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Influence of energy dissipation on plane harmonic waves through a piezo-thermo-elastic medium

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Abstract. The concept of thermo-elasticity proposed by Green and Naghdi is employed to study the plane harmonic waves through a piezo-electric thermo-elastic medium. An analytical technique of normal modes is adopted to find the exact solution of the problem. The theoretical results obtained are represented graphically for the particular material. It is found that energy dissipation reduces the amplitude of waves propagating through the medium. The results fully agree with physical interpretation of the problem.

1 Introduction

Piezo-electric materials are used to transduce electrical and mechanical energy. Wave propagation is one of the most realistic models of piezo-electric and elastic solids, which have been extensively used in many engineering and industrial applications, such as sensors, actuators, intelligent structures, radio, and computer technology and ultrasonic.

The models of heat propagation were proposed by Green and Naghdi [1–3], where the type-II considered undamped thermo-elastic waves in an elastic material, model which is also named as the theory of thermo-elasticity with no energy dissipation. The type-III model includes the type-I and type-II ones as special cases. The uniqueness of the solution to the governing equations of the Green-Naghdi (GN) type-II model is presented in [4]. Chandrasekharaiah [5] analyzed the one-dimensional problem related to thermal waves in the context of the Green-Naghdi model, using the Laplace transform method. Othman and Song [6] extended the work and examined the influence of the magnetic field on the reflected waves generated in a rotating medium in the context of thermo-elasticity of type II. Recently, some authors discussed different type of problems [7–11].

Aoudi [12] studied the problem of a piezo-electric material with temperature-dependent elastic properties. In his article, he used the heat conduction equation by Lord and Shulman [13], while the problem is assumed to be one-dimensional. Recently, Fatemah [14] and Othman *et al.* [15] studied the problems related to the piezo-electric material in the context of different heat theories. The analysis on the piezo-electric material in the context of Green-Naghdi equations has never been done before. This work aims to analyze the response of the piezo-electric material during the small deformation in the medium. Deformation in the medium produces heat that propagates through it and, to study the conduction of heat waves, we have considered the GN theory. The harmonic wave solution is used to obtain the analytical response of each wave propagating through the medium.

2 Basic equations

The basic equations for the selected material were presented in [12], while *Hook's law* for the piezo-electric material is represented by

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{ijk}E_k - \beta_{ij}T. \tag{1}$$

The equation of motion can be represented as

$$\sigma_{ij,j} = \rho\ddot{u}_i. \tag{2}$$

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The piezo-electric material has no free charge, because of the Gauss's equations and electric field relations,

$$D_{i,i} = 0 \tag{3}$$

$$D_i = e_{jik}\varepsilon_{jk} + \in_{ij} E_j + p_i T, \tag{4}$$

where $E_i = -\varphi_{,i}$ is the electric field and D_i is the electric displacement. The heat conduction equation proposed by Green and Naghdi [3] is

$$K_{ij}\dot{T}_{,ij} + K^*_{ij}T_{,ji} = \rho c_e \ddot{T} + T_\circ[\beta_{ij}\ddot{u}_{i,j} - p_i\dot{\varphi}_{,i}], \tag{5}$$

where T is the temperature of the medium above the reference temperature T_0, u_i, σ_{il}, e_{il}, β_{il} and e_{ilk} are the components of displacement vector, stress tensor, strain tensor, coupling constant and piezo-electric moduli, respectively, mass density and dielectric moduli are represented by ρ and D_i, the specific heat is C_E, K^* and K are the thermal conductivity and the material characteristic, respectively. The constitutive relations are

$$\begin{aligned}
\sigma_{xx} &= C_{11}\varepsilon_{xx} + C_{13}\varepsilon_{zz} - e_{31}E_z - \beta_1 T,\\
\sigma_{zz} &= C_{13}\varepsilon_{xx} + C_{33}\varepsilon_{zz} - e_{33}E_z - \beta_3 T,\\
\sigma_{zx} &= 2C_{44}\varepsilon_{zx} - e_{15}E_x,\\
D_{\mathrm{x}} &= e_{15}(u_{,z} + w_{,x}) + \in_{11} E_x,\\
D_{\mathrm{z}} &= e_{31}u_{,x} + e_{33}w_{,z} + \in_{33} E_z + \mathrm{p}_3\mathrm{T}.
\end{aligned}$$

3 Formulation

The half-space $x_3 \geq 0$, with x_3 pointing vertically into the medium, is chosen for analysis of the plane waves through the medium. The Cartesian coordinate system is selected to show the mathematical representation of the problem. The plane strain is represented as

$$T(x,z,t), \qquad \vec{u} = (x,z,t) = (u,0,w), \quad \text{and} \quad \varphi = (x,z,t).$$

The above basic equations become

$$C_{11}u_{,xx} + C_{44}u_{,zz} + (C_{13} + C_{44})w_{,xz} + (e_{31} + e_{15})\varphi_{,xz} - \beta_1 T_{,x} = \rho\ddot{u}, \tag{6}$$

$$(C_{44} + C_{13})u_{,xz} + C_{44}w_{,xx} + C_{33}w_{,zz} + e_{15}\varphi_{,xx} + e_{33}\varphi_{,zz} - \beta_3 T_{,z} = \rho\ddot{w}, \tag{7}$$

$$K_1\dot{T}_{,xx} + K_3\dot{T}_{,zz} + K_1^*T_{,xx} + K_3^*T_{,zz} - \rho c_e\ddot{T} = T_0[\beta_{11}\ddot{u}_{,x} + \beta_{33}\ddot{w}_{,z} - p_3\dot{\varphi}_{,z}], \tag{8}$$

