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Lump solutions and interaction phenomenon to the third-order nonlinear evolution equation

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Abstract. In this work, the lump solution and the kink solitary wave solution from the $(2+1)$-dimensional third-order evolution equation, using the Hirota bilinear method are obtained through symbolic computation with `Maple`. We have assumed that the lump solution is centered at the origin, when $t = 0$. By considering a mixing positive quadratic function with exponential function, as well as a mixing positive quadratic function with hyperbolic cosine function, interaction solutions like lump-exponential and lump-hyperbolic cosine are presented. A completely non-elastic interaction between a lump and kink soliton is observed, showing that a lump solution can be swallowed by a kink soliton.

1 Introduction

As we all know, in nonlinear science fields, solitary wave solutions of nonlinear evolution partial differential equations play an important role in many natural sciences, such as mathematics, biology, chemistry, and particulary in almost all branches of physics like fluid mechanics [1], plasma physics [2,3], optical fibers [4–6], oceanography [7] and so on. Solitary wave solutions can provide some useful information on the relevant nonlinear phenomena and experimental results.

However, for finding solitary wave solutions of nonlinear evolution partial differential equations, many effective methods have been developed, such as the Darboux transformation method [8–10], the inverse scattering transformation [11], the Lie group method [12,13], the variable separation method [14], the Bäcklund transformation [15,16], the Hirota bilinear method [11,15,17], the homogeneous balance method [18,19], the Painlevé analysis method [20–22], the Lucas Ricatti expansion method [23], the F-expansion method [24], the exp-function method [25], the variable method [26], the extended homoclinic test function [27], the multiple exp-function method [28], the three-wave method [29], the Jacobi elliptic function method [30], the Adomian decomposition method [31] and so on.

The study on lump solutions has attracted much attention ever since lump solutions were discovered [32,33]. Particular examples of lump solutions are found for many integrable equations, such as the Kadomtsev-Petviashvili I (KPI) equation [33–35], the three-dimensional three-wave resonant interaction [36], the B-KP equation, which is a subclass of the KP hierarchy of B type [37], the Davey-Stewartson-II equation [35] and the Ishimori-I equation [38].

Recently, interaction solutions have attracted much more attention and have already got good results [32,39–45]. Ma [40,41] obtained explicit interaction solutions through the Wronkskian technique. Cheng and Zhang [46] have solved the $(4+1)$-dimensional nonlinear Fokas equation and have obtained two classes of lump-type solutions based on the Hirota bilinear method. Zhang and Ma [47] have presented a class of lump solutions, namely, the bright lump wave and the bright-dark lump wave solutions of the $(2+1)$-dimensional bilinear Sawada-Kotera (SK) equation using the Hirota bilinear method. Futhermore, for the $(2+1)$-dimensional bilinear p-SK equation, where p is a prime number

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that has been introduced by Ma to generalize bilinear operators [48], Zhang and Ma [47] have shown that, for the case of $p = 3$, all three families of rational solutions exhibit the bright-dark lump wave structure. By using the generalized N-fold Darboux transformations, Zhang, Liu and Wen [49] have investigated the $(2+1)$-dimensional NLS equation, with a rich variety of evolution behaviors such as the bright-line solitons and breathers. Moreover, several patterns for first-, second-, and higher-order rogue wave solutions fixed at space, on the one hand, and first-, second-, and third-order rogue waves fixed at time, on the other hand, have been revealed. Wen and Zhang [50] have implemented the N-fold iteration of Darboux transformation to construct linear rogue wave and parabolic rogue wave solutions for the $(2+1)$-dimensional derivative NLS equation. With the help of the higher-order NLS equation with variable coefficients, Zhang and Chen [51] have derived a family of the first-, second-, third-, and fourth-order rogue wave solutions. The interaction about lump solitons has become the hotspot of the research. Another advantage of lump solutions is that, the lump solutions give two types of interaction like elastic interaction [32,39,42,43] and non-elastic interaction [44,52], when certain conditions are satisfied.

