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Axisymmetric Stokes flow past a composite spheroidal shell of immiscible fluids

M. Krishna Prasad[a] and G. Manpreet Kaur[b]

Department of Mathematics, National Institute of Technology, Raipur-492010, Chhattisgarh, India

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Abstract. We study the flow of an incompressible Newtonian fluid past a composite spheroidal shell whose shape deviates slightly from that of a sphere. A composite particle referred to in this paper is a spheroidal liquid core covered with a porous layer. The Brinkman equation is used for the flow inside the porous medium and the Stokes equation is used for the flow in the fluid region. We assume that the external and internal viscous fluids are immiscible and the viscosity of the porous medium is different than the viscosity of pure liquid. The Ochoa-Tapia and Whitaker's stress jump boundary condition for tangential stress is applied on the porous-fluid interface. Velocity and pressure distributions are found and the drag force acting on the spheroidal shell is evaluated. The analytical solution is obtained by dividing the flow into three regions. Both type of spheroids, oblate and prolate are considered. Numerical results of the normalized hydrodynamic drag force acting on the spheroidal shell are tabulated and represented graphically for different values of the parameters characterizing the stress jump coefficient, separation parameter, permeability, deformation parameter, and viscosity ratios. The analysis of the flow pattern is done by plotting streamlines and several renowned cases are deduced.

1 Introduction

The motion of solid particles or fluid droplets in an immiscible fluid at low Reynolds numbers continues to receive much attention from researchers in the fields of chemical, biomedical, and environmental engineering and science. The area of fluid flow relative to fluid droplets is important in many practical applications such as raindrop formation, mechanics and rheology of emulsions, liquid-liquid extraction, motion of blood cells in an artery or vein, extraction of crude oil from petroleum products, and sedimentation phenomena. The creeping flow motion of a single spherical drop of radius a in an unbounded medium of viscosity μ was first analyzed independently by Hadamard [1] and Rybczynski [2]. Assuming continuous velocity and continuous tangential shear stress across the interface between the fluid phases in the absence of surface active agents, they found that the force exerted on the fluid sphere by the surrounding fluid is

$$\mathcal{F} = -6\,\pi\,\mu\,a\frac{3\mu^*+2}{3\mu^*+3}U, \tag{1}$$

where U is the migration velocity of the drop and μ^* is the internal to external viscosity ratio. The above formula reduces to the drag force exerted on a no-slip solid sphere (Stokes' law [3]) when the viscosity of the drop is infinite and to the case of a perfect-slip gas bubble when the viscosity approaches zero. This problem is also treated by Happel and Brenner in their book [4].

Bart [5] investigated the motion of a spherical droplet settling normal to a plane interface between two immiscible viscous fluids using spherical bipolar coordinates. He obtained the exact solution by assuming that the sphere moves perpendicularly to the plane at low Reynolds number conditions. Hetsroni and Haber [6] examined the problem of a single spherical droplet submerged in an unbounded viscous fluid of a different viscosity. They assumed the droplet to be spherical as a first approximation and later determined the equation of the interface using an iterative method. Wacholder and Weihs [7] also utilized bipolar coordinates to study the motion of a fluid sphere through another fluid

[a] e-mail: `madaspra.maths@nitrr.ac.in`
[b] e-mail: `manpreet.kaur22276@yahoo.com`

normal to a rigid or free plane surface and their calculations agree with the results obtained by Bart. They obtained exact solutions for the forces on a fluid particle in the presence of another fluid particle when both are falling along their line of centres, or of a plane surface normal to the settling velocity vector. Lee and Keh [8] used a combined analytical-numerical method with the boundary collocation technique to examine the quasisteady creeping flow of a spherical drop in an immiscible fluid within a spherical cavity. They obtained the wall-corrected drag force exerted on the drop with good convergence. Choudhuri and Sri Padmavati [9] studied an arbitrary unsteady Stokes flow in and around a liquid sphere. They have discussed the flow inside and outside the liquid sphere generated due to an oscillating singularity inside the sphere and found that the deformation is mainly affected by two parameters: the ratio of the viscosities of the two fluids and the capillary number.

All results cited above concern viscous fluids. For micropolar fluids, Niefer and Kaloni [10] discussed the problems of the flow of a viscous fluid past a micropolar fluid sphere and the flow of a micropolar fluid past a viscous fluid drop using non-zero spin boundary condition. They obtained the expression for drag in each case and found that the viscosity ratios and the spin parameter have significant effect upon the drag in each case. Ramkissoon [11] investigated the Stokes flow due to the translation of a spherical fluid particle in an unbounded non-Newtonian fluid medium. He determined the flow field within the Newtonian fluid sphere and of the micropolar fluid outside the sphere and evaluated drag force exerted on the sphere. Ramkissoon and Majumdar [12] examined the problem of symmetrical micropolar fluid flow past a Newtonian fluid spheroid whose shape varies slightly from that of a sphere. These two problems are solved by using no-spin microrotation boundary condition. Saad [13] discussed the problem of flow of micropolar fluid past a viscous fluid sphere and the flow of a viscous fluid past a micropolar fluid sphere using cell models. It is found that the normalized hydrodynamic drag is a decreasing function of the spin parameter. Recently, Krishna Prasad and Kaur [14] analytically studied the Stokes axisymmetric flow of an incompressible micropolar fluid past a viscous fluid spheroid whose shape deviates slightly from that of a sphere. They observed that the drag force is an increasing function of classical viscosity ratio of internal fluid to that of the surrounding fluid. The drag force decreases with increasing spin parameter and concluded that the spin parameter has a significant influence on the drag.

Fluid flow past porous particles of arbitrary shapes has been the subject of numerous studies over the years as they are of great importance in geophysical, industrial, and engineering applications. Some of the applications are the flow through packed beds, extraction of energy from the geothermal regions, filtration of solids from liquids, sedimentation problem, flow of oil through underground porous rocks, etc. In general, while formulating the problem of a viscous flow past a porous particle, one has to consider the corresponding governing equations in both the porous and clear fluid regions. For the steady viscous flows, the governing equations in the clear fluid region are the Stokes equations and Darcy law or Brinkman equation for the flow within the porous region. Joseph and Tao [15] solved the coupled problem of the streaming of a viscous liquid past a permeable sphere. They used Darcy law for the flow inside the porous region and Stokes equations for the non-porous region. Assuming continuity of the normal velocity and pressure at the surface of the porous sphere, and no-slip condition for tangential component of velocity of the free fluid, they found that the drag on a permeable sphere is same as the drag on an impermeable sphere of reduced radius. Sutherland and Tan [16] calculated the flow field, described by Darcy law, near and within a permeable sphere. They assumed continuity of normal velocity, pressure, and tangential velocity at the sphere surface.

Darcy law, however, is proved to be inadequate for high porosity flows and for porous medium of complicated structure. To model such flows, Brinkman [17] and Debye and Bueche [18] independently introduced modified Darcy equation, which is known as Brinkman equation. The applicability of the Brinkman equation has been stated in Ooms *et al.* [19] and Neale *et al.* [20]. Using the Brinkman equation for the flow inside the permeable sphere, Qin and Kaloni [21] obtained a Cartesian-tensor solution for the flow of an incompressible viscous fluid past a porous sphere and evaluated the drag force exerted on the porous sphere. There were several features in their analytical solutions, which cannot be seen from Darcy equation. Bhatt and Sacheti [22] discussed the problem of slow viscous flow past a porous spherical shell using Brinkman equation. Srinivasacharya [23] considered the creeping flow past a porous approximate sphere. Zlatanovski [24] investigated the problem of creeping axisymmetric flow past a porous prolate spheroidal particle. Keh and Chou [25] studied the motion of composite particle composed of a solid core and a surrounding porous shell in concentric spherical cavity containing viscous fluid. Keh and Lu [26] examined the problem of translation and rotation motions of a porous spherical shell in concentric spherical cavity containing viscous fluid. Saad [27] analytically solved the flow problems of an incompressible axisymmetrical quasisteady translation and steady rotation of a porous spheroid in a concentric spheroidal container. All these authors have used continuity of the velocity, pressure, and tangential stresses at the porous-liquid interface.

Ochoa-Tapia and Whitaker [28,29] investigated boundary conditions on the porous-liquid interface by applying volume average technique and developed the momentum transfer condition, which is known as stress jump boundary condition. They showed that the equations require a discontinuity in shear stress, but continuity in velocity components and normal stress. In the last few years, many authors have investigated various flow problems using this stress jump boundary condition and reported significant changes in the results. Kuznetsov [30] used this stress jump boundary condition at the porous-liquid interface to study the flow in parallel plates and cylindrical channels partially filled with a porous medium. Bhattacharyya and Raja Sekhar [31] have examined the Stokes flow of a viscous fluid in a sphere with internal singularities, enclosed by a porous spherical shell. Yadav *et al.* [32] presented the general solution of the problem of the flow of an incompressible viscous fluid past a porous spherical particle enclosing a solid core. Prakash *et al.* [33] investigated the overall bed permeability of an assemblage of porous particles using stress jump condition. They obtained the expression for the drag force and used it to estimate the overall bed permeability. Srinivasacharya and Krishna Prasad [34–36] studied the flow of viscous fluid past a porous approximate sphere, porous approximate sphere with an impermeable core, and porous approximate spherical shell, respectively. Saad [37] studied the axisymmetric flow of an incompressible viscous fluid past an assemblage of porous spherical shell using four known boundary conditions-Happel, Kuwabara, Kvashnin and Cunningham (Mehta-Morse condition) on the cell surface. Ashmawy [38] investigated the problem of the rotary oscillation of a composite sphere, consisting of a solid core surrounded by a porous shell in an incompressible viscous fluid bounded by a concentric spherical cavity. Ashmawy [39] also studied the steady rotational motion of an axially symmetric porous particle about its axis of symmetry in a viscous fluid by using a combined analytical-numerical technique. All these authors have used continuity of the velocity, pressure, and stress jump boundary condition for tangential stresses at the porous-liquid interface. They have shown that the stress jump condition has a significant impact on the hydrodynamic drag force and torque force. Hence, calculations without considering the stress jump condition can result in a significant loss of accuracy.

At present, researchers are focusing much attention on microcapsules [40]. Microcapsules represent porous shells that may contain either solid particles or liquids. The properties of porous media differ from the properties of a dispersion medium. They are important for modern nanotechnologies and are characterized by the diverse values of their parameters. Hence, the theoretical prediction of the hydrodynamic behavior of microcapsules presents not only mathematical interest, but can also be useful for the formation and application of encapsulated media. Artificial capsules are widely used in the pharmaceutical, cosmetics, and food industries for controlling the release of active substances, aromas, and flavors. Capsule technology finds important applications in the engineering of artificial organs, and in cell therapy where living cells are encapsulated for the treatment of disease such as diabetes and liver failure. In all of these cases, the motion of the drop-shell system relative to the external flow occurs either at the material production stage or in the practical use of capsules. The motion of capsules in the flow of liquid is of great applied and theoretical interest. These studies are expected to have significant implications in engineering and biological applications [41,42]. Vasin and Kharitonova [40] investigated the problem of a liquid flow that is uniform at infinity around a spherical porous capsule. Vasin and Kharitonova [41] solved the problem of the infinite uniform flow of liquid around the spherical drop coated with the porous layer. Jaiswal and Gupta [43] examined the problem of flow of an incompressible Newtonian fluid around the sphere filled with Reiner-Rivlin liquid and coated with the porous layer.

The objective of the present work is to extend the analysis in Vasin and Kharitonova [41] to the composite spheroidal shell and to determine the correction to eq. (1) for the motion of the composite spheroidal shell. Here, we examine the Stokes flow of an incompressible viscous fluid around a spheroid coated with porous layer. It is assumed that the external and internal viscous fluids are immiscible. We have used the Brinkman equation for the flow within the porous region and Stokes model for the flow inside and outside the spheroidal shell. The flow examined is axially symmetric in Nature. The expressions for the stream function and the pressure for flows inside and outside the spheroidal shell and in the porous region are obtained. The drag force acting on the porous spheroidal shell is also evaluated. The results of many existing situations are also shown as the special and limiting cases of the present study.

2 Formulation of the problem

Consider the axisymmetric Stokes flow of an incompressible Newtonian viscous fluid of viscosity μ_3 past a porous spheroidal shell of radius r_a having viscosity μ_2, with a liquid core of radius r_b ($r_b < r_a$) containing an incompressible Newtonian viscous immiscible fluid of viscosity μ_1 which is held fixed in a uniform stream of velocity U (See fig. 1). The following assumptions are considered to be valid:

i) the flows are steady and axisymmetric;

ii) the flow outside and inside the porous spheroidal shell are governed by Stokes equation and the flow within annular region, *i.e.*, the porous region, is governed by the Brinkman model;

iii) the physical properties (density ρ and viscosities μ_1, μ_2 and μ_3) are constants;

iv) the liquid outside the spheroidal shell (region III) penetrates into the porous layer (region II) and is not mixed with liquid inside the cavity (region I);

v) the spheroidal shell contains the liquid which cannot flow out of the porous layer, but is subjected to the action of viscous forces from the side of the flowing liquid;

vi) the shape of the particle deforms slightly from that of a sphere;

vii) there is no interfacial mass transfer (the radial velocity is zero) at the interface $r = r_b$, and

viii) there are no surface-active materials.

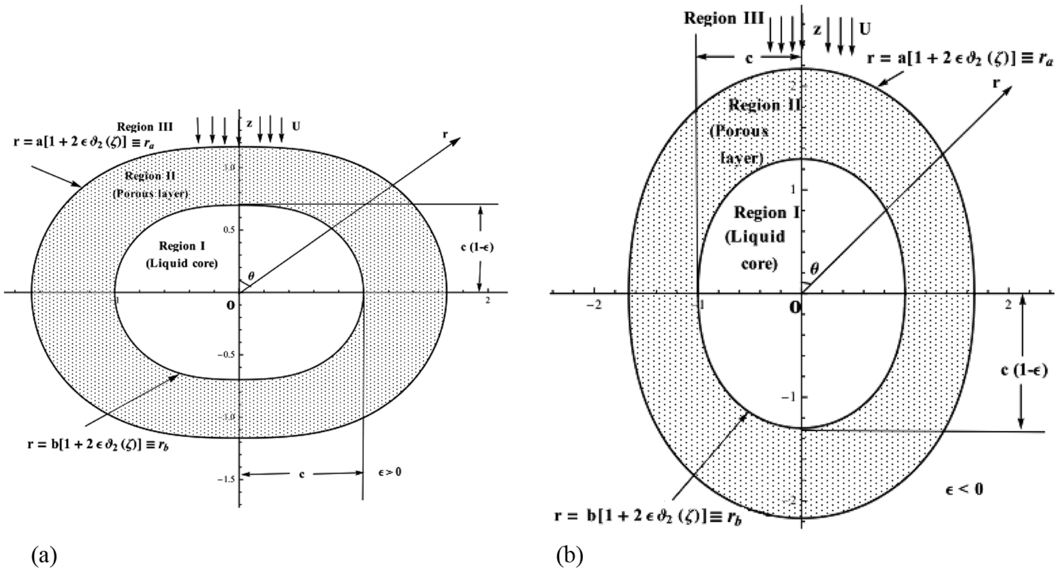


Fig. 1. The physical situation and the coordinate system. (a) Oblate spheroidal shell ($\epsilon > 0$). (b) Prolate spheroidal shell ($\epsilon < 0$).

The equations of motion for regions I and III are

$$\nabla \cdot \boldsymbol{q}^{(i)} = 0, \tag{2a}$$

$$\nabla p^{(i)} + \mu_i \nabla \times \nabla \times \boldsymbol{q}^{(i)} = 0, \quad i = 1, 3 \tag{2b}$$

and the equations of motion for region II are

$$\nabla \cdot \boldsymbol{q}^{(2)} = 0, \tag{3a}$$

$$\nabla p^{(2)} + \frac{\mu_3}{k} q^{(2)} + \mu_2 \nabla \times \nabla \times \boldsymbol{q}^{(2)} = 0, \tag{3b}$$

where $\boldsymbol{q}^{(i)}$ is the volumetric average of the velocity, $p^{(i)}$ is the average of the pressure, μ_i is the dynamic viscosity coefficient in regions I, II, and III, respectively, and k is the permeability of the porous medium.

Let (r, θ, ϕ) denote a spherical polar coordinate system with unit base vectors $(\boldsymbol{e}_r, \boldsymbol{e}_\theta, \boldsymbol{e}_\phi)$. Since the flow of the fluid is in the meridian plane and the flow is axially symmetric, all the physical quantities are independent of ϕ. Hence, we assume the velocity vectors as

$$\boldsymbol{q}^{(i)} = q_r^{(i)}(r, \theta)\, \boldsymbol{e}_r + q_\theta^{(i)}(r, \theta)\, \boldsymbol{e}_\theta, \quad i = 1, 2, 3. \tag{4}$$

Introducing the following non-dimensional variables:

$$r = a\,\tilde{r}, \qquad \nabla = \frac{\tilde{\nabla}}{a}, \qquad \boldsymbol{q}^{(i)} = U\,\tilde{\boldsymbol{q}}^{(i)}, \qquad p^{(i)} = \frac{\mu_3\, U}{a} \tilde{p}^{(i)}, \tag{5}$$

in (2) and (3) and dropping tildes, we get

$$\nabla \cdot \boldsymbol{q}^{(1)} = 0, \tag{6a}$$

$$\nabla p^{(1)} + \gamma_1^2 \nabla \times \nabla \times \boldsymbol{q}^{(1)} = 0, \tag{6b}$$

$$\nabla \cdot \boldsymbol{q}^{(2)} = 0, \tag{7a}$$

$$\nabla p^{(2)} + \alpha^2 \gamma_2^2\, q^{(2)} + \gamma_2^2 \nabla \times \nabla \times \boldsymbol{q}^{(2)} = 0, \tag{7b}$$

$$\nabla \cdot \boldsymbol{q}^{(3)} = 0, \tag{8a}$$

$$\nabla p^{(3)} + \nabla \times \nabla \times \boldsymbol{q}^{(3)} = 0, \tag{8b}$$

where $\alpha^2 = \frac{a^2}{k} \frac{\mu_3}{\mu_2}$, $\gamma_1^2 = \frac{\mu_1}{\mu_3}$ and $\gamma_2^2 = \frac{\mu_2}{\mu_3}$.

Let the surface of the spheroid be $r = a[1 + f(\theta)]$ which deviates slightly from the sphere $r = a$. The orthogonality relations of the Gegenbauer functions $\vartheta_m(\zeta)$, $\zeta = \cos\theta$, permit us, under general circumstances, to assume the expansion $f(\theta) = \sum_{m=2}^{\infty} \alpha_m \vartheta_m(\zeta)$, where the Gegenbauer function is related to the Legendre function $P_n(\zeta)$ by the relation

$$\vartheta_n(\zeta) = \frac{P_{n-2}(\zeta) - P_n(\zeta)}{2n-1}, \quad n \geq 2. \tag{9}$$

Therefore, we assume the surface of the external spheroid r_a and internal spheroid r_b as

$$r = a\left[1 + \sum_{m=2}^{\infty} \alpha_m \vartheta_m(\zeta)\right], \tag{10}$$

$$r = b\left[1 + \sum_{m=2}^{\infty} \alpha_m \vartheta_m(\zeta)\right], \tag{11}$$

and assume that the coefficients α_m are sufficiently small so that their squares and higher powers may be neglected [4]. Then, we have

$$(r/a)^y \approx 1 + y\,\alpha_m \vartheta_m(\zeta), \tag{12}$$

where y is positive or negative.

In view of the incompressibility condition $\nabla \cdot \boldsymbol{q}^{(i)} = 0$, $i = 1, 2, 3$, we introduce the stream functions $\psi^{(i)}(r, \theta)$, $i = 1, 2, 3$ for the flow regions I, II, and III, respectively, then the velocity components in terms of stream functions are

$$q_r^{(i)} = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi^{(i)}}{\partial \theta}, \quad q_\theta^{(i)} = \frac{1}{r\sin\theta}\frac{\partial \psi^{(i)}}{\partial r} \quad i = 1, 2, 3. \tag{13}$$

Eliminating pressure from (6)–(8), and substituting (13) in the resulting equations, we get the following dimensionless equations for $\psi^{(i)}(r, \theta)$, $i = 1, 2, 3$:

$$E^4 \psi^{(1)} = 0, \tag{14}$$

$$E^2(E^2 - \alpha^2)\psi^{(2)} = 0, \tag{15}$$

$$E^4 \psi^{(3)} = 0, \tag{16}$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}.$$

3 Boundary conditions

To determine the velocity and pressure of the flow fields outside and within the porous spheroidal shell, we assume the continuity of the normal and the tangential velocity components, continuity of the normal stress, and stress jump condition at the external fluid-porous interface $r = r_a$. Since we assume that the fluids outside and inside the spheroidal shell are immiscible. Thus, the equilibrium theory of interfacial tension acting at the junction between the two immiscible fluids is applicable to our problem. This produces a discontinuity in the normal stresses at internal porous-liquid interface $r = r_b$ and does not in any way affect the tangential stress. Hence, to determine the flow velocity inside the cavity region, we assume the kinematical condition of the mutual impenetrability, continuity of the tangential velocity components, and stress jump condition at the porous-liquid interface $r = r_b$. These conditions are physically realistic and mathematically consistent [4, 11–13, 28, 29, 41].

The boundary conditions at the liquid-porous interface of the spheroid $r = a[1 + \alpha_m \vartheta_m(\zeta)]$ are

$$\left(\boldsymbol{q}^{(3)} - \boldsymbol{q}^{(2)}\right) \cdot \boldsymbol{n} = 0, \tag{17}$$

$$\left(\boldsymbol{q}^{(3)} - \boldsymbol{q}^{(2)}\right) \cdot \boldsymbol{s} = 0, \tag{18}$$

$$\left(\boldsymbol{n} \cdot \mathsf{t}^{(3)}\right) \cdot \boldsymbol{n} = \left(\boldsymbol{n} \cdot \mathsf{t}^{(2)}\right) \cdot \boldsymbol{n}, \tag{19}$$

$$\boldsymbol{n} \cdot \left(\mathsf{t}^{(2)} - \mathsf{t}^{(3)}\right) \cdot \boldsymbol{s} = \sigma \frac{\mu_3}{\sqrt{k}} \boldsymbol{q}^{(2)} \cdot \boldsymbol{s}. \tag{20}$$

The boundary conditions at the porous-liquid interface of the spheroid $r = b[1 + \alpha_m\,\vartheta_m(\zeta)]$ are

$$\boldsymbol{q}^{(1)} \cdot \boldsymbol{n} = 0, \tag{21}$$

$$\boldsymbol{q}^{(2)} \cdot \boldsymbol{n} = 0, \tag{22}$$

$$\left(\boldsymbol{q}^{(1)} - \boldsymbol{q}^{(2)}\right) \cdot \boldsymbol{s} = 0, \tag{23}$$

$$\boldsymbol{n} \cdot \left(\mathsf{t}^{(2)} - \mathsf{t}^{(1)}\right) \cdot \boldsymbol{s} = \sigma \frac{\mu_3}{\sqrt{k}} \boldsymbol{q}^{(2)} \cdot \boldsymbol{s}, \tag{24}$$

where

$$\boldsymbol{n} = \boldsymbol{e}_r - \alpha_m\, a\, \nabla\, \vartheta_m(\zeta) = \boldsymbol{e}_r - \alpha_m \sin\theta\, P_{m-1}(\zeta) \boldsymbol{e}_\theta,$$

and $\boldsymbol{s}$ are the unit normal and arbitrary tangential vectors at the surface of the spheroid.

Substituting the expression for the unit normal and arbitrary tangential vectors into (17)–(24) give the approximate boundary conditions (up to $O(\alpha_m)$):

$$q_r^{(3)} - q_r^{(2)} = \left(q_\theta^{(3)} - q_\theta^{(2)}\right) \alpha_m \sin\theta\, P_{m-1}(\zeta), \tag{25}$$

$$q_\theta^{(3)} = q_\theta^{(2)}, \tag{26}$$

$$t_{rr}^{(3)} - t_{rr}^{(2)} = 2\,\alpha_m \left(t_{r\theta}^{(3)} - t_{r\theta}^{(2)}\right) \sin\theta\, P_{m-1}(\zeta), \tag{27}$$

$$t_{r\theta}^{(2)} + \alpha_m \left(t_{rr}^{(2)} - t_{\theta\theta}^{(2)}\right) \sin\theta\, P_{m-1}(\zeta) - t_{r\theta}^{(3)} - \alpha_m \left(t_{rr}^{(3)} - t_{\theta\theta}^{(3)}\right) \sin\theta\, P_{m-1}(\zeta) = \sigma \frac{\mu_3}{\sqrt{k}} \left(\alpha_m \sin\theta\, P_{m-1}(\zeta)\, q_r^{(2)} + q_\theta^{(2)}\right), \tag{28}$$

$$q_r^{(1)} = q_\theta^{(1)}\, \alpha_m \sin\theta\, P_{m-1}(\zeta), \tag{29}$$

$$q_r^{(2)} = q_\theta^{(2)}\, \alpha_m \sin\theta\, P_{m-1}(\zeta), \tag{30}$$

$$q_\theta^{(1)} = q_\theta^{(2)}, \tag{31}$$

$$t_{r\theta}^{(2)} + \alpha_m \left(t_{rr}^{(2)} - t_{\theta\theta}^{(2)}\right) \sin\theta\, P_{m-1}(\zeta) - t_{r\theta}^{(1)} - \alpha_m \left(t_{rr}^{(1)} - t_{\theta\theta}^{(1)}\right) \sin\theta\, P_{m-1}(\zeta) = \sigma \frac{\mu_3}{\sqrt{k}} q_\theta^{(2)}. \tag{32}$$

The boundary conditions in terms of stream functions $\psi^{(i)}$, $i = 1, 2, 3$ in dimensionless form are as follows: on the surface $r = 1 + \alpha_m\,\vartheta_m(\zeta)$

$$\frac{\partial\,\psi^{(3)}}{\partial\zeta} - \frac{\partial\,\psi^{(2)}}{\partial\zeta} = \alpha_m\, r \left(\frac{\partial\,\psi^{(3)}}{\partial r} - \frac{\partial\,\psi^{(2)}}{\partial r}\right) P_{m-1}(\zeta), \tag{33}$$

$$\frac{\partial\,\psi^{(3)}}{\partial r} = \frac{\partial\,\psi^{(2)}}{\partial r}\,, \tag{34}$$

$$\begin{aligned} &-(p^{(3)} - p^{(2)}) - \frac{2}{r^2}\left[\frac{2}{r}\left(\frac{\partial\,\psi^{(3)}}{\partial\zeta} - \gamma_2^2 \frac{\partial\,\psi^{(2)}}{\partial\zeta}\right) - \left(\frac{\partial^2\psi^{(3)}}{\partial r\,\partial\zeta} - \gamma_2^2 \frac{\partial^2\psi^{(2)}}{\partial r\,\partial\zeta}\right)\right] - \frac{2\,\alpha_m}{r}\left\{2\,r \frac{\partial}{\partial r}\left[\frac{1}{r}\left(\frac{\partial\psi^{(3)}}{\partial r} - \gamma_2^2 \frac{\partial\psi^{(2)}}{\partial r}\right)\right]\right. \\ &\left. +\, E^2\left(\gamma_2^2\,\psi^{(2)} - \psi^{(3)}\right)\right\} P_{m-1}(\zeta) = 0, \end{aligned} \tag{35}$$

$$\begin{aligned} &2\,r \frac{\partial}{\partial r}\left[\frac{1}{r}\left(\gamma_2^2 \frac{\partial\psi^{(2)}}{\partial r} - \frac{\partial\psi^{(3)}}{\partial r}\right)\right] + E^2\left(\psi^{(3)} - \gamma_2^2\,\psi^{(2)}\right) \\ &+ 2\,\alpha_m\left[\frac{4}{r}\left(\gamma_2^2 \frac{\partial^2\psi^{(2)}}{\partial r\,\partial\zeta} - \frac{\partial^2\psi^{(3)}}{\partial r\,\partial\zeta}\right) - \frac{6}{r^2}\left(\gamma_2^2 \frac{\partial\psi^{(2)}}{\partial\zeta} - \frac{\partial\psi^{(3)}}{\partial\zeta}\right) + \frac{P_1(\zeta)}{r\,\vartheta_2(\zeta)}\left(\gamma_2^2 \frac{\partial\psi^{(2)}}{\partial r} - \frac{\partial\psi^{(3)}}{\partial r}\right)\right] \vartheta_2(\zeta)\, P_{m-1}(\zeta) = \\ &\alpha\,\sigma\,\gamma_2\left(\frac{\partial\psi^{(2)}}{\partial r} + 2\,\frac{\alpha_m}{r}\frac{\partial\psi^{(2)}}{\partial\zeta}\vartheta_2(\zeta)\, P_{m-1}(\zeta)\right). \end{aligned} \tag{36}$$

On the surface $r=\eta[1+\alpha_m\,\vartheta_m(\zeta)]$, where $\eta=b/a$

$$\frac{\partial\,\psi^{(1)}}{\partial\zeta}=\alpha_m\,r\,\frac{\partial\,\psi^{(1)}}{\partial r}P_{m-1}(\zeta), \tag{37}$$

$$\frac{\partial\,\psi^{(2)}}{\partial\zeta}=\alpha_m\,r\,\frac{\partial\,\psi^{(2)}}{\partial r}P_{m-1}(\zeta), \tag{38}$$

$$\frac{\partial\,\psi^{(1)}}{\partial r}=\frac{\partial\,\psi^{(2)}}{\partial r}\,, \tag{39}$$

$$\begin{aligned}&2\,r\frac{\partial}{\partial r}\left[\frac{1}{r}\left(\gamma_2^2\frac{\partial\psi^{(2)}}{\partial r}-\gamma_1^2\frac{\partial\psi^{(1)}}{\partial r}\right)\right]+E^2\left(\gamma_1^2\,\psi^{(1)}-\gamma_2^2\psi^{(2)}\right)\\&+2\,\alpha_m\left[\frac{4}{r}\left(\gamma_2^2\frac{\partial^2\psi^{(2)}}{\partial r\,\partial\zeta}-\gamma_1^2\frac{\partial^2\psi^{(1)}}{\partial r\,\partial\zeta}\right)-\frac{6}{r^2}\left(\gamma_2^2\frac{\partial\psi^{(2)}}{\partial\zeta}-\gamma_1^2\frac{\partial\psi^{(1)}}{\partial\zeta}\right)+\frac{P_1(\zeta)}{r\,\vartheta_2(\zeta)}\left(\gamma_2^2\frac{\partial\psi^{(2)}}{\partial r}-\gamma_1^2\frac{\partial\psi^{(1)}}{\partial r}\right)\right]\vartheta_2(\zeta)\,P_{m-1}(\zeta)=\\&\alpha\,\sigma\,\gamma_2\,\frac{\partial\psi^{(2)}}{\partial r}\,.\end{aligned} \tag{40}$$

4 Solution of the problem

For region I, the solution of (14) is

$$\psi^{(1)}=\left[a_2\,r^2+b_2\,r^4\right]\vartheta_2(\zeta)+\sum_{n=3}^{\infty}\left[A_n\,r^n+B_n\,r^{n+2}\right]\vartheta_n(\zeta), \tag{41}$$

for region II, the solution of (15) is

$$\begin{aligned}\psi^{(2)}=&\left[c_2\,r^2+d_2\,r^{-1}+e_2\,\sqrt{r}\,K_{3/2}(\alpha\,r)+f_2\,\sqrt{r}\,I_{3/2}(\alpha\,r)\right]\vartheta_2(\zeta)\\&+\sum_{n=3}^{\infty}\left[C_n\,r^n+D_n\,r^{-n+1}+E_n\,\sqrt{r}\,K_{n-1/2}(\alpha\,r)+F_n\,\sqrt{r}\,I_{n-1/2}(\alpha\,r)\right]\vartheta_n(\zeta),\end{aligned} \tag{42}$$

and for region III, the solution of (16) is

$$\psi^{(3)}=\left[r^2+g_2\,r^{-1}+h_2\,r\right]\vartheta_2(\zeta)+\sum_{n=3}^{\infty}\left[G_n\,r^{-n+1}+H_n\,r^{-n+3}\right]\vartheta_n(\zeta), \tag{43}$$

where $I_{n-1/2}(\alpha\,r)$ and $K_{n-1/2}(\alpha\,r)$ are the modified Bessel functions of the first kind and second kind of order $n-1/2$, respectively.