$$\because D_{i,i} = 0 \tag{9}$$

$$\therefore (e_{15} + e_{31})u_{,xz} + e_{15}w_{,xx} + e_{33}w_{,zz} - \in_{11}\varphi_{,xx} - \in_{33}\varphi_{,zz} + \mathrm{p}_3\mathrm{T}_{,z} = 0, \tag{10}$$

non-dimensionalizing the governing equation with the help of following variables:

$$\begin{aligned}
&(x', z') = \frac{\omega^*}{v_p}(x,z), \qquad (u', w') = \frac{\rho\omega^* v_p}{\beta_1 T_0}(u,w), \qquad T' = \frac{T}{T_0}\,,\\
&\sigma'_{ij} = \frac{\sigma_{ij}}{\beta_1 T_0}\,, \qquad \varphi' = \varepsilon_p\varphi, \qquad t' = \omega^* t,\\
&D'_i = \frac{D_i}{\beta_1 T_0}\,,
\end{aligned} \tag{11}$$

where $\omega^* = \frac{c_e c_{11}}{K_{11}}$, $\varepsilon_p = \frac{\omega^* e_{33}}{v_p\beta_1 T_0}$, $\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3$, $\beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3$.

The system of equations in non-dimensional form (removing prime for simplicity) is

$$u_{,xx} + \delta_1 u_{,zz} + \delta_2 w_{,xz} + \delta_3\varphi_{,xz} + \delta_4 T_{,x} = \delta_5\ddot{u}, \tag{12}$$

$$\delta_2 u_{,xz} + \delta_1 w_{,xx} + \delta_6 w_{,zz} + \delta_7\varphi_{,xx} + \delta_5\varphi_{,zz} + \delta_8 T_{,z} = \delta_5\ddot{w}, \tag{13}$$

$$\delta_9 u_{,xz} + \delta_{10}w_{,xx} + \delta_{11}w_{,zz} + \delta_{12}\varphi_{,xx} + \delta_{13}\varphi_{,zz} + \delta_{14}T_{,z} = 0, \tag{14}$$

$$\delta_{15}\dot{T}_{,xx} + \delta_{16}\dot{T}_{,zz} + \delta_{17}T_{,zz} + \delta_{18}T_{,zz} - \ddot{T} = [\delta_{19}\dot{u}_{,x} + \delta_{20}\dot{w}_{,z} + \delta_{21}\dot{\varphi}_{,z}], \tag{15}$$

where

$$\delta_1=\frac{c_{44}}{c_{11}},\qquad \delta_2=\frac{(c_{13}+c_{44})}{c_{11}},\qquad \delta_3=\frac{(e_{31}+e_{15})\rho v_p^2}{c_{11}e_{33}},\qquad \delta_4=\frac{-\rho v_p^2}{c_{11}},$$
$$\delta_5=\frac{\rho v_p^2}{c_{11}},\qquad \delta_6=\frac{c_{33}}{c_{11}},\qquad \delta_7=\frac{e_{15}\rho v_p^2}{c_{11}e_{33}},\qquad \delta_8=\frac{-\beta_3\rho v_p^2}{c_{11}\beta_1},$$
$$\delta_9=\frac{(e_{15}+e_{31})}{c_{11}},\qquad \delta_{10}=\frac{e_{15}}{c_{11}},\qquad \delta_{11}=\frac{e_{33}}{c_{11}},\qquad \delta_{12}=\frac{-\in_{11}\rho v_p^2}{e_{33}c_{11}},$$
$$\delta_{13}=\frac{-\in_{33}\rho v_p^2}{e_{33}c_{11}},\qquad \delta_{14}=\frac{p_3\rho v_p^2}{c_{11}\beta_1},\qquad \delta_{15}=\frac{K_1\omega^*}{\rho c_e v_p^2},\qquad \delta_{16}=\frac{K_3\omega^*}{\rho c_e v_p^2},$$
$$\delta_{17}=\frac{K_1^*}{\rho c_e v_p^2},\qquad \delta_{18}=\frac{K_3^*}{\rho c_e v_p^2},\qquad \delta_{19}=\frac{\beta_1^2 T_0}{\rho^2 c_e v_p^2},\qquad \delta_{20}=\frac{\beta_1\beta_3 T_0}{\rho^2 c_e v_p^2},$$
$$\delta_{21}=-\frac{p_3\beta_1 T_0}{\rho c_e\omega^* e_{33}}.$$

4 Harmonic solution

Time harmonic wave solution is selected for each variable to analyze the harmonic behavior of waves propagating through the medium,

$$[u,w,\varphi,T](x,z,t)=[u^*,w^*,\varphi^*,T^*](z)e^{ia(x-ct)}. \tag{16}$$

with $\mathrm{D}=\frac{\mathrm{d}}{\mathrm{d}z}$, $c=\frac{\omega}{a}$, the frequency ω, the wave number a, and the amplitudes u^*, w^*, φ^* and T^*, the equations become

$$(\mathrm{D}^2+A_1)u^*+A_2\mathrm{D}w^*+A_3\mathrm{D}\varphi^*+A_4T^*=0, \tag{17}$$
$$A_5\mathrm{D}u^*+(\mathrm{D}^2+A_6)w^*+(A_7+A_8\mathrm{D}^2)\varphi^*+A_9\mathrm{D}T^*=0, \tag{18}$$
$$A_{10}\mathrm{D}u^*+(A_{11}+\mathrm{D}^2)w^*+(A_{12}+A_{13}\mathrm{D}^2)\varphi^*+A_{14}\mathrm{D}T^*=0, \tag{19}$$
$$A_{15}u^*+A_{16}\mathrm{D}w^*+A_{17}\mathrm{D}\varphi^*+(\mathrm{D}^2+A_{18})T^*=0, \tag{20}$$

