectile. The amplitude for this process, also called the inter-nucleus term, A_n, is

$$A_n = \langle \psi_f | T_{PT,PT} | \psi_i \rangle = \frac{2}{\sqrt{N}} \begin{cases} I_{31} & \text{Singlet}, \\ 0 & \text{Triplet}. \end{cases} \tag{39}$$

Proton-nucleus collision, $A_{Te_P,PT}$

There is a possibility that the incoming proton, leaving the helium atom after recoil by scattering from the nucleus, also during the collision disturbs the electronic cloud, a double interaction term. In this case an electron leaves the electronic cloud with the reflecting proton from the target nucleus and forms an atom with the projectile. This mechanism is represented by $T_{Te_P,PT}$ and its amplitude is shown as

$$A_{Te_P,PT} = \langle \psi_f | T_{Te_P,PT} | \psi_i \rangle = \frac{1}{\sqrt{N}} \begin{cases} I_{33} + I_{42} & \text{Singlet}, \\ I_{33} - I_{42} & \text{Triplet}, \end{cases} \tag{40}$$

which is symbolized in fig. 1.

Proton-nucleus collision $A_{Pe_T,PT}$

Another double interaction mechanism, denoted by $T_{Pe_T,PT}$, could be described similarly to the previous one, but the transition matrix is written differently. This mechanism is denoted by fig. 2 where its amplitude is simplified as

$$A_{Pe_T,PT} = \langle \psi_f | T_{Pe_T,PT} | \psi_i \rangle = \frac{1}{\sqrt{N}} \begin{cases} I_{34} + I_{43} & \text{Singlet}, \\ I_{34} - I_{43} & \text{Triplet}. \end{cases} \tag{41}$$

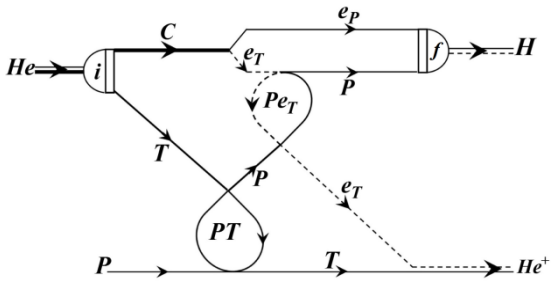

Fig. 2. The graphical description of interaction $T_{Pe_T,PT}$.

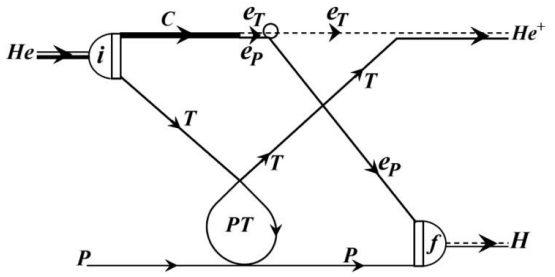

Fig. 3. The graphical description of interaction $T_{e_Pe_T,PT}$.

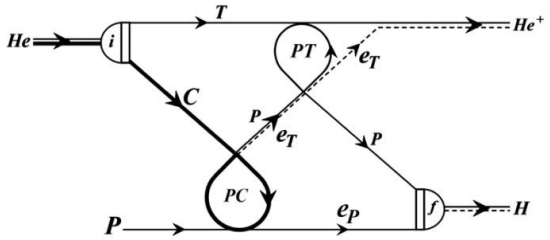

Fig. 4. The graphical description of interaction $T_{PT,PC}$.

Electron-electron interaction, $A_{e_Pe_T,PT}$

The other probable possibility for charge transfer is that the spherical symmetry of the electronic cloud is destroyed, and it is divided into two parts as a result of the proton-nucleus collision. The next interaction would be between the two parts of the electronic cloud, leaving one attached to the projectile and the other to the target's nucleus. This mechanism is described by the transition matrix $T_{e_Pe_T,PT}$. The amplitude for this mechanism to take place simplifies as

$$A_{e_Pe_T,PT} = \langle \psi_f | T_{e_1e_2,PT} | \psi_i \rangle = \frac{2}{\sqrt{N}} \begin{cases} I_{35} & \text{Singlet}, \\ 0 & \text{Triplet}, \end{cases} \tag{42}$$

which is described in fig. 3.