The expression for pressures in regions I, II, and III are given as

$$p^{(1)}=-\gamma_1^2\left[10\,b_2\,r\,P_1(\zeta)+\sum_{n=3}^{\infty}\frac{4\,n+2}{n-1}B_n\,r^{n-1}\,P_{n-1}(\zeta)\right], \tag{44}$$

$$p^{(2)}=\alpha^2\,\gamma_2^2\left\{\left[c_2\,r-\frac{d_2}{2}r^{-2}\right]P_1(\zeta)+\sum_{n=3}^{\infty}\left[\frac{1}{n-1}C_n\,r^{n-1}-\frac{1}{n}D_n\,r^{-n}\right]P_{n-1}(\zeta)\right\}, \tag{45}$$

$$p^{(3)}=-h_2\,r^{-2}\,P_1(\zeta)+\sum_{n=3}^{\infty}\frac{6-4\,n}{n}H_n\,r^{-n}\,P_{n-1}(\zeta). \tag{46}$$

We first develop the solution corresponding to the boundaries $r=1+\alpha_m\,\vartheta_m(\zeta)$ and $r=\eta[1+\alpha_m\,\vartheta_m(\zeta)]$. The comparison of eqs. (41)–(43) with those obtained in case of flow of liquid around the encapsulated drop of another liquid [41], indicates that the terms involving A_n, B_n, C_n, D_n, E_n, F_n, G_n, and H_n for $n>2$ are the extra terms here which are not present in the case of a sphere. The body that we are considering is spheroid which deviates slightly from that of a sphere and the flow generated is not expected to be very different from the one generated by the flow past a spherical shell. For the solution $r=1+\sum_{m=2}^{\infty}\alpha_m\,\vartheta_m(\zeta)$ and $r=\eta[1+\sum_{m=2}^{\infty}\alpha_m\,\vartheta_m(\zeta)]$, we employ the same technique for each m and obtain the expression for stream functions in all the regions. Thus, velocity components are determined.

5 Application to a porous spheroidal shell

As a particular example of the above analysis, we now consider the particular case of flow past a prolate or an oblate spheroidal shell. The surface of the spheroid is represented in the Cartesian frame (x, y, z) by the equation

$$\frac{x^2+y^2}{c^2}+\frac{z^2}{c^2\,(1-\epsilon)^2}=1, \tag{47}$$

where c is equatorial radius and ϵ is so small that squares and higher powers of it are neglected. The polar equation of the spheroidal surface (47) is

$$r=1+2\,\epsilon\,\vartheta_2(\zeta), \tag{48}$$

and $a=c(1-\epsilon)$. For the case $0<\epsilon\le 1$, the spheroid is an oblate, and for the case $\epsilon<0$, it is a prolate. When $\epsilon=0$, eq. (47) describes a sphere of radius c.

To apply the above results, we must take $m=2$, $\alpha_m=2\,\epsilon$. Therefore, the stream functions are given by

$$\psi^{(1)}=\left[(a_2+A_2)\,r^2+(b_2+B_2)\,r^4\right]\vartheta_2(\zeta)+\left[A_4\,r^4+B_4\,r^6\right]\vartheta_4(\zeta), \tag{49}$$

$$\begin{aligned}\psi^{(2)}=&\left[(c_2+C_2)\,r^2+(d_2+D_2)\,r^{-1}+(e_2+E_2)\sqrt{r}\,K_{3/2}(\alpha\,r)+(f_2+F_2)\sqrt{r}\,I_{3/2}(\alpha\,r)\right]\vartheta_2(\zeta)\\&+\left[C_4\,r^4+D_4\,r^{-3}+E_4\sqrt{r}\,K_{7/2}(\alpha\,r)+F_4\sqrt{r}\,I_{7/2}(\alpha\,r)\right]\vartheta_4(\zeta),\end{aligned} \tag{50}$$

$$\psi^{(3)}=\left[r^2+(g_2+G_2)\,r^{-1}+(h_2+H_2)\,r\right]\vartheta_2(\zeta)+\left[G_4\,r^{-3}+H_4\,r^{-1}\right]\vartheta_4(\zeta). \tag{51}$$

6 Drag on the body

The drag force acting on the porous spheroidal shell by the external viscous fluid is given by

$$\mathcal{F}=\int(\boldsymbol{n}\cdot \mathrm{t}^{(3)})\cdot\boldsymbol{k}\,\mathrm{d}S, \tag{52}$$

where $\boldsymbol{n}=\boldsymbol{e}_r-\epsilon\sin 2\theta\,\boldsymbol{e}_\theta$, $\mathrm{d}S=2\,\pi\,a^2(1+2\,\epsilon\sin^2\theta)\sin\theta\,\mathrm{d}\theta$, $\mathrm{t}^{(3)}$ is the stress tensor of the region III, and $\boldsymbol{k}$ is the unit vector in the z-direction and the integral is taken over the surface of the body $r=1+2\,\epsilon\,\vartheta_2(\zeta)$. We have

$$\mathcal{F}=4\,\pi\,a\,U\,\mu_3(h_2+H_2), \tag{53}$$

where h_2 and H_2 are constants whose expressions are given in appendix B.

6.1 Special cases

1) If $\gamma_1\to\infty$ and $\gamma_2=1$, we get drag force acting on the porous spheroid with an impermeable core in an unbounded medium, which is given as

$$\begin{aligned}\mathcal{F}=&-12\,\pi\,a\,\mu_3\,U\,\alpha\frac{(3\,\eta^2\,\lambda_1+\alpha\,(2+\eta^3)\,\lambda_2)}{2\,\Delta}\\&\times\left[1-\epsilon\frac{3\,\alpha\,\eta^2\,\lambda_1+\left(1+\alpha^2(2+\eta^3)\right)\lambda_2+2\,\alpha^2\,z_5+(\alpha+\sigma)\lambda_3-\frac{4}{5}\alpha\,\Delta_1}{\Delta}\right],\end{aligned} \tag{54}$$

where

$$\lambda_1=-z_1\,\alpha\,\sigma+z_6\,(\alpha+\sigma)\,,\qquad \lambda_2=-z_5\,\alpha\,\sigma+z_4\,(\alpha+\sigma)\,,$$

$$\lambda_3=3\,\eta^2\,z_1-3\,\sqrt{\eta}(z_2+\eta\,z_3)+2\,z_4+\alpha\,\eta^3\,z_5,$$

$$\begin{aligned}\Delta=&\,3\,\eta^2\,\left(\alpha+\sigma\,(1-\alpha^2)\right)\,z_1+\left(-3\,\sqrt{\eta}\,(z_2+\eta\,z_3)+(3+\alpha^2\,(2+\eta^3))\,z_4+3\,\alpha\,\eta^2\,z_6\right)\\&\times(\alpha+\sigma)+\left(\alpha^2\,(2+\eta^3)\,(1-\alpha\,\sigma)+\alpha\,\sigma\,(-1+\eta^3)\right)z_5.\end{aligned}$$

$$z_1=T_1\,S_6+T_6\,S_1,\qquad z_2=T_1\,S_2+T_2\,S_1,\qquad z_3=T_6\,S_5+T_5\,S_6,$$

$$z_4=T_2\,S_5+T_5\,S_2,\qquad z_5=T_1\,S_5-T_5\,S_1,\qquad z_6=T_2\,S_6-T_6\,S_2,$$

$$z_7=T_6^2\,S_1^2+T_1^2\,S_6^2,\qquad z_8=T_1\,T_6\,S_5\,S_2+T_5\,T_2\,S_1\,S_6,$$

$$z_9=T_2\,T_6\,S_1\,S_5+T_1\,T_5\,S_2\,S_6,\qquad z_{10}=T_5\,T_6\,S_1\,S_2+T_1\,T_2\,S_5\,S_6,$$

$$
\begin{aligned}
\Delta_1 = \Big[&\alpha^2 \Big(-\left(-5+4\,\eta^3+\eta^6\right)\sigma^2+\alpha^2\left(2+\eta^3\right)^2\left(1+\sigma^2\right)\Big) z_5^2 - 3\alpha\sqrt{\eta}\left(\alpha^2\left(2+\eta^3\right)\right.\\
&\left.-3\alpha\sigma-\left(5+\eta^3\right)\sigma^2\right) z_5(z_2+\eta\, z_3)+\alpha\eta\Big(\alpha^3\left(2+\eta^3\right)^2+18\eta\sigma+2\alpha^2\left(4+6\eta+4\eta^3+3\eta^4+\eta^6\right)\sigma\\
&+\alpha\left(4\sigma^2+4\eta^3\sigma^2+6\eta^4\sigma^2+\eta^6\sigma^2+3\eta\left(6+\sigma^2\right)\right)\Big) z_8+3\,\alpha\,\eta^{3/2}\left(\alpha^2\left(2+\eta^3\right)+\alpha\left(4+3\eta+2\eta^3\right)\sigma\right.\\
&\left.+\left(2+3\eta+\eta^3\right)\sigma^2\right) z_6\left(z_2+\eta\, z_3\right)+6\alpha\eta^2\left(2+\eta^3\right)\left(-\sigma^2+\alpha^2\left(1+\sigma^2\right)\right) z_1\, z_5\\
&-3\eta^{3/2}\left(\alpha^3\left(2+\eta^3\right)\sigma-3\eta\sigma^2+\alpha^2\left(3\eta+2\sigma^2+\eta^3\sigma^2\right)\right) z_1\left(z_2+\eta\, z_3\right)\\
&+9\eta^3\left(-\eta\sigma^2+\alpha^2\left(\eta+\sigma^2+\eta\sigma^2\right)\right) z_1^2+18\,\alpha\,\sigma\,\eta^3(1+\eta)(\alpha+\sigma)T_6\, S_2\, z_1+9(\alpha+\sigma)^2\left(z_4^2+\eta^3\, z_6^2\right)\\
&+3\sqrt{\eta}\left(-6\,\alpha\,\sigma+\alpha^3\left(2+\eta^3\right)\sigma-3\sigma^2+\alpha^2\left(-3+\left(2+\eta^3\right)\sigma^2\right)\right) z_4(z_2+\eta\, z_3)\\
&+\alpha\eta\Big(\alpha^3\left(2+\eta^3\right)^2+18\,\eta\,\sigma+2\,\sigma\,\alpha^2(-1+\eta)^2\left(4+2\eta+2\eta^3+\eta^4\right)\\
&+\alpha\left(4\sigma^2+4\eta^3\sigma^2-6\eta^4\sigma^2+\eta^6\sigma^2-3\eta\left(-6+7\sigma^2\right)\right)\Big) z_9-2\,\alpha\Big(9+\alpha^2\left(2+\eta^3\right)^2\Big)\sigma(\alpha+\sigma)z_4\, z_5\\
&-12\alpha^2\eta^2\left(2+\eta^3\right)\sigma(\alpha+\sigma)z_{10}-18\,\alpha\,\sigma\,\eta^3(1+\eta)(\alpha+\sigma)T_2\, S_6\, z_1\Big)\Big] \,/\, \left(3\,\eta^2\,\lambda_1+\alpha\left(2+\eta^3\right)\lambda_2\right).
\end{aligned}
$$

When $\epsilon = 0$, the result agrees with the drag acting on the porous sphere with an impermeable core case derived by Srinivasacharya and Krishna Prasad [35].

2) If $\gamma_1 \to 0$, $\gamma_2 = 1$ and $\eta \to 0$, we get drag force acting on the porous spheroid in an unbounded medium, which is given as

$$
\mathcal{F} = 4\,\pi\,a\,\mu_3\,U\left[\frac{3\,\alpha^2\left(T_1\,\alpha\,\sigma - T_2(\alpha+\sigma)\right)}{\Delta_2}+\frac{3\,\epsilon\,\alpha^2\,\Delta_3}{5\,\Delta_2^2}\right], \tag{55}
$$

where

$$
\begin{aligned}
\Delta_3 &= \left(T_2^2\left(-3+10\,\alpha^2\right)(\alpha+\sigma)^2+2\,T_1\,T_2\,\alpha\left(13\,\alpha\,\sigma-2\alpha^3\sigma+8\,\sigma^2+\alpha^2\left(5-2\sigma^2\right)\right)\right.\\
&\quad\left.+T_1^2\,\alpha^2\left(-10\alpha\,\sigma-5\,\sigma^2+2\,\alpha^2\left(-4+\sigma^2\right)\right)\right),\\
\Delta_2 &= T_2\left(3+2\,\alpha^2\right)(\alpha+\sigma)-T_1\,\alpha\left(-2\alpha+\sigma+2\,\alpha^2\,\sigma\right).
\end{aligned}
$$

When $\epsilon = 0$, the result agrees with the drag exerted on the porous sphere case derived by Srinivasacharya and Krishna Prasad [34].

3) If $\sigma \to 0$ we get the drag acting on the porous spheroid with continuity of tangential stress in unbounded medium

$$
\begin{aligned}
\mathcal{F} = 4\,\pi\,a\,\mu_3\,U\Bigg[&\frac{3\,\alpha^2(-\alpha\cosh\alpha+\sinh\alpha)}{\alpha(3+2\,\alpha^2)\cosh\alpha-3\sinh\alpha}\\
&+\frac{3\,\alpha^2\,\epsilon(\alpha^2(-3+10\,\alpha^2)\cosh^2\alpha-(3+8\,\alpha^4)\sinh^2\alpha+\alpha(3-5\alpha^2)\sinh 2\,\alpha)}{5\left(\alpha(3+2\,\alpha^2)\cosh\alpha-3\sinh\alpha\right)^2}\Bigg],
\end{aligned} \tag{56}
$$

which agrees with result obtained by Saad [37]. When $\epsilon = 0$, we will get the drag acting on the porous sphere with continuity of tangential stress which agrees with the result obtained by Brinkman [17] and Neale *et al.* [20].

4) If $\gamma_2 = 1$, $\eta \to 1$ and $\alpha \to 0$, we get drag force acting on the fluid spheroid in an unbounded medium, which is given as

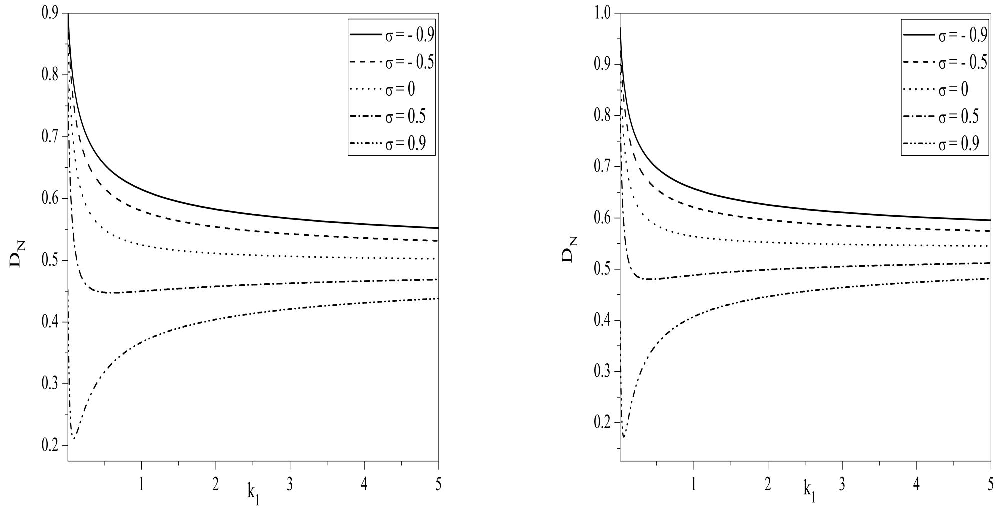
$$
\mathcal{F} = -2\,\pi\,a\,\mu_3\,U\left[\frac{2+3\,\gamma_1^2}{1+\gamma_1^2}-\frac{\epsilon}{5}\frac{(3\,\gamma_1^4+9\,\gamma_1^2-2)}{(1+\gamma_1^2)^2}\right]. \tag{57}
$$

which agrees with the drag force acting on the fluid spheroid when both the fluids are Newtonian as in the case derived by Krishna Prasad and Kaur [14]. When $\epsilon = 0$, we get the drag force exerted on the fluid sphere by the surrounding fluid which agrees with the result obtained by Hadamard [1] and Rybczynski [2]. If $\gamma_1 \to \infty$, the drag force acting on the solid spheroid is obtained, which agrees with the previous result by Happel and Brenner [4].

5) If $\gamma_2 \to \infty$, we get drag force acting on the solid spheroid in an unbounded medium, which is given as

$$
\mathcal{F}_\infty = -6\,\pi\,a\,\mu_3\,U\left(1-\frac{\epsilon}{5}\right), \tag{58}
$$

which is the well-known Stokes result for flow past a solid spheroid in an unbounded medium [4].



(a) (b)

Fig. 2. Variations of the drag coefficient *versus* permeability for different values of the stress jump coefficient with $\gamma_1 = 1.2$, $\gamma_2 = 1$, and $\eta = 0.6$. (a) for $\epsilon = 0.1$; (b) for $\epsilon = -0.1$.

7 Results and discussion

It is convenient to normalize the drag force $\mathcal{F}$ exerted on the porous spheroidal shell with respect to the drag force $\mathcal{F}_\infty$ acting on the solid spheroid in an infinite expanse of the fluid. With the aid of eqs. (53) and (58) this becomes

$$D_N = \frac{\mathcal{F}}{\mathcal{F}_\infty}\,.$$

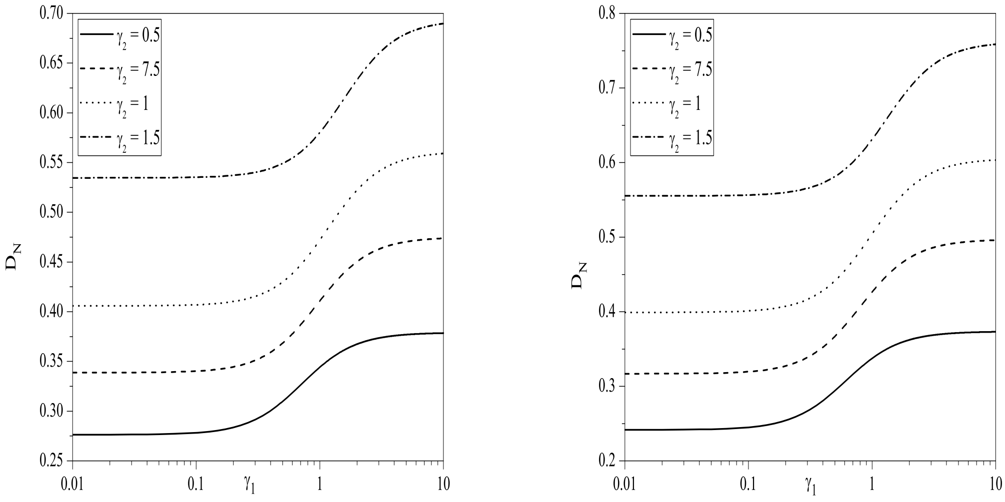
The considered parameters of the problem are as follows:

i) The ratio of viscosity of the internal fluid to that of the external fluid, $\gamma_1^2 (= \frac{\mu_1}{\mu_3})$.
ii) The ratio of an effective viscosity to that of viscosity of the external fluid, $\gamma_2^2 (= \frac{\mu_2}{\mu_3})$.
iii) The separation parameter, $\eta (= \frac{b}{a})$.
iv) The deformation parameter, ϵ.
v) The permeability parameter, $k_1 (= \frac{1}{\gamma_2^2\alpha^2} = \frac{k}{a^2})$.
vi) The stress jump coefficient, σ.

The drag coefficient D_N is numerically computed for different values of the considered parameters and is presented in figs. 2–4 and table 1. In fig. 2, the plots of variation of the drag coefficient D_N *versus* permeability k_1 have been shown for an oblate and a prolate spheroid. The values of σ are taken in the range -1 to 1 as proposed by Ochoa-Tapia and Whitaker [28,29]. As expected, the drag force exerted on the porous spheroidal shell decreases monotonically with an increase in σ. This is because when σ is negative, the shear stress of external free flow region becomes greater than in the porous region which generates a significant drag force on the porous surface. Also, the D_N is decreasing as k_1 is increasing when there is a jump in the stress at the boundary. However, the drag coefficient corresponding to the positive values of σ shows a mixed behavior. For positive values of σ, a sudden decrease followed by a consistent increase in the drag coefficient is reported. We also note that the results for the prolate spheroidal shell are similar to those for the oblate spheroidal shell.

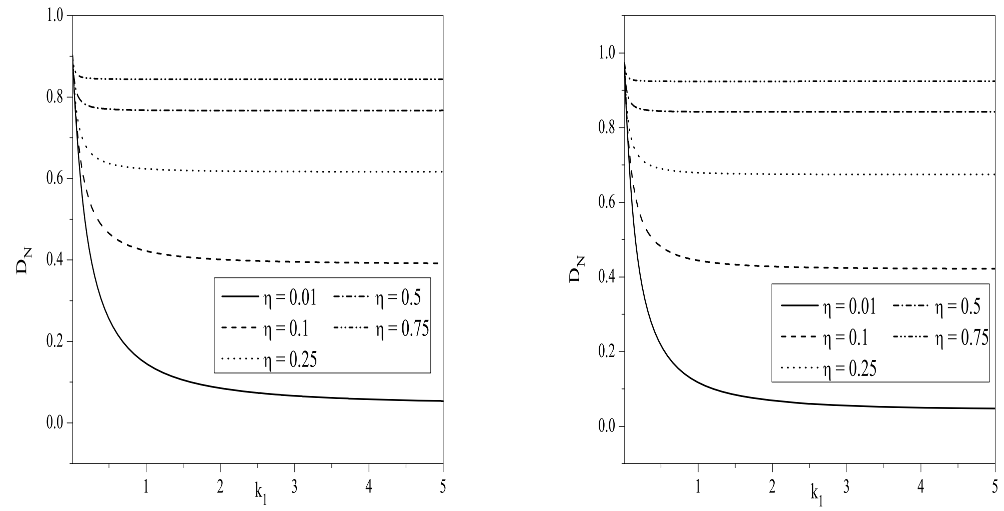
The influence of the viscosity ratios γ_1 and γ_2 on the drag coefficient for an oblate spheroid and a prolate spheroid is depicted in fig. 3. As $\gamma_1 \to \infty$, the problem reduces to the flow past a porous spheroid with an impermeable core. It is seen that the effect of γ_2 is to enhance the drag coefficient. The D_N also increases on increasing γ_1 keeping the other parameters fixed. This figure apparently indicates that an increase in the viscosity of the porous region (μ_2) and in the viscosity of the internal fluid (μ_1) gives rise to the drag coefficient.

Figure 4 exhibits the variation of the drag coefficient D_N with the permeability parameter k_1 for different values of the separation parameter η. The separation parameter η represents the extent of closeness between the liquid core and the porous particle. It can be perceived from fig. 4 that D_N increases as the separation parameter η increases. Interestingly, the effect of permeability on the drag coefficient D_N is stronger when η is smaller. As η tends to 1, *i.e.*,



(a) (b)

Fig. 3. Variations of the drag coefficient *versus* γ_1 for different values of γ_2 with $\sigma = 0.3$, $k_1 = 0.75$, and $\eta = 0.6$. (a) for $\epsilon = 0.1$; (b) for $\epsilon = -0.1$.



(a) (b)

Fig. 4. Variations of the drag coefficient *versus* permeability for different values of η with $\gamma_1 = 2$, $\gamma_2 = 3$, and $\sigma = 0.3$. (a) for $\epsilon = 0.1$; (b) for $\epsilon = -0.1$.

the distance between the liquid core and the porous particle decreases, the porous particle with liquid core becomes a fluid spheroid with radius r_b, and the problem reduces to the Stokes flow of an incompressible viscous fluid past an immiscible fluid spheroid. In this case, the porous region is absent and the drag remains constant for a fixed value of the viscosity ratio γ_1. As η tends to 0, *i.e.*, the distance between the liquid core and the porous particle increases, the porous particle with liquid core becomes a porous spheroid with radius r_a, and the problem reduces to the Stokes flow of an incompressible viscous fluid past a porous spheroid. In this case, the liquid core is absent and the drag coefficient D_N decreases as the permeability k_1 increases.