where

$$A_1=-\frac{(a^2-a^2c^2\delta_5)}{\delta_1},\qquad A_2=\frac{ia\delta_2}{\delta_1},\qquad A_3=\frac{A_2\delta_3}{\delta_2},\qquad A_4=\frac{A_2\delta_4}{\delta_2},$$
$$A_5=\frac{ia\delta_2}{\delta_6},\qquad A_6=-\frac{(a^2\delta_1-a^2c^2\delta_5)}{\delta_6},\qquad A_7=-\frac{a^2\delta_7}{\delta_6},$$
$$A_8=\frac{\delta_5}{\delta_6},\qquad A_9=\frac{\delta_8}{\delta_6},\qquad A_{10}=\frac{ia\delta_9}{\delta_{11}},\qquad A_{11}=-\frac{a^2\delta_{10}}{\delta_{11}},$$
$$A_{12}=-\frac{a^2\delta_{12}}{\delta_{11}},\qquad A_{13}=\frac{\delta_{13}}{\delta_{11}},\qquad A_{14}=\frac{\delta_{14}}{\delta_{11}},\qquad A_{15}=\frac{ia\delta_{19}}{\delta_{18}+iac\delta_{16}}(iac),$$
$$A_{16}=\frac{\delta_{20}}{\delta_{18}+iac\delta_{16}}(iac),\qquad A_{17}=\frac{\delta_{21}}{\delta_{18}+iac\delta_{16}}(iac),$$
$$A_{18}=-\frac{(a^2\delta_{17}-ia^3c\delta_{15}-a^2c^2)}{\delta_{18}+iac\delta_{16}}.$$

The non-trivial solution of eqs. (17)–(20) gives the following differential equation:

$$\left(\mathrm{D}^{10}-\coprod_1\mathrm{D}^8+\coprod_2\mathrm{D}^6-\coprod_3\mathrm{D}^4+\coprod_4 D^2-\coprod_5\right)\{u^*,w^*,\varphi^*,T^*\}(z)=0, \tag{21}$$

where

$$\coprod_1 = -\frac{\eta_2}{\eta_1}, \qquad \coprod_2 = \frac{\eta_3}{\eta_1}, \qquad \coprod_3 = -\frac{\eta_4}{\eta_1}, \qquad \coprod_4 = \frac{\eta_5}{\eta_1}, \qquad \coprod_5 = -\frac{\eta_6}{\eta_1}$$

$$\Lambda_1 = A_1 + A_{18}, \qquad \Lambda_2 = A_4 A_{15} - A_1 A_{18}, \qquad \Lambda_3 = A_4 A_{16} - A_2 A_{18},$$
$$\Lambda_4 = A_{17} A_4 - A_3 A_{18}, \qquad \Lambda_5 = A_4 A_5 - A_1 A_9, \qquad \Lambda_6 = A_4 - A_2 A_9,$$
$$\Lambda_7 = A_4 A_{16}, \qquad \Lambda_8 = A_4 A_7, \qquad \Lambda_9 = A_4 A_8 - A_9 A_3 \qquad \Lambda_{10} = A_4 A_{10} - A_1 A_{14},$$
$$\Lambda_{11} = A_4 - A_2 A_{14}, \qquad \Lambda_{12} = A_4 A_{11}, \qquad \Lambda_{13} = A_{13} A_4 - A_3 A_{14}$$
$$\Lambda_{14} = A_4 A_{12}, \qquad \Lambda_{15} = -\Lambda_{11} \Lambda_9 + \Lambda_6 \Lambda_{14}, \qquad \Lambda_{16} = -\Lambda_9 \Lambda_{12} + \Lambda_5 \Lambda_{11} - \Lambda_6 \Lambda_{10} + \Lambda_7 \Lambda_{14},$$
$$\Lambda_{17} = \Lambda_5 \Lambda_{12} - \Lambda_{10} \Lambda_7, \qquad \Lambda_{18} = -\Lambda_6 \Lambda_{13} + \Lambda_9 \Lambda_{11},$$
$$\Lambda_{19} = -\Lambda_6 \Lambda_{14} - \Lambda_7 \Lambda_{13} + \Lambda_9 \Lambda_{12} + \Lambda_8 \Lambda_{11} \qquad \Lambda_{20} = -\Lambda_7 \Lambda_{14} + \Lambda_8 \Lambda_{12},$$
$$\Lambda_{21} = \Lambda_{11} + A_{14} \Lambda_2, \qquad \Lambda_{22} = \Lambda_{12} + \Lambda_1 \Lambda_{11} - A_2 \Lambda_{10} - \Lambda_3 A_{14},$$
$$\Lambda_{23} = -\Lambda_1 \Lambda_{12} + \Lambda_2 \Lambda_{11} - \Lambda_3 \Lambda_{10}, \qquad \Lambda_{24} = \Lambda_2 \Lambda_{12}, \qquad \Lambda_{25} = \Lambda_{13} A_2 - \Lambda_3 \Lambda_{11},$$
$$\Lambda_{26} = -\Lambda_{13} \Lambda_3 + A_2 \Lambda_{14} - A_3 \Lambda_{12} + A_4 \Lambda_{11}, \qquad \Lambda_{27} = -\Lambda_3 \Lambda_{14} + \Lambda_4 \Lambda_{12}.$$
$$\eta_1 = \Lambda_{15} \Lambda_{25} + \Lambda_{18} \Lambda_{21}, \qquad \eta_2 = \Lambda_{15} \Lambda_{26} + \Lambda_{16} \Lambda_{25} + \Lambda_{18} \Lambda_{22} + \Lambda_{19} \Lambda_{21},$$
$$\eta_3 = \Lambda_{15} \Lambda_{17} + \Lambda_{16} \Lambda_{26} + \Lambda_{17} \Lambda_{25} - \Lambda_{18} \Lambda_{23} + \Lambda_{19} \Lambda_{22} + \Lambda_{20} \Lambda_{21},$$
$$\eta_4 = \Lambda_{16} \Lambda_{27} + \Lambda_{17} \Lambda_{26} - \Lambda_{18} \Lambda_{24} - \Lambda_{19} \Lambda_{23} + \Lambda_{20} \Lambda_{22}, \qquad \eta_5 = \Lambda_{17} \Lambda_{27} - \Lambda_{19} \Lambda_{24} - \Lambda_{20} \Lambda_{23},$$
$$\eta_6 = -\Lambda_{20} \Lambda_{24}.$$