In this paper, we focus our attention on the $(2+1)$-dimensional third-order evolution equation and present lump solutions and some interaction solutions using symbolic computation with the aid of `Maple`. The $(2+1)$-dimensional third-order evolution equation has a Hirota bilinear form, and so, we will do a search for two positive quadratic function solutions and two positive quadratics interaction function solutions containing a set of free parameters. Then, following an appropriate choice of those free parameters, we seek interaction solutions from the $(2+1)$-dimensional third-order equation. The remainder of the paper is organized as follows: In sect. 2, lump solutions of the $(2+1)$-dimensional third-order equation are studied. In sect. 3, a kink solitary wave solutions of $(2+1)$-dimensional third-order equation is discussed. In sect. 4, a non-elastic interaction solutions lump-exponential and lump-hyperbolic cosine of the $(2+1)$-dimensional third-order equation are presented and the process of interaction is shown. Section 5 concludes the paper.

2 Lump solutions of the $(2+1)$-dimensional new third-order equation

The $(2+1)$-dimensional third-order equation,

$$\eta_t + \eta_x + \frac{\alpha}{2}\left(3\eta_x\eta + a\eta_y\right) + \varepsilon\left(\frac{1}{6}(1-3\tau)\eta_{xxx} - \frac{1}{4}(1+2\tau)\eta_{xyy}\right) = 0, \tag{1}$$

was derived, by Fokou *et al.* [53], as a model for the unidirectional propagation of long waves over shallow water, via asymptotic expantion around simple wave motion of the Euler equations up to first order in the small-wave amplitude. In this equation, η is the surface elevation, α represents the measure of the ratio of wave amplitude to undisturbed fluid depth, ε represent the square of the ratio of fluid deph to wave length, x and y are the horizontal coordinates, t is the time variable, τ is the Bond number, a is the velocity components in the horizontal y-direction and subscripts denote partial derivatives with respect to the space x, y and the time t variables. The pulse solitary wave solution of this equation has been found [53] using the Hirota's bilinear method. In this paper, we propose a new test method for solving this equation. The general form of solution of eq. (1) is given by

$$\eta(x,y,t) = R(\ln(f(x,y,t)))_x, \tag{2}$$

where $R = \frac{1}{18}\frac{\varepsilon(6\tau(k_1^3+k_1k_2^2)+3k_1k_2^2-2k_1^3)-6\alpha a k_2-12(k_3-k_1)}{\alpha k_1^2}$ is constant and $f = f(x,y,t)$ is a real function to be determined. Substituting eq. (2) into eq. (1), by the aid of the Hirota bilinear operator D, we obtain the following bilinear form:

$$\begin{aligned}&\left(D_tD_x + \frac{a}{2}aD_yD_x + \frac{\varepsilon}{6}(1-3t)D_x^4 - \frac{1}{4}(1+2t)D_x^2D_y^2\right)\times(f\cdot f) = f_{xt}f - f_xf_t + f_{xx}f - f_x^2 + \frac{a}{2}af_{xy}f - f_xf_y + \varepsilon\\ &\times\left(\frac{1}{6}(1-3t)(f_{xxxx}f - 4f_{xxx}f_x + 3f_{xx}^2) - \frac{1}{4}(1+2t)(f_{xxyy}f - 2f_{xyy}f_x - 2f_{xxy}f_y + 2f_{xy}^2 + f_{xx}f_{yy})\right) = 0.\end{aligned} \tag{3}$$

It is clear that if f solves the bilinear equation (3), then $\eta = \eta(x,y,t)$ is a solution to eq. (1) through the transformation given by eq. (2). With regard to eq. (3), we first choose the test function in the following form:

$$f = g^2 + h^2 + a_9, \tag{4}$$

with

$$g(x,y,t) = a_1x + a_2y + a_3t + a_4, \tag{5}$$

$$h(x,y,t) = a_5x + a_6y + a_7t + a_8, \tag{6}$$

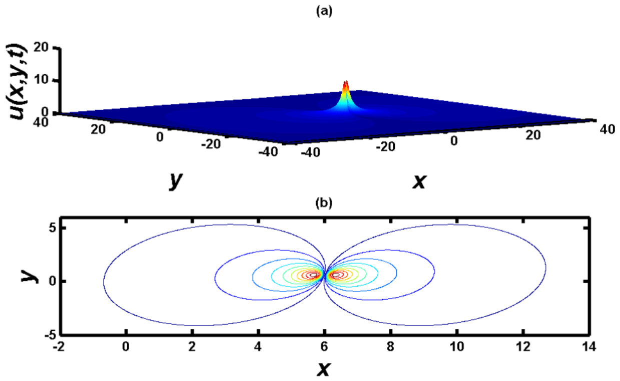

Fig. 1. A lump solution η of eq. (1) with the parameters (9), $\alpha = 0.2$, $\varepsilon = 0.1$, $t = 6$, $\tau = 0.0$.