Table 1 shows the numerical results of the drag coefficient D_N for different values of the permeability k_1 and the deformation parameter ϵ for the case of $\sigma = -0.5$ (discontinuity in the shear stress), $\sigma = 0$ (continuity in the shear stress), and $\sigma = 0.5$ (discontinuity in the shear stress) keeping the values of γ_1, γ_2, and η as fixed. The numerical result shows that the drag coefficient D_N decreases as the deformation parameter ϵ increases for non-positive values

Table 1. Drag coefficient for different values of the permeability parameter k_1, deformation parameter ϵ and stress jump coefficient σ with $\gamma_2 = 0.5$, $\gamma_1 = 1.5$, and $\eta = 0.6$.

k_1	D_N				
	$\epsilon = -0.15$	$\epsilon = -0.1$	$\epsilon = 0$	$\epsilon = 0.1$	$\epsilon = 0.15$
			$\sigma = -0.5$		
0.5	0.580171	0.576052	0.567727	0.559283	0.555016
1	0.524928	0.520968	0.512983	0.504908	0.500837
3	0.465859	0.461986	0.454200	0.446363	0.442425
5	0.447635	0.443825	0.436173	0.428479	0.424616
10	0.429488	0.425784	0.418349	0.410877	0.407128
			$\sigma = 0$		
0.5	0.463420	0.463193	0.462489	0.461450	0.460804
1	0.428413	0.427143	0.424431	0.421493	0.419939
3	0.401783	0.399367	0.394449	0.389414	0.386853
5	0.396094	0.393391	0.387918	0.382356	0.379541
10	0.391743	0.388809	0.382891	0.376905	0.373887
			$\sigma = 0.5$		
0.5	0.268458	0.274171	0.285108	0.295392	0.300290
1	0.287907	0.290133	0.294291	0.298054	0.299788
3	0.321751	0.320979	0.319298	0.317431	0.316429
5	0.334812	0.333314	0.330217	0.326981	0.325312
10	0.349062	0.346943	0.342630	0.338217	0.335974

of the stress jump coefficient σ. This behavior, however, changes for positive values of stress jump coefficient. For relatively smaller values of k_1, the drag is an increasing function of the deformation parameter whereas for large values of permeability the drag coefficient is a decreasing function of the deformation parameter.

The streamline patterns have been plotted in fig. 5 using different values of k_1, γ_1, γ_2, η, and σ (taking $\sigma = 0$, *i.e.*, there is no jump in the shear stress at the porous-liquid interface, and $\sigma = -0.5$ and 0.5, *i.e.*, there is a jump in the shear stress at the porous-liquid interface) for a prolate spheroid ($\epsilon = -0.1$), a sphere ($\epsilon = 0$), and an oblate spheroid ($\epsilon = 0.1$), respectively. Figure 5 shows the streamline patterns of the present problem. We observe that the fluid outside the spheroidal shell penetrates throughout the porous layer and flows around the internal sphere (spheroid), *i.e.*, outside the region I, and the fluid inside the cavity region (region I) at the same time starts executing a circulatory motion.

8 Conclusion

In this work, the steady axisymmetric Stokes flow of a viscous fluid around a porous spheroidal shell of another immiscible fluid is investigated theoretically. The Stokes and the Brinkman equations for the flow field applicable to these axisymmetric motions are analytically solved, and the hydrodynamic drag force exerted on the porous spheroidal shell is evaluated. Numerical evaluations of the normalized hydrodynamic drag are performed and the results are illustrated graphically for various values of the considered parameters. The results show that the drag coefficient not only changes with the permeability of the porous region but also with the stress jump coefficient. As the stress jump coefficient increases, the drag coefficient decreases. It is also observed that the drag coefficient increases on increasing the separation parameter and viscosity ratios.

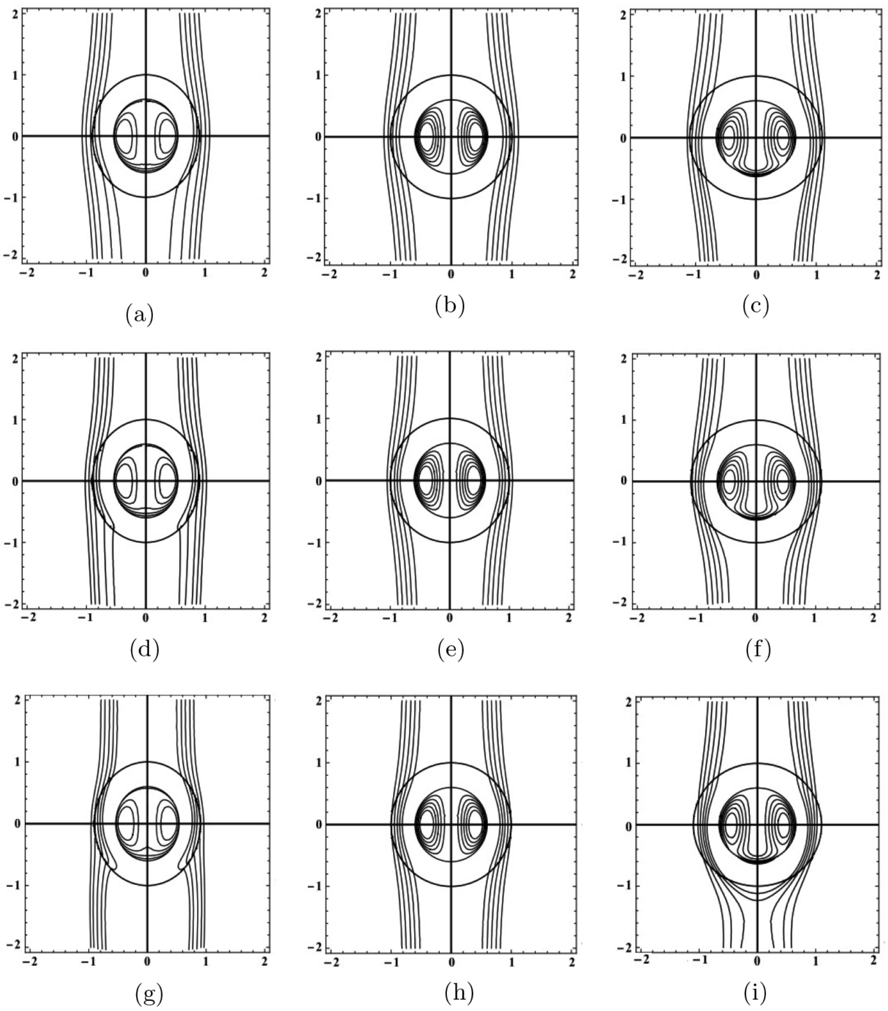


Fig. 5. Streamlines for different values of ϵ with $\gamma_1 = 1.2$, $\gamma_2 = 0.2$, $k_1 = 1$, and $\eta = 0.6$. (a) $\epsilon = -0.1$, $\sigma = -0.5$; (b) $\epsilon = 0$, $\sigma = -0.5$; (c) $\epsilon = 0.1$, $\sigma = -0.5$; (d) $\epsilon = -0.1$, $\sigma = 0$; (e) $\epsilon = 0$, $\sigma = 0$; (f) $\epsilon = 0.1$, $\sigma = 0$; (g) $\epsilon = -0.1$, $\sigma = 0.5$; (h) $\epsilon = 0$, $\sigma = 0.5$; (i) $\epsilon = 0.1$, $\sigma = 0.5$.

Appendix A.

Applying the boundary conditions (33)–(40) to the first order in α_m, we obtain the following system of algebraic equations:

$$[1 + g_2 + h_2 - c_2 - d_2 - e_2 S_2 - f_2 T_2]\, P_1(\zeta) + \alpha_m\, [2 - g_2 + h_2 - 2c_2 + d_2 + e_2 w_1 + f_2 w_2]\, (P_1(\zeta)\vartheta_m(\zeta) + \vartheta_2(\zeta) P_{m-1}(\zeta))$$
$$+ \sum_{n=3}^{\infty} [G_n + H_n - C_n - D_n - E_n\, S_3 - F_n\, T_3]\, P_{n-1}(\zeta) = 0, \tag{A.1}$$

$$[2 - g_2 + h_2 - 2c_2 + d_2 + e_2 w_1 + f_2 w_2]\, \vartheta_2(\zeta) + \alpha_m \left[2 + 2g_2 - 2c_2 - 2d_2 - e_2(\alpha^2 + 2) S_2 - f_2(\alpha^2 + 2) T_2\right] \vartheta_2(\zeta)\vartheta_m(\zeta)$$
$$+ \sum_{n=3}^{\infty} [(1 - n)\, G_n + (3 - n)\, H_n - n\, C_n - (1 - n)\, D_n - E_n\, w_{19} - F_n\, w_{20}]\, \vartheta_n(\zeta) = 0, \tag{A.2}$$

$$\left[3\, h_2 + 6\, g_2 + \gamma_2^2\, \alpha^2\, c_2 - \gamma_2^2\, (\alpha^2/2 + 6)\, d_2 - 2\, \gamma_2^2\, w_3\, e_2 - 2\, \gamma_2^2\, w_4\, f_2\right] P_1(\zeta)$$
$$+ \alpha_m \left[-6\, h_2 - 12\, g_2 + \gamma_2^2\, \alpha^2\, c_2 + \gamma_2^2 \left(\alpha^2 + 12\right)\, d_2 + 4\, \gamma_2^2\, w_3\, e_2 + 4\, \gamma_2^2\, w_4\, f_2\right] P_1(\zeta)\, \vartheta_m(\zeta)$$
$$- 2\, \alpha_m \left[6\, g_2 - \gamma_2^2\, (6\, d_2 + w_5\, e_2 + w_6\, f_2)\right] (P_1(\zeta)\, \vartheta_m(\zeta) + \vartheta_2(\zeta)\, P_{m-1}(\zeta))$$
$$+ \sum_{n=3}^{\infty} \left[2\, (n^2 + n - 3)/n\, H_n + 2\, (n + 1)\, G_n + \gamma_2^2(\alpha^2/(n - 1) + 2\, (n - 2))\, C_n - \gamma_2^2(\alpha^2/n + 2\, (n + 1))\, D_n + 2\, \gamma_2^2\, w_{21}\, E_n\right.$$
$$\left. + 2\, \gamma_2^2\, w_{22}\, F_n\right] P_{n-1}(\zeta) = 0, \tag{A.3}$$

$$\begin{aligned}
&\left[-6\,g_2-2\,\alpha\,\sigma\,\gamma_2\,c_2+\left(6\,\gamma_2^2+\alpha\,\sigma\,\gamma_2\right)\,d_2+w_{11}\,e_2+w_{12}\,f_2\right]\vartheta_2(\zeta)\\
&+\alpha_m\left[\gamma_2^2\left(-18\,d_2-(6+\alpha^2)\,w_3\,e_2-(6+\alpha^2)\,w_4\,f_2\right)+18\,g_2-\gamma_2\,\sigma\,\alpha\left(2\,c_2+2\,d_2+(\alpha^2+2)\,S_2\,e_2+(\alpha^2+2)\,T_2\,f_2\right)\right]\\
&\times\vartheta_2(\zeta)\vartheta_m(\zeta)+2\alpha_m\left[\gamma_2^2\left(9d_2+3w_3e_2+3w_4f_2\right)-9g_2-3h_2+\gamma_2\sigma\alpha\left(c_2+d_2+e_2S_2+f_2T_2\right)\right]P_1(\zeta)\vartheta_2(\zeta)P_{m-1}(\zeta)\\
&+\sum_{n=3}^{\infty}\left[n\left(2\,\gamma_2^2\,(n-2)-\gamma_2\,\alpha\,\sigma\right)C_n+(n-1)\left(2\,\gamma_2^2\,(n+1)+\gamma_2\,\alpha\,\sigma\right)D_n+w_{25}\,E_n+w_{26}\,F_n\right]\vartheta_n(\zeta)=0,
\end{aligned}\tag{A.4}$$

$$\left[a_n\,\eta^2+b_2\,\eta^4\right]P_1(\zeta)+\alpha_m\left[2\,a_2\,\eta^2+4\,b_2\,\eta^4\right]\left(P_1(\zeta)\,\vartheta_m(\zeta)+\vartheta_2(\zeta)\,P_{m-1}(\zeta)\right)+\sum_{n=3}^{\infty}\left[A_n\,\eta^n+B_n\,\eta^{n+2}\right]P_{n-1}(\zeta)=0,\tag{A.5}$$

$$\begin{aligned}
&\left[c_2\,\eta^2+d_2\,\eta^{-1}+e_2\,\eta^{1/2}\,S_6+f_2\,\eta^{1/2}\,T_6\right]P_1(\zeta)+\alpha_m\left[2\,c_2\,\eta^2-d_2\,\eta^{-1}-e_2\,\eta^{1/2}\,w_7-f_2\,\eta^{1/2}\,w_8\right]\left(P_1(\zeta)\,\vartheta_m(\zeta)\right.\\
&\left.+\vartheta_2(\zeta)\,P_{m-1}(\zeta)\right)+\sum_{n=3}^{\infty}\left[C_n\,\eta^n+D_n\,\eta^{1-n}+E_n\,\eta^{1/2}\,S_7+F_n\,\eta^{1/2}\,T_7\right]P_{n-1}(\zeta)=0,
\end{aligned}\tag{A.6}$$

$$\begin{aligned}
&\left[2\,a_2\,\eta+4\,b_2\,\eta^3-2\,c_2\,\eta+d_2\,\eta^{-2}+e_2\,\eta^{-1/2}\,w_7+f_2\,\eta^{-1/2}\,w_8\right]\vartheta_2(\zeta)\\
&+\alpha_m\left[2\,a_2\,\eta+12\,b_2\,\eta^3-2\,c_2\,\eta-2\,d_2\,\eta^{-2}-e_2\,\eta^{-1/2}\,(\alpha^2\,\eta^2+2)\,S_6-f_2\,\eta^{-1/2}\,(\alpha^2\,\eta^2+2)\,T_6\right]\vartheta_2(\zeta)\,\vartheta_m(\zeta)\\
&+\sum_{n=3}^{\infty}\left[n\,A_n\,\eta^{n-1}+(n+2)\,B_n\,\eta^{n+1}-n\,C_n\,\eta^{n-1}+(n-1)\,D_n\,\eta^{-n}-E_n\,\eta^{-1/2}\,w_{27}-F_n\,\eta^{-1/2}\,w_{28}\right]\vartheta_n(\zeta)=0,
\end{aligned}\tag{A.7}$$

$$\begin{aligned}
&\left[-6\,\gamma_1^2\,\eta^2\,b_2-2\,\gamma_2\,\alpha\,\sigma\,\eta\,c_2+\left(6\,\gamma_2^2\,\eta^{-3}+\gamma_2\,\alpha\,\sigma\,\eta^{-2}\right)d_2+w_{13}\,e_2+w_{14}\,f_2\right]\vartheta_2(\zeta)\\
&+\alpha_m\left[\gamma_2^2\left(-18\,d_2\,\eta^{-3}-\eta^{-3/2}\left(6+\alpha^2\,\eta^2\right)w_{15}\,e_2-\eta^{-3/2}\left(6+\alpha^2\,\eta^2\right)w_{16}\,f_2\right)\right.\\
&\left.-12\,\gamma_1^2\,\eta^2\,b_2-\gamma_2\,\sigma\,\alpha\left(2\,c_2\,\eta+2\,\eta^{-2}\,d_2+\eta^{-1/2}\left(\alpha^2\,\eta^2+2\right)S_6\,e_2+\eta^{-1/2}\left(\alpha^2\,\eta^2+2\right)T_6\,f_2\right)\right]\vartheta_2(\zeta)\,\vartheta_m(\zeta)\\
&+2\,\alpha_m\left[\gamma_2^2\left(9\,d_2\,\eta^{-3}+3\,\eta^{-3/2}\,w_{15}\,e_2+3\,\eta^{-3/2}\,w_{16}\,f_2\right)+6\,\gamma_1^2\,\eta^2\,b_2\right]P_1(\zeta)\,\vartheta_2(\zeta)\,P_{m-1}(\zeta)\\
&+\sum_{n=3}^{\infty}\left[-2\,\gamma_1^2\,n\,(n-2)\,A_n\,\eta^{n-2}-2\,\gamma_1^2\,(n^2-1)\,B_n\,\eta^n+n\,\eta^{n-1}\left(2\,\gamma_2^2\,(n-2)\,\eta^{-1}-\gamma_2\,\alpha\,\sigma\right)C_n\right.\\
&\left.+\,(n-1)\,\eta^{-n}\left(2\,\gamma_2^2\,(n+1)\,\eta^{-1}+\gamma_2\,\alpha\,\sigma\right)D_n+w_{31}\,E_n+w_{32}\,F_n\right]\vartheta_n(\zeta)=0.
\end{aligned}\tag{A.8}$$

On solving the leading terms of eqs. (A.1)–(A.8) we will get the values of a_2, b_2, c_2, d_2, e_2, f_2, g_2, and h_2. Since the expressions are very lengthy we are not presenting them here except h_2. To obtain the remaining arbitrary constants A_n, B_n, C_n, D_n, E_n, F_n, G_n, H_n we require the following identities:

$$\vartheta_m(\zeta)\vartheta_2(\zeta)=\frac{-(m-2)(m-3)}{2(2m-1)(2m-3)}\vartheta_{m-2}(\zeta)+\frac{m(m-1)}{(2m+1)(2m-3)}\vartheta_m(\zeta)-\frac{(m+1)(m+2)}{2(2m-1)(2m+1)}\vartheta_{m+2}(\zeta),\tag{A.9}$$

$$\begin{aligned}
\vartheta_m(\zeta)P_1(\zeta)+P_{m-1}(\zeta)\vartheta_2(\zeta)=&\frac{-(m-2)(m-3)}{2(2m-1)(2m-3)}P_{m-3}(\zeta)+\frac{m(m-1)}{(2m+1)(2m-3)}P_{m-1}(\zeta)\\
&-\frac{(m+1)(m+2)}{2(2m-1)(2m+1)}P_{m+1}(\zeta),
\end{aligned}\tag{A.10}$$

$$\begin{aligned}
P_1(\zeta)\vartheta_2(\zeta)P_{m-1}(\zeta)=&\frac{-(m-1)(m-2)(m-3)}{2(2m-1)(2m-3)}\vartheta_{m-2}(\zeta)+\frac{m(m-1)}{2(2m+1)(2m-3)}\vartheta_m(\zeta)\\
&+\frac{m(m+1)(m+2)}{2(2m-1)(2m+1)}\vartheta_{m+2}(\zeta),
\end{aligned}\tag{A.11}$$

$$\vartheta_m(\zeta)P_1(\zeta)=\frac{(m-2)}{(2m-1)(2m-3)}P_{m-3}(\zeta)+\frac{1}{(2m+1)(2m-3)}P_{m-1}(\zeta)-\frac{(m+1)}{(2m-1)(2m+1)}P_{m+1}(\zeta).\tag{A.12}$$

Using these in (A.1)–(A.8), we get $A_n,\ B_n,\ C_n,\ D_n,\ E_n,\ F_n,\ G_n,\ H_n = 0$, for $n \neq m-2, m, m+2$ and when $n = m-2, m, m+2$, we have the following system:

$$
\begin{aligned}
&\xi_1\,\bar{a}_n + G_n + H_n - C_n - D_n - E_n\,S_3 - F_n\,T_3 = 0,\\
&\xi_2\,\bar{a}_n + (1-n)\,G_n + (3-n)\,H_n - n\,C_n - (1-n)\,D_n - E_n\,w_{19} - F_n\,w_{20} = 0,\\
&\xi_3\,\bar{c}_n - 2\,\xi_4\,\bar{a}_n + 2\,(n^2+n-3)/n\,H_n + 2\,(n+1)\,G_n + \gamma_2^2(\alpha^2/(n-1) + 2\,(n-2))\,C_n\\
&-\gamma_2^2(\alpha^2/n + 2\,(n+1))\,D_n + 2\,\gamma_2^2\,w_{21}\,E_n + 2\,\gamma_2^2\,w_{22}\,F_n = 0,\\
&\xi_5\,\bar{a}_n + 2\,\xi_6\,\bar{b}_n + n\left(2\,\gamma_2^2\,(n-2) - \gamma_2\,\alpha\,\sigma\right)C_n + (n-1)\left(2\,\gamma_2^2\,(n+1) + \gamma_2\,\alpha\,\sigma\right)D_n + w_{25}\,E_n + w_{26}\,F_n = 0,\\
&\xi_7\,\bar{a}_n + A_n\,\eta^n + B_n\,\eta^{n+2} = 0,\\
&\xi_8\,\bar{a}_n + C_n\,\eta^n + D_n\,\eta^{1-n} + E_n\,\eta^{1/2}\,S_7 + F_n\,\eta^{1/2}\,T_7 = 0,\\
&\xi_9\,\bar{a}_n + n\,A_n\,\eta^{n-1} + (n+2)\,B_n\,\eta^{n+1} - n\,C_n\,\eta^{n-1} + (n-1)\,D_n\,\eta^{-n} - E_n\,\eta^{-1/2}\,w_{27} - F_n\,\eta^{-1/2}\,w_{28} = 0,\\
&\xi_{10}\,\bar{a}_n + 2\,\xi_{11}\,\bar{b}_n - 2\,\gamma_1^2\,n\,(n-2)\,A_n\,\eta^{n-2} - 2\,\gamma_1^2\,(n^2-1)\,B_n\,\eta^n + n\,\eta^{n-1}\left(2\,\gamma_2^2\,(n-2)\,\eta^{-1} - \gamma_2\,\alpha\,\sigma\right)C_n\\
&+(n-1)\,\eta^{-n}\left(2\,\gamma_2^2\,(n+1)\,\eta^{-1} + \gamma_2\,\alpha\,\sigma\right)D_n + w_{31}\,E_n + w_{32}\,F_n = 0,
\end{aligned}
\tag{A.13}
$$

where

$$
\begin{aligned}
\xi_1 &= 2 - g_2 + h_2 - 2\,c_2 + d_2 + e_2\,w_1 + f_2\,w_2,\\
\xi_2 &= 2 + 2\,g_2 - 2\,c_2 - 2\,d_2 - e_2\,(\alpha^2+2)\,S_2 - f_2\,(\alpha^2+2)\,T_2,\\
\xi_3 &= -6\,h_2 - 12\,g_2 + \gamma_2^2\,\alpha^2\,c_2 + \gamma_2^2\left(\alpha^2+12\right)\,d_2 + 4\,\gamma_2^2\,w_3\,e2 + 4\,\gamma_2^2\,w_4\,f_2,\\
\xi_4 &= 6\,g_2 - \gamma_2^2\,(6\,d_2 + w_5\,e_2 + w_6\,f_2)\,,\\
\xi_5 &= \gamma_2^2\left(-18\,d_2 - (6+\alpha^2)\,w_3\,e_2 - (6+\alpha^2)\,w_4\,f_2\right) + 18\,g_2\\
&\quad - \gamma_2\,\sigma\,\alpha\left(2\,c_2 + 2\,d_2 + (\alpha^2+2)\,S_2\,e_2 + (\alpha^2+2)\,T_2\,f_2\right),\\
\xi_6 &= \gamma_2^2\,(9\,d_2 + 3\,w_3\,e_2 + 3\,w_4\,f_2) - 9\,g_2 - 3\,h_2 + \gamma_2\,\sigma\,\alpha\,(c_2 + d_2 + e_2\,S_2 + f_2\,T_2)\,,\\
\xi_7 &= 2\,a_2\,\eta^2 + 4\,b_2\,\eta^4,\\
\xi_8 &= 2\,c_2\,\eta^2 - d_2\,\eta^{-1} - e_2\,\eta^{1/2}\,w_7 - f_2\,\eta^{1/2}\,w_8,\\
\xi_9 &= 2\,a_2\,\eta + 12\,b_2\,\eta^3 - 2\,c_2\,\eta - 2\,d_2\,\eta^{-2} - e_2\,\eta^{-1/2}\,(\alpha^2\,\eta^2+2)\,S_6\\
&\quad - f_2\,\eta^{-1/2}\,(\alpha^2\,\eta^2+2)\,T_6,\\
\xi_{10} &= \gamma_2^2\left(-18\,d_2\,\eta^{-3} - \eta^{-3/2}\left(6+\alpha^2\,\eta^2\right)\,w_{15}\,e_2 - \eta^{-3/2}\left(6+\alpha^2\,\eta^2\right)w_{16}\,f_2\right)\\
&\quad - 12\,\gamma_1^2\,\eta^2\,b_2 - \gamma_2\,\sigma\,\alpha\left(2\,c_2\,\eta + 2\,\eta^{-2}\,d_2 + \eta^{-1/2}\left(\alpha^2\,\eta^2+2\right)S_6\,e_2\right.\\
&\quad \left. + \,\eta^{-1/2}\left(\alpha^2\,\eta^2+2\right)T_6\,f_2\right),\\
\xi_{11} &= \gamma_2^2\left(9\,d_2\,\eta^{-3} + 3\,\eta^{-3/2}\,w_{15}\,e_2 + 3\,\eta^{-3/2}\,w_{16}\,f_2\right) + 6\,\gamma_1^2\,\eta^2\,b_2,
\end{aligned}
$$

$$
\begin{array}{llll}
S_1 = K_{1/2}(\alpha), & S_2 = K_{3/2}(\alpha), & S_3 = K_{n-1/2}(\alpha), & S_4 = K_{n+1/2}(\alpha),\\
S_5 = K_{1/2}(\alpha\,\eta), & S_6 = K_{3/2}(\alpha\,\eta), & S_7 = K_{n-1/2}(\alpha\,\eta), & S_8 = K_{n+1/2}(\alpha\,\eta),\\
T_1 = I_{1/2}(\alpha), & T_2 = I_{3/2}(\alpha), & T_3 = I_{n-1/2}(\alpha), & T_4 = I_{n+1/2}(\alpha),\\
T_5 = I_{1/2}(\alpha\,\eta), & T_6 = I_{3/2}(\alpha\,\eta), & T_7 = I_{n-1/2}(\alpha\,\eta), & T_8 = I_{n+1/2}(\alpha\,\eta),
\end{array}
$$

$$
\begin{array}{ll}
w_1 = S_2 + \alpha\,S_1, & w_2 = T_2 - \alpha\,T_1,\\
w_3 = 3\,S_2 + \alpha\,S_1, & w_4 = 3\,T_2 - \alpha\,T_1,\\
w_5 = (6+\alpha^2)\,S_2 + 2\,\alpha\,S_1, & w_6 = (6+\alpha^2)\,T_2 - 2\,\alpha\,T_1,\\
w_7 = S_6 + \alpha\,\eta\,S_5, & w_8 = T_6 - \alpha\,\eta\,T_5,\\
w_9 = (6+\alpha^2\,\eta^2)\,S_6 + 2\,\alpha\,\eta\,S_5, & w_{10} = (6+\alpha^2\,\eta^2)\,T_6 - 2\,\alpha\,\eta\,T_5,\\
w_{11} = \gamma_2^2\,w_5 + \gamma_2\,\alpha\,\sigma\,w_1, & w_{12} = \gamma_2^2\,w_6 + \gamma_2\,\alpha\,\sigma\,w_2,\\
w_{13} = \gamma_2^2\,\eta^{-3/2}\,w_9 + \gamma_2\,\alpha\,\sigma\,\eta^{-1/2}\,w_7, & w_{14} = \gamma_2^2\,\eta^{-3/2}\,w_{10} + \gamma_2\,\alpha\,\sigma\,\eta^{-1/2}\,w_8,\\
w_{15} = 3\,S_6 + \alpha\,\eta\,S_5, & w_{16} = 3\,T_6 - \alpha\,\eta\,T_5,
\end{array}
$$

$$
\begin{aligned}
&w_{17} = (12+\alpha^2)\, S_2 + 4\,\alpha\, S_1, && w_{18} = (12+\alpha^2)\, T_2 - 4\,\alpha\, T_1,\\
&w_{19} = n\, S_3 - \alpha\, S_4, && w_{20} = n\, T_3 + \alpha\, T_4\\
&w_{21} = (n-2)\, S_3 - \alpha\, S_4, && w_{22} = (n-2)\, T_3 + \alpha\, T_4,\\
&w_{23} = \left(2\,n\,(n-2) + \alpha^2\right) S_3 + 2\,\alpha\, S_4, && w_{24} = \left(2\,n\,(n-2) + \alpha^2\right) T_3 - 2\,\alpha\, T_4,\\
&w_{25} = \gamma_2^2\, w_{23} - \gamma_2\,\alpha\,\sigma\, w_{19}, && w_{26} = \gamma_2^2\, w_{24} - \gamma_2\,\alpha\,\sigma\, w_{20},\\
&w_{27} = n\, S_7 - \alpha\,\eta\, S_8, && w_{28} = n\, T_7 + \alpha\,\eta\, T_8,\\
&w_{29} = \left(2\,n(n-2) + \alpha^2\,\eta^2\right) S_7 + 2\,\alpha\,\eta\, S_8, && w_{30} = \left(2\,n(n-2) + \alpha^2\,\eta^2\right) T_7 - 2\,\alpha\,\eta\, T_8,\\
&w_{31} = \gamma_2^2\,\eta^{-3/2}\, w_{29} - \gamma_2\,\alpha\,\sigma\,\eta^{-1/2}\, w_{27}, && w_{32} = \gamma_2^2\,\eta^{-3/2}\, w_{30} - \gamma_2\,\alpha\,\sigma\,\eta^{-1/2}\, w_{28},
\end{aligned}
$$

$$
\bar{a}_{m-2} = \frac{-\alpha_m(m-2)(m-3)}{2(2\,m-1)(2\,m-3)}\,, \qquad \bar{a}_m = \frac{\alpha_m\, m(m-1)}{(2\,m+1)(2\,m-3)}\,, \qquad \bar{a}_{m+2} = \frac{-\alpha_m(m+1)(m+2)}{2(2\,m-1)(2\,m+1)}\,,
$$

$$
\bar{b}_{m-2} = (m-1)\bar{a}_{m-2}, \qquad \bar{b}_m = \bar{a}_m/2, \qquad \bar{b}_{m+2} = m\,\bar{a}_{m+2},
$$

$$
\bar{c}_{m-2} = \frac{\alpha_m(m-2)}{(2\,m-1)(2\,m-3)}\,, \qquad \bar{c}_m = \frac{\alpha_m}{(2\,m+1)(2\,m-3)}\,, \qquad \bar{c}_{m+2} = \frac{-\alpha_m(m+1)}{(2\,m-1)(2\,m+1)}\,.
$$

Appendix B.