Projectile-electron cloud interaction, $A_{PT,PC}$

A main interaction among the possible double interaction mechanisms, for charge transfer collisions, is between the projectile and the electronic cloud which is shown graphically in fig. 4 and described by $T_{PT,PC}$. Here, the next double interaction involves the projectile to initially interact with the target's nucleus while one electron bounds with it and the second one bounds with the target's nucleus.

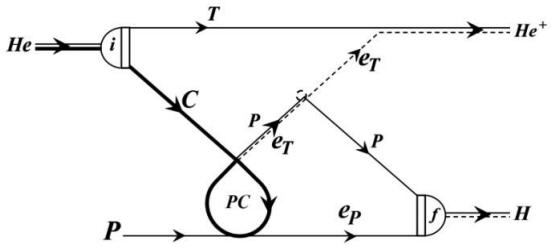

Fig. 5. The graphical description of interaction $T_{Pe_T,PC}$.

Table 1. The ε from $|i\rangle$ to $|f_1\rangle$ and to $|f_2\rangle$ in the first-order amplitudes.

I_{31}	$\varepsilon_{PT} = \mu_n(2\varepsilon_i - k_{1i}^2 - k_{2i}^2)(2\varepsilon_f - k_{2i}^2 - \lvert\vec{k}_{1i} - \vec{V}\rvert^2)/2E_{PT}\lvert\vec{k}_{1i} + \vec{T}\rvert^2$
I_{32}	$\varepsilon_{Pe_2} = (2\varepsilon_i - k_{1i}^2 - k_{2i}^2)(2\varepsilon_f - k_{1P}^2 - k_{2T}^2)/\lvert\vec{k}_{1P} - \vec{J}\rvert^2$
I_{41}	$\varepsilon_{Pe_2} = (2\varepsilon_i - k_{1i}^2 - k_{2i}^2)(2\varepsilon_f - k_{2P}^2 - k_{2T}^2)/\lvert\vec{k}_{2P} - \vec{J}\rvert^2$

This mechanism is present in a three-body treatment of charge transfer, when it is assumed that one electron is inactive. The amplitude for this process to occur, $A_{PT,PC}$, can be written as

$$A_{PT,PC} = \langle\psi_f|T_{PT,PC}|\psi_i\rangle = \frac{1}{\sqrt{N}}\begin{cases}(I_{36} + I_{44}) & \text{Singlet,}\\ 0 & \text{Triplet.}\end{cases} \tag{43}$$

Projectile-electron cloud interaction, $A_{Pe_T,PC}$

The other probable possibility for charge transfer is that the spherical symmetry of the electronic cloud is divided into two parts by the projectile in the first interaction. The next interaction would then be between the projectile and the part of the electronic cloud being attached to the target's nucleus, leaving the other part of the cloud to the projectile which is symbolized in fig. 5. The amplitude for this interaction resulting in a charge transfer $A_{Pe_T,PC}$ is

$$A_{Pe_T,PC} = \langle\psi_f|T_{Pe_T,PC}|\psi_i\rangle = \frac{1}{2\sqrt{N}}\begin{cases}I_{32} + I_{37} + I_{41} + I_{45} & \text{Singlet,}\\ I_{32} + I_{37} - (I_{41} + I_{45}) & \text{Triplet.}\end{cases} \tag{44}$$

6 Results

In order to calculate each amplitude/sub-amplitude, the critical part is the calculation of the two-body transition matrix elements. The matrix elements of the two-body transition operator at energy E in momentum space, as derived by Nutt [55] and Schwinger [54], is given by eq. (I-6) in paper I [53].