Equation (21), in factorized form, reads

$$\sum_{n=1}^{5} (\mathrm{D}^2 - \xi_n^2) u^*(z) = 0. \tag{22}$$

The characteristic equation of (22) can be written as

$$(\lambda^2 - \xi_1^2)(\lambda^2 - \xi_2^2)(\lambda^2 - \xi_3^2)(\lambda^2 - \xi_4^2)(\lambda^2 - \xi_5^2) = 0. \tag{23}$$

By using the boundary conditions $z \to \infty$ on the solution, we get

$$u^* = \sum_{n=1}^{5} M_n e^{-\xi_n z}, \tag{24}$$

$$\varphi^* = \sum_{n=1}^{5} H_{1n} M_n e^{-\xi_n z}, \tag{25}$$

$$w^* = \sum_{n=1}^{5} H_{2n} M_n e^{-\xi_n z}, \tag{26}$$

$$T^* = \sum_{n=1}^{5} H_{3n} M_n e^{-\xi_n z}, \tag{27}$$

where ξ_n^2 represents the roots of eq. (23), and the constitutive equations become

$$\sigma_{xx}^* = \sum_{n=1}^{5} H_{4n} M_n e^{-\xi_n z}, \tag{28}$$

$$\sigma_{zz}^* = \sum_{n=1}^{5} H_{5n} M_n e^{-\xi_n z}, \tag{29}$$

$$\sigma_{xz}^* = \sum_{n=1}^{5} H_{6n} M_n e^{-\xi_n z}, \tag{30}$$

$$D_x^* = \sum_{n=1}^{5} H_{7n} M_n e^{-\xi_n z}, \tag{31}$$

$$D_z^* = \sum_{n=1}^{5} H_{8n} M_n e^{-\xi_n z}, \tag{32}$$

where

$$H_{1n}=\frac{\xi_n^5\Lambda_{15}+\xi_n^3\Lambda_{16}+\xi_n\Lambda_{17}}{\xi_n^4\Lambda_{18}+\xi_n^2\Lambda_{19}+\Lambda_{20}},$$

$$H_{2n}=\frac{(-A_{14}\xi_n^3+\xi_n\Lambda_{10})-(\xi_n^2\Lambda_{13}+\Lambda_{14})H_{1n}}{\xi_n^2\Lambda_{11}+\Lambda_{12}},$$

$$H_{3n}=\frac{1}{A_4}[A_3\xi_nH_{1n}+A_2\xi_nH_{2n}-(\xi_n^2+A_1)],$$

$$H_{4n}=(r_1-\xi_nr_2H_{2i}-\xi_ir_3H_{1i}-H_{3i}),$$

$$H_{5n}=r_4-\xi_nr_5H_{2i}-\xi_nH_{1i}-r_6H_{3i},$$

$$H_{6n}=r_7H_{2i}-r_8\xi_n+r_9H_{3n},$$

$$H_{7n}=-r_{10}\xi_n+r_{11}H_{2n}-r_{12}H_{1n},$$

$$H_{8n}=r_{13}-r_{14}\xi_nH_{2n}+r_{15}\xi_nH_{1n}+r_{16}H_{3n},$$

$$r_1=\frac{C_{11}ia}{v_p^2\rho},\quad r_2=\frac{C_{13}}{v_p^2\rho},\quad r_3=\frac{e_{31}}{e_{33}},\quad r_4=\frac{C_{13}ia}{v_p^2\rho},\quad r_5=\frac{C_{33}}{v_p^2\rho},\quad r_6=\frac{\beta_3}{\beta_1},\quad r_7=\frac{C_{44}ia}{v_p^2\rho},\quad r_8=\frac{C_{44}}{v_p^2\rho},$$

$$r_9=\frac{iae_{15}}{e_{33}},\quad r_{10}=-\frac{e_{15}}{v_p^2\rho},\quad r_{11}=\frac{iae_{15}}{v_p^2\rho},\quad r_{12}=\frac{\in_{11}ia}{e_{33}},\quad r_{13}=\frac{e_{31}ia}{v_p^2\rho},\quad r_{14}=\frac{e_{33}}{v_p^2\rho},\quad r_{15}=\frac{\in_{33}}{e_{33}},\quad r_{16}=\frac{p_3}{\beta_1}.$$

5 Boundary conditions

The boundary conditions, assumed on the surface $z=0$, are the following:

1) *Mechanical boundary conditions.*
   A periodic force with magnitude f_1^* is acting vertically into the medium,
   $$\sigma_{zz}(x,0,t)=-f_1^*\exp ia(x-ct),\qquad \sigma_{xx}(x,0,t)=0. \tag{33}$$
   The tangential stress is assumed to be negligible
   $$\sigma_{xz}(x,0,t)=0. \tag{34}$$
2) *Thermal boundary conditions.*
   Before any deformation, the medium is assumed to be in the state of equilibrium without any source of heat supply:
   $$T=0. \tag{35}$$
3) The normal component of the electric field is assumed to be zero:
   $$\frac{\partial\varphi}{\partial z}=0, \tag{36}$$
   where f_1^* is constant. Using eq. (24) in (26), we can obtain the following relations:

$$\sum_{n=1}^{5}H_{5n}M_n=-f_1, \tag{37}$$

$$\sum_{n=1}^{5}H_{4n}M_n=0, \tag{38}$$

$$\sum_{n=1}^{5}H_{6n}M_n=0, \tag{39}$$

$$\sum_{n=1}^{5}H_{3n}M_n=0, \tag{40}$$

$$\sum_{n=1}^{5}k_nH_{1n}M_n=0; \tag{41}$$

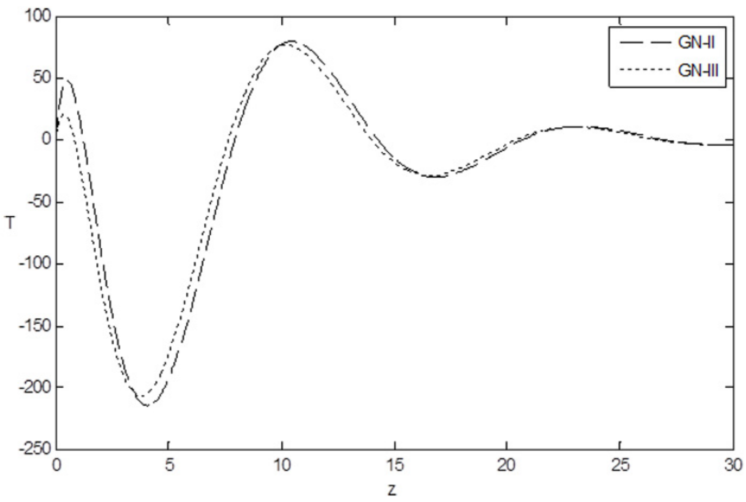

Fig. 1. Temperature distribution function.

solving eqs. (37)–(41) for M_n $(n = 1, \ldots, 5)$, as follows:

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} H_{51} & H_{52} & H_{53} & H_{54} & H_{55} \\ H_{41} & H_{42} & H_{43} & H_{44} & H_{45} \\ H_{61} & H_{62} & H_{63} & H_{64} & H_{65} \\ H_{31} & H_{32} & H_{33} & H_{34} & H_{35} \\ \xi_1 H_{11} & \xi_2 H_{12} & \xi_3 H_{13} & \xi_4 H_{14} & \xi_5 H_{15} \end{pmatrix}^{-1} \begin{pmatrix} -f_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{42}$$

6 Discussion

The numerical problem is solved for a particular material, cadmium selenide [7], and results are obtained and represented graphically:

$$c_{11} = 7.41 \times 10^{10}\,\mathrm{Nm}^{-2}, \qquad c_{12} = 4.52 \times 10^{10}\,\mathrm{Nm}^{-2},$$
$$c_{13} = 3.93 \times 10^{10}\,\mathrm{Nm}^{-2}, \qquad c_{33} = 8.36 \times 10^{10}\,\mathrm{Nm}^{-2},$$
$$c_{44} = 1.32 \times 10^{10}\,\mathrm{Nm}^{-2}, \qquad T_\circ = 298\,\mathrm{K},$$
$$\rho = 5504\,\mathrm{Kg\,m}^{-3}, \qquad e_{13} = -0.160\,\mathrm{C\,m}^{-2},$$
$$e_{33} = 0.347\,\mathrm{C\,m}^{-2}, \qquad e_{15} = -0.138\,\mathrm{C\,m}^{-2},$$
$$\beta_1 = 0.621 \times 10^{6}\,\mathrm{Nk}^{-1} \cdot \mathrm{m}^{-2}, \qquad \beta_3 = 0.551 \times 10^{6}\,\mathrm{NK}^{-1} \cdot \mathrm{m}^{-2},$$
$$p_3 = -2.94 \times 10^{-6}\,\mathrm{CK}^{-1} \cdot \mathrm{m}^{-2}, \qquad K_1 = K_3 = 9\,\mathrm{Wm}^{-1} \cdot \mathrm{K}^{-1},$$
$$K_1^* = K_3^* = 0.9\,\mathrm{Wm}^{-1} \cdot \mathrm{K}^{-1}, \qquad \in_{11} = 8.26 \times 10^{-11}\,\mathrm{C}^2\mathrm{N}^{-1} \cdot \mathrm{m}^{-2},$$
$$\in_{33} = 9.03 \times 10^{-11}\,\mathrm{C}^2\mathrm{N}^{-1} \cdot \mathrm{m}^{-2}, \qquad C_e = 260\,\mathrm{J} \cdot \mathrm{Kg}^{-1}\mathrm{K}^{-1}.$$

The computations were carried out for the non-dimensional form of the field variables against the vertical component of distance during $t = 0.1$ and $x = 1.5$.

Figure 1 presents the curves for the non-dimensional temperature distribution function against vertical distance for different Green-Naghdi theories, *i.e.* the type-II and type-III models. From graphical observation, it is found that the absolute amplitude of the temperature distribution function in the context of GN-III is lower as compared to that found in GN-II, indicating that the energy dissipation has a decreasing effect on the heat waves propagating along the depth of the medium. Both curves converge to zero as the distance from surface $z = 0$ increases.