where a_i $(1 \leq i \leq 9)$ are all real parameters to be determined later. By substituting eq. (4) into eq. (3), we obtain polynomials, which are functions of the variables x, y and t. Equating the coefficients of all power of x, y, and t, we get the algebraic equation which, after solving, gives the following relations between the parameters a_i:

$$a_3 = -a_1 - \frac{1}{2}\alpha a a_2, \tag{7}$$

$$a_7 = -a_5 - \frac{1}{2}\alpha a a_6. \tag{8}$$

Substituting eqs. (7) and (8) into eq. (4), we obtain

$$f = (a_1 x + a_2 y - (a_1 + 1/2\,\alpha\, a a_2)\, t + a_4)^2 + (a_5 x + a_6 y - (1/2\,\alpha\, a a_6 + a_5)\, t + a_8)^2 + a_9. \tag{9}$$

Introducing eq. (9) into eq. (2), we obtain

$$\eta(x, y, t) = 2\,\frac{R(g a_1 + h a_5)}{f}\,, \tag{10}$$

where

$$g = (a_1 x + a_2 y - (a_1 + 1/2\,\alpha\, a a_2)\, t + a_4)\,, \tag{11}$$

$$h = (a_5 x + a_6 y - (1/2\,\alpha\, a a_6 + a_5)\, t + a_8)\,. \tag{12}$$

In a class of lump solutions, parameters a_1, a_2, a_4, a_5, a_6, a_8 and a_9 are involved in the solution η. In this work, we choose the parameters $a_4 = a_8 = 0$, which implies the lump is centered at origin, when $t = 0$. By choosing appropriate values of the remainder parameters, we set

$$a_1 = 1, \qquad a_2 = 2, \qquad a_5 = 3, \qquad a_6 = -1, \qquad a_9 = 1. \tag{13}$$

Their plots, when $t = 6$, are depicted in fig. 1. Figure 1(a) shows that the amplitude of the lump is 10. Figure 1(b) shows contour plot of the lump solutions at $t = 6$.

3 Kink solitary wave solution of the $(2 + 1)$-dimensional third-order equation

Here, we seek the solitary wave solutions of eq. (1). We choose the test function f in the following form:

$$f = 1 + \exp(k_1 x + k_2 y + k_3 t). \tag{14}$$

Substituting eq. (14) into eq. (3) with the help of `Maple`, and equating the coefficient of $\exp(k_1 x + k_2 y + k_3 t)$, we obtain, after solving, the following dispersion relation:

$$k_3 = \frac{1}{2}\varepsilon k_1 \left(-\frac{1}{3}k_1^2 + \tau k_1^2 - 1 + \tau k_2^2 + \frac{1}{2}k_2^2\right) - \frac{1}{2}\alpha a k_2. \tag{15}$$

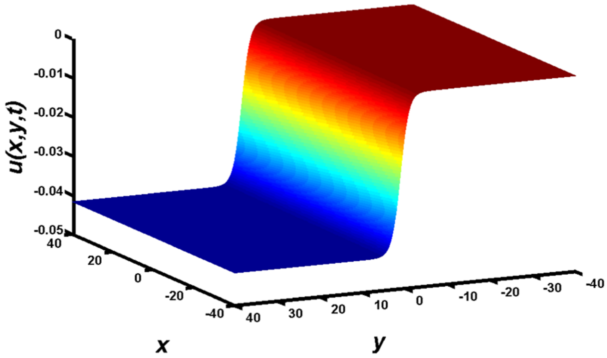

Fig. 2. A solitary wave solution η of eq. (1) with $k_1 = 1/8$, $k_2 = 1$, $\alpha = 0.2$, $\varepsilon = 0.1$, $t = 6$, $\tau = 0.0$.