There are several exact or approximate forms for the off-shell Coulomb T-matrix. However, we will not discuss these different forms for calculating the two-body transition matrix, here, as they have already been discussed elsewhere [52, 59] in detail. The ε, which is defined by eq. (I-7) in paper I for each integral in the different transition amplitudes, is simplified and presented in tables 1 to 3. Nonetheless, when we calculated the transition matrix making use of the different forms, there was no significant difference between the various results. Therefore, we chose the Haar approximation [60,61], which was the faster one numerically. In addition, the wave function in final channel is considered as

$$\Phi_{He^+}\left(\vec{k}_{2T}\right)\Phi_H\left(\vec{k}_{1P}\right) = \phi_{100}\left(\vec{k}_{2T}; Z_{He}\right)\phi_{100}\left(\vec{k}_{1P}; Z_H\right), \tag{45}$$

where $\phi_{100}(\vec{k}; Z)$ is the ground state hydrogenic wave function in momentum space and with nuclear charge Z.

The wave function for the helium atom in the initial channel is $\varphi_{He}(\vec{r}_1, \vec{r}_2) = (Z_e^3/\pi) \times \exp[-Z_e(r_1 + r_2)]$, where Z_e is to be determined. Hence, in momentum space we have

$$\varphi_{He}\left(\vec{k}_1, \vec{k}_2\right) = \frac{2^3 Z_e^5}{\pi^2(Z_e^2 + k_1^2)^2(Z_e^2 + k_2^2)^2}\,. \tag{46}$$

Table 2. The ε from $|i\rangle$ to $|f_1\rangle$ in the second-order amplitudes.

I_{33}	$\varepsilon_{Te_1} = \frac{(2\varepsilon_f - k_{1P}^2 - k_{2T}^2)}{E_{Te_1}\lvert\vec{k}_{1P} - \vec{k}_{1i} + \vec{V}\rvert^2 G_0(E_{Te_1})}$	$\varepsilon_{PT} = \frac{\mu_n(2\varepsilon_i - k_{1i}^2 - k_{2i}^2)}{E_{PT}\lvert\vec{J} - \vec{k}_{1P}\rvert^2 G_0(E_{PT})}$
I_{34}	$\varepsilon_{Pe_2} = \frac{2\varepsilon_f - k_{1P}^2 - k_{2T}^2}{E_{Pe_2}\lvert\vec{k}_{2i} - \vec{k}_{2T}\rvert^2 G_0(E_{Pe_2})}$	$\varepsilon_{PT} = \frac{\mu_n(2\varepsilon_i - k_{1i}^2 - k_{2i}^2)}{E_{PT}\lvert\vec{k}_{2T} - \vec{T}\rvert^2 G_0(E_{PT})}$
I_{35}	$\varepsilon_{e_1e_2} = \frac{(\varepsilon_f - 1/2(k_{1P}^2 + k_{2T}^2))}{G_0(E_{e_1e_2})E_{e_1e_2}\lvert\vec{k}_{1i} - \vec{k}_{1P} - \vec{V}\rvert^2}$	$\varepsilon_{PT} = \frac{\mu_n(\varepsilon_i - k_{1i}^2 - k_{2i}^2)}{G_0(E_{PT})E_{PT}\lvert\vec{k}_{1P} - \vec{J}\rvert^2}$
I_{36}	$\varepsilon_{Pe_1} = \frac{2\varepsilon_i - k_{1i}^2 - k_{2i}^2}{G_0(E_{Pe_1})E_{Pe_1}\lvert-\vec{k}_{1P} + \vec{k}_{1i} - \vec{V}\rvert^2}$	$\varepsilon_{PT} = \frac{\mu_n(2\varepsilon_f - k_{1P}^2 - k_{2T}^2)}{G_0(E_{PT})E_{PT}\lvert\vec{k}_{1i} + \vec{T}\rvert^2}$
I_{37}	$\varepsilon_{Pe_2} = \frac{(2\varepsilon_f - k_{1P}^2 - k_{2T}^2)}{G_0(E_{Pe_2})E_{Pe_2}\lvert\vec{k}_{1i} + \vec{T}\rvert^2}$	$\varepsilon_{Pe_1} = \frac{(2\varepsilon_i - k_{1i}^2 - k_{2i}^2)}{G_0(E_{Pe_1})E_{Pe_1}\lvert\vec{k}_{1f} - \vec{k}_{1i} + \vec{V}\rvert^2}$

Table 3. The ε from $|i\rangle$ to $|f_2\rangle$ in the second-order amplitudes.