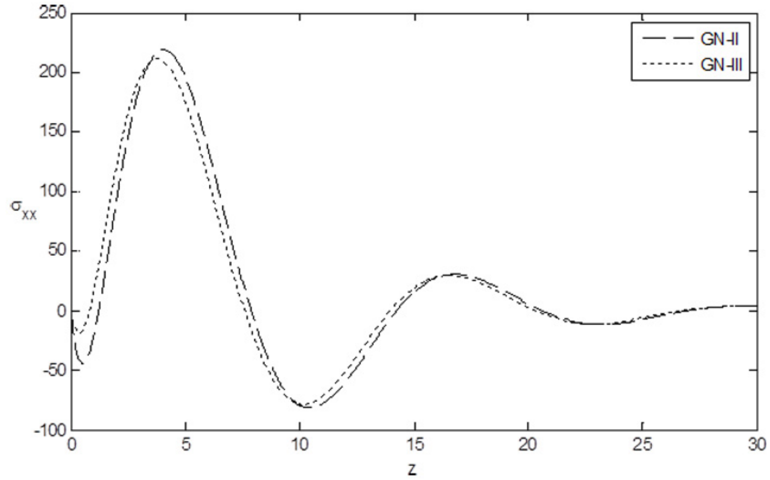

Fig. 2. Horizontal component of the normal stress distribution.

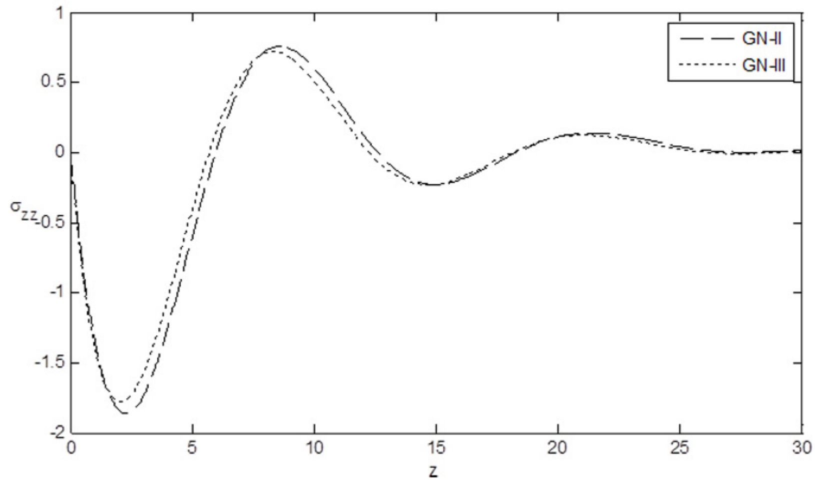

Fig. 3. Vertical component of normal stress distribution.

The graphical analysis of the horizontal component of the stress distribution function against the depth of the medium is shown in fig. 2. It can be seen that the curves without energy dissipation have a higher amplitude for the non-dimensional variable as compared to the curves with energy dissipation. Starting point for both curves is the same, which satisfies the boundary condition. All curves converge to zero as the vertical distance from the surface increases.

Figure 3 gives the graphical representation of the vertical component of the non-dimensional stress distribution function. It is found that the dissipation has a decreasing effect on the absolute amplitude of the normal stress σ_{zz}. From observation of the figure, it is seen that the greater is the distance from the surface of the medium the lower is the effect of energy dissipation.

The horizontal component of the displacement distribution function is presented in fig. 4. The amplitude of waves generated through the type-II theory presents higher curves as compared to those obtained for the type-III one. Initially, the absolute amplitude increases at $0 \le z \le 1.7$ and it decreases for $z > 1.7$. Finally, all curves converge to zero.

The vertical component of the displacement distribution function (fig. 5) has different starting points and has a very slow rate of convergence toward zero. Like the curves in the other figures, the amplitude value in the case of energy dissipation is less than the amplitude in the case without energy dissipation.

Figures 6–11 show the 3D curves for each field variable. From these curves it is clear that the curves will propagate harmonically along the horizontal component of the medium while they will damp out along the vertical component of the distance from the surface of the medium.

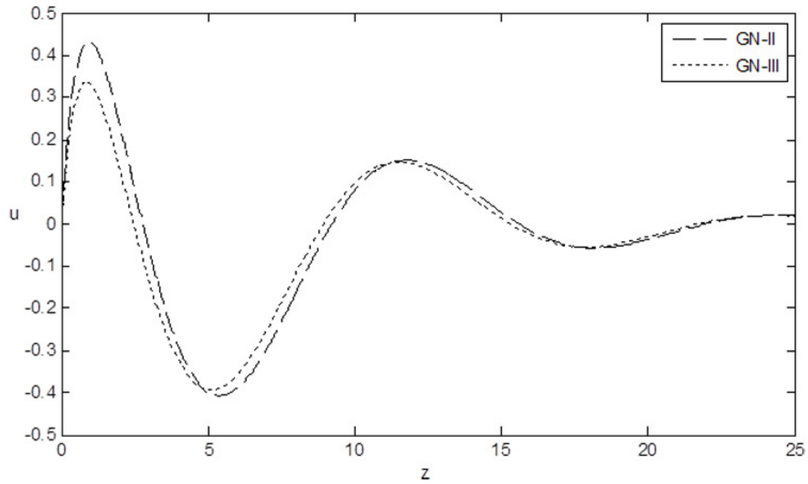

Fig. 4. Horizontal component of the displacement distribution function.