By introducing eq. (15) and eq. (14) into eq. (2), we obtain the following solution of eq. (1):

$$\eta(x, y, t) = \frac{Rk_1 \mathrm{e}^{k_1 x + k_2 y + k_3 t}}{1 + \mathrm{e}^{k_1 x + k_2 y + k_3 t}}\,. \tag{16}$$

This solution shows that the asymptotic behavior of η can be obtained. When $t \to -\infty$ and $k_3 < 0$, the solution $\eta \to k_1 R$, and when $t \to +\infty$ and $k_3 < 0$, the solution $\eta \to 0$. This behavior shows that the solution (16) is the kink solitary wave solution (see fig. 2).

4 Interaction between lump solution and kink solitary wave

In this section, we study the interaction between a lump solution and the kink solitary wave solution of a $(2+1)$-dimensional third-order equation. We choose two different cases of stripe soliton.

4.1 First case

In the first case, we choose the $f(x, y, t)$ function as a positive quadratic function with exponential function, that is

$$f = g^2 + h^2 + k \exp(l) + a_9, \tag{17}$$

where g and h are defined by eqs. (7) and (8), and $l(x, y, t) = k_1 x + k_2 y + k_3 t$, k_i are the constant parameters to be determined later. Putting eq. (17) into eq. (3), with the help of `Maple`, we obtain the following set of constraining equations for the parameters:

$$a_1 = -\frac{a_5 a_6}{a_2}\,, \tag{18}$$

$$a_3 = -a_1 - \frac{1}{2}\alpha a a_2, \tag{19}$$

$$a_7 = -a_5 - \frac{1}{2}\alpha a a_6, \tag{20}$$

$$k_3 = \frac{1}{2}\varepsilon k_1 \left(-\frac{1}{3}k_1^2 + \tau k_1^2 - 1 + \tau k_2^2 + \frac{1}{2}k_2^2\right) - \frac{1}{2}\alpha a k_2. \tag{21}$$

Substituting eqs. (18)–(21) into eq. (2), we obtain the exact interaction solution of η, which is

$$\eta(x, y, t) = \frac{R(2\,(g)a_1 + 2\,(h)a_5 + k_1 \mathrm{e}^{(l)})}{1 + g^2 + h^2 + \mathrm{e}^{(l)} + a_9}\,, \tag{22}$$

where the functions g, h and l are now known. To illustrate the interaction phenomena between lump solution and kink soliton, we select the following parameters:

$$a_2 = 2, \qquad a_5 = 3, \qquad a_6 = -1, \qquad a_4 = a_8 = 0, \qquad a_9 = 1, \qquad k_1 = \frac{1}{2}, \quad k_2 = 1. \tag{23}$$

Figures 3 and 4 show that the interaction between lump solution and kink soliton is completely non-elastic. This is confirmed by the deformation of the waves after their interaction during its propagation in space time. We also observe, in these figures, that when the value of the parameter k increases, the amplitude of the lump wave decreases and, when k decreases, the amplitude of the lump wave increases.

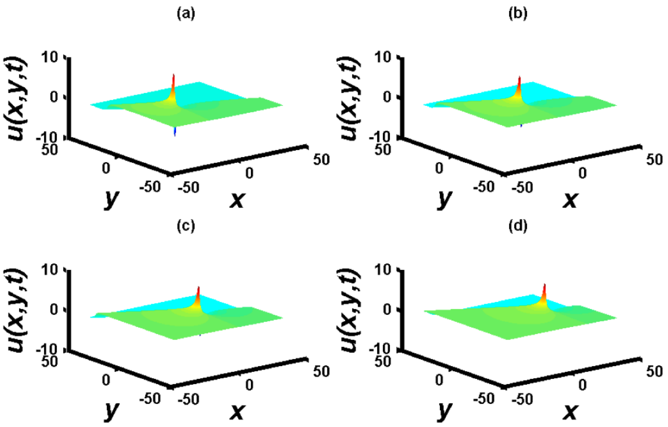

Fig. 3. Profiles of interaction between a lump and kink solution η of eq. (1) with the parameters of eq. (23) and other parameters: $\alpha = 0.2$, $\varepsilon = 0.1$, $\tau = 0.0$, $k = 2.0$, $a = 1$ at (a) $t = -10$; (b) $t = 0$; (c) $t = 10$; (d) $t = 20$.