I_{22}	$\varepsilon_{Te_1} = \frac{(2\varepsilon_f - k_{2P}^2 - k_{1T}^2)}{E_{Te_1}\lvert\vec{k}_{1T} - \vec{k}_{1i}\rvert^2 G_0(E_{Te_1})}$	$\varepsilon_{PT} = \frac{\mu_n(2\varepsilon_i - k_{1i}^2 - k_{2i}^2)}{E_{PT}\lvert\vec{k}_{2i} + \vec{T}\rvert^2 G_0(E_{PT})}$
I_{23}	$\varepsilon_{Pe_2} = \frac{(2\varepsilon_f - k_{1T}^2 - k_{2P}^2)}{E_{Pe_2}\lvert\vec{k}_{2i} - \vec{V} - \vec{k}_{2P}\rvert^2 G_0(E_{Pe_2})}$	$\varepsilon_{PT} = \frac{\mu_n(2\varepsilon_i - k_{1i}^2 - k_{2i}^2)}{E_{PT}\lvert\vec{k}_{2i} - \vec{T}\rvert^2 G_0(E_{PT})}$
I_{24}	$\varepsilon_{Pe_1} = \frac{2\varepsilon_i - k_{1i}^2 - k_{2i}^2}{E_{Pe_1}\lvert\vec{k}_{1T} - \vec{k}_{1i}\rvert^2 G_0(E_{Pe_1})}$	$\varepsilon_{PT} = \frac{\mu_n(2\varepsilon_f - k_{1T}^2 - k_{2P}^2)}{E_{PT}\lvert\vec{k}_{1i} - \vec{J}\rvert^2 G_0(E_{PT})}$
I_{25}	$\varepsilon_{Pe_2} = \frac{(2\varepsilon_f - k_{1P}^2 - k_{2T}^2)}{G_0(E_{Pe_2})E_{Pe_2}\lvert\vec{k}_{2i} - \vec{V} - \vec{k}_{2P}\rvert^2}$	$\varepsilon_{Pe_1} = \frac{(2\varepsilon_i - k_{1i}^2 - k_{2i}^2)}{G_0(E_{Pe_1})E_{Pe_1}\lvert\vec{k}_{2i} + \vec{T}\rvert^2}$

Note that Z_e is calculated to take the value of 27/16, in order to reproduce the experimental values of the ground state energy when making use of a variational method. However, here Z_e took the value of 2 in order for the calculated differential cross section to better match the magnitude of the measured values, especially in the forward direction and at the Thomas peak.

6.1 Singlet forms of the partial amplitudes

The singlet forms of the partial amplitudes are shown in fig. 6. As it is apparent from these figures and as was expected, the dominant term belongs to the Born amplitude term, A_e. The phase of the Born term, as is shown in fig. 7, is equal to π. Hence each singlet partial amplitude, whose phase difference with respect to the Born term is in the range of $(-\pi/2, \pi/2)$ or in the range $(-\pi, -\pi/2)$ and $(\pi/2, \pi)$, might have a destructive or a constructive effect, respectively, on the total amplitude. Figure 7 also shows explicitly the phase of the singlet forms of partial amplitudes as well as the Born amplitude. One should note that the amplitude terms $A_{Pe_2,PC}$, $A_{e_1e_2,PT}$ and $A_{e_1e_2,PC}$, which are shown in fig. 6(b), have very small values and so they will have a minute effect on the total electron transfer cross sections. In addition, their phase values would be largely irrelevant. Their presence, however, is important to our understanding of the physics behind the many-body interactions.