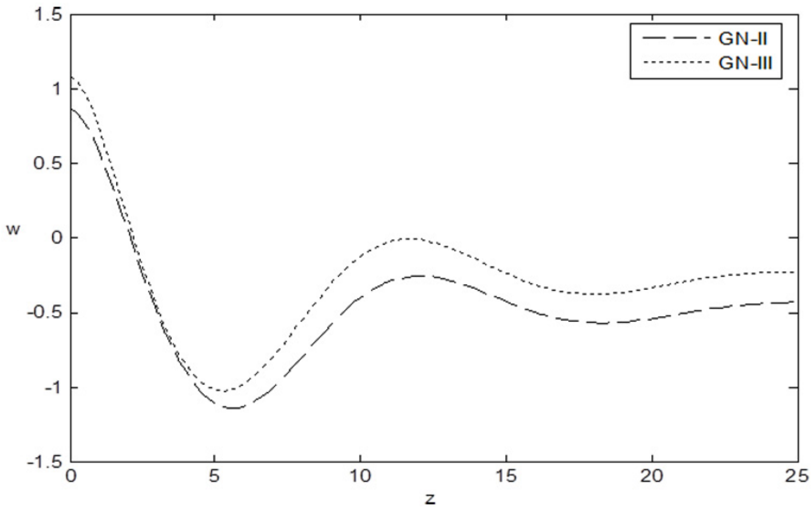

Fig. 5. Vertical component of the displacement distribution function.

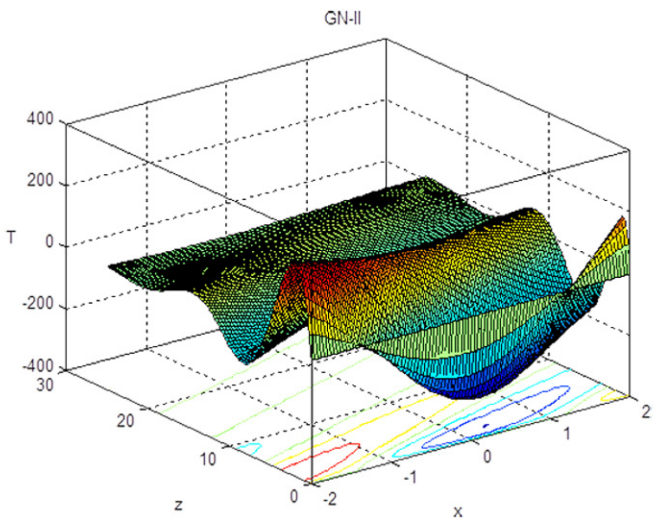

Fig. 6. 3D temperature distribution.

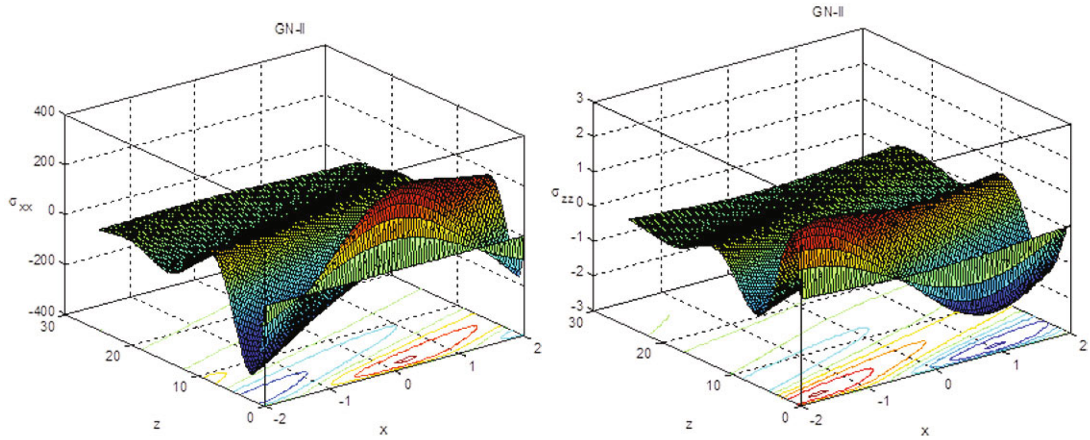

Fig. 7. 3D normal stress distribution.

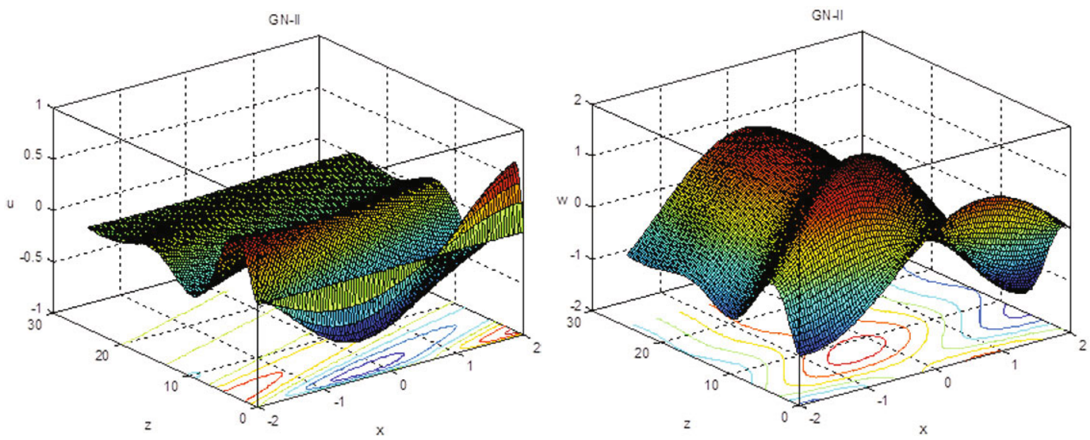

Fig. 8. 3D displacement distribution function.