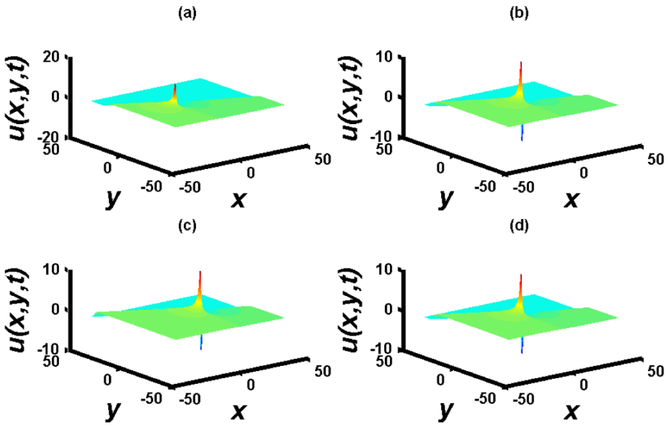

Fig. 4. Profiles of interaction between lump and kink solution η of eq. (1) with the parameters of eq. (23) and other parameters: $\alpha = 0.2$, $\varepsilon = 0.1$, $\tau = 0.0$, $k = 1/4$, $a = 1$ at (a) $t = -10$; (b) $t = 0$; (c) $t = 10$; (d) $t = 20$.

4.2 Second case

Here, we choose the function $f(x, y, t)$ as a positive quadratic function add with hyperbolic cosine function; therefore the form is the following:

$$f = g^2 + h^2 + k\cosh(l) + a_9, \tag{24}$$

where g, h and l have been defined in the previous section.

Again, substituting eq. (24) into eq. (3), with the help of `Maple`, we obtain the following relation between the parameters:

$$a_1 = -\frac{a_5 a_6}{a_2}, \tag{25}$$

$$a_3 = -a_1 - \frac{1}{2}\alpha a a_2, \tag{26}$$

$$a_7 = -a_5 - \frac{1}{2}\alpha a a_6, \tag{27}$$

$$k_3 = \frac{1}{2}\varepsilon k_1\left(-\frac{1}{3}k_1^2 + \tau k_1^2 - 1 + \tau k_2^2 + \frac{1}{2}k_2^2\right) - \frac{1}{2}\alpha a k_2. \tag{28}$$

Substitution in eqs. (25)–(28) of g, h and l gives

$$g(x, y, t) = a_1 x + a_2 y + (-a_1 - 1/2\,\alpha\, a a_2)\, t + a_4, \tag{29}$$

$$h(x, y, t) = a_5 x + a_6 y + (-a_5 - 1/2\,\alpha\, a a_6)\, t + a_8, \tag{30}$$

$$l(x, y, t) = k_1 x + k_2 y + \left(\frac{1}{3}k_1\varepsilon(-2k_1^2 + 6\tau(k_1^2 + k_2^2) + 3(k_2^2 - 1)) - \frac{1}{2}\alpha a k_2\right) t. \tag{31}$$

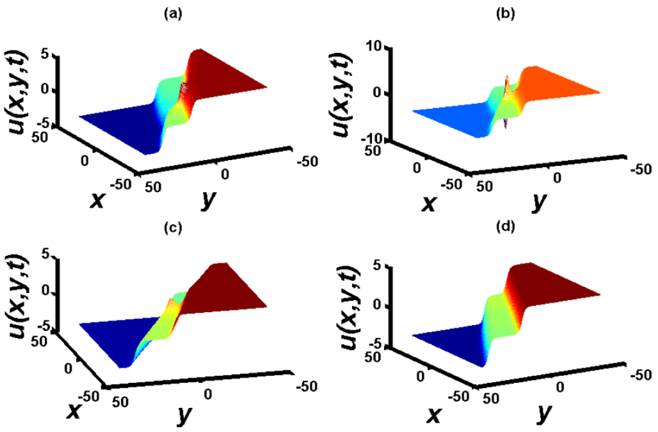

Fig. 5. Profiles of interaction between lump and hyperbolic solution η of eq. (1) with the parameters of eq. (23) and other parameters: $\alpha = 0.2$, $\varepsilon = 0.1$, $\tau = 0.0$, $k = 2.0$, $a = 1$ at (a) $t = -10$; (b) $t = 0$; (c) $t = 10$; (d) $t = 40$.