Taking these phase results into account, the singlet term $T_{PT,PC}$ is constructive and amplifies the main amplitude, A_e, as the two are almost in phase. Otherwise, the phase of the singlet terms of $T_{PT,PT}$ and $T_{Pe_2,PT}$ are in the range $(-\pi/2, \pi/2)$, hence they add up destructively with A_e. As the phase of the term $T_{Te_1,PT}$ is either $-\pi$ or π and it is comparable in value to A_e, the two add up constructively. The phase of the singlet amplitude $T_{Te_1,PC}$ is in range $(0, \pi/2)$ below the scattering angle 0.47 mrad, where the Thomas peak appears, while it is in the range $(-\pi, -\pi/2)$ for scattering angles which are larger than 0.47 mrad. Hence, it is partially destructive for scattering angles in the range of 0 to 0.47 mrad and it is constructive at other angles, see fig. 7. As these terms are comparable with the main amplitude, A_e, so their being constructive or destructive can significantly affect the final result.

6.2 Triplet forms of the partial amplitudes

As shown in fig. 8, among the five non-zero interaction amplitudes, the dominant triplet amplitudes belong to the interaction terms $T_{Te_1,PC}$ and $T_{Te_1,PT}$, where $T_{Te_1,PC}$, as a Thomas peak term, is investigated in paper I [53] while the term $T_{Pe_2,PC}$ has a minute effect on the total triplet amplitude. One should note that the amplitude is zero for the projectile-nucleus, T_{PT}, $T_{PT,PC}$ and also $T_{e_Pe_T,PC}$ amplitudes, where last one is investigated in paper I [53] as a term which results in Thomas peak. The phase of the non-zero triplet terms are plotted in fig. 9. The Born amplitude, A_e, is

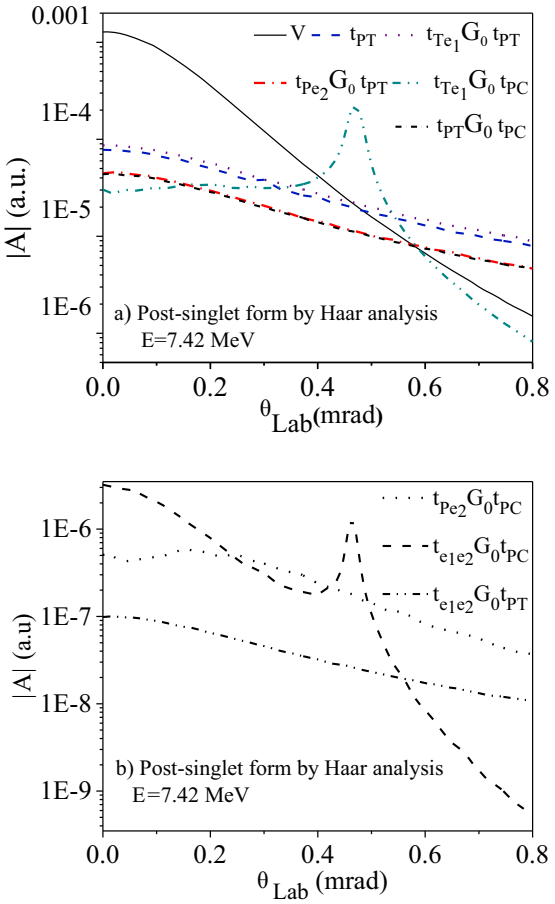

Fig. 6. Singlet amplitudes of the terms (a) V, T_{PT}, $T_{Te_1,PT}$, $T_{Pe_2,PT}$, $T_{Te_1,PC}$ and $T_{PT,PC}$ and (b) the terms $T_{Pe_2,PC}$, $T_{e_1e_2,PT}$ and $T_{e_1e_2,PC}$ for $Z_e = 27/16$. See legend in the figure and text for future detail.