The expression for h_2 appearing in eq. (53) is given as

$$
h_2 = \frac{1}{\Delta_4}\left(x_1\, z_5 + x_2\,(z_2 + \eta\, z_3) - x_3\, z_6 + x_4\, z_1 + x_5\, z_4\right), \tag{B.1}
$$

where

$$
\begin{aligned}
x_1 &= -3\,\alpha^3\,\gamma_2\,\eta^{7/2}\left(2\,\alpha\,\gamma_2\left(\gamma_2^2-1\right)\left(2+\eta^3\right) + 8\left(\gamma_2^2-1\right)\left(\eta^3-1\right)\sigma + \alpha^2\,\gamma_2^2\left(\eta^3+2\right)\sigma\right) y_1,\\
x_2 &= -36\,\alpha^2\,\gamma_2\left(\gamma_2^2-1\right)\eta^4(\alpha\,\gamma_2 - 2\sigma) y_1,\\
x_3 &= -3\,\alpha^2\,\gamma_2^2\,\eta^{9/2}\left(54\left(\gamma_2^2-1\right)\, y_2\,\eta + \alpha^4\,\gamma_2^4\left(\eta^3+2\right) + \alpha^3\,\gamma_2^3\left(\eta^3 + 3\,\eta^2 + 2\right)\sigma\right.\\
&\quad \left. +3\,\alpha\,\gamma_2\left(8 + 3\,\gamma_1^2\eta - 18\,\eta^2 + 2\,\gamma_2^2\left(9\,\eta^2 + \eta - 4\right)\right)\sigma - 3\,\alpha^2\,\gamma_2^2\left(4 - 3\,\gamma_1^2\eta + 6\,\eta^3 - 2\,\gamma_2^2\left(3\eta^3 + \eta + 2\right) - \eta^2\sigma^2\right)\right),\\
x_4 &= -3\,\alpha^2\,\gamma_2\,\eta^{9/2}\left(24\left(\gamma_2^2-1\right)\, y_2\,\eta\,\sigma + \alpha^4\,\gamma_2^4\left(\eta^3+2\right)\sigma + \alpha^2\,\gamma_2^2\left(8 + 9\,\gamma_1^2\,\eta - 6\,\eta^2 - 8\,\eta^3\right.\right.\\
&\quad \left. +\gamma_2^2\left(8\,\eta^3 + 6\,\eta^2 + 6\,\eta - 8\right)\right)\sigma + 6\,\alpha\,\gamma_2\left(\gamma_2^2-1\right)\eta\left(3\,\gamma_1^2 + 2\,\gamma_2^2 + 4\,\eta\,\sigma^2\right)\\
&\quad \left. +\alpha^3\,\gamma_2^3\left(2\,\gamma_2^2\left(2+\eta^3\right) + 3\,\eta^2\sigma^2 - 4 - 2\,\eta^3\right)\right),\\
x_5 &= 3\,\alpha^2\,\gamma_2\,\eta^{7/2}\left(\alpha^3\,\gamma_2^3\left(\eta^3+2\right) + 6\,\alpha\,\gamma_2\left(\gamma_2^2-1\right)\left(3\,\eta^3+2\right) - 24\left(\gamma_2^2-1\right)\sigma + \alpha^2\,\gamma_2^2\left(\eta^3+2\right)\sigma\right) y_1,\\
\Delta_4 &= x_6\,(z_2 + \eta\, z_3) + x_7\, z_1 + x_8\, z_5 - x_9\, z_6 + x_{10}\, z_4,\\
x_6 &= 6\,\alpha\,\eta^4\left(-18 - 3\left(\alpha^2-6\right)\gamma_2^2 + 4\,\alpha^2\,\gamma_2^4 + 9\,\alpha\,\gamma_2\,\sigma - 8\,\alpha\,\gamma_2^3\sigma\right) y_1,\\
x_7 &= -2\,\alpha\,\eta^{9/2}\left(54\left(\gamma_2^2-1\right)\, y_2\,\eta - \alpha^5\,\gamma_2^5\left(\eta^3+2\right)\sigma - \alpha^3\,\gamma_2^3\left(9\,\gamma_1^2\eta - 9\left(\eta^3 + \eta^2 - 1\right)\right.\right.\\
&\quad \left. +\gamma_2^2\left(8\,\eta^3 + 6\,\eta^2 + 6\,\eta - 8\right)\right)\sigma + 3\,\alpha\,\gamma_2\eta\left(-3\,\gamma_1^2\left(8\,\gamma_2^2 - 9\right) - 2\left(8\,\gamma_2^4 + 9\,\eta - 9\,\gamma_2^2(1+\eta)\right)\right)\sigma\\
&\quad + \alpha^4\,\gamma_2^4\left(-2\,\gamma_2^2\left(2+\eta^3\right) + 3\left(2 + \eta^3 - \eta^2\sigma^2\right)\right) - 3\,\alpha^2\,\gamma_2^2\left(-6 - 9\,\gamma_1^2\eta + 4\,\gamma_2^4\,\eta + 6\,\eta^3 - 9\,\eta^2\,\sigma^2\right.\\
&\quad \left.\left. +\gamma_2^2\left(6 + 6\left(\gamma_1^2-1\right)\eta - 6\,\eta^3 + 8\,\eta^2\,\sigma^2\right)\right)\right),\\
x_8 &= 2\,\alpha^2\,\eta^{7/2}\left(18\left(\eta^3-1\right) + 2\alpha^2\,\gamma_2^4\left(\eta^3+2\right) - 3\,\gamma_2^2\left(6\left(\eta^3-1\right) + \alpha^2\left(\eta^3+2\right)\right) - 9\alpha\,\gamma_2\left(\eta^3-1\right)\sigma\right.\\
&\quad \left. +\alpha\,\gamma_2^3\left(8\left(\eta^3-1\right) + \alpha^2\left(\eta^3+2\right)\right)\sigma\right) y_1,\\
x_9 &= 2\,\alpha^2\,\gamma_2^2\,\eta^{9/2}\left(-54\left(1 + 3\,\gamma_1^2\eta\right) + \gamma_2^4\left(108\,\eta + \alpha^4\left(2+\eta^3\right) + 6\,\alpha^2\left(2 + \eta + 3\eta^3\right)\right) + 9\,\alpha\,\gamma_2\left(3 + \gamma_1^2\eta - 6\eta^2\right)\sigma\right.\\
&\quad \left. +\ \alpha\gamma_2^3\left(6\left(9\,\eta^2 + \eta - 4\right) + \alpha^2\left(\eta^3 + 3\,\eta^2 + 2\right)\right)\sigma - 3\,\gamma_2^2\left(-18\left(1 - 2\eta + 3\,\gamma_1^2\eta\right) + \alpha^2\left(3 - 3\,\gamma_1^2\,\eta + 6\,\eta^3 - \eta^2\,\sigma^2\right)\right)\right),\\
x_{10} &= -2\,\alpha\,\eta^{7/2}\, y_1\left(-54 + \gamma_2\left(\gamma_2\left(54 + \alpha^4\,\gamma_2^2\left(2+\eta^3\right) + 3\,\alpha^2\left(-3 + 4\,\gamma_2^2 + 6\left(\gamma_2^2-1\right)\eta^3\right)\right)\right.\right.\\
&\quad \left.\left. +\alpha\left(27 + \gamma_2^2\left(-24 + \alpha^2\left(2+\eta^3\right)\right)\right)\sigma\right)\right).
\end{aligned}
$$

The expression for H_2 appearing in eq. (53) is given as

$$H_2 = \frac{1}{5\,\Delta_4^2}\left(x_{11}\,z_4\,z_2 + x_{12}\,z_3\,z_4 + x_{13}\,z_1\,z_2 + x_{14}\,z_1\,z_3 + x_{15}\,z_2\,z_5 + x_{16}\,z_3\,z_5 + x_{17}\,z_9 + x_{18}\,z_2^2 + x_{19}\,z_3^2 - x_{20}\,z_2\,z_6\right.$$
$$- x_{21}\,z_3\,z_6 - x_{22}\,z_1\,z_5 + x_{23}\,z_1\,z_6 + x_{24}\,z_4\,z_6 + x_{25}\,z_1^2 + x_{26}\,z_{11} + x_{27}\,z_{12} + x_{28}\,z_6^2 + x_{29}\left(z_5^2 + T_5\,T_2\,S_5\,S_2\right)$$
$$\left.-x_{30}\,z_4\,z_5 + x_{31}\,z_{10} + x_{32}\,T_5\,T_2\,S_5\,S_2\right) + \frac{1}{5\,\Delta_4}(x_{33}\,z_5 + x_{34}\,z_2 - x_{35}\,z_6 + x_{36}\,z_4 + x_{37}\,z_3 + x_{38}\,z_1), \tag{B.2}$$

where

$$z_{11} = T_5\,T_2\,S_1\,S_6 + T_1\,T_6\,S_5\,S_2, \qquad z_{12} = T_2^2\,S_5^2 + T_5^2\,S_2^2,$$

$$x_{11} = -24\,\alpha^3\,\gamma_2^2\epsilon\,\eta^{15/2}\,y_1\left(y_{11} + 12\,\alpha^2\,\gamma_2\,\sigma\left(-y_2\left(21 - 45\,\gamma_2^2 + 32\,\gamma_2^4\right) - 18\,\eta\left(3 - 10\,\gamma_2^2 + 7\,\gamma_2^4\right)\right.\right.$$
$$\left.-24\,y_2\,\eta^3\left(\gamma_2^2-1\right)^2 + 108\,\eta^4\left(\gamma_2^2-1\right)^2 + 2\,y_3\,\eta\,\sigma^2\right) + 24\,\alpha\left(9\,y_2\left(\gamma_2^2-1\right)\left(3 - 7\,\gamma_2^2 + 6\,\eta^3\left(\gamma_2^2-1\right)\right) + y_4\right)$$
$$+\,\alpha^3\left(12\left(\gamma_2^2\,y_2\left(33 - 45\,\gamma_2^2 + 8\,\gamma_2^4\right) + 3\,y_2\,\eta^3\left(\gamma_2^2-1\right)^2\left(9 + 4\,\gamma_2^2\right) + 81\,\gamma_2^2\,\eta^5\left(\gamma_2^2-1\right)^2\right)\right.$$
$$\left.-\gamma_2^2\left(36\eta\left(7{+}8\eta^3\right){+}32\gamma_2^4\left(2{+}12\eta{+}\eta^3{+}9\eta^4\right){-}6\gamma_2^2\left(5{+}90\eta{+}4\eta^3{+}96\eta^4\right){+}\gamma_1^2\left(48\gamma_2^2\left(2{+}\eta^3\right){-}9\left(5{+}4\eta^3\right)\right)\right)\sigma^2\right)$$
$$+\,y_7 + \alpha^4\,\gamma_2\,\sigma\left(324\,\eta^4 + 6\,\gamma_1^2\left(-18 + 9\,\gamma_2^2 - 8\,\gamma_2^4 - 4\,\eta^3\left(9 - 9\,\gamma_2^2 + \gamma_2^4\right)\right) + 16\,\gamma_2^6\left(-2 + 6\eta - \eta^3 + 9\eta^4\right)\right.$$
$$+\,4\,\gamma_2^4\left(9 + 9\eta\left(3+\eta^2\right)\left(-5 + \eta + 3\eta^2\right) - 4\,\eta\,\sigma^2\left(2+\eta^3\right)\right) + 3\,\gamma_2^2\left(-12\left(2 - 11\,\eta + 3\,\eta^2 + 4\,\eta^3 + 14\,\eta^4 + 3\,\eta^5\right)\right.$$
$$\left.\left.+\eta\left(5+4\eta^3\right)\sigma^2\right)\right) + \alpha^5\,\gamma_2^2\left(16\,\gamma_2^6\left(2+\eta^3\right) - 36\,\eta\,\sigma^2\left(1+2\eta^3\right) + 2\gamma_2^4\left(3 + 6\eta^2\left(9 + \eta + 9\eta^3\right) - 4\,\eta\,\sigma^2\left(2+\eta^3\right)\right)\right.$$
$$+\,3\,\gamma_2^2\left(-12\left(2 + \eta^2\left(3 + \eta + 3\eta^3\right)\right) + (4 + 6\,\eta + 2\,\eta^2 + 2\eta^3 + 24\eta^4 + \eta^5)\sigma^2\right) + 3\gamma_1^2\left(8\,\gamma_2^4\left(2+\eta^3\right) + \gamma_2^2\left(3 + 6\eta^3\right)\right.$$
$$\left.\left.\left.\left.+3\left(2+\eta^3\right)\left(\sigma^2-6\right)\right)\right)\right)\right),$$

$$x_{12} = -24\,y_1\,\alpha^3\,\gamma_2^2\,\epsilon\,\eta^{17/2}\left(y_{11} + y_{14} + 24\alpha\left(y_4 + 9\,y_2\left(\gamma_2^2-1\right)\left(6 - 7\,\gamma_2^2 + 6\,\eta^3\left(\gamma_2^2-1\right)\right)\right)\right.$$
$$+\,\alpha^3\left(12\left(y_2\,\gamma_2^2\left(24 - 33\,\gamma_2^2 + 8\,\gamma_2^4\right) + 3\,y_2\,\eta^3\left(9 - 20\,\gamma_2^2 + 7\,\gamma_2^4 + 4\,\gamma_2^6\right) + 81\,\gamma_2^2\,\eta^5\left(\gamma_2^2-1\right)^2\right)\right.$$
$$+\,\gamma_2^2\left(72\,\eta\left(1-4\eta^3\right) - 32\,\gamma_2^4\left(2 + 12\,\eta + \eta^3 + 9\,\eta^4\right) + 6\,\gamma_2^2\left(5 + 42\,\eta + 4\,\eta^3 + 96\,\eta^4\right)\right.$$
$$\left.\left.+\gamma_1^2\left(-48\,\gamma_2^2\left(2+\eta^3\right) + 9\left(5 + 4\,\eta^3\right)\right)\right)\sigma^2\right) + \alpha^4\,\gamma_2\sigma\left(324\,\eta^4 + 16\,\gamma_2^6\left(-2 + 6\,\eta - \eta^3 + 9\eta^4\right)\right.$$
$$-\,6\,\gamma_1^2\left(18 + 36\,\eta^3 + 4\,\gamma_2^4\left(2+\eta^3\right) - 21\,\gamma_2^2\left(1+2\eta^3\right)\right) + 4\,\gamma_2^4\left(21 + 3\,\eta\left(-33 + 9\,\eta + 14\,\eta^2 + 21\,\eta^3 + 9\,\eta^4\right)\right.$$
$$\left.\left.-4\eta\left(2+\eta^3\right)\sigma^2\right) + 3\gamma_2^2\left(-12\left(2 - 8\eta + 3\eta^2 + 4\eta^3 + 20\eta^4 + 3\eta^5\right) + \eta\left(5+4\eta^3\right)\sigma^2\right)\right) + \alpha^5\gamma_2^2\left(2\gamma_2^2\left(8\gamma_2^4\left(2+\eta^3\right)\right.\right.$$
$$\left.\left.-18\left(2 + \eta^2\left(3 + \eta + 3\eta^3\right)\right) + 9\,\gamma_2^2\left(3 + 2\eta^2\left(3 + \eta + 3\eta^3\right)\right)\right) + \left(-8\,\gamma_2^4\,\eta\left(2+\eta^3\right) - 36\left(\eta + 2\eta^4\right)\right.$$
$$\left.\left.+3\,\gamma_2^2\left(4 + 14\,\eta + 2\,\eta^2 + 2\,\eta^3 + 28\,\eta^4 + \eta^5\right)\right)\sigma^2 + 3\,\gamma_1^2\left(8\,\gamma_2^4\left(2+\eta^3\right) + 9\,\gamma_2^2\left(3+2\eta^3\right) + 3\left(2+\eta^3\right)\left(-6+\sigma^2\right)\right)\right)$$
$$+\,12\,\alpha^2\,\gamma_2\,\sigma\left(3\,\gamma_1^2\left(6 + 21\,\gamma_2^2 - 32\,\gamma_2^4 - 24\,\eta^3\left(\gamma_2^2-1\right)^2\right) + 2\left(-8\,\gamma_2^6\left(4+3\eta^3\right) + 9\eta\left(-6 + 6\,\eta^3 + \sigma^2\right)\right.\right.$$
$$\left.\left.\left.-3\,\gamma_2^2\left(-2 + \eta\left(-39 + 4\eta^2(2+9\eta) + 9\sigma^2\right)\right) + \gamma_2^4\left(21 + \eta\left(-63 + 6\,\eta^2\left(8 + 9\,\eta\right) + 16\,\sigma^2\right)\right)\right)\right)\right),$$

$$x_{13} = 24\,\alpha^3\,\gamma_2^2\,\epsilon\,\eta^{17/2}\left(y_{17} + \alpha^2\left(y_{18} + y_9\,\alpha^3\,\gamma_2^2 + \alpha^5\,\gamma_2^4\left(y_{13} + 3\,\gamma_2^2\,\eta\left(1 + 12\,\eta + 3\,\eta^2 - \eta^3 + 6\,\eta^4\right)\sigma^2\right)\right.\right.$$
$$+\,\alpha^4\,\gamma_2^3\,\sigma\left(y_8 + \gamma_2^2\left(27\,\gamma_1^2\left(-1 + 8\,\eta + 2\,\eta^2 - \eta^3 + 4\,\eta^4\right) + \gamma_2^2\left(-12 + 3\,\eta(40 + \eta(12 + \eta(29\,\eta - 5)))\right.\right.\right.$$
$$\left.\left.\left.\left.-8\,\eta\left(4 + \eta^2(2\,\eta-3)\right)\sigma^2\right) + 9\,\eta\left(-16 - 11\,\eta^3 + \left(5 + 2\,\eta\left(2 + \eta\left(4\,\eta^2 + \eta - 2\right)\right)\right)\sigma^2\right)\right)\right)\right)$$
$$+\,\alpha\left(y_{20} + 27\,\gamma_1^4\,\eta\left(16\,\gamma_2^6 + 108\,\eta^2 + \gamma_2^4\left(33 + 108\,\eta^2 - 16\,\sigma^2\right) + 9\,\gamma_2^2\left(-5 - 24\,\eta^2 + \sigma^2\right)\right)\right.$$
$$\left.+4\,\gamma_2^2\left(y_{21} + 9\,\gamma_2^4\left(40 - 15\,\eta - 154\,\eta^3 + (-12 - 5\,\eta - 4\,\eta^2 + 90\,\eta^3 + 8\,\eta^4)\sigma^2\right)\right)\right)$$
$$+\,\alpha^3\,\gamma_2^2\left(y_{19} + 3\,\gamma_1^2\left(-72\,\eta\left(1+2\eta^3\right)\sigma^2 + \gamma_2^4\left(48 + 36\,\eta + 51\,\eta^3 - 16\left(2 + (-3+\eta)\eta^2\right)\sigma^2\right) + 3\,\gamma_2^2\left(-3\left(8 + 7\,\eta^3\right)\right.\right.\right.$$
$$\left.\left.\left.\left.+\left(11 + 4\,\eta\left(6 + \eta\left(12\,\eta^2 + \eta - 6\right)\right)\right)\sigma^2\right)\right) - 27\,\eta\,\sigma^2\left(-12\,\eta^4 + \gamma_2^2\left(12 + \eta^2\left(5 + 8\,\eta\left(1 + 3\,\eta\right) - \sigma^2\right)\right)\right)\right)$$
$$+\,2\,\alpha^2\,\gamma_2\,\sigma\left(y_{22} + 54\,\gamma_1^4\left(-3 + 2\,\gamma_2^4\eta - 6\,\eta^3 + \gamma_2^2\left(3 - 3\,\eta + 6\,\eta^3\right)\right) + 3\,\gamma_1^2\left(324\,\eta^4 + 16\,\gamma_2^6(-4 + 3\,\eta)\right.\right.$$
$$\left.\left.\left.-9\,\gamma_2^2\left(14 + \eta^2\left(15 + 4\,\eta + 72\,\eta^2 - 3\,\sigma^2\right)\right) + 3\,\gamma_2^4\left(62 - 24\,\eta + 33\,\eta^2 + 10\,\eta^3 + 108\,\eta^4 - 16\,\sigma^2\right)\right)\right)\right),$$

$$\begin{aligned}
x_{14} = {} & 24\,\alpha^3\,\gamma_2^2\,\epsilon\,\eta^{19/2}\,\bigl(y_{27}+\alpha^2\,\bigl(y_{23}+y_9\,\alpha^3\,\gamma_2^2+\alpha^5\,\gamma_2^4\,\bigl(y_{13}+9\,\gamma_2^2\,\eta\,\bigl(3+4\,\eta+\eta^2+\eta^3+2\,\eta^4\bigr)\,\sigma^2\bigr)\\
&+\alpha^4\,\gamma_2^3\sigma\,\bigl(y_8+\gamma_2^2\,\bigl(9\,\gamma_1^2\,\bigl(5+24\,\eta+6\,\eta^2+\eta^3+12\,\eta^4\bigr)+9\,\eta\,\bigl(-3\,\bigl(8+5\,\eta^3\bigr)+\sigma^2\,\bigl(5+4\,\eta+2\,\eta^3+8\,\eta^4\bigr)\bigr)\\
&+\gamma_2^2\,\bigl(36+3\,\eta\,\bigl(56+12\,\eta+3\,\eta^2+37\,\eta^3\bigr)-8\,\eta\,\bigl(4+\eta^2(-3+2\,\eta)\bigr)\,\sigma^2\bigr)\bigr)\bigr)+\alpha^3\,\gamma_2^2\,\bigl(324\,\eta^5\,\sigma^2\\
&+4\,\gamma_2^6\,\bigl(57+9\,\eta+42\,\eta^3+4\,\bigl(-4-8\,\eta+6\,\eta^2+\eta^3+2\,\eta^4\bigr)\,\sigma^2\bigr)+6\,\gamma_2^4\,\bigl(-18\,\bigl(3+2\,\eta^3\bigr)\\
&+\sigma^2\,\bigl(12+58\,\eta+33\,\eta^3+50\,\eta^4+54\,\eta^5\bigr)-8\,\eta^3\,\sigma^4\bigr)-27\,\gamma_2^2\,\eta\,\sigma^2\,\bigl(8+\eta^2\,\bigl(9+12\,\eta(1+2\,\eta)-\sigma^2\bigr)\bigr)\\
&+3\,\gamma_1^2\,\bigl(-72\,\eta\,\bigl(1+2\,\eta^3\bigr)\,\sigma^2+\gamma_2^4\,\bigl(96+36\,\eta+75\,\eta^3-16\,\bigl(2+\eta^2(\eta-3)\bigr)\,\sigma^2\bigr)+3\,\gamma_2^2\,\bigl(-48-33\,\eta^3\\
&+\bigl(11+4\,\eta\,\bigl(6+\eta^2+12\,\eta^3\bigr)\bigr)\,\sigma^2\bigr)\bigr)\bigr)+\alpha\,\bigl(y_{24}+27\,\gamma_1^4\,\eta\,\bigl(16\,\gamma_2^6+108\,\eta^2+\gamma_2^4\,\bigl(57+108\,\eta^2-16\,\sigma^2\bigr)\\
&+9\,\gamma_2^2\,\bigl(-9-24\,\eta^2+\sigma^2\bigr)\bigr)+4\,\gamma_2^2\,\bigl(y_{25}+9\,\gamma_2^4\,\bigl(52-27\,\eta-166\,\eta^3+(-12-5\,\eta+28\,\eta^2+90\,\eta^3+8\,\eta^4)\sigma^2\bigr)\bigr)\bigr)\\
&+2\,\alpha^2\,\gamma_2\,\sigma\,\bigl(y_{26}+2\,\gamma_2^2\,\bigl(8\,\gamma_2^6(3\,\eta-8)+27\,\eta\,\bigl(19\,\eta^3-4\bigr)-9\,\eta\,\bigl(-2+\eta^2(9+2\eta)\bigr)\,\sigma^2\\
&+\gamma_2^4\,\bigl(147+3\,\eta(-42+57\,\eta+32\,\eta^2+159\,\eta^3)-8\,\eta(-4+9\,\eta-7\,\eta^2+4\,\eta^3)\sigma^2\bigr)\\
&+9\,\gamma_2^2\,\bigl(-9+\eta\,\bigl(26-6\,\sigma^2+\eta\,\bigl(-2\,\eta\,(6+55\,\eta)+\eta(7+6\,\eta)\sigma^2+3\,\bigl(\sigma^2-9\bigr)\bigr)\bigr)\bigr)\bigr)\bigr)\bigr)\bigr)\,,
\end{aligned}$$

$$\begin{aligned}
x_{15} = {} & 24\,\alpha^4\,\gamma_2^2\,\epsilon\,\eta^{15/2}\,y_1\,\bigl(72\,\alpha\,y_2\,\bigl(\gamma_2^2-1\bigr)\,\bigl(3-12\eta^3+\gamma_2^2\,\bigl(13\eta^3-7\bigr)\bigr)+y_{10}+\alpha^5\,\gamma_2^4\,\bigl(3\,\bigl(y_2+6\,\eta^2\,\bigl(\gamma_2^2-1\bigr)\bigr)\,\bigl(2+\eta^3\bigr)\\
&+\eta\,\bigl(8\gamma_2^2\,\bigl(2{+}\eta^3\bigr){+}3\,\bigl({-}11{+}2\eta{-}4\eta^3{+}\eta^4\bigr)\bigr)\,\sigma^2\bigr){+}\alpha^4\gamma_2^3\sigma\,\bigl(16\gamma_2^4(1{+}\eta)\,\bigl(2{+}\eta^3\bigr)\,{+}3\gamma_1^2\,\bigl(8\gamma_2^2\,\bigl(2{+}\eta^3\bigr)\,{-}3\,\bigl(11{+}4\eta^3\bigr)\bigr)\\
&+\gamma_2^2\,\bigl(-66+3\,\eta(-32+12\,\eta-8\,\eta^2+11\,\eta^3+24\eta^4)-16\,\eta\,\bigl(2+\eta^3\bigr)\,\sigma^2\bigr)+9\eta\,\bigl(8-4\eta-8\eta^4+\sigma^2+\eta^3\,\bigl(\sigma^2-5\bigr)\bigr)\bigr)\\
&+4\alpha^2\,\gamma_2\sigma\,\bigl(3\,\gamma_1^2\,\bigl(-27-3\,\gamma_2^2\,\bigl(\eta^3-19\bigr)+8\,\gamma_2^4\,\bigl(\eta^3-4\bigr)\bigr)+2\,\bigl(8\,\gamma_2^6\,\bigl(\eta^3-4\bigr)+\gamma_2^4\,\bigl(57-63\,\eta-3\,\eta^3+117\,\eta^4\\
&-16\,\eta\,\sigma^2\,\bigl(\eta^3-1\bigr)\bigr)+9\,\gamma_2^2\,\bigl(-3+10\,\eta-25\,\eta^4+3\,\eta\,\sigma^2\,\bigl(\eta^3-1\bigr)\bigr)+9\,\eta\,\bigl(-3+\sigma^2-\eta^3\,\bigl(\sigma^2-12\bigr)\bigr)\bigr)\bigr)\\
&+\alpha^3\,\gamma_2^2\,\bigl(2\,\bigl(16\,\gamma_2^6\,\bigl(2+\eta^3\bigr)+54\,\eta\,\bigl(3\eta^4-\sigma^2\bigr)+3\,\gamma_2^2\,\bigl(-3\,\bigl(-8+5\,\eta^3+36\,\eta^5\bigr)+\bigl(3+38\,\eta+3\,\eta^3-2\eta^4\bigr)\,\sigma^2\bigr)\\
&+\gamma_2^4\,\bigl(-96+33\eta^3+162\eta^5{+}16\,\bigl(-2+\eta(\eta-2)\,\bigl(2+\eta{+}\eta^2\bigr)\bigr)\,\sigma^2\bigr)\bigr){+}\,3\gamma_1^2\,\bigl(16\gamma_2^4\,\bigl(2+\eta^3\bigr){+}\,9\,\bigl(8+\sigma^2{+}\eta^3\,\bigl(\sigma^2-5\bigr)\bigr)\\
&-\gamma_2^2\,\bigl(32\,\bigl(3+\sigma^2\bigr)+\eta^3\,\bigl(16\sigma^2-33\bigr)\bigr)\bigr)\bigr)\bigr)\,,
\end{aligned}$$