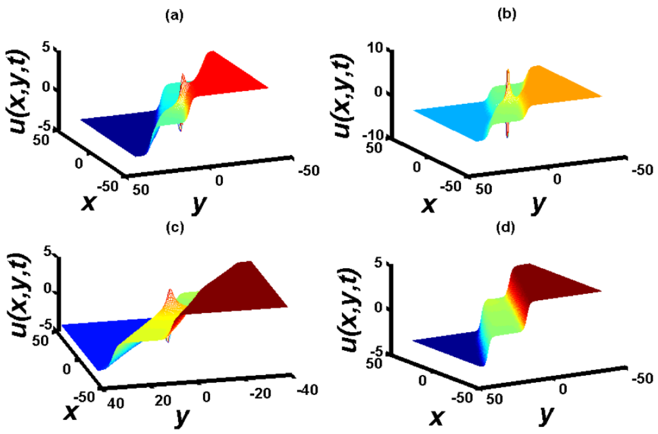

Fig. 6. Profiles of interaction between lump and hyperbolic solution η of eq. (1) with the parameters of eq. (23) and other parameters: $\alpha = 0.2$, $\varepsilon = 0.1$, $\tau = 0.0$, $k = 1/4$, $a = 1$ at (a) $t = -10$; (b) $t = 0$; (c) $t = 10$; (d) $t = 40$.

Putting eqs. (29)–(31) into eq. (24), we obtain the expression of $f(x, y, t)$, which is

$$f(x,y,t) = \left(a_1 x + a_2 y - \frac{1}{2}(2a_1 + \alpha a a_2)t + a_4\right)^2 + \left(a_5 x + a_6 y - \frac{1}{2}(2a_5 + \alpha a a_6)t + a_8\right)^2 + \cosh\left(-k_1 x - k_2 y - \frac{1}{3}k_1 \varepsilon \left(-2k_1^2 + 6\tau(k_1^2 + k_2^2) + 3(k_2^2 - 1) - \frac{1}{2}\alpha a k_2\right) t\right). \tag{32}$$

Substituting eq. (32) into eq. (2), we can achieve a new exact interaction solution of the third-order equation

$$\eta(x,y,t) = \frac{R(2\,g a_1 + 2\,h a_5 + \sinh(l)k_1)}{f}, \tag{33}$$

where the expressions of g, h and f are given above. Figures 5 and 6 show the plot of solution (33) with the parameters of eq. (23). These figures present the dynamic graphs of interaction between the lump solution and one stripe soliton. In this interaction, a deformation of the wave appears. So, it is a non-elastic collision. As depicted in figs. 5 and 6, for various values $k = 1/4$ and $k = 2$, the amplitude of wave changes. We observe an increase in the amplitude of the lump wave when k decreases (see fig. 5). The opposite phenomenon is observed in fig. 6. At $t = 0$, the lump tangles with the kink soliton (see figs. 5(b) and 6(b)), then the kink begins to swallow a lump step by step, as shown in fig. 5(c), (d) and figs. 6(c) and (d), which shows that the energy of the lump is transfered into the kink soliton gradually.

5 Conclusion

In this paper, the $(2+1)$-dimensional third-order nonlinear evolution equation has been studied by using the Hirota bilinear method. The lump solutions, the kink solitary wave solutions, and the mixed lump-exponential solitary wave solutions, mixed lump-hyperbolic cosine solutions of this equation were obtained. The spatio-temporal deformation of kink solitary wave and a lump solution have been studied. The non-elastic interactions between a lump and kink soliton are obtained. However, in figs. 5 and 6, the energy of lump is transfered into the kink soliton gradually during their propagation. The method can also be extended to other types of nonlinear evolution equations of the nonlinear dynamics.

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