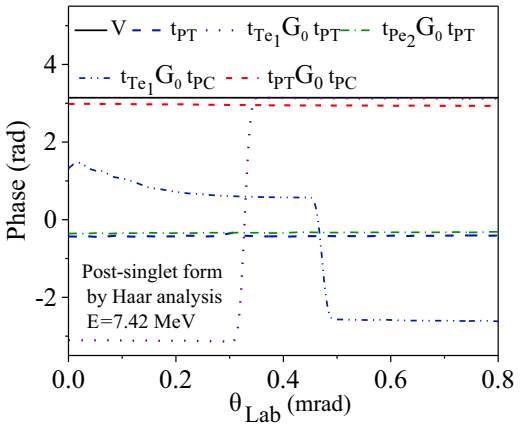

Fig. 7. Singlet amplitude phase of the terms V, T_{PT}, $T_{Te_1,PT}$, $T_{Pe_2,PT}$, $T_{Te_1,PC}$ and $T_{PT,PC}$ for $Z_e = 27/16$. See legend in the figure for further detail and text.

not zero in post-triplet form, opposite of prior one, but this amplitude is not dominate term anymore. The destructive or constructive effect of these terms on each other is quite complex, as their phase varies more than the dominant terms in the singlet case. It should be noted that the amplitudes of the triplet interactions are comparable to or even dominate with the other terms except the singlet interaction A_e. The final total singlet and triplet amplitudes for the charge transfer from the ground state of atomic helium into the ground state of the final atomic hydrogen atom, in the collision of energetic protons with atomic helium, are shown in fig. 10.

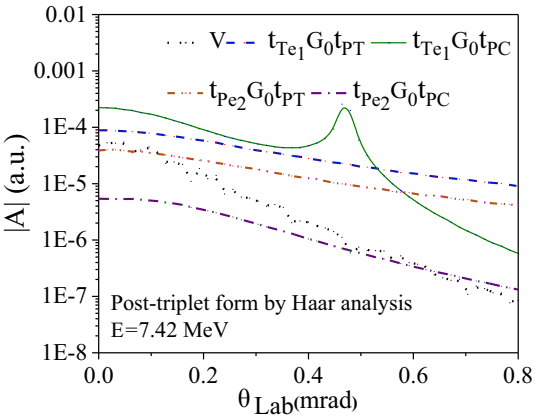

Fig. 8. Triplet amplitude for the terms V, $T_{Te_1,PT}$, $T_{Pe_1,PC}$, $T_{Pe_2,PT}$ and $T_{Pe_2,PC}$ for $Z_e = 27/16$. See legend in figure and text for further details.

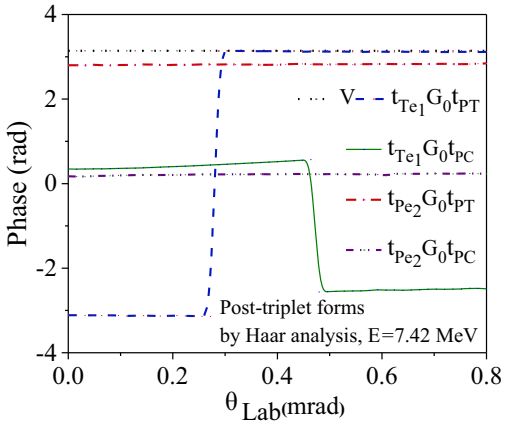

Fig. 9. Phase of the triplet amplitudes for the terms V, $T_{Te_1,PT}$, $T_{Te_1,PC}$, $T_{Pe_2,PT}$ and $T_{Pe_2,PC}$ for $Z_e = 27/16$. See legend in figure and text for further details.

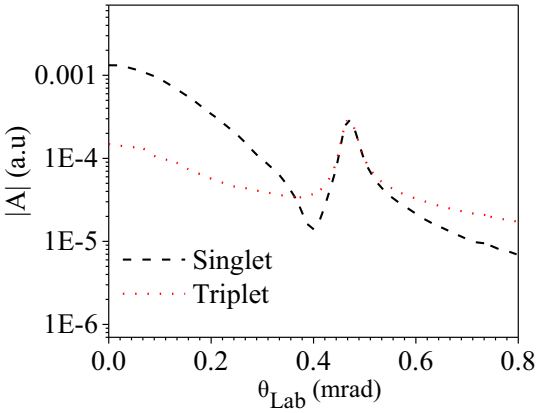

Fig. 10. The final singlet and triplet amplitudes in the charge transfer between protons and helium atoms. The collision energy is 7.42 MeV and $Z_e = 27/16$. See legend in figure and text for further details.