$$\begin{aligned}
x_{16} = {} & 24\,\alpha^4\,\gamma_2^2\,\epsilon\,\eta^{17/2}\,y_1\,\bigl(72\,\alpha\,y_2\,\bigl(\gamma_2^2-1\bigr)\,\bigl(6-15\,\eta^3+\gamma_2^2\,\bigl(13\,\eta^3-7\bigr)\bigr)+y_{10}+\alpha^5\,\gamma_2^4\,\bigl(3\,\bigl(y_2+6\,\eta^2\,\bigl(\gamma_2^2-1\bigr)\bigr)\,\bigl(2+\eta^3\bigr)\\
&+\eta\,\bigl(-9+6\,\eta+3\,\eta^4+8\,\gamma_2^2\,\bigl(2+\eta^3\bigr)\bigr)\,\sigma^2\bigr)+\alpha^4\,\gamma_2^3\,\sigma\,\bigl(16\,\gamma_2^4(1+\eta)\,\bigl(2+\eta^3\bigr)+3\,\gamma_1^2\,\bigl(-9+8\,\gamma_2^2\,\bigl(2+\eta^3\bigr)\bigr)\\
&+\gamma_2^2\,\bigl(-18-48\,\eta+36\,\eta^2+57\,\eta^4+72\,\eta^5-16\,\eta\,\sigma^2\,\bigl(2+\eta^3\bigr)\bigr)+9\,\eta\,\bigl(-4\,\eta-8\,\eta^4+\sigma^2+\eta^3\,\bigl(\sigma^2-9\bigr)\bigr)\bigr)\\
&+4\,\alpha^2\,\gamma_2\,\sigma\,\bigl(3\,\gamma_1^2\,\bigl(-27\,\eta^3+8\,\gamma_2^4\,\bigl(\eta^3-4\bigr)+3\,\gamma_2^2\,\bigl(11+7\,\eta^3\bigr)\bigr)+2\,\bigl(8\,\gamma_2^6\,\bigl(\eta^3-4\bigr)+9\,\gamma_2^2\eta\,\bigl(13-3\,\eta^2-28\,\eta^3\\
&+3\,\sigma^2\,\bigl(\eta^3-1\bigr)\bigr)+\gamma_2^4\,\bigl(33+3\,\eta\,\bigl(-21+\eta^2(7+39\,\eta)\bigr)-16\,\eta\,\sigma^2\,\bigl(\eta^3-1\bigr)\bigr)+9\,\eta\,\bigl(-6+\sigma^2-\eta^3\,\bigl(\sigma^2-15\bigr)\bigr)\bigr)\bigr)\\
&+\alpha^3\,\gamma_2^2\,\bigl(2\,\bigl(16\,\gamma_2^6\,\bigl(2+\eta^3\bigr)+54\,\eta^4\,\bigl(3\,\eta-\sigma^2\bigr)+3\,\gamma_2^2\,\bigl(-27\,\bigl(\eta^3+4\,\eta^5\bigr)+\bigl(3+22\,\eta+3\,\eta^3+14\,\eta^4\bigr)\,\sigma^2\bigr)\\
&+\gamma_2^4\,\bigl(-48+57\,\eta^3+162\,\eta^5+16\,\bigl(-2+\eta\,(\eta-2)\,\bigl(2+\eta+\eta^2\bigr)\bigr)\,\sigma^2\bigr)\bigr)+3\,\gamma_1^2\,\bigl(16\,\gamma_2^4\,\bigl(2+\eta^3\bigr)+9\,\bigl(\sigma^2+\eta^3\,\bigl(\sigma^2-9\bigr)\bigr)\\
&-\gamma_2^2\,\bigl(48+32\,\sigma^2+\eta^3\,\bigl(16\,\sigma^2-57\bigr)\bigr)\bigr)\bigr)\bigr)\,,
\end{aligned}$$

$$\begin{aligned}
x_{17} = {} & -8\,\alpha^4\,\gamma_2^2\,\epsilon\,\eta^8\,\bigl(y_{34}-y_{31}-y_{35}-144\,y_2\,y_5\,\eta+\alpha^2\,\bigl(3\,y_{32}-8748\,\gamma_1^4\,\eta+\gamma_2^2\,\bigl(27\,\bigl(9\,\gamma_1^4\,\eta\,\bigl(25+32\,\eta^3\bigr)\\
&-4\,\gamma_2^6\,\bigl(28-19\,\eta+32\,\eta^3+40\,\eta^4+51\,\eta^6\bigr)-3\,\gamma_2^2\,\bigl(16+\eta\,\bigl(48+48\,\eta^2+68\,\eta^5+3\,\gamma_1^4\,\bigl(40\,\eta^3-19\bigr)\bigr)\bigr)\bigr)\\
&+4\,\gamma_2^4\,\bigl(40+\eta\,\bigl(25+6\,\gamma_1^4\,\bigl(3\,\eta^3-2\bigr)+2\,\eta^2\,\bigl(34+16\,\eta+51\,\eta^3\bigr)\bigr)\bigr)\bigr)-6\,\alpha\,\gamma_2\,\bigl(108\,\eta\,\bigl(1+6\,\eta+3\,\eta^3+2\,\eta^6\bigr)\\
&-9\,\gamma_1^4\,\eta\,\bigl(-87+6\,\eta^2\,\bigl(6-5\,\eta+3\,\eta^3\bigr)+4\,\gamma_2^2\,\bigl(8+7\,\eta^3\bigr)\bigr)-16\,\gamma_2^6\,\bigl(-8+8\,\eta-6\,\eta^2-4\,\eta^3+7\,\eta^4+9\,\eta^5+6\,\eta^6\bigr)\\
&-6\,\gamma_2^2\,\bigl(-8+\eta\,\bigl(96+\alpha^2\,\bigl(11+\eta^3\,\bigl(8+6\,\eta+8\eta^3+3\,\eta^4\bigr)\bigr)+\eta\,(75+28\,\eta+102\,\eta^2+96\,\eta^3+25\,\eta^4+72\,\eta^5)\bigr)\bigr)\\
&+3\gamma_1^2\,\bigl(3\,\bigl(8+\eta\,\bigl(-72+\eta\,\bigl(-75-34\eta-96\eta^3{-}28\eta^4{+}216\eta^5\bigr)\bigr)\bigr)-8\gamma_2^4\,\bigl(-8+16\eta-6\eta^2{-}4\eta^3{+}14\eta^4+9\eta^5+6\eta^6\bigr)\\
&+3\,\gamma_2^2\,\bigl(-28+\alpha^2\,\bigl(3+2\,\eta^2+11\,\eta^3-24\,\eta^4+4\,\eta^5+4\,\eta^6-12\,\eta^7\bigr)+\eta\,\bigl(116-57\,\eta+20\,\eta^2+40\,\eta^3+120\,\eta^4\\
&+4\,\eta^5\,(11-27\,\eta)\bigr)\bigr)\bigr)+\gamma_2^4\,\bigl(6\,(-28+100\,\eta-57\,\eta^2+14\,\eta^3+68\,\eta^4+120\,\eta^5+41\,\eta^6+36\,\eta^7)\\
&+\alpha^2(14+78\,\eta+12\,\eta^2+62\,\eta^3+78\,\eta^4+60\,\eta^5+23\,\eta^6+60\,\eta^7+18\,\eta^8)\bigr)\bigr)\,\sigma+3\,\bigl(-1296\eta^3\,\bigl(-3+\eta^3\bigr)\\
&+64\,\gamma_2^6\,\bigl(4-\eta+30\,\eta^2-4\,\eta^3+\eta^4+18\,\eta^5\bigr)+144\,\gamma_2^2(1+2\,\eta\,(1-\eta-29\,\eta^2-\eta^3+4\,\eta^4+9\,\eta^5))\\
&+\gamma_2^4\,\bigl(-432+\eta\,\bigl(48(1-\eta)\,(-5-29\,\eta+70\,\eta^2+75\,\eta^3+27\,\eta^4\bigr)+\alpha^4\,\bigl(-2+\eta\,(-4+\eta\,(-6+\eta\,(-50\\
&+\eta\,(68+\eta\,(-12+5\,\eta\,(-4+7\,\eta)))))))\bigr)\bigr)\bigr)\,\sigma^2+2\,\alpha\,\gamma_2\,\eta\,\bigl(108\,\bigl(1+\eta^2\,\bigl(-1-\eta+4\,\eta^3\bigr)\bigr)\\
&+4\,\gamma_2^4\,\bigl(12(4-2\,\eta+15\,\eta^2-4\,\eta^3+2\,\eta^4+9\,\eta^5)+\alpha^2(-16+6\,\eta+24\,\eta^2-4\,\eta^3+3\,\eta^4+21\,\eta^5+2\eta^6)\bigr)\bigr)\\
&+3\,\gamma_2^2\,\bigl(-12\,\bigl(9+\eta\,\bigl(-2+\eta\,\bigl(12+\eta\,\bigl(-9+2\,\eta+24\,\eta^2\bigr)\bigr)\bigr)\bigr)+\alpha^2\,\bigl(19+\eta\,\bigl(-18+6\,\gamma_1^2\,\bigl(2+\eta^3\bigr)\\
&-87\,\eta+\eta^2+36\,\eta^3-30\,\eta^4-2\,\eta^5+18\,\eta^6\bigr)\bigr)\bigr)\bigr)\,\sigma^3-144\,y_3\,\eta^3\,\sigma^4\bigr)\bigr)\bigr)\,,
\end{aligned}$$

$$
\begin{aligned}
x_{18} = {} & 288\alpha^3\gamma_2^2\epsilon\eta^8 \left(\alpha\left(-27+9\left(10+\alpha^2\right)\gamma_2^2-3\left(21+5\alpha^2\right)\gamma_2^4+4\alpha^2\gamma_2^6\right)-2\gamma_2\left(18\left(\gamma_2^2-1\right)+\alpha^2\left(9-15\gamma_2^2+8\gamma_2^4\right)\right)\sigma\right.\\
& \left. +\alpha y_3\sigma^2\right) y_1^2,\\
x_{19} = {} & 288\alpha^3\gamma_2^2\epsilon\eta^{10}\left(\alpha^3\gamma_2^4\left(4\gamma_2^2-3\right)-9\alpha\left(9-16\gamma_2^2+7\gamma_2^4\right)-36\gamma_2\sigma\left(\gamma_2^2-1\right)+\alpha^2\gamma_2\sigma\left(9+6\gamma_2^2-16\gamma_2^4\right)+\alpha y_3\sigma^2\right)y_1^2,\\
x_{20} = {} & 24\,\alpha^3\,\gamma_2^2\,\epsilon\,\eta^{17/2}\Big(y_{40}+\alpha^2\Big(324\,\eta\left(\gamma_2^2-1\right)^2\left(21\,\gamma_1^2\,\gamma_2^2\,\eta^2+17\,\gamma_2^4\,\eta^2+3\,\gamma_1^4\left(9+4\,\gamma_2^2+9\,\eta^2\right)\right)-y_{37}\\
& +3\,\alpha^5\,\gamma_2^5\sigma\left(\gamma_2^2\left(4+13\,\eta+2\,\eta^3+5\,\eta^4\right)+\eta\left(2+\eta\left(4+\eta\left(3+\eta+2\,\eta^2\right)\right)\right)\sigma^2\right)\\
& +\alpha^4\,\gamma_2^4\left(6\,\gamma_2^2\,(2+15\,\eta+24\,\eta^2+4\,\eta^3+15\,\eta^4+18\,\eta^5)\sigma^2+\gamma_2^4\left(60+21\,\eta^3-8\,\eta\left(2+\eta^2(\eta-3)\right)\sigma^2\right)\right.\\
& -3\eta\sigma^2\left(12+6\eta\left(6+\eta\left(3+4\eta+6\eta^2\right)\right)+\eta\left(2+\eta^3\right)\sigma^2\right)+9\gamma_1^2\left(3\gamma_2^2\left(3+\eta^3\right)+\left(2+\eta\left(8+\eta\left(6+\eta+4\eta^2\right)\right)\right)\sigma^2\right)\Big)\\
& +\alpha^2\,\gamma_2^2\left(12\,\gamma_2^2\left(54\,\eta^3+8\,\gamma_2^6\left(2+\eta+3\,\eta^3\right)+\gamma_2^4\left(-9+6\,\eta+33\,\eta^3\right)-3\,\gamma_2^2\left(5+6\,\eta+37\,\eta^3\right)\right)+y_{38}\right.\\
& +3\gamma_1^2\left(12\left(27\eta^3+4\gamma_2^6\left(2+2\eta+3\eta^3\right)+3\,\gamma_2^4\left(-3+2\,\eta+4\eta^3\right)-3\,\gamma_2^2\left(1+6\,\eta+17\,\eta^3\right)\right)+\left(-216\,\eta\left(1+2\,\eta+\eta^3\right)\right.\right.\\
& \left.\left.\left.-16\gamma_2^4\left(2+\eta^2\left(3+\eta\right)\right)+3\,\gamma_2^2\left(13+84\,\eta+144\,\eta^2+5\,\eta^3+72\,\eta^4\right)\right)\sigma^2\right)\right)+\alpha^3\,\gamma_2^3\,\sigma\left(324\,\eta^4+27\,\gamma_1^4\left(4+3\,\eta+2\,\eta^3\right)\right.\\
& +16\,\gamma_2^6(-2+6\,\eta+6\,\eta^2-\eta^3+9\,\eta^4)+18\,\gamma_2^4(8+8\,\eta+4\,\eta^2+9\,\eta^3+20\,\eta^4)-36\,\gamma_2^2\left(2+7\,\eta+6\,\eta^2+4\,\eta^3+23\,\eta^4\right)\\
& +\eta\left(-108\,\eta\left(1+2\,\eta+\eta^3\right)-8\,\gamma_2^4\left(4+\eta^2(3+2\,\eta)\right)+3\,\gamma_2^2(17+48\,\eta+72\,\eta^2+7\,\eta^3+36\,\eta^4)\right)\sigma^2\\
& -6\,\gamma_1^2\left(18+4\,\gamma_2^4\left(2+(-6+\eta)\eta^2\right)-3\,\gamma_2^2(11+42\,\eta+6\,\eta^2+13\,\eta^3+36\,\eta^4)+3\,\eta\left(6\left(6+\eta\left(3+2\,\eta+6\,\eta^2\right)\right)\right.\right.\\
& \left.\left.\left.+\left(2+\eta^3\right)\sigma^2\right)\right)\right)-12\alpha\gamma_2\sigma\left(9\gamma_1^4\left(2\gamma_2^4\eta+9\left(1+2\eta+\eta^3\right)-9\gamma_2^2\left(1+2\eta+\eta^3\right)\right)+y_{39}+\gamma_2^2\left(8\gamma_2^6(8+\eta(1+6(-3+\eta)\eta))\right.\right.\\
& -9\eta\left(-6+36\eta+39\eta^3+2\sigma^2-8\eta^2\sigma^2\right)+\gamma_2^4\left(-3(57-18\eta+12\eta^2+41\eta^3+117\eta^4)+8\eta(-4+\eta(2+9\eta))\sigma^2\right)\\
& \left.\left.+3\,\gamma_2^2\left(41+\eta\left(18\left(\sigma^2-2\right)+\eta\left(168+25\,\eta+234\,\eta^2-4\,\sigma^2(1+12\,\eta)\right)\right)\right)\right)\right)\Big)\Big),\\
x_{21} = {} & 24\alpha^3\gamma_2^2\epsilon\eta^{19/2}\Big(y_{44}+\alpha^2\Big(324\eta^3\left(\gamma_2^2-1\right)^2\left(27\gamma_1^4+21\gamma_1^2\gamma_2^2+17\gamma_2^4\right)-y_{37}+3\alpha^5\gamma_2^5\sigma\left(\gamma_2^2\left(4+21\eta+2\eta^3+9\eta^4\right)\right.\\
& \left.+\eta\left(2+\eta\left(4+\eta\left(3+\eta+2\,\eta^2\right)\right)\right)\sigma^2\right)+y_{41}+\alpha^3\,\gamma_2^3\,\sigma\left(324\,\eta^4+27\,\gamma_1^4\left(4+3\,\eta+2\,\eta^3\right)\right.\\
& +16\gamma_2^6(-2+6\eta+6\eta^2-\eta^3+9\eta^4)-36\gamma_2^2(2+10\eta+6\eta^2+4\eta^3+29\eta^4)+6\gamma_2^4(32+48\eta+36\eta^2+31\eta^3+96\eta^4)\\
& +\eta\left(-108\eta\left(1+2\,\eta+\eta^3\right)-8\,\gamma_2^4\left(4+\eta^2(3+2\eta)\right)+3\,\gamma_2^2(17+48\,\eta+84\,\eta^2+7\,\eta^3+36\,\eta^4)\right)\sigma^2\\
& \left.-6\gamma_1^2\left(18+4\gamma_2^4\left(2+\eta^2(\eta-6)\right)-9\gamma_2^2(5+14\eta+6\eta^2+5\eta^3+12\eta^4)+3\eta\left(6\left(6+\eta\left(3+2\eta+6\eta^2\right)\right)+\sigma^2\left(2+\eta^3\right)\right)\right)\right)\\
& +\alpha^2\gamma_2^2\left(12\gamma_2^2\left(54\eta^3+8\gamma_2^6\left(2+\eta+3\eta^3\right)+3\gamma_2^4\left(5+6\eta+23\eta^3\right)-3\gamma_2^2\left(11+6\eta+49\eta^3\right)\right)+y_{42}+12\gamma_2^2\left(3-4\gamma_2^2\right)\eta^3\sigma^4\right.\\
& +27\,\gamma_1^4\left(8\,\gamma_2^4\,\eta-18\left(2+\eta+2\,\eta^3\right)+18\,\gamma_2^2\left(2+\eta+2\,\eta^3\right)-\left(2+\eta^3\right)\sigma^2\right)+3\,\gamma_1^2\left(324\,\eta^3+48\,\gamma_2^6\left(2+2\,\eta+3\,\eta^3\right)\right.\\
& -216\,\eta\left(1+2\,\eta+\eta^3\right)\sigma^2+4\,\gamma_2^4\left(9+54\,\eta+90\,\eta^3-4\left(2+\eta^2(3+\eta)\right)\sigma^2\right)+3\,\gamma_2^2\left(-12\left(4+6\,\eta+23\,\eta^3\right)\right.\\
& \left.\left.\left.+(13+84\eta+168\eta^2+5\eta^3+72\eta^4)\sigma^2\right)\right)\right)-12\alpha\gamma_2\sigma\left(9\gamma_1^4\left(2\gamma_2^4\eta+9\left(1+2\eta+\eta^3\right)-3\gamma_2^2\left(3+7\eta+3\eta^3\right)\right)+y_{43}\right.\\
& +\gamma_2^2\left(8\,\gamma_2^6\left(8+\eta\left(1+6\,\eta\left(\eta-3\right)\right)\right)+\gamma_2^4\left(-3(41-14\,\eta+84\,\eta^2+41\,\eta^3+117\,\eta^4)+8\,\eta\left(-4+\eta\left(2+9\,\eta\right)\right)\sigma^2\right)\right.\\
& \left.\left.-9\eta\left(2\left(\sigma^2-6\right)+\eta\left(36+39\eta^2-8\eta\sigma^2\right)\right)+3\gamma_2^2\left(23+\eta\left(18\left(\sigma^2-3\right)+\eta\left(240+25\eta+234\eta^2-4(1+12\eta)\sigma^2\right)\right)\right)\right)\right)\Big)\Big),\\
x_{22} = {} & -16\,y_1\,\alpha^5\,\gamma_2^2\,\epsilon\,\eta^8\Big(-y_{45}+12\,y_4\,\eta\left(\eta^3-1\right)+4\,\alpha^5\,\gamma_2^7\,\sigma\left(2+\eta^3\right)^2+\alpha^6\,\gamma_2^6\left(2+\eta^3\right)^2\left(\sigma^2-3\right)\\
& +\alpha^4\,\gamma_2^4\left(\left(2+\eta^3\right)\left(4\,\gamma_2^4\left(2+\eta^3\right)+9\left(2+13\,\eta^3\right)\right)-9\left(-1+\eta^2\left(8+3\,\eta+4\,\eta^3+\eta^4\right)\right)\sigma^2\right)\\
& +\alpha^2\gamma_2^2\left(9\gamma_2^2\left(40-42y_2\eta+16\eta^3-39y_2\eta^4+160\eta^6+2\left(-6+4\gamma_1^2\left(\eta+2\eta^4\right)-3\eta^3(-4+\eta(1+2\eta(3+\eta)))\right)\sigma^2\right)\right.\\
& +9\left(y_2\left(42\eta+39\eta^4\right)-12\left(1+\eta^3+7\eta^6\right)+\left(4+\eta^2\left(-6-8\eta-9\eta^2\gamma_1^2+24\eta^3+4\eta^4\right)\right)\sigma^2\right)+4\gamma_2^4\left(3y_2\eta\left(2+\eta^3\right)\right.\\
& \left.\left.-9\left(7+\eta^3+19\,\eta^6\right)+4\left(4+3\,\eta+3\,\eta^2-8\,\eta^3+6\,\eta^4+6\,\eta^5+4\,\eta^6\right)\sigma^2\right)\right)\\
& +3\,\alpha^3\,\gamma_2^3\,\sigma\left(-12+\gamma_2^2\left(36+\eta\left(-48-3\,\eta\left(42+36\,\eta+8\,\eta^2+39\,\eta^3+12\,\eta^4\right)+8\,\eta\left(1+2\,\eta^3\right)\sigma^2\right)\right)\right.\\
& \left.+3\,\eta^2\left(42+\eta\left(32+\eta^2\left(39+8\,\eta-3\,\sigma^2\right)\right)\right)\right)+6\,\alpha\,\gamma_2\sigma\left(16\,\gamma_2^6\left(\eta+2\,\eta^4\right)+9\,\eta\left(6\,\eta\left(4-7\,\eta^3\right)+3\,\gamma_1^2\left(4\,\eta^3-1\right)\right.\right.\\
& \left.+2\,\eta\,\sigma^2\left(\eta^3-1\right)\right)+2\,\gamma_2^4\left(-12+12\,\gamma_1^2\left(\eta+2\,\eta^4\right)+\eta^2\left(-3(-39-8\,\eta+18\,\eta^2+57\,\eta^3+4\,\eta^4)+16\,\sigma^2\left(\eta^3-1\right)\right)\right)\\
& \left.+6\,\gamma_2^2\left(4+\eta\left(-3+\eta\left(-75+9\,\sigma^2+\eta\left(-8+\eta\left(-27\,\gamma_1^2+4\left(3+\eta\left(30+\eta\right)\right)-9\,\eta\,\sigma^2\right)\right)\right)\right)\right)\right)\Big),
\end{aligned}
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\begin{aligned}
x_{23} = {} & -16\,\alpha^5\,\gamma_2^3\,\epsilon\,\eta^9 \Big(-y_{49} + 2\,\alpha^6\,\gamma_2^7\left(\gamma_2^2-3\right)\left(2+\eta^3\right)^2 + \alpha^7\,\gamma_2^8\left(2+\eta^3\right)^2\sigma + 3\,\alpha^2\,\gamma_2\left(24\,\gamma_2^8\,\eta\left(8+\eta+6\,\eta^3\right)\right.\\
& + 81\eta\left(6\gamma_1^4 - \left(\gamma_1^4 - 16\gamma_1^2\eta + 12\eta^2\right)\sigma^2\right) + 2\gamma_2^6\left(-9\left(14+\eta\left(71-4\gamma_1^2\left(4+\eta+3\eta^3\right)+4\eta\left(1+\eta\left(4+10\eta+3\eta^3\right)\right)\right)\right)\right.\\
& +2(16-8\,\eta-21\,\eta^2+68\,\eta^3+35\,\eta^4+72\,\eta^5)\sigma^2\big) + y_{47} + 3\,\gamma_2^4\left(6\left(20+\eta\left(61+3\,\gamma_1^4\,\eta+34\,\eta^2+32\,\eta^3+24\,\eta^5\right.\right.\right.\\
& -6\gamma_1^2\left(17+2\eta+10\eta^3\right)\big)\big) - \left(36+\eta\left(19-4\gamma_1^2\left(-4+\eta\left(3+42\eta+4\eta^2\right)\right)+4\eta(78+102\eta+47\eta^2+48\eta^3)\right)\right)\sigma^2\\
& +8\eta^4\sigma^4\big)\big) + \alpha^4\gamma_2^3\left(2\gamma_2^4\left(-9\left(13+8\eta+31\eta^3+4\eta^4+10\eta^6\right)+(-32+12\eta+96\eta^2+10\eta^3+15\eta^4+84\eta^5+4\eta^6)\sigma^2\right)\right.\\
& + 3\gamma_2^2\left(6\left(11+26\eta^3+8\eta^6\right)+\left(7+6\gamma_1^2\eta\left(2+\eta^2(3+\eta)\right)-\eta^2(138+59\eta+18\eta^2+60\eta^3+2\eta^4)\right)\sigma^2+3\eta^4\sigma^4\right)\\
& -27\,\eta^2\,\sigma^2\left(6\,\gamma_1^2+\eta\left(\sigma^2-6\right)\right)\big) + 3\,\alpha^5\,\gamma_2^4\,\sigma\left(-9\,\eta^3\,\sigma^2+\gamma_2^2\left(-25+\eta^2\left(4\left(\sigma^2-6\right)+\eta\left(-37+\eta\left(-2\,\eta\,(6+5\,\eta)\right.\right.\right.\right.\right.\\
& +\sigma^2\,(3+2\,\eta)\big)\big)\big)\big)\big)\big) + y_{48} + 3\,\alpha^3\,\gamma_2^2\,\sigma\left(4\,\gamma_2^6\left(-16+32\,\eta+27\,\eta^2-2\,\eta^3+28\,\eta^4+18\,\eta^5+12\,\eta^6\right)\right.\\
& - 27\,\eta\left(\gamma_1^2-4\,\eta\right)\left(3\,\gamma_1^2+2\,\eta\,\sigma^2\right)+\gamma_2^4\left(84+\eta\left(12\,\gamma_1^2\,(16+3\,\eta+3\,\eta^2+14\,\eta^3)\right.\right.\\
& -3(83+8\,\eta(30+7\,\eta+5\,\eta^2+15\,\eta^3+4\,\eta^4))+4\,\eta\,(-4+3\,\eta+21\,\eta^2+4\,\eta^3)\sigma^2\big)\big)\\
& + 3\,\gamma_2^2\left(-8+\eta\left(9\,\gamma_1^4\,\eta+2\,\eta\left(105+23\,\eta+48\,\eta^3+8\,\eta^4-(7+36\,\eta+15\,\eta^2+2\,\eta^3)\sigma^2\right)\right.\right.\\
& -6\,\gamma_1^2\left(20+\eta\left(18+\eta\left(6+10\,\eta-\sigma^2\right)\right)\right)\big)\big)\big)\Big)\,,
\end{aligned}
$$