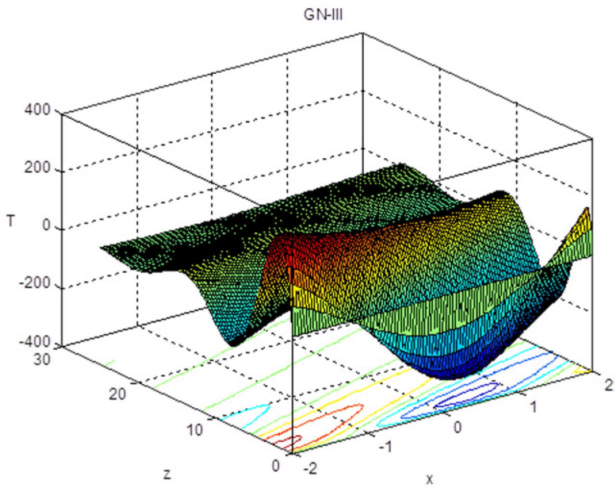

Fig. 9. 3D temperature distribution.

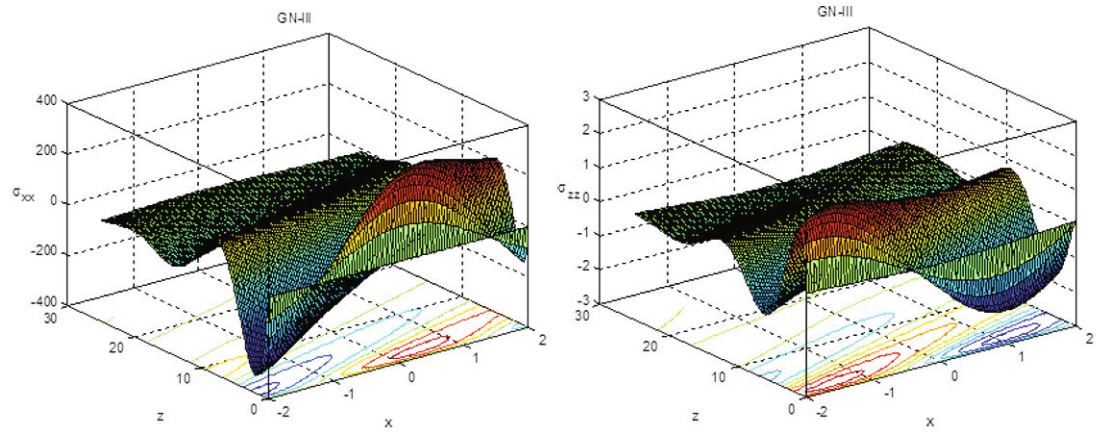

Fig. 10. 3D normal stress distribution.

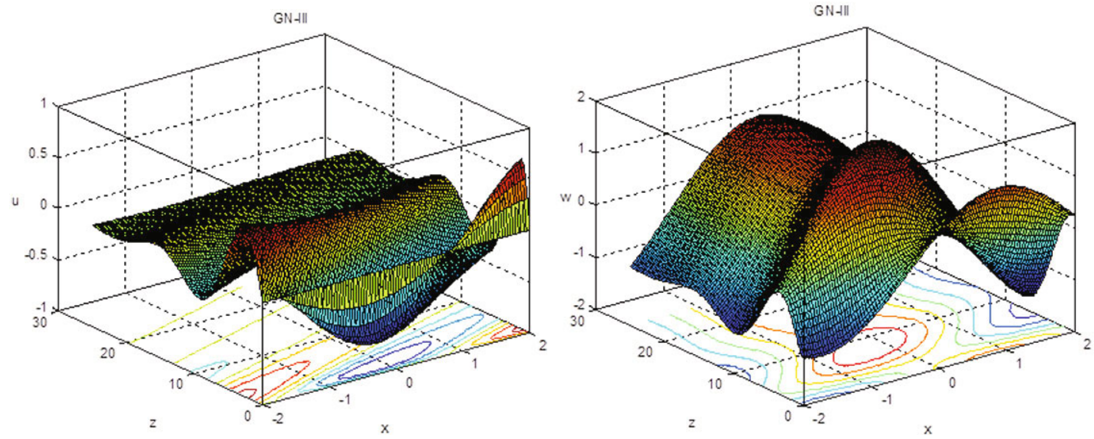

Fig. 11. 3D displacement distribution function.

7 Conclusion

By using the above analysis and graphical representations the following conclusions can be drawn:

1) The initial point for each curve is the same, satisfying the physical assumption related to the boundary condition of the problem.
2) The amplitude of each curve in each figure converges to zero as the distance from the surface $z = 0$ increases, satisfying the condition of the surface waves.
3) According to the physics of the problem, energy dissipation reduces the temperature of the medium, which, in turn, reduces the intensity of internal energies. Graphically, the amplitudes of the waves have a decreasing effect on energy dissipation. This fully agrees with the physics of the problem.
4) The effect of energy dissipation is directly proportional to the distance from the surface $z = 0$ of the medium. At higher values of the vertical distance both curves move with the same and small amplitude.
5) From the set of 3D curves it is observed that the horizontal distance also plays a very important role in the propagation of waves. In all cases the curves are of normal mode form so that their propagation abilities and properties could be studied.

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