The relationship between the differential cross section and the total FWL amplitude is defined [36] as

$$\left(\frac{\mathrm{d}\sigma_{fi}}{\mathrm{d}\Omega}\right) = \frac{\nu_i \nu_f K_f}{(2\pi)^2 K_i} |(A_{FWL})_{fi}|^2, \tag{47}$$

where ν_i and ν_f are the ratios of the mass of the projectile to the target before the collision, and of the hydrogen atom to the helium ion after collision, respectively.

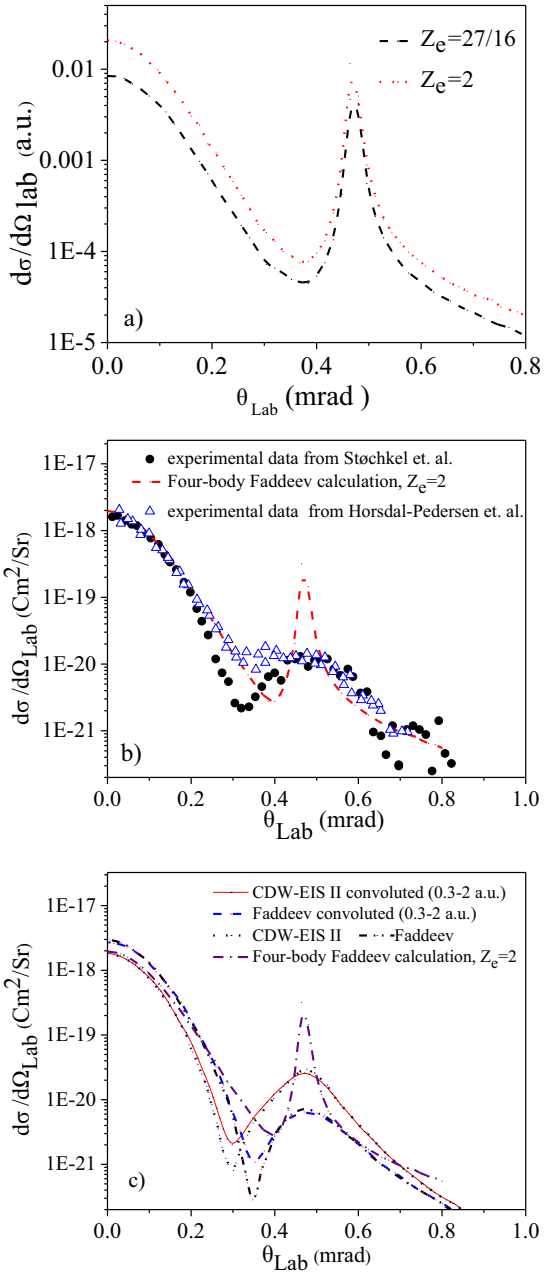

Fig. 11. The differential cross section in proton-helium collisions at 7.42 MeV. (a) The calculated cross section for Z_e equal to 2 and 27/16. (b) Comparison between experiment [17,22] and the present work. (c) Comparison between the different theories. The CDW-EIS II [12], CDW-EIS II convoluted [22] and Faddeev convoluted [25] theories are compared. The collision energy is 7.42 MeV. See legend in figure and text for further details.