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\begin{aligned}
x_{24} = {} & 16\,y_1\,\alpha^4\,\gamma_2^3\,\epsilon\,\eta^8\Big(y_{50}+\alpha^7\,\gamma_2^5\left(3+\gamma_2^2\right)\left(2+\eta^3\right)^2+9\,\alpha^5\,\gamma_2^5\left(-11+2\,\eta\left(2+\gamma_1^2\right)+9\,\eta^3+\eta^4\,\left(2+\gamma_1^2\right)+8\,\eta^6\right)\\
& - 1296\,\gamma_2^2\left(\gamma_2^2-1\right)\sigma+6\,\alpha^4\,\gamma_2^2\left(-18\,\eta^2\left(3+2\,\eta+3\,\eta^3+\eta^4\right)+3\,\gamma_2^2(-3+4\,\eta+9\,\eta^2+5\,\eta^3+2\,\eta^4+12\,\eta^5+4\,\eta^6)\right.\\
& +2\,\gamma_2^4(-4+2\,\eta+12\,\eta^2+4\,\eta^3+\eta^4+9\,\eta^5+3\,\eta^6)\big)\,\sigma+18\,\alpha\,\gamma_2\left(18\left(\gamma_2^2-1\right)\left(3-2\,y_2\,\eta+\gamma_2^2(2\,y_2\,\eta-7)\right)\right.\\
& + \left(9\left(2+\eta\left(\gamma_1^2-8\,\eta\right)\right)-8\,\gamma_2^4\left(9\,\eta^2+\eta-4\right)+6\,\gamma_2^2\left(-9+\eta-2\,\gamma_1^2\,\eta+24\,\eta^2\right)\right)\sigma^2\big)+3\,\alpha^3\,\gamma_2\left(48\,\gamma_2^6\left(1+3\,\eta^3\right)\right.\\
& - 108\,\eta\left(1+\eta^3\right)\left(y_2+\eta\,\sigma^2\right)+3\,\gamma_2^2\left(96+156\,\eta^3+6\,y_2\,\eta\left(3+4\,\eta^3\right)+\left(5+4\,\eta+24\,\eta^2+\eta^3+2\,\eta^4+24\,\eta^5\right)\sigma^2\right)\\
& +2\,\gamma_2^4\left(6\left(-30-51\,\eta^3+y_2\eta\left(4+3\,\eta^3\right)\right)+\left(-16+2\,\eta+12\,\eta^2-8\,\eta^3+\eta^4+18\,\eta^5\right)\sigma^2\right)\big)\\
& + 18\,\alpha^2\,\sigma\left(2\,\gamma_2^6\left(2+3\,\eta^3\right)\left(-8+\eta(2+9\,\eta)\right)-54\,\eta\left(-3\,\eta^4+\gamma_1^2\left(1+\eta^3\right)\right)+3\,\gamma_2^2\left(-2+\eta\left(-12+12\,\gamma_1^2\left(1+\eta^3\right)\right.\right.\right.\\
& +\eta\left(12-2\eta\left(-1+6\eta+45\eta^2\right)+\sigma^2\right)\big)\big)+2\gamma_2^4\left(15+\eta\left(12+\gamma_1^2\left(6+9\eta^3\right)+\eta\left(3\eta(7+\eta(4+9\eta))-2\left(18+\sigma^2\right)\right)\right)\right)\big)\Big)\,,
\end{aligned}
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\begin{aligned}
x_{25} = {} & 8\,\alpha^4\,\gamma_2^2\,\epsilon\,\eta^9\Big(-36\,y_2\left(-y_5+9\,y_2\left(21-40\,\gamma_2^2+19\,\gamma_2^4\right)\right)\eta^2+4\,\alpha^7\,\gamma_2^9\left(2+\eta^3\right)^2\sigma+y_{54}\\
& + 6\,\alpha^5\,\gamma_2^3\,\sigma\left(-3\,\gamma_2^2\left(2-\eta^2\left(42+16\,\eta+39\,\eta^3+4\,\eta^4\right)+3\,\gamma_1^2\,\eta\left(8+\eta^2(3+4\,\eta)\right)\right)\right.\\
& +\gamma_2^2\left(-6+16\,\eta+42\,\eta^2+24\,\eta^3+8\,\eta^4+39\,\eta^5+6\,\eta^6\right)\big)+\eta^2\left(54\,\eta+9\,\gamma_1^2\left(\gamma_2^2\,\eta-3\right)-9\,\gamma_2^2\,(3+6\,\eta+2\,\eta^2+\eta^3)\right.\\
& +2\,\gamma_2^4(4+3\,\eta+3\,\eta^2+8\,\eta^3)\big)\,\sigma^2\big)+3\,\alpha^2\,\eta\left(-2916\,\gamma_1^4\left(\gamma_2^2-1\right)^2+3\,\eta\left(3\,\gamma_1^2\,\gamma_2+2\,\gamma_2^3\right)^2\left(117-117\,\gamma_2^2+4\,\gamma_2^4\right)\right.\\
& - 72y_2\gamma_2^2\eta^3\left(21-40\gamma_2^2+19\gamma_2^4\right)+\gamma_2^2\left(27\gamma_1^4\eta\left(16\gamma_2^2-9\right)+12\gamma_1^2\left(-18+18\eta^2(6+\eta)-27\gamma_2^2(-2+\eta+6\eta^2+2\eta^3)\right.\right.\\
& +16\gamma_2^4(-2+3\eta+3\eta^2+2\eta^3)\big)+4\left(-567\eta^3+16\gamma_2^6\left(-4+\eta\left(3+6\eta+4\eta^2\right)\right)+36\gamma_2^2(-1-2\eta+6\eta^2+31\eta^3+2\eta^4)\right.\\
& -9\,\gamma_2^4(-12-5\,\eta+36\,\eta^2+69\,\eta^3+8\,\eta^4)\big)\big)\,\sigma^2+12\,y_3\,\gamma_2^2\,\eta^3\,\sigma^4\big)+\alpha^6\,\gamma_2^4\Big(4\,\gamma_2^6\left(2+\eta^3\right)^2-27\,\eta^3\,\sigma^4\\
& +9\,\gamma_2^2\left(4+28\,\eta^3+13\,\eta^6-\left(-1+\eta^2(16+3\,\eta+3\,\eta^2+8\,\eta^3+\eta^4)\right)\sigma^2+\eta^4\,\sigma^4\right)\Big)+\alpha^4\,\gamma_2^2\left(-243\,\eta\left(\gamma_1^4-8\,\gamma_1^2\,\eta\right.\right.\\
& +4\eta^2\big)\,\sigma^2+4\gamma_2^6\left(-9\left(7+42\eta+3\eta^2+\eta^3+39\eta^4+19\eta^6\right)+(16+24\eta+33\eta^2+4\eta^3+57\eta^4+48\eta^5+16\eta^6)\sigma^2\right)\\
& + y_{52}+9\gamma_2^2\left(-12\left(1+\eta^3+7\eta^6\right)+\left(4+\eta^2(204+208\eta+117\eta^2+48\eta^3+4\eta^4)\right)\sigma^2-9\eta^4\sigma^4+9\gamma_1^4\eta^2\left(\sigma^2-3\right)\right.\\
& -18\gamma_1^2\eta\left(-14-13\eta^3+(2+12\eta+4\eta^2+\eta^3)\sigma^2\right)\big)\big)+6\alpha^3\gamma_2\sigma\left(486\gamma_1^2\left(\gamma_1^2-2\eta\right)\eta+8\gamma_2^8\eta(4+3\eta+3\eta^2+8\eta^3)\right.\\
& + y_{53}+3\,\gamma_2^4\left(8+\eta\left(18\,\gamma_1^4\,\eta+6(25+58\,\eta)+2\,\eta\left(\eta\,(109+4\,\eta\,(6+\eta\,(60+\eta)))\right)-9(-2+\eta\,(1+\eta)\,(1+2\eta))\sigma^2\right)\right.\\
& -3\,\gamma_1^2\left(72+132\,\eta+117\,\eta^2+36\,\eta^3-16\,\sigma^2\,\eta^2\right)\big)\big)\big)\Big)\,,
\end{aligned}
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\begin{aligned}
x_{26} = {}& 8\,\alpha^3\,\gamma_2^2\,\epsilon\,\eta^8\left(y_{58} + 72\,y_2\,\alpha\left(y_5 + 9\,y_2\,\gamma_2^2 - 9\,y_2\,\gamma_2^4\right)\eta - 2592\,y_2^2\,\gamma_2\,\eta\,\sigma\left(\gamma_2^2 - 1\right)\right.\\
&+ 3\,\alpha^7\,\gamma_2^4\,\eta\,\sigma^2\left(-\gamma_2^2(98 - 4\,\eta + 42\,\eta^2 + 98\,\eta^3 + 68\,\eta^4 + 12\,\eta^5 + 20\,\eta^6 + 35\,\eta^7) - \eta\left(4 - 9\,\eta + 4\,\eta^3 + \eta^6\right)\sigma^2\right) - y_{56}\\
&+ \alpha^5\,\gamma_2^2\left(36\,\gamma_2^2\left(27\,\eta^8 + \gamma_2^2\left(22 + 70\,\eta^3 + 7\,\eta^6 - 54\,\eta^8\right) + \gamma_2^4\left(-26 - 14\,\eta - 80\,\eta^3 - 4\,\eta^4 - 11\,\eta^6 + 27\,\eta^8\right)\right)\right.\\
&+ \left(-972\,\eta^3\left(\eta^5 - 1\right) + 9\,\gamma_2^2\,\eta\left(-16 - 144\,\eta + 129\,\eta^2 + 4\,\eta^3\left(41 + 24\,\eta^2 + 8\,\eta^3 + 54\,\eta^4\right)\right)\right.\\
&+ 8\,\gamma_2^6\left(-32 + \eta\left(-42 + 96\,\eta + 82\,\eta^2 - 21\,\eta^3 + 84\,\eta^4 + 31\,\eta^5 + 36\,\eta^6\right)\right)\\
&\left.\left.-3\,\gamma_2^4\left(8 + \eta\left(-204 - 24\,\eta + 995\,\eta^2 + 408\,\eta^3 + 240\,\eta^4 + 365\,\eta^5 + 192\,\eta^6 + 324\,\eta^7\right)\right)\right)\sigma^2\right.\\
&+ 27\,\gamma_1^4\left(-6\,\gamma_2^2\,\eta\left(7 + 2\,\eta^2\left(6 + \eta + 3\,\eta^3\right)\right) - \left(4 - 9\,\eta + 4\,\eta^3 + \eta^6\right)\sigma^2\right) + 6\,\gamma_1^2\left(-18\,\gamma_2^2\left(-11 - 2\,\eta^3\left(16 + \eta^3\right)\right.\right.\\
&\left.+\gamma_2^2\left(13 + \eta\left(14 + \eta^2\left(37 + 4\left(\eta + \eta^3\right)\right)\right)\right)\right) + \left(108\,\eta^2\left(-3 + 2\,\eta^2 + \eta^5\right)\right.\\
&+ 4\,\gamma_2^4\left(-16 + 6\,\eta + 48\,\eta^2 - 4\,\eta^3 + 3\,\eta^4 + 42\,\eta^5 + 2\,\eta^6\right) - 3\,\gamma_2^2\left(1 - 18\,\eta - 6\,\eta^2 + 7\,\eta^3 + 72\,\eta^4\right.\\
&\times\left.\left.\left.\left. 60\,\eta^5 + \eta^6 + 36\,\eta^7\right)\right)\sigma^2\right)\right) + 3\,\alpha^3\left(27\,\gamma_1^4\,\eta\left(108 + \gamma_2^2\left(129 - 321\,\gamma_2^2 + 80\,\gamma_2^4 + 24\left(4 - 5\,\gamma_2^2 + \gamma_2^4\right)\eta^3\right)\right)\right.\\
&+ 4\,\gamma_2^2\left(3\,\gamma_2^2\left(8\,\gamma_2^6\,\eta\left(10 + 3\,\eta^3\right) + 3\,\gamma_2^2\left(40 + 43\,\eta + 68\,\eta^3 + 32\,\eta^4 - 6\,\eta^6\right) + 9\left(-4 + 12\,\eta - 12\,\eta^3 + \eta^6\right)\right.\right.\\
&\left.+3\,\gamma_2^4\left(-28 - 107\,\eta - 32\,\eta^3 - 40\,\eta^4 + 3\,\eta^6\right)\right) + 2\left(-162\,\eta^6 + 8\,\gamma_2^6\left(4 - \eta - 42\,\eta^2 - 4\,\eta^3 + \eta^4 + 18\,\eta^5\right)\right.\\
&+ 9\,\gamma_2^2\left(2 + 4\,\eta - 22\,\eta^2 + \eta^3 - 4\,\eta^4 + 16\,\eta^5 + 36\,\eta^6\right)\\
&\left.\left.-3\,\gamma_2^4\left(18 + 10\,\eta - 168\,\eta^2 - 9\,\eta^3 + 2\,\eta^4\left(-5 + 48\,\eta + 27\,\eta^2\right)\right)\right)\sigma^2 + 6\,y_3\,\eta^3\,\sigma^4\right)\\
&+ 12\,\gamma_1^2\,\gamma_2^2\left(24\,\gamma_2^6\,\eta\left(10 + 3\,\eta^3\right) + 54\left(-1 + 6\,\eta - 3\,\eta^3 + \eta^6\right) + 18\left(1 + \eta^2\left(-11 - \eta + 8\,\eta^3\right)\right)\sigma^2\right.\\
&+ \gamma_2^4\left(9\left(-14 - 107\,\eta - 16\,\eta^3 - 40\,\eta^4 + 6\,\eta^6\right) + 16\left(2 - \eta - 21\,\eta^2 - 2\,\eta^3 + \eta^4 + 9\,\eta^5\right)\sigma^2\right)\\
&\left.\left.\left.+3\,\gamma_2^2\left(60 + 3\eta\left(43 + 2\eta^2\left(17 + 16\,\eta - 6\,\eta^3\right)\right) + 2(-9 + 2\,\eta + 84\,\eta^2 + 9\,\eta^3 - 2\,\eta^4 - 48\,\eta^5)\sigma^2\right)\right)\right) + y_{55}\right),\\
x_{27} = {}& 8\,\alpha^3\,\gamma_2^2\,\epsilon\,\eta^7\,y_1^2\left(\alpha^7\,\gamma_2^4\left(3 + \gamma_2^2\right)\left(2 + \eta^3\right)^2 - 1296\,\gamma_2\,\sigma\left(\gamma_2^2 - 1\right) + 2\,\alpha^6\,\gamma_2^3\left(3 + \gamma_2^2\right)\left(2 + \eta^3\right)^2\sigma\right.\\
&- 36\,\alpha^2\,\gamma_2\left(3 - 15\,\gamma_2^2 + 16\,\gamma_2^4 + 24\,\eta^3\left(\gamma_2^2 - 1\right)^2\right)\sigma + 6\,\alpha^4\,\gamma_2\left(-18\,\eta^3\left(2 + \eta^3\right) + 3\,\gamma_2^2\left(-3 + 8\,\eta^3 + 4\,\eta^6\right)\right.\\
&\left.+\gamma_2^4\left(-8 + 8\,\eta^3 + 6\,\eta^6\right)\right)\sigma + 36\,\alpha\left(9\left(\gamma_2^2 - 1\right)\left(3 - 7\,\gamma_2^2 + 12\,\eta^3\left(\gamma_2^2 - 1\right)\right) + y_3\,\sigma^2\right)\\
&+ \alpha^5\,\gamma_2^2\left(-99\,\gamma_2^2 + 48\,\gamma_2^4 + 12\left(-18 + 9\,\gamma_2^2 + 8\,\gamma_2^4\right)\eta^3 + 36\left(-3 + 2\,\gamma_2^2 + \gamma_2^4\right)\eta^6 + \left(3 + \gamma_2^2\right)\left(2 + \eta^3\right)^2\sigma^2\right)\\
&+ 3\,\alpha^3\left(12\left(24\,\gamma_2^2 - 30\,\gamma_2^4 + 4\,\gamma_2^6 + 12\,\gamma_2^2\left(\gamma_2^2 - 1\right)^2\eta^3 + 9\,\eta^6\left(\gamma_2^2 - 1\right)^2\left(3 + \gamma_2^2\right)\right)\right.\\
&\left.\left.+\gamma_2^2\left(-16\,\gamma_2^2\left(2 + \eta^3\right) + 3\left(5 + 4\,\eta^3\right)\right)\sigma^2\right)\right),\\
x_{28} = {}& 8\,\alpha^4\,\gamma_2^2\,\epsilon\,\eta^9\left(y_{62} - 2916\,\eta\left(\gamma_2^2 - 1\right)^2\left(27\,\gamma_1^4 + 24\,\gamma_1^2\,\gamma_2^2 + 19\,\gamma_2^4 - y_2^2\,\eta\left(3 + \gamma_2^2\right)\right) + \alpha^8\,\gamma_2^8\left(3 + \gamma_2^2\right)\left(2 + \eta^3\right)^2 + y_{59}\right.\\
&+ 9\alpha^2\gamma_2^2\left(1944\gamma_1^2\eta^4 + 144\gamma_2^8\eta\left(2 + \eta + 3\eta^3\right) + 9\gamma_2^2\left(12\left(-1 + 3\gamma_1^4\eta(2\eta - 3) + \eta^3(12\eta - 5) - 2\gamma_1^2\eta\left(-8 + 6\eta + 15\eta^3\right)\right)\right.\right.\\
&+ \left.\left(4 + \eta^2\left(40 + \gamma_1^4 + 12(14 - 15\eta)\eta + \gamma_1^2(48\eta - 68)\right)\right)\sigma^2\right) + 3\gamma_2^4\left(12\left(10 + \eta\left(35 + 9\gamma_1^4\eta + 6\gamma_1^2\left(-10 + 4\eta + 3\eta^3\right)\right)\right.\right.\\
&\left.\left.-6\,\eta\left(2 + 5\,\eta\left(2\,\eta - 1\right)\right)\right)\right) + \left(-36 + \eta\left(-1 + 4\,\gamma_1^2\left(-4 + \eta + 18\,\eta^2\right) + 4\,\eta\left(-5 + 3\,\eta\left(9\,\eta - 19\right)\right)\right)\right)\sigma^2\Big)\\
&\left.+4\,\gamma_2^6\left(9\left(-7 + \eta\left(-43 + 6\,\gamma_1^2\left(2 + 2\,\eta + 3\,\eta^3\right) + \eta\left(8 + 3\,\eta\left(4\,\eta - 5\right)\right)\right)\right) + (16 - 8\,\eta - 71\,\eta^2 + 36\,\eta^3 + 81\,\eta^4)\sigma^2\right)\right)\\
&+ \alpha^6\,\gamma_2^6\left(3\left(4 + \eta^2\left(24 - 5\,\eta + 9\,\eta^2 + 12\,\eta^3 + \eta^4\right)\right)\sigma^2 + \gamma_2^2\left(9\left(-11 + 4\left(2 + \gamma_1^2\right)\eta + 9\,\eta^3 + 2\left(2 + \gamma_1^2\right)\eta^4 + 8\,\eta^6\right)\right.\right.\\
&\left.\left.+ \left(4 + \eta^2\left(24 + 4\,\eta + 9\,\eta^2 + 12\,\eta^3 + \eta^4\right)\right)\sigma^2\right)\right) + y_{60} + 3\,\alpha^4\,\gamma_2^4\left(12\,\gamma_2^6\left(4 + \eta\left(16 + \eta\left(1 + 3\,\eta\left(4 + 4\,\eta + 3\,\eta^3\right)\right)\right)\right)\right.\\
&+ \gamma_2^4\left(9\left(-40 + \eta\left(15 + 4\gamma_1^2\left(8 + \eta + 6\eta^3\right) + 4\eta\left(1 + \eta\left(-17 + 8\eta + 3\eta^3\right)\right)\right)\right) + 8\left(-4 + \eta + 6\eta^2 + \eta^3 + 14\eta^4 + 9\eta^5\right)\sigma^2\right)\\
&+ 3\gamma_2^2\left(3\left(32 + \eta\left(-48 + 3\gamma_1^4\eta + 52\eta^2 - 48\eta^3 - 60\eta^5 + 12\gamma_1^2\left(2 + \eta + 4\eta^3\right)\right)\right) + \left(5 + \eta\left(8 + 2\gamma_1^2\left(2 + \eta^2(6 + \eta)\right)\right.\right.\right.\\
&\left.\left.\left.+\eta\left(12 + \eta\left(-83 + 76\eta + 48\eta^2\right)\right)\right)\right)\sigma^2 + \eta^4\sigma^4\right) + 18\eta\left(18\,\eta^5 - 6\,\eta\left(2 - 3\,\eta + 3\,\eta^2 + 2\,\eta^3\right)\sigma^2\right.\\
&\left.\left.\left.+\gamma_1^2\left(-36\left(1 + \eta^3\right) + \left(2 - 6\,\eta + 6\,\eta^2 + \eta^3\right)\right)\right)\sigma^2\right)\right) + y_{61}\Big),\\
x_{29} = {}& 8\,\alpha^5\,\gamma_2^2\,\epsilon\,\eta^7\,y_1^2\left(-144\,\gamma_2\,\sigma\left(\gamma_2^2 - 1\right)\left(\eta^3 - 1\right)^2 + 4\,\alpha^4\,\gamma_2^3\,\sigma\left(\gamma_2^2 - 3\right)\left(2 + \eta^3\right)^2\right.\\
&+ 4\,\alpha^2\,\gamma_2\left(\eta^3 - 1\right)\left(9 + 18\,\eta^3 - 27\,\gamma_2^2\left(1 + \eta^3\right) + 8\,\gamma_2^4\left(2 + \eta^3\right)\right)\sigma + \alpha^5\,\gamma_2^4\left(2 + \eta^3\right)^2\left(\sigma^2 - 3\right)\\
&+ 4\,\alpha\left(\eta^3 - 1\right)\left(9\left(1 - \gamma_2^2\right)\left(3 - 21\,\eta^3 + \gamma_2^2\left(19\,\eta^3 - 7\right)\right) + y_3\,\sigma\left(\eta^3 - 1\right)^2\right)\\
&\left.+\alpha^3\gamma_2^2\left(\left(2 + \eta^3\right)\left(4\gamma_2^4\left(2 + \eta^3\right) + 9\left(2 + 13\eta^3\right) - 9\gamma_2^2\left(2 + 13\eta^3\right)\right) + \left(\eta^3 - 1\right)\left(-9\left(1 + \eta^3\right) + 16\gamma_2^2\left(2 + \eta^3\right)\right)\sigma^2\right)\right),
\end{aligned}
$$

$$x_{30} = 16\,\alpha^4\,\gamma_2^2\,\epsilon\,\eta^7\,y_1^2\left(432\,\gamma_2\left(\gamma_2^2-1\right)\left(\eta^3-1\right)\sigma+\alpha^6\,\gamma_2^5\left(2+\eta^3\right)^2\sigma+6\,\alpha^2\,\gamma_2\left(-12+42\,\gamma_2^2-32\,\gamma_2^4\right.\right.$$
$$\left.+\left(-39+60\,\gamma_2^2-16\,\gamma_2^4\right)\eta^3+24\,\eta^6\left(\gamma_2^2-1\right)^2\right)\sigma+\alpha^4\,\gamma_2^3\left(-75-84\,\eta^3-30\,\eta^6+4\,\gamma_2^2\left(2+\eta^3\right)\left(2+7\,\eta^3\right)\right)\sigma$$
$$+\alpha^5\,\gamma_2^4\left(2+\eta^3\right)^2\left(-6+2\,\gamma_2^2+\sigma^2\right)+12\,\alpha\left(9(1-\gamma_2^2)\left(-3+18\,\eta^3-6\,\eta^6+\gamma_2^2\left(7-19\,\eta^3+6\,\eta^6\right)\right)\right.$$
$$\left.-y_3\,\sigma^2\left(\eta^3-1\right)\right)+\alpha^3\,\gamma_2^2\left(6\left(33-39\,\gamma_2^2+8\,\gamma_2^4+\left(51-66\,\gamma_2^2+16\,\gamma_2^4\right)\eta^3+6\left(4-5\,\gamma_2^2+\gamma_2^4\right)\eta^6\right)\right.$$
$$\left.\left.+\left(21+12\,\eta^3-6\,\eta^6+8\,\gamma_2^2\left(\eta^3-4\right)\left(2+\eta^3\right)\right)\sigma^2\right)\right),$$

$$x_{31} = -32\,y_1\,\alpha^4\,\gamma_2^2\,\epsilon\,\eta^8\left(-y_{63}-18\,y_4\,\eta-1944\,y_2\,\eta^4\left(\gamma_2^2-1\right)^2+2\,\alpha^6\,\gamma_2^6\left(\gamma_2^2-3\right)\left(2+\eta^3\right)^2+\alpha^7\,\gamma_2^7\left(2+\eta^3\right)^2\sigma\right.$$
$$-3\,\alpha^5\,\gamma_2^5\left(25+\eta^2\left(12+37\,\eta+6\,\eta^3+10\,\eta^4\right)\right)\sigma+\alpha^4\,\gamma_2^4\left(18\left(11+26\,\eta^3+8\,\eta^6-\gamma_2^2\left(13-2\,\eta\left(\gamma_1^2-2\right)+31\,\eta^3\right.\right.\right.$$
$$\left.\left.-\eta^4\left(\gamma_1^2-2\right)+10\,\eta^6\right)\right)+\left(21-3\,\eta^2\left(42+\eta\left(5+2\,\eta^2\left(15+\eta\right)\right)\right)+2\,\gamma_2^2\left(-32+6\,\eta+48\,\eta^2-8\,\eta^3+3\,\eta^4\right.\right.$$
$$\left.\left.\left.+42\,\eta^5+4\,\eta^6\right)\right)\sigma^2\right)+3\,\alpha^2\,\gamma_2^2\left(9\left(y_2\,\eta\left(17+16\,\eta^3\right)-12\left(1+3\,\eta^3+2\,\eta^6\right)+\left(4-2\,\gamma_1^2\,\eta\left(\eta^3-1\right)\right.\right.\right.$$
$$\left.\left.+\eta^2\left(-13-4\,\eta+16\,\eta^3\right)\right)\sigma^2\right)+4\,\gamma_2^4\left(-63+3\,\eta\left(y_2-6\,\eta^2\right)\left(4+3\,\eta^3\right)+4\left(4-\eta-3\,\eta^2-4\,\eta^3+\eta^4+9\,\eta^5\right)\sigma^2\right)$$
$$\left.+6\gamma_2^2\left(60+102\eta^3+72\eta^6-3y_2\eta\left(11+10\eta^3\right)+2\left(-9+\eta\left(1+2\gamma_1^2\left(\eta^3-1\right)-\eta\left(-15+\eta\left(-9+\eta+24\eta^2\right)\right)\right)\right)\sigma^2-1\right)\right)$$
$$+9\,\alpha\,\gamma_2\sigma\left(32\,\gamma_2^6\,\eta\left(3\,\eta^3\right)-9\,\eta\left(\gamma_1^2\left(13-16\,\eta^3\right)+2\,\eta\left(-18+12\,\eta^3+\sigma^2\right)\right)+6\,\gamma_2^2\left(8+\eta\left(-13+6\,\gamma_1^2\left(5-8\,\eta^3\right)\right.\right.\right.$$
$$\left.\left.+\eta\left(-111+8\eta(-1+\eta(2+9\eta))+9\sigma^2\right)\right)\right)-2\gamma_2^4\left(24+\eta\left(-60+\gamma_1^2\left(24-72\eta^3\right)+\eta\left(-171+12\eta(-2+\eta(8+9\eta))\right.\right.\right.$$
$$\left.\left.\left.\left.+16\,\sigma^2\right)\right)\right)\right)+3\,\alpha^3\,\gamma_2^3\,\sigma\left(4\,\gamma_2^4\left(-16+16\,\eta+12\,\eta^2-8\,\eta^3+14\,\eta^4+9\,\eta^5+12\,\eta^6\right)\right.$$
$$+3\left(-8+\eta\left(16\eta^4(3+\eta)+\eta(51+46\eta)-6\gamma_1^2\left(7+5\eta^3\right)-2\eta\sigma^2\left(\eta^3-1\right)\right)\right)-2\gamma_2^2\left(-42+\eta\left(42-6\gamma_1^2\left(8+7\eta^3\right)\right.\right.$$
$$\left.\left.\left.\left.\left.+\eta\left(99+6\,\eta\left(8+5\,\eta+15\,\eta^2+8\,\eta^3\right)+4\,\sigma^2-4\eta^3\,\sigma^2\right)\right)\right)\right)\right),$$

$$x_{32} = 8\,\alpha^3\,\gamma_2^2\,\epsilon\,\eta^7\,y_1^2\left(-2592\,\gamma_2\,\sigma\left(\gamma_2^2-1\right)+24\,\alpha^6\,\gamma_2^3\left(2+\eta^3\right)^2\sigma-72\,\alpha^2\,\gamma_2\left(5+20\,\eta^3+2\,\eta^6+8\,\gamma_2^4\left(2+3\,\eta^3\right)\right.\right.$$
$$\left.-\gamma_2^2\left(17+44\,\eta^3+2\,\eta^6\right)\right)\sigma+4\,\alpha^4\,\gamma_2\left(9-99\,\eta^3-72\,\eta^6+2\gamma_2^4\left(-4+8\,\eta^3+5\,\eta^6\right)+9\,\gamma_2^2\left(-6+8\,\eta^3+7\,\eta^6\right)\right)\sigma$$
$$+\alpha^7\,\gamma_2^4\left(2+\eta^3\right)^2\left(9+2\,\gamma_2^2-\sigma^2\right)+72\,\alpha\left(9\left(\gamma_2^2-1\right)\left(3-7\,\gamma_2^2+12\,\eta^3\left(\gamma_2^2-1\right)\right)+y_3\sigma^2\right)$$
$$+2\,\alpha^3\left(36\,\gamma_2^6\left(2+3\,\eta^3\right)^2+54\left(1-8\,\eta^3+25\,\eta^6\right)-18\left(-1+\eta^3\right)^2\sigma^2+2\,\gamma_2^4\left(9\left(-53+37\,\eta^3\left(\eta^3-2\right)\right)\right.\right.$$
$$\left.\left.+8\left(-8+\eta^3-2\eta^6\right)\sigma^2\right)+9\gamma_2^2\left(76+148\eta^3-260\eta^6+\left(11-8\eta^3+6\eta^6\right)\sigma^2\right)\right)+\alpha^5\gamma_2^2\left(4\gamma_2^4\left(2+\eta^3\right)\left(10+17\eta^3\right)\right.$$
$$\left.\left.-9\left(2+\eta^3\right)\left(2+37\,\eta^3\right)+3\left(5+8\,\eta^3+5\,\eta^6\right)\sigma^2+\gamma_2^2\left(9\left(-18+52\,\eta^3+29\,\eta^6\right)-2\left(-20+4\,\eta^3+7\,\eta^6\right)\sigma^2\right)\right)\right),$$

$$x_{33} = -16\,\alpha^3\,\gamma_2\,\epsilon\,\eta^{7/2}\left(\alpha\,\gamma_2\left(2\,\gamma_2^2-3\right)\left(2+\eta^3\right)+2\,\sigma\left(4\,\gamma_2^2-3\right)\left(\eta^3-1\right)+\alpha^2\,\gamma_2^2\left(2+\eta^3\right)\sigma\right)y_1,$$

$$x_{34} = -96\,\alpha^2\,\gamma_2\,\epsilon\,\eta^4\left(\alpha\,\gamma_2\left(2\,\gamma_2^2-3\right)+\left(3-4\,\gamma_2^2\right)\sigma\right)y_1,$$

$$x_{35} = -16\,\alpha^2\,\gamma_2^2\,\epsilon\,\eta^{9/2}\left(54\,y_2\,\eta\left(\gamma_2^2-1\right)+\alpha^4\,\gamma_2^4\left(2+\eta^3\right)+\alpha^3\,\gamma_2^3\left(2+3\,\eta^2+\eta^3\right)\sigma+3\,\alpha\,\gamma_2\left(3\left(2+\gamma_1^2\,\eta-6\,\eta^2\right)\right.\right.$$
$$\left.\left.+2\,\gamma_2^2\left(-4+\eta+9\,\eta^2\right)\right)\sigma+3\,\alpha^2\,\gamma_2^2\left(-6+3\,\gamma_1^2\,\eta-6\,\eta^3+2\,\gamma_2^2\left(2+\eta+3\,\eta^3\right)+\eta^2\,\sigma^2\right)\right),$$

$$x_{36} = 16\,\alpha^2\,\gamma_2\,\epsilon\,\eta^{7/2}\left(\alpha^3\,\gamma_2^3\left(2+\eta^3\right)+6\,\alpha\,\gamma_2\left(-3\left(1+\eta^3\right)+\gamma_2^2\left(2+3\,\eta^3\right)\right)+6\left(3-4\,\gamma_2^2\right)\sigma+\alpha^2\,\gamma_2^2\left(2+\eta^3\right)\sigma\right)y_1,$$

$$x_{37} = -48\,\alpha^2\,\gamma_2\,\epsilon\,\eta^5 y_1\left(4\,\gamma_2^2-3\right)\left(\alpha\,\gamma_2-2\,\sigma\right),$$

$$x_{38} = -16\alpha^2\gamma_2\epsilon\eta^{9/2}\left(6y_2\eta\sigma\left(4\gamma_2^2-3\right)+\alpha^4\gamma_2^4\left(2+\eta^3\right)\sigma+\alpha^2\gamma_2^2\left(6+9\gamma_1^2\eta-9\eta^2-6\eta^3+\gamma_2^2\left(-8+6\eta+6\eta^2+8\eta^3\right)\right)\sigma\right.$$
$$\left.+3\,\alpha\,\gamma_2\,\eta\left(4\,\gamma_2^4+\gamma_1^2\left(6\,\gamma_2^2-9\right)-6\,\eta\,\sigma^2+\gamma_2^2\left(8\,\eta\,\sigma^2-6\right)\right)+\alpha^3\,\gamma_2^3\left(2\,\gamma_2^2\left(2+\eta^3\right)-3\left(2+\eta^3-\eta^2\,\sigma^2\right)\right)\right),$$

$$y_1 = 3\,\gamma_1^2+\gamma_2(2\,\gamma_2+\alpha\,\eta\,\sigma),\qquad y_2 = 3\,\gamma_1^2+2\,\gamma_2^2,\qquad y_3 = 9-27\,\gamma_2^2+16\,\gamma_2^4,$$

$$y_4 = \sigma^2\left(y_2\,y_3-36\,\gamma_2^2\left(\gamma_2^2-1\right)\eta\right),\qquad y_5 = \sigma^2\left(y_2\,y_3-72\,\gamma_2^2\left(\gamma_2^2-1\right)\eta\right),\qquad y_6 = \sigma^2\left(y_2\,y_3-18\,\gamma_2^2\left(\gamma_2^2-1\right)\eta\right),$$

$$y_7 = \alpha^6\,\gamma_2^3\,\sigma\left(9\,\gamma_1^2\left(2+\eta^3\right)+8\,\gamma_2^4\,\eta\left(2+\eta^3\right)+3\,\gamma_2^2\left(4+\eta\left(1+2\,\eta\left(2+\eta+\eta^2+\eta^3\right)\right)\right)+3\,\eta\left(2+\eta^3\right)\left(\sigma^2-6\right)\right),$$

$$y_8 = 27\gamma_1^4\left(2+\eta^3\right)+16\gamma_2^6(1+\eta)\left(2+\eta^3\right)-36\eta^2\left(1+2\eta^3\right)\sigma^2,\qquad y_9 = 27\gamma_1^4\left(3\gamma_2^2\left(4+\eta+2\eta^3\right)+\left(2+\eta^3\right)\left(\sigma^2-6\right)\right),$$

$$y_{10} = 288\,\gamma_2\,y_2\,\sigma\left(\gamma_2^2-1\right)\left(\eta^3-1\right)+3\,\alpha^6\,\gamma_2^5\,\eta(1+\eta)\left(2+\eta^3\right)\sigma-8\,\alpha\,y_4\left(\eta^3-1\right),$$

$$y_{11} = -864\,\gamma_2\left(\gamma_2^2-1\right)y_2\,\sigma+3\,\alpha^7\,\gamma_2^4\,\eta\left(2+\eta^3\right)\left(\gamma_2^2\,\eta+\sigma^2\right),$$

$$y_{12} = 864\,\gamma_2\left(\gamma_2^2-1\right)y_2^2\,\eta\,\sigma+3\,\alpha^8\,\gamma_2^5\,\eta\left(2+\eta^3\right)\sigma\left(\gamma_2^2+\eta\,\sigma^2\right)+24\,\alpha\,y_2\,\eta\left(9\,y_2\left(12-25\,\gamma_2^2+13\,\gamma_2^4\right)-y_5\right),$$

$$y_{13} = 3\,\eta^2\,\sigma^2\left(2+\eta^3\right)\left(\sigma^2-6\right)+9\,\gamma_1^2\left(2+\eta^3\right)\left(\gamma_2^2+2\,\eta\,\sigma^2\right)+2\,\gamma_2^4\left(2+\eta^3\right)\left(3+4\,\eta\,\sigma^2\right),$$

$$
\begin{aligned}
y_{14} &= \alpha^6\,\gamma_2^3\,\sigma\left(9\,\gamma_1^2\left(2+\eta^3\right)+8\,\gamma_2^4\,\eta\left(2+\eta^3\right)+3\,\gamma_2^2\left(4+\eta\left(9+2\,\eta\left(2+\eta+3\,\eta^2+\eta^3\right)\right)\right)+3\,\eta\left(2+\eta^3\right)\left(\sigma^2-6\right)\right),\\
y_{15} &= 864\,\gamma_2\,y_2^2\,\eta\,\sigma\left(\gamma_2^2-1\right)+3\,\alpha^8\,\gamma_2^5\,\eta\left(2+\eta^3\right)\sigma\left(\gamma_2^2+\eta\,\sigma^2\right)+24\,\alpha\,y_2\,\eta\left(9\,y_2\left(15-28\,\gamma_2^2+13\,\gamma_2^4\right)-y_5\right),\\
y_{16} &= \left(48\,\alpha^5\,\gamma_1^2\,\gamma_2^8+32\,\alpha^5\,\gamma_2^{10}+24\,\alpha^6\,\gamma_1^2\,\gamma_2^7\,\sigma-108\,\alpha^6\,\gamma_1^2\,\gamma_2^3\,\eta\,\sigma+18\,\alpha^6\,\gamma_1^2\,\gamma_2^3\,\eta\,\sigma^3\right)\left(2+\eta^3\right)+32\,\alpha^4\,\gamma_2^7\,\eta^3\,\sigma^3\left(3-4\,\eta\right),\\
y_{17} &= \left(96\,\alpha^4\,\gamma_1^2\,\gamma_2^7\,\eta^2\,\sigma+64\,\alpha^4\,\gamma_2^9\,\eta^2\,\sigma\right)\left(3+\eta\right)+y_{16}+y_{12},\\
y_{18} &= 12\,\gamma_2\left(-4\,y_6\eta^2+y_2\left(16\,\gamma_2^6\,\eta+432\,\eta^2+6\,\gamma_2^4\left(-4+\eta\left(-1+4\,\gamma_1^2+78\,\eta+4\,\eta^2\right)\right)\right.\right.\\
&\quad\left.\left.-3\,\gamma_2^2\left(-8+\eta\left(3\,\gamma_1^2+4\,\eta(75+2\,\eta)\right)\right)\right)\right)\sigma,\\
y_{19} &= 4\,\gamma_2^6\left(33+9\,\eta+30\,\eta^3+4\left(-4-8\,\eta+6\,\eta^2+\eta^3+2\,\eta^4\right)\sigma^2\right)+6\,\gamma_2^4\left(-6\left(5+4\,\eta^3\right)\right.\\
&\quad\left.+\left(12+74\,\eta-24\,\eta^2+21\,\eta^3+34\,\eta^4+54\,\eta^5\right)\sigma^2-8\,\eta^3\,\sigma^4\right),\\
y_{20} &= 12\,\gamma_1^2\,\gamma_2^2\left(-54+48\,\gamma_2^6\,\eta+297\,\eta^3+9\,\gamma_2^4\left(-14+11\,\eta+35\,\eta^3\right)-9\,\gamma_2^2\left(-20+15\,\eta+68\,\eta^3\right)\right.\\
&\quad\left.+\left(18-18\,\eta^3+16\,\gamma_2^4(2-3\,\eta+3\,\eta^2-2\,\eta^3)+9\,\gamma_2^2\left(-6+\eta\left(3-2\,\eta+6\,\eta^2\right)\right)\right)\sigma^2\right),\\
y_{21} &= 48\,\gamma_2^8\,\eta-6\,\eta^3\,\sigma^2\left(y_3\,\sigma^2-108\right)+9\,\gamma_2^2\left(-12+75\,\eta^3+2\left(2+4\,\eta-77\,\eta^3-4\,\eta^4\right)\sigma^2\right)+\gamma_2^6\left(9\left(-28+11\,\eta+79\,\eta^3\right)\right.\\
&\quad\left.+16\left(4+\eta\left(-3+6\,\eta-4\,\eta^2\right)\right)\sigma^2\right),\\
y_{22} &= 2\gamma_2^2\left(8\gamma_2^6(3\eta-8)+\gamma_2^4\left(3(65-54\eta+33\eta^2+16\eta^3+159\eta^4)\right)+16(2-3\eta)\eta\sigma^2\right)-9\eta\left(6-2\sigma^2+\eta^3\left(2\sigma^2-51\right)\right)\\
&\quad+9\,\gamma_2^2\left(-15+\eta\left(20-15\,\eta-6\,\eta^2-104\,\eta^3+(-6+3\,\eta-\eta^2+6\,\eta^3)\sigma^2\right)\right)\right),\\
y_{23} &= 12\,\gamma_2\left(-4\,y_6\,\eta^2+y_2\left(16\,\gamma_2^6\,\eta+27\,\eta\left(20\,\eta-3\,\gamma_1^2\right)+6\,\gamma_2^4\left(-4+\eta\left(7+4\,\gamma_1^2+78\,\eta+4\,\eta^2\right)\right)\right.\right.\\
&\quad\left.\left.-3\,\gamma_2^2\left(-8+\eta\left(18-21\,\gamma_1^2+8\,\eta(42+\eta)\right)\right)\right)\right)\sigma,\\
y_{24} &= 12\,\gamma_1^2\,\gamma_2^2\left(-108+48\,\gamma_2^6\,\eta+351\,\eta^3+9\,\gamma_2^4\left(-14+19\,\eta+35\,\eta^3\right)-9\,\gamma_2^2\left(-26+27\,\eta+74\,\eta^3\right)+\left(18-18\,\eta^2\left(9+\eta\right)\right.\right.\\
&\quad\left.\left.+16\,\gamma_2^4(2-3\,\eta+3\,\eta^2-2\,\eta^3)+9\,\gamma_2^2(-6+3\,\eta+14\,\eta^2+6\,\eta^3)\right)\sigma^2\right),\\
y_{25} &= 48\,\gamma_2^8\,\eta-6\,\eta^3\,\sigma^2\left(y_3\,\sigma^2-135\right)+\gamma_2^6\left(9\left(-28+19\,\eta+79\,\eta^3\right)+16\left(4+\eta\left(-3+6\,\eta-4\,\eta^2\right)\right)\sigma^2\right)\\
&\quad+9\,\gamma_2^2\left(-24+87\,\eta^3-4\left(-1-2\,\eta+9\,\eta^2+43\,\eta^3+2\,\eta^4\right)\sigma^2\right),\\
y_{26} &= 3\,\gamma_1^2\left(324\,\eta^4+16\,\gamma_2^6\left(3\,\eta-4\right)+18\,\gamma_1^2\left(-3+3\,\gamma_2^2+2\,\gamma_2^4\,\eta+6\left(\gamma_2^2-1\right)\eta^3\right)\right.\\
&\quad\left.+3\,\gamma_2^4\left(46+\eta^2\left(57+2\,\eta\left(13+54\,\eta\right)-16\,\sigma^2\right)\right)-9\,\gamma_2^2\left(8+\eta^2\left(27+10\,\eta+72\,\eta^2-3\,\sigma^2\right)\right)\right),\\
y_{27} &= \left(96\,\alpha^4\,\gamma_1^2\,\gamma_2^7\,\eta^2\,\sigma+64\,\alpha^4\,\gamma_2^9\,\eta^2\,\sigma\right)\left(3+\eta\right)+y_{16}+y_{15},\\
y_{28} &= \alpha^2\,\gamma_2^4\left(32\,\gamma_2\,y_2\,\sigma\left(\eta^3-4\right)+\alpha\left(2+\eta^3\right)\left(48\,\gamma_1^2\,\gamma_2^2+32\,\gamma_2^4+\alpha\left(3\,y_2\,\alpha+2\,\alpha\,\eta\left(9\left(\gamma_2^2-1\right)\eta+4\,\gamma_2^2\,\sigma^2\right)\right.\right.\right.\\
&\quad\left.\left.\left.+8\,\gamma_2\,\sigma\left(3\,\gamma_1^2+2\,\gamma_2^2\left(1+\eta\right)-2\,\eta\,\sigma^2\right)\right)\right)\right),\\
y_{29} &= \alpha^{-1}\left(-1152\,\alpha^2\,\gamma_2^7\,\eta\,\sigma\left(3\,\gamma_1^2+\gamma_2^2\right)\left(5+3\,\eta^3\right)-1296\,y_2^2\,\eta\left(\gamma_2^2-1\right)\left(\alpha\left(10\,\gamma_2^2-9\right)+4\,\gamma_2\,\sigma\right)\right.\\
&\quad-12\,\alpha^3\,\gamma_2^2\,\eta\,\sigma^2\left(4\,\gamma_2^2-3\right)\left(\eta^3-1\right)\left(3\,\gamma_1^2+\alpha\,\gamma_2\,\eta\,\sigma\right)^2+\alpha^5\,\gamma_2^2\left(2+\eta^3\right)\left(-2\,\alpha\,\gamma_2\left(27\,\gamma_1^4\,\gamma_2^2\,\eta+12\,\gamma_1^2\left(-27\,\eta^4\right.\right.\right.\\
&\quad\left.+\gamma_2^4(2+3\eta+3\eta^2+7\eta^3)\right)+4\gamma_2^6(4+9\eta+6\eta^2+14\eta^3+9\eta^4)\big)\,\sigma-6\left(8\gamma_2^8\left(2+\eta+3\eta^3\right)+12\gamma_1^2\gamma_2^6\left(2+2\eta+3\eta^3\right)\right.\\
&\quad\left.+9\,\gamma_1^4\,\eta\left(2\,\gamma_2^4-18\,\eta^2+\gamma_2^2\sigma^2\right)\right)+2\,\alpha^3\,\gamma_2^5\,\eta\,\sigma\left(-2\left(3+\sigma^2\right)+\eta\left(6+\eta\left(3\,\eta\left(2+\eta\right)-(3+\eta)\sigma^2\right)\right)\right)\\
&\quad+\alpha^2\gamma_2^2\left(108\,\eta^5\,\sigma^2+\gamma_2^4\left(-6+33\,\eta^3+108\,\eta^5-8\left(1+2\,\eta+3\,\eta^2+2\,\eta^3+7\,\eta^4\right)\sigma^2\right)\right.\\
&\quad\left.\left.\left.+6\,\gamma_2^2\left(\gamma_1^2\left(9\,\eta^3-\left(2+\eta^2\left(6+\eta\right)\right)\sigma^2\right)-\eta^3\left(18\,\eta^2+\sigma^4\right)\right)\right)\right)\right),\\
y_{30} &= \alpha^5\gamma_2^3\left(2+\eta^3\right)^2\left(-54\gamma_1^4\sigma-2\alpha^2\gamma_2^2\sigma\left(2\gamma_2^4(1+\eta)+3\gamma_1^2\left(\gamma_2^2+3\eta\right)+3\eta^2\sigma^2\right)+\alpha^3\gamma_2^3\eta\left(-3\eta\sigma^2+\gamma_2^2\left(3\eta-2\sigma^2\right)\right)\right.\\
&\quad\left.-\alpha\,\gamma_2\left(27\,\gamma_1^4+8\,\gamma_2^6+12\,\gamma_1^2\left(\gamma_2^4+3\,\eta\,\sigma^2\right)\right)\right),\\
y_{31} &= -288\,\alpha^2\,\gamma_2^8\,\eta\left(3\,\gamma_1^2+\gamma_2^2\right)\left(3\,\eta^3-2\right)+108\,\alpha^3\,\gamma_2\eta\left(-3+2\,\eta^2+\eta^5\right)\sigma\left(3\,\gamma_1^2+\alpha\,\gamma_2\,\eta\,\sigma\right)^2\\
&\quad-3\,\alpha^4\,\gamma_2^2\,\sigma^2\left(4-9\,\eta+4\,\eta^3+\eta^6\right)\left(3\,\gamma_1^2+\alpha\,\gamma_2\,\eta\,\sigma\right)^2-972\,\alpha^3\,\gamma_2\eta^2\,\sigma\left(\eta^5-1\right)\left(6\,\gamma_1^2+\alpha\,\gamma_2\,\eta\,\sigma\right)\\
&\quad-972\left(\gamma_2^2-1\right)^2\eta^4\left(-4\,y_2^2+\alpha\,\gamma_1^4\left(9\,\alpha\,\eta^2+8\,\gamma_2\sigma\right)\right)+y_{30}+y_{29},\\
y_{32} &= 12\gamma_1^2\gamma_2^2\left(9\left(-6\left(1+6\eta+3\eta^3+5\eta^6\right)-\gamma_2^4\left(14-19\eta+16\eta^3+40\eta^4+30\eta^6\right)+\gamma_2^2\left(20+25\eta+34\eta^3+32\eta^4+60\eta^6\right)\right)\right.\\
&\quad+2\left(9+9\,\eta^2\left(-2-\eta+8\,\eta^3\right)+8\,\gamma_2^4\left(2-\eta+15\,\eta^2-2\,\eta^3+\eta^4+9\,\eta^5\right)\right.\\
&\quad\left.\left.-3\,\gamma_2^2\left(9-2\,\eta+24\,\eta^2-9\,\eta^3+2\,\eta^4+48\,\eta^5\right)\right)\sigma^2\right),\\
y_{33} &= \left(-144\,\eta+9\,\eta^2\left(144+75\,\eta+4\,\eta^2\left(5+24\,\eta^2+8\,\eta^3-54\,\eta^4\right)\right)\right.\\
&\quad+8\,\gamma_2^4\left(-32-42\,\eta+96\,\eta^2-26\,\eta^3-21\,\eta^4+84\,\eta^5+31\,\eta^6+36\,\eta^7\right)\\
&\quad\left.+3\,\gamma_2^2\left(64+132\,\eta-696\,\eta^2+163\,\eta^3+24\,\eta^4-240\,\eta^5-371\,\eta^6-192\,\eta^7+324\,\eta^8\right)\right)\sigma^2,
\end{aligned}
$$

$$
\begin{aligned}
y_{34} = {} & \alpha^4\gamma_2^4 (y_{33} + 162\,\gamma_1^4\,\eta\,(-1 + 2\,\eta^2\,(6 - \eta + 3\,\eta^3)) + 36\,(-27\,\eta^8 - \gamma_2^4\,(26 + 2\,\eta + 44\,\eta^3 + 4\,\eta^4 + 29\,\eta^6 + 27\,\eta^8) \\
& + \gamma_2^2\,(22 + 34\,\eta^3 + 25\,\eta^6 + 54\,\eta^8)) + 6\,\gamma_1^2\,(18\,(11 + 20\,\eta^3 + 14\,\eta^6) \\
& + 9(5 - 6\,\eta - 58\,\eta^2 - \eta^3 + 24\,\eta^4 + \eta^5\,(3\,\eta - 4)\,(5 + 4\eta))\sigma^2 + 2\,\gamma_2^2\,(-9\,(13 + 2\,\eta + 25\,\eta^3 + 4\,\eta^4 + 16\,\eta^6) \\
& + 2(-16 + 6\,\eta + 48\,\eta^2 - 4\,\eta^3 + 3\,\eta^4 + 42\,\eta^5 + 2\,\eta^6)\sigma^2))) \,,
\end{aligned}
$$

$$
\begin{aligned}
y_{35} = {} & 72\alpha\gamma_2\sigma\,(9\gamma_1^4\,(3 + 12\gamma_2^2 - 20\gamma_2^4)\,\eta + 12\gamma_1^2\,(9\eta^2\,(-9 + 3\eta^3 + \sigma^2) - 3\gamma_2^2\,(1 + \eta\,(-1 + \eta\,(-57 + \eta(-1 + 2\eta(2 + 9\eta)) \\
& + 9\,\sigma^2))) + \gamma_2^4\,(3 + 12\,\eta - 90\,\eta^2 + 3\,\eta^3\,(1 + \eta)\,(9\,\eta - 1) + 16\,\sigma^2\,\eta^2)) + 4\,\gamma_2^2\,(18\,\eta^2\,(-9 + 3\,\eta^3 + \sigma^2 + \eta\,\sigma^2) \\
& + 2\,\gamma_2^4\,(3 + 6\,\eta - 90\,\eta^2 - 3\,\eta^3 + 12\,\eta^4 + 27\,\eta^5 + 16\,\sigma^2\,\eta^2) - 3\,\gamma_2^2\,(2 + \eta\,(-1 + 2\,\eta\,(-57 - \eta + 2\,\eta^2 \\
& + 18\,\eta^3 + 3\,(3 + \eta)\sigma^2))))) \,,
\end{aligned}
$$

$$
y_{36} = 3888\,(\gamma_2^2 - 1)^2\,y_2^2\,\eta - 864\,\alpha\,\gamma_2\,(\gamma_2^2 - 1)\,y_2\,(\gamma_2^2 + (\gamma_2^2 - 1)\,y_2\,\eta - 9\,(\gamma_2^2 - 1)\,\eta^2)\,\sigma + 3\,\alpha^8\,\gamma_2^6\,\eta\,(1 + \eta)\,(2 + \eta^3)\,\sigma^2,
$$

$$
\begin{aligned}
y_{37} = {} & 12\,\gamma_2^2\,\Big(-2\,y_2\,y_3 + (-27\,\gamma_1^4 + 36\,\gamma_2^2\,(\gamma_1^2 - 2)\,(1 + \gamma_1^2) + 12\,(5 + 4\,\gamma_1^2)\,\gamma_2^4 + 16\,\gamma_2^6)\,\eta \\
& + 144\,(\gamma_2^2 - 1)^2\;y_2\,\eta^2 - 324\,\eta^3\,(\gamma_2^2 - 1)^2\Big)\,\sigma^2,
\end{aligned}
$$

$$
\begin{aligned}
y_{38} = {} & (972\,(\eta^3 + \eta^5) - 16\,\gamma_2^6(4 + 24\,\eta + 6\,\eta^2 - 25\,\eta^3 + 18\,\eta^4) - 36\,\gamma_2^2\,\eta\,(25 + 24\,\eta + 42\,\eta^2 + 17\,\eta^3 + 54\,\eta^4) \\
& + 3\gamma_2^4(28 + 408\eta + 288\eta^2 + 47\eta^3 + 300\eta^4 + 324\eta^5))\,\sigma^2 + 12\gamma_2^2\,(3 - 4\gamma_2^2)\,\eta^3\sigma^4 + 27\gamma_1^4\,(8\gamma_2^4\eta - 18\,(2 + \eta + 2\eta^3) \\
& + 6\,\gamma_2^2\,(6 + \eta + 6\,\eta^3) - (2 + \eta^3)\,\sigma^2) \,,
\end{aligned}
$$

$$
\begin{aligned}
y_{39} = {} & 3\,\gamma_1^2\,(-162\,(\eta^2 + \eta^4) + 8\,\gamma_2^6(4 + \eta - 9\,\eta^2 + 3\,\eta^3) - \gamma_2^4\,(81 + 72\,\eta + 18\,\eta^2 + 57\,\eta^3 + 162\,\eta^4 - 8\,\sigma^2\,\eta^2) \\
& + 3\,\gamma_2^2\,(19 + 24\,\eta + 84\eta^2 + 11\,\eta^3 + 108\,\eta^4 - 2\,\sigma^2\,\eta^2) \,,
\end{aligned}
$$

$$
\begin{aligned}
y_{40} = {} & \alpha^2\,\gamma_2^2\,(27\,\alpha^2\,\gamma_1^4\,(\alpha^2\,\gamma_2^2\,(2 + \eta^3) + (4 + 3\,\eta + 2\,\eta^3)\,\sigma^2) + 3\,\gamma_1^2\,(648\,(6\,\eta - 1) \\
& + 2\,\gamma_2^2\,(108\,(10 - 7\,\gamma_2^2 + 2\,(-14 + \gamma_2^2 + 4\gamma_2^4)\,\eta) + \alpha^4\,(4\,\gamma_2^4 - 9)\,(2 + \eta^3)) + 3\,\alpha^5\,\gamma_2^3(1 + 2\,\eta)\,(2 + \eta^3)\,\sigma \\
& + 6\,\alpha^3\,\gamma_2\,\eta\,(4 + 3\,\eta + 2\,\eta^3)\,\sigma^3) + \gamma_2^2\,(1296\,(3\,\eta - 1) + 4\,\gamma_2^2\,(\alpha^4\,(4\,\gamma_2^4 - 9)\,(2 + \eta^3) \\
& + 108\,(10 - 14\,\eta + \gamma_2^2\,(-7 + \eta + 4\,\gamma_2^2\,\eta))) + 2\,\alpha^5\,\gamma_2\,\eta\,\sigma\,(4\,\gamma_2^4 - 9)\,(2 + \eta^3) + 3\,\alpha^4\,\eta^2\,(4 + 3\,\eta + 2\,\eta^3)\,\sigma^4)) + y_{36},
\end{aligned}
$$