In an unpolarized system, the particle's spins are randomly oriented. Hence the probability of producing triplet states is three times that of generating singlet states, so the FWL amplitude is related to the singlet and triplet amplitudes as

$$|(A_{FWL})_{fi}|^2 = \frac{1}{4}\,|(A_{FWL})_{fi}|_s^2 + \frac{3}{4}\,|(A_{FWL})_{fi}|_t^2\,. \tag{48}$$

Therefore, we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{4}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_s + \frac{3}{4}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_t. \tag{49}$$

The differential cross sections for charge transfer, in the collision of protons with helium atoms, are shown in fig. 11. In fig. 11(a), we compare the Z_e dependence of the differential cross sections for proton-helium collisions at 7.42 MeV

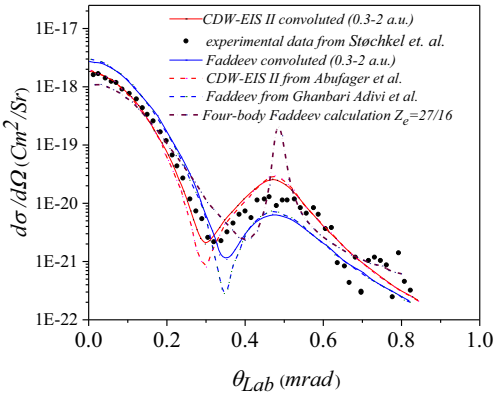

Fig. 12. The differential cross sections in proton-helium collisions at 7.42 MeV for $Z_e = 27/16$, are compared with the experimental works of Støchkel [22] and the other available theoretical results [6,12].

impact energy. The cross sections are higher in magnitude at all calculated scattering angles when the helium nucleus charge is higher. Indeed the only difference between two $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{lab}}$ is related to the charge of the target nucleus. By studying figs. 11(b), (c) and 12, it can be seen that the differential cross section for $Z_e = 2$ has a better compatibility with the experimental and other theoretical data compared with the case for $Z_e = 27/16$. This is especially true in the scattering angular range from 0 to 0.3 mrad. Hence, we can conclude that $Z_e = 2$ is better than $Z_e = 27/16$ in order to optimize the differential cross sections. This seems to suggest that at larger incident proton energies the nucleus charge is not screened significantly by the target electron cloud.

Certainly it is apparent from fig. 11(b) that our result, at the small scattering angles, converges to the experimental data, which is also the case for almost all the other theoretical works. The angular resolution of the earlier measurement [17] is not as good as the most recent one [22], as shown in fig. 11(b), therefore, the older results show a shallower minimum below the Thomas peak. Note, that the later measurement is normalized to the earlier one at forward angles. Finally, we emphasize that the differential cross sections which are reported here are obtained from the post-form of the transition matrix.

7 Concluding remarks

The present work is a four-body treatment of the charge transfer collision between protons and helium atoms, which has been well studied previously. However, the goal of this work is to better understand the scattering mechanisms behind the atomic collisions, specifically for charge transfer.

The fully quantum mechanical Faddeev-Watson-Lovelace formalism was chosen for this task, as it separates the interaction amplitude in such a way that each term represent a specific sub-interaction in the collision. It was concluded that in a multi-body treatment of charge transfer, there are other terms leading to the Thomas "like" mechanisms which show an exactly similar peak at 0.47 mrad. Additionally momentum conditions were determined for every sub-interaction, some of which could be described classically but some others had no classical counterpart.

The other important point in this investigation was to consider the electrons in an atomic species as an electron cloud, and we also established the Pauli exclusion principle in the formalism as the electrons are fermions. This principle was taken into account for both the wave functions and the operators describing the electrons. The crucial point is that we had to use the value $Z_e = 2$ for the charge of the helium nucleus, to arrive at a better result at forward angles, where the two available experimental works agree in value. Note that in order to reproduce the ground state energy of the helium atom, one has to take into account the screening of one electron with respect to the other. This seems not to be true here. This point needs further study and we believe new physics may result from it.

There is a final point that should be emphasized here, namely that a plane wave was assumed for the projectile and the outgoing atom. This makes the model best suited for higher projectile energies. Therefore, a comparison with the experimental results at lower energies [23] was not provided in this work. Even at high energies, a boundary corrected spherical wave is best suited for the outgoing atom is created at the scattering center. Lastly, it should be highlighted that the present method is more suitable for describing double charge transfer, transfer excitation and transfer ionization. We intend to further this work to study these new scattering channels as well as making use of boundary corrected plane waves and/or spherical wave for the incoming and outgoing species, respectively.

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