$$
\begin{aligned}
y_{41} = {} & \alpha^4\,\gamma_2^4\,(6\,\gamma_2^2(2 + 19\,\eta + 24\,\eta^2 + 10\,\eta^3 + 17\,\eta^4 + 18\,\eta^5)\sigma^2 + \gamma_2^4\,(108 + 45\,\eta^3 - 8\,\eta\,(2 + (-3 + \eta)\eta^2)\,\sigma^2) \\
& - 3\eta\sigma^2\,(12 + 6\eta\,(6 + \eta\,(3 + 4\eta + 6\eta^2)) + \eta\,(2 + \eta^3)\,\sigma^2) + 9\gamma_1^2\,(\gamma_2^2\,(17 + 7\eta^3) + (2 + 8\eta + 6\eta^2 + \eta^3 + 4\eta^4)\,\sigma^2)) \,,
\end{aligned}
$$

$$
\begin{aligned}
y_{42} = {} & (972\,(\eta^3 + \eta^5) - 16\,\gamma_2^6\,(4 + 24\,\eta + 6\,\eta^2 - 25\,\eta^3 + 18\,\eta^4) - 36\,\gamma_2^2\,\eta\,(16 + 24\,\eta + 60\,\eta^2 + 17\,\eta^3 + 54\,\eta^4) \\
& + 3\,\gamma_2^4(28 + 312\,\eta + 336\,\eta^2 + 263\,\eta^3 + 300\,\eta^4 + 324\,\eta^5))\,\sigma^2,
\end{aligned}
$$

$$
\begin{aligned}
y_{43} = {} & 3\,\gamma_1^2\,(-162\,(\eta^2 + \eta^4) + 8\,\gamma_2^6(4 + \eta - 9\,\eta^2 + 3\,\eta^3) - \gamma_2^4\,(57 + \eta\,(84 + \eta\,(126 + 3\,\eta\,(19 + 54\,\eta) - 8\,\sigma^2))) \\
& + \gamma_2^2\,(30 + 3\,\eta\,(24 + \eta\,(120 + 11\,\eta + 108\,\eta^2 - 2\,\sigma^2)))) \,,
\end{aligned}
$$

$$
\begin{aligned}
y_{44} = {} & \alpha^2\,(108\,y_2\,(\gamma_2^2 - 1)\;(2\,\gamma_2^2\,(6 - 7\,\gamma_2^2) + y_2\;(-9 + 1\,\gamma_2^2 + 4\,\gamma_2^4)\,\eta) \\
& + 3\,\alpha^2\,\gamma_2^2\,(2 + 3\,\eta + \eta^3)\,\sigma^2\,(3\,\gamma_1^2 + \alpha\,\gamma_2\,\eta\,\sigma)^2 + \alpha^2\,\gamma_2^2\,(2 + \eta^3)\,(\alpha^3\,\gamma_2^3\,(2\,\eta\,(4\,\gamma_2^4 - 9) + 9\,\gamma_1^2(1 + 2\,\eta))\,\sigma + 27\,\gamma_1^4\,\sigma^2 \\
& + 18\,\alpha\,\gamma_1^2\,\gamma_2\,\eta\,\sigma^3 + \alpha^2\,\gamma_2^2\,(27\,\gamma_1^4 - 36\,\gamma_2^2 + 16\,\gamma_2^6 + 6\,\gamma_1^2\,(4\,\gamma_2^4 - 9) + 3\,\eta^2\,\sigma^4))) + y_{36},
\end{aligned}
$$

$$
\begin{aligned}
y_{45} = {} & \alpha^{-1}\,(108\,y_2\,(\gamma_2^2 - 1)\,\eta\,(\alpha\,(12 - 21\,\eta^3 + \gamma_2^2\,(19\,\eta^3 - 13)) + 4\,\gamma_2\,\sigma\,(\eta^3 - 1)) \\
& + \alpha^4\,\gamma_2^3\,(2 + \eta^3)\,(-4\,(-27\,\gamma_1^2\,\eta + 9\,\gamma_1^2\,\gamma_2^2\,\eta + \gamma_2^4\,(-8 + 6\,\eta + 3\,\eta^2 + 8\,\eta^3))\,\sigma + 3\,\alpha^2\,\gamma_2^2\,\sigma\,(8 + \eta^2\,(3 + 4\,\eta - \sigma^2)) \\
& + \alpha\,\gamma_2\,(-9\,\gamma_1^2\,\eta\,(\sigma^2 - 3) + \gamma_2^2\,(9\,(2 + 2\,\eta + 13\,\eta^3) - 2\,(-8 + \eta\,(3 + 6\,\eta + 8\,\eta^2))\,\sigma^2)))) \,,
\end{aligned}
$$

$$
\begin{aligned}
y_{46} = {} & 18\,\eta\,\Big(18\,\gamma_2\,\eta\,\Big(y_2^2\,(4 - 5\,\gamma_2^2 + \gamma_2^4) - 12\,y_2\,\eta^2\,(\gamma_2^2 - 1)^2\Big) + \gamma_2\,(-2\,y_2\,y_3 + (-27\,\gamma_1^4 + 36\,\gamma_2^2\,(\gamma_1^2 - 2)\,(1 + \gamma_1^2) \\
& + 12\,(5 + 4\,\gamma_1^2)\,\gamma_2^4 + 16\,\gamma_2^6)\,\eta)\,\sigma^2) \,,
\end{aligned}
$$

$$
\begin{aligned}
y_{47} = {} & 9\,\gamma_2^2\,(-6\,(2 + 6\,\eta^3 + 4\,\eta^6 + 3\,\gamma_1^4\eta\,(3 + \eta) - \gamma_1^2\eta\,(29 + 16\,\eta^3)) + (4 + \eta\,(3\,\gamma_1^4\,\eta - 4\,\gamma_1^2(2 + \eta\,(3 + \eta)\,(12 + \eta)) \\
& + 2\,\eta\,(59 + 106\,\eta + 24\,\eta^2 + 16\,\eta^3)))\,\sigma^2 - 2\,\eta^4\,\sigma^4) \,,
\end{aligned}
$$

$$
\begin{aligned}
y_{48} = {} & 18\,\alpha\,\sigma\,(324\,\gamma_1^2\,(\gamma_1^2 - 3\,\eta)\,\eta + 8\,\gamma_2^8\,\eta\,(-4 + 7\,\eta + 9\,\eta^2 + 12\,\eta^3) + 2\,\gamma_2^6\,(-12 + \eta\,(6\,\gamma_1^2\,(-4 + 14\,\eta + 9\,\eta^2 + 12\,\eta^3) \\
& - 3\,(34 + 115\,\eta + 4\,\eta^2\,(14 + 8\,\eta + 9\,\eta^2)) + 4\,\eta\,(-4 + \eta\,(2 + 9\,\eta))\sigma^2)) - 9\,\gamma_2^2\,\eta\,(3\,\gamma_1^4\,(12 + 5\,\eta) \\
& + 2\,\eta\,(36 + 12\,\eta^3 + \sigma^2 - 4\,\eta^2\,\sigma^2) - \gamma_1^2\,(35 + 2\,\eta\,(108 + \eta\,(24 + 8\,\eta - \sigma^2)))) + 6\,\gamma_2^4\,(4 + \eta\,(41 + 21\,\gamma_1^4\,\eta + \\
& \eta\,(213 + 9\,\sigma^2 + 2\,\eta\,(22 - \sigma^2 + 4\,\eta\,(2 + 9\,\eta - 3\,\sigma^2))) - 2\,\gamma_1^2\,(21 + 96\,\eta + 45\,\eta^2 + 24\,\eta^3 - 2\,\sigma^2\,\eta^2)))) \,,
\end{aligned}
$$

$$\begin{aligned}
y_{49} = {} & \alpha^{-2}\,\gamma_2^{-1}\left(324\,\alpha\,\gamma_2^2\,\eta\left(\gamma_2^2-1\right)\left(\alpha\left(-45\,\gamma_2^2+43\,\gamma_2^4+\gamma_1^2\left(-54+51\,\gamma_2^2\right)\right)-4\,y_2\,\gamma_2\,\sigma\right)\right.\\
& +324\left(\gamma_2^2-1\right)^2\eta\left(18\,y_2^2\,\eta+\alpha\left(81\,\alpha\,\gamma_1^4-4\,y_2\,\gamma_2\left(y_2-9\,\eta\right)\eta\sigma+2\,\alpha\,\gamma_2^2\,\eta^2\,\sigma^2(9\,\eta-4\,y_2)\right)\right)\\
& +\alpha^6\gamma_2^6\left(2+\eta^3\right)\left(108\gamma_1^2\eta+\gamma_2\left(-12\gamma_2\left(3\gamma_1^2\eta+\gamma_2^2\left(2+2\eta+3\eta^3\right)\right)-2\alpha\left(9\gamma_1^2\eta+2\gamma_2^2(2+3\eta+3\eta^2+7\eta^3)\right)\sigma\right.\right.\\
& \left.\left.\left.-\alpha^2\,\gamma_2\left(2+\eta^2\left(6+\eta\right)\right)\sigma^2\right)\right)\right)-y_{46},\\
y_{50} = {} & \alpha^{-1}\,\gamma_2^{-1}\Big(324\,\eta\left(\gamma_2^2-1\right)^2\left(\alpha\,\gamma_2\,\eta\left(-15\,\alpha\,\gamma_2\,\eta+\alpha^3\,\gamma_2\left(3+\gamma_2^2\right)\eta^4+18\,\sigma\right)+y_2\left(18+3\,\alpha^2\left(3+\gamma_2^2\right)\eta^3-4\,\alpha\,\gamma_2\sigma\right)\right)\\
& +\alpha^4\,\gamma_2^2\left(2+\eta^3\right)\left(\alpha^3\,\gamma_2^3\left(3+\gamma_2^2\right)\left(4+\eta^2\left(3+2\,\eta\right)\right)\sigma+9\,\gamma_1^2\,\eta\,\sigma^2\left(3+\gamma_2^2\right)+3\,\alpha\,\gamma_2\,\eta\,\sigma\left(3+\gamma_2^2\right)\left(6\,\gamma_1^2+\eta\,\sigma^2\right)\right.\\
& \left.+\alpha^2\,\gamma_2^2\left(27\,\eta\left(\gamma_1^2-4\,\eta^2\right)+6\,\gamma_2^4\left(4+\eta+6\,\eta^3\right)+\left(3+\gamma_2^2\right)\left(2+\eta^2\left(6+\eta\right)\right)\sigma^2\right)\right)\Big),\\
y_{51} = {} & 36\,\alpha\,\gamma_2\,\eta\left(2\,y_6\,\eta^2+y_2\left(16\,\gamma_2^6\,\eta+54\left(\gamma_1^2-7\,\eta\right)\eta-6\,\gamma_2^4\left(-4+9\,\eta-4\,\gamma_1^2\,\eta+57\,\eta^2+4\,\eta^3\right)\right.\right.\\
& \left.\left.+3\,\gamma_2^2\left(-8-27\,\gamma_1^2\,\eta+12\,\eta+240\,\eta^2+8\,\eta^3\right)\right)\right)\sigma+\alpha^8\,\gamma_2^8\left(2+\eta^3\right)^2\left(\sigma^2-3\right),\\
y_{52} = {} & 9\,\gamma_2^4\left(40+2\,\eta\left(84+8\,\eta^2+78\,\eta^3+80\,\eta^5-9\,\gamma_1^2\left(14+2\,\eta+13\,\eta^3\right)\right)-\left(12+\eta\left(-4\,\gamma_1^2\left(4+\eta\left(3+6\,\eta+8\,\eta^2\right)\right)\right.\right.\right.\\
& \left.\left.\left.+3\left(3+\eta\right)\left(3+23\,\eta+7\,\eta^2+12\,\eta^3+4\,\eta^4\right)\right)\right)\sigma^2+16\,\eta^4\,\sigma^4\right),\\
y_{53} = {} & 2\,\gamma_2^6\left(-12+\eta\left(6\,\gamma_1^2(4+6\,\eta+3\,\eta^2+8\,\eta^3)-3(81+96\,\eta+109\,\eta^2+36\,\eta^3+114\,\eta^4+4\,\eta^5)\right.\right.\\
& \left.\left.+8\,\eta(-4+6\,\eta+3\,\eta^2+4\,\eta^3)\sigma^2\right)\right)+9\,\gamma_2^2\,\eta\left(-18\,\gamma_1^4\left(3+\eta\right)+4\,\eta\left(-3\left(5+7\,\eta^3\right)+\left(-1+\eta^2\left(3+\eta\right)\right)\sigma^2\right)\right.\\
& \left.+3\,\gamma_1^2\left(22+72\,\eta+39\,\eta^2+8\,\eta^3-3\,\sigma^2\,\eta^2\right)\right),\\
y_{54} = {} & \alpha^{-1}\,\gamma_2\left(-108\,\eta\left(\gamma_2^2-1\right)\left(\alpha^3\,\gamma_2\left(-33\,\gamma_2^2+29\,\gamma_2^4+6\,\gamma_1^2\left(-6+5\,\gamma_2^2\right)\right)+12\,y_2^2\,\eta\,\sigma\right)\right.\\
& +\alpha^5\,\gamma_2^5\left(2+\eta^3\right)\left(24\,\gamma_2^2\,y_2\,\eta+8\,\alpha\,\gamma_2\left(9\,\gamma_1^2\,\eta+\,\gamma_2^2\left(-4+6\,\eta+3\,\eta^2+4\,\eta^3\right)\right)\sigma\right.\\
& +6\,\alpha^3\,\gamma_2\,\sigma\left(-4+\eta^2\left(-3-2\,\eta+\sigma^2\right)\right)+\alpha^2\left(18\,\gamma_1^2\,\eta\left(\sigma^2-3\right)+\gamma_2^2\left(-9\left(2+4\,\eta+13\,\eta^3\right)\right.\right.\\
& \left.\left.\left.\left.+4\left(-4+\eta\left(3+6\,\eta+4\,\eta^2\right)\right)\sigma^2\right)\right)\right)\right)+y_{51},\\
y_{55} = {} & 6\,\alpha^2\,\gamma_2\,\sigma\left(6\left(-9\,\gamma_1^4\left(33-84\,\gamma_2^2+56\,\gamma_2^4\right)\eta+12\,\gamma_1^2\left(8\,\gamma_2^6\,\eta\left(3\,\eta^3-7\right)+9\,\eta^2\left(\sigma^2-6\,\eta^3\right)\right.\right.\right.\\
& \left.+3\gamma_2^2\left(2+\eta\left(-11+\eta\left(3+2\eta(-1+2\eta(2+9\eta))-9\sigma^2\right)\right)\right)+\gamma_2^4\left(-6+84\eta-9\eta^2-6\eta^3(1+\eta)(9\eta-1)+16\sigma^2\eta^2\right)\right)\\
& +4\,\gamma_2^2\left(8\,\gamma_2^6\,\eta\left(3\,\eta^3-7\right)+18\,\eta^2\left(-6\,\eta^3+(1+\eta)\sigma^2\right)-2\,\gamma_2^4\left(6-42\,\eta+9\,\eta^2+6\,\eta^3\left(-1+4\,\eta+9\,\eta^2\right)-16\,\sigma^2\,\eta^2\right)\right.\\
& \left.\left.+3\gamma_2^2\left(4+\eta\left(-11+2\eta\left(3-2\eta+4\eta^2+36\eta^3-3(3+\eta)\sigma^2\right)\right)\right)\right)\right)+\alpha^2\left(9\gamma_1^4\eta\left(4\gamma_2^4\left(8+7\eta^3\right)+18\left(-3+2\eta^2+\eta^5\right)\right.\right.\\
& \left.-3\,\gamma_2^2\left(-1+2\,\eta^2\left(6+5\,\eta+3\,\eta^3\right)\right)\right)+2\,\gamma_2^2\left(8\,\gamma_2^6\left(-8+8\,\eta+30\,\eta^2-4\,\eta^3+7\,\eta^4+9\,\eta^5+6\,\eta^6\right)\right.\\
& +9\,\eta\left(-6\left(1-6\,\eta+3\,\eta^3+2\,\eta^6\right)+\left(2+\eta^2\left(-11-2\,\eta+8\,\eta^3\right)\right)\sigma^2\right)\\
& +\gamma_2^4\left(84-3\,\eta\left(40+321\,\eta+50\,\eta^2+68\,\eta^3+120\,\eta^4+23\,\eta^5+36\,\eta^6\right)\right.\\
& \left.+8\,\eta\left(4-2\,\eta-21\,\eta^2-4\,\eta^3+2\,\eta^4+9\,\eta^5\right)\sigma^2\right)+3\,\gamma_2^2\left(-8+\eta\left(24+129\,\eta+64\,\eta^2+102\,\eta^3+96\,\eta^4\right.\right.\\
& \left.\left.\left.+7\,\eta^5+72\,\eta^6+2\left(-9+2\,\eta+42\,\eta^2+9\,\eta^3-2\,\eta^4-24\,\eta^5\right)\sigma^2\right)\right)\right)+3\,\gamma_1^2\left(-324\,\eta^2\left(-1+\eta^5\right)\right.\\
& +8\,\gamma_2^6\left(-8+16\,\eta+30\,\eta^2-4\,\eta^3+14\,\eta^4+9\,\eta^5+6\,\eta^6\right)\\
& -3\,\gamma_2^4(-28-4\,\eta+321\,\eta^2+4\,\eta^3\left(11+10\,\eta+30\,\eta^2+5\,\eta^3+27\,\eta^4\right))\\
& \left.\left.\left.+3\,\gamma_2^2\left(-8-72\,\eta+129\,\eta^2+58\,\eta^3+4\,\eta^6+216\,\eta^7+44\,\sigma^2\,\eta^2+96\,\eta^5+8\sigma^2\,\eta^5\right)\right)\right)\right),\\
y_{56} = {} & 2\,\alpha^6\,\gamma_2^3\,\sigma\left(3\,\gamma_2^2\left(-66\,\eta+6\,\eta^4\left(-44+6\,\eta-8\,\eta^3+3\,\eta^4\right)+3\,\gamma_1^2\left(41+\eta^2(42+41\,\eta+4\,\eta^2\left(18+3\,\eta+2\,\eta^2+9\,\eta^3\right))\right)\right.\right.\\
& \left.+\gamma_2^2\left(86+78\,\eta+84\,\eta^2+86\,\eta^3+294\,\eta^4-12\,\eta^5+17\,\eta^6+60\,\eta^7-18\,\eta^8\right)\right)+\eta\left(-54\,\eta^2\left(-3+2\,\eta^2+\eta^5\right)\right.\\
& +9\,\gamma_1^2\left(4-9\,\eta+4\,\eta^3+\eta^6\right)-4\,\gamma_2^4\left(-16+6\,\eta+24\,\eta^2-4\,\eta^3+3\,\eta^4+21\,\eta^5+2\,\eta^6\right)\\
& \left.\left.+3\,\gamma_2^2\left(5-18\,\eta-3\,\eta^2+11\,\eta^3+2\,\eta^4\left(18+15\,\eta+\eta^2+9\,\eta^3\right)\right)\right)\sigma^2\right),\\
y_{57} = {} & \alpha^3\,\gamma_2^2\left(-12\,\eta\,\sigma^2\left(11+2\,\eta^3\right)\left(9\,\gamma_1^4\left(4\,\gamma_2^2-3\right)+24\,\alpha\,\gamma_1^2\,\gamma_2^3\,\eta\,\sigma+\alpha^2\,\gamma_2^2\,\eta^2\,\sigma^2\left(4\,\gamma_2^2-3\right)\right)\right.\\
& +\alpha^4\,\gamma_2^2\left(2+\eta^3\right)^2\left(-27\,\gamma_1^4+6\,\gamma_1^2\left(2\,\gamma_2^4+\alpha\,\gamma_2\left(\gamma_2^2-3\,\eta\right)\sigma-6\,\eta\,\sigma^2\right)+\gamma_2\left(8\,\gamma_2^5+4\,\alpha\,\gamma_2^4(1+\eta)\sigma-3\,\alpha^2\,\gamma_2\,\eta^2\,\sigma^2\right.\right.\\
& \left.\left.\left.-6\,\alpha\,\eta^2\,\sigma^3+\alpha^2\,\gamma_2^3\,\eta\left(3\,\eta+2\,\sigma^2\right)\right)\right)\right),
\end{aligned}$$

$$
\begin{aligned}
y_{58} &= \alpha \Big(12\,\alpha^2\,\gamma_2^2\,\eta\,\sigma^2\left(4\,\gamma_2^2-3\right)\left(10+3\,\eta^3\right)\left(3\,\gamma_1^2+\alpha\,\gamma_2\,\eta\,\sigma\right)^2+\alpha^4\,\gamma_2^2\left(2+\eta^3\right)\left(3\left(4\left(9\,\gamma_1^4\,\eta\left(\gamma_2^4+9\,\eta^2\right)\right.\right.\right.\\
&\quad +4\,\gamma_2^8\left(2+\eta+3\,\eta^3\right)+6\,\gamma_1^2\,\gamma_2^6\left(2+2\,\eta+3\,\eta^3\right)\big)+\alpha^2\,\gamma_2^4\left(-36\,\eta^5-6\,\gamma_1^2\left(8+\eta^3\right)+\gamma_2^2\left(-34-5\,\eta^3+36\,\eta^5\right)\right)\big)\\
&\quad +2\,\alpha\,\gamma_2\left(27\,\gamma_1^4\left(-2+\gamma_2^2\,\eta-\eta^3\right)+12\,\gamma_1^2\left(27\,\eta^4+\gamma_2^4\left(2+3\,\eta+3\,\eta^2+7\,\eta^3\right)\right)+\gamma_2^4\left(3\,\alpha^2\,\eta\left(-10+2\,\eta-2\,\eta^3+\eta^4\right)\right.\right.\\
&\quad \left.\left.+4\,\gamma_2^2\left(4+9\,\eta+6\,\eta^2+14\,\eta^3+9\,\eta^4\right)\right)\right)\sigma+2\,\gamma_2^2\left(27\,\gamma_1^4\eta+\alpha^2\left(54\,\eta^5+3\,\gamma_1^2\,\gamma_2^2\left(2+\eta^2\left(6+\eta\right)\right)\right.\right.\\
&\quad \left.\left.+4\,\gamma_2^4\left(1+2\,\eta+3\,\eta^2+2\,\eta^3+7\,\eta^4\right)\right)\right)\sigma^2+2\,\alpha\,\gamma_2^3\,\eta\left(18\,\gamma_1^2\,\eta+\alpha^2\,\gamma_2^2\left(2+\eta^2\left(3+\eta\right)\right)\right)\sigma^3+6\,\alpha^2\,\gamma_2^4\,\eta^3\,\sigma^4\big)\\
&\quad -972\left(\gamma_2^2-1\right)^2\,\eta^4\left(4\,y_2^2+\alpha\,\gamma_1^4\left(9\,\alpha\,\eta^2-8\,\gamma_2\sigma\right)\right)\Big)+y_{57},\\
y_{59} &= 324\,\alpha\,\gamma_2^3\left(\gamma_2^2-1\right)\left(4\,\gamma_2^4\,\eta\left(-4+\eta+9\,\eta^2\right)+3\,\eta\left(3\,\gamma_1^4\,\eta+6\left(7-6\,\eta\right)\eta+\gamma_1^2\left(-4+6\,\eta\left(-7+6\,\eta\right)\right)\right)\right.\\
&\quad \left.+\gamma_2^2\left(-4+\eta\left(-11+6\,\eta\left(-19+12\,\eta\right)+6\,\gamma_1^2\left(-4+\eta\left(2+9\eta\right)\right)\right)\right)\right)\sigma,\\
y_{60} &= 6\,\alpha^5\,\gamma_2^5\,\sigma\left(2\,\gamma_2^4\left(-4+4\,\eta+24\,\eta^2+7\,\eta^3+2\,\eta^4+18\,\eta^5+3\,\eta^6\right)+3\,\eta\left(-6\,\eta\left(6+2\,\eta+6\,\eta^3+\eta^4\right)\right.\right.\\
&\quad \left.+3\,\gamma_1^2(4-3\,\eta+3\,\eta^2+2\,\eta^3)+\eta\left(2-3\,\eta+3\,\eta^2+\eta^3\right)\sigma^2\right)+\gamma_2^2\left(-9+\eta\left(24+3\,\gamma_1^2\left(4+\eta^2\left(3+2\,\eta\right)\right)\right.\right.\\
&\quad \left.\left.\left.+3\,\eta\left(9+11\,\eta+4\,\eta^2+24\,\eta^3+4\,\eta^4\right)+\eta\left(2+\eta^2\left(3+\eta\right)\right)\sigma^2\right)\right)\right),\\
y_{61} &= 18\,\alpha^3\,\gamma_2^3\,\sigma\left(-108\,\eta\left(-3\,\eta^4+\gamma_1^2\left(1-3\,\eta+3\,\eta^2+\eta^3\right)\right)+4\,\gamma_2^6\left(-8+4\,\eta+19\,\eta^2+6\,\eta^3+6\,\eta^4+27\,\eta^5\right)\right.\\
&\quad +\gamma_2^4\left(30+\eta\left(21+12\,\gamma_1^2\left(2+\eta+9\,\eta^2+3\,\eta^3\right)+6\,\eta\left(-76+31\,\eta+8\,\eta^2+18\,\eta^3\right)+2\,\eta\left(-4+\eta+9\,\eta^2\right)\sigma^2\right)\right)\\
&\quad \left.+3\gamma_2^2\left(-2+\eta\left(-24+3\gamma_1^4\eta-2\eta(-66+35\eta+12\eta^2+90\eta^3)+\eta(-1+4\eta(-4+3\eta))\sigma^2+\gamma_1^2\left(12+\eta^2\sigma^2\right)\right)\right)\right),\\
y_{62} &= \alpha\,\gamma_2\left(2\,\alpha^5\,\gamma_2^5\left(2+\eta^3\right)\left(27\,\gamma_1^2\,\eta+6\left(-9\,\eta^3+\gamma_2^4\left(2+\eta+3\,\eta^3\right)\right)+\alpha\,\gamma_2\left(3+\gamma_2^2\right)\left(2+\eta^2\left(3+\eta\right)\right)\sigma\right)\right.\\
&\quad +27\,\eta\left(\eta-1\right)\left(324\,\gamma_1^2\left(\gamma_2^2-1\right)\left(\gamma_1^2-6\,\eta\right)\sigma+9\,\alpha\,\gamma_2\left(-36\,\gamma_1^4+\left(\gamma_1^4-24\,\gamma_1^2\,\eta+36\,\eta^2\right)\sigma^2\right)+\alpha^3\,\gamma_2^3\left(9\,\gamma_1^4+\eta^2\,\sigma^4\right)\right.\\
&\quad \left.\left.+6\,\alpha^2\,\gamma_2^2\,\sigma\left(3\,\gamma_1^4-6\,\eta^2\,\sigma^2+\gamma_1^2\,\eta\left(8\,\gamma_2^2\left(4+\eta\right)+\sigma^2\right)\right)\right)\right),\\
y_{63} &= \alpha^{-1}\left(-162\,y_2\left(\gamma_2^2-1\right)\eta\left(\alpha\left(-18+19\,\gamma_2^2\right)+4\,\gamma_2\,\sigma\right)-\alpha^5\,\gamma_2^4\left(2+\eta^3\right)\left(12\,\gamma_2^4\left(2+\eta+3\,\eta^3\right)\right.\right.\\
&\quad \left.\left.+2\,\alpha\,\gamma_2^3(4+3\,\eta+3\,\eta^2+14\,\eta^3)\sigma+\alpha^2\,\gamma_2^2\left(2+\eta^2\left(3+\eta\right)\right)\sigma^2+9\,\gamma_1^2\,\eta\left(\sigma^2-6\right)+3\,\alpha\,\gamma_2\,\eta\,\sigma\left(3\,\gamma_1^2+\eta\,\sigma^2\right)\right)\right).
\end{aligned}
$$

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