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Determining the amplitude of Mercury's long period librations with the BepiColombo radio science experiment*

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Abstract. The Mercury Orbiter Radio science Experiment (MORE) is one of the experiments on-board the ESA/JAXA BepiColombo mission to Mercury. A crucial goal of MORE is to determine the gravity field and rotational state of Mercury in order to enable a better understanding of the planet's geophysics. The authors have recently reported on the results of a set of simulations of the MORE gravimetry and rotation experiments, carried out with the dedicated ORBIT14 software. Since that time, the launch date has been postponed twice, leading to a shift of more than one year in the orbital phase of the mission. Actually, the updated schedule results in a more suitable planetary configuration to determine the amplitude of the forced librations in longitude induced by Jupiter. In fact, the amplitude can be considerably enhanced due to a near-resonance with the free librations period, a key parameter to constrain the interior structure of Mercury. We show that the newest launch date allows the measurement of the long period librations amplitude forced by Jupiter with an accuracy of some tenth of arcseconds, a significant improvement with respect to the results with the previous mission schedule.

1 Introduction

BepiColombo is a joint ESA/JAXA mission for the exploration of the planet Mercury, including the ESA-led Mercury Planetary Orbiter (MPO), devoted to the study of the planet's surface and internal composition [1]. At the beginning of 2016, the launch date was officially postponed from July 2017 to April 2018, with orbit insertion around Mercury at the end of 2024 and the beginning of MPO scientific operations in orbit scheduled for March 2025. Recently, the most likely scenario foresees the launch in October 2018, with the arrival at Mercury in December 2025 and beginning of MPO scientific operations in orbit in March 2026, providing nevertheless for a one-year nominal duration of the orbital mission, with a possible further one-year extension. On-board the MPO spacecraft, the Mercury Orbiter Radio science Experiment (MORE) will enable a better understanding of both Mercury geophysics and fundamental physics. The main goals of the BepiColombo radio science experiment concern the measurement of the gravity field and the rotational state of Mercury (*gravimetry* and *rotation experiments*, see *e.g.* [2–7]) and a precise test of General Relativity (*relativity experiment*, see *e.g.* [8–13]). To perform accurate radio science, the radio tracking observables (range, range-rate) collected through the on-board transponder [14] will be supported by the readings of non-gravitational accelerations from the on-board Italian Spring Accelerometer (ISA) (see, *e.g.* [15]) and the optical observations from the on-board high-resolution camera HRIC, part of the SYMBIO-SYS payload (see, *e.g.* [16]).

A comprehensive discussion on the MORE gravimetry and rotation experiments has been recently presented by the authors in [6]: the adopted dynamical models has been detailed, together with a sensitivity analysis of both the downgrading effects due to systematic errors in the accelerometer readings and the possible advantages of supporting the tracking observations with the optical images from the on-board camera. Hence, we refer to that paper for all the details, pointing out that the results of simulations therein were based upon the 2017 launch scenario. During the last year the launch date has been officially postponed twice, first to April 2018, then to October 2018, but neither the MPO orbital configurations or the nominal mission duration have been significantly changed [17–19]. For the sake of completeness, the details on the MPO target orbit for the three launch scenario are summarized in table 1.

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Table 1. MPO target orbit for the three launch options: July 2017, April 2018, October 2018. Last column shows the following MPO orbital parameters: altitude of apohermion and perihermion, inclination, right ascension of the ascending node, argument of perihermion.

Launch date	Orbit insertion	Start of MPO sci. operations	Initial MPO orbital elements
10 July 2017	1 January 2024	10 April 2024	$1500 \times 480\,\text{km}$, $i = 90°$
			$\Omega = 67.8°$, $\omega = 16°$
5 April 2018	18 December 2024	24 March 2025	$1500 \times 480\,\text{km}$, $i = 90°$
			$\Omega = 67.8°$, $\omega = 16°$
16 October 2018	5 December 2025	14 March 2026	$1500 \times 480\,\text{km}$, $i = 90°$
			$\Omega = 67.8°$, $\omega = 16°$

Since the MPO orbital period is expected to be about 2.3 hours in any launch scenario, the periods related to the gravity field perturbations are less than some thousands of seconds, to be compared with a nominal one-year duration of the mission. Thus the results for the gravity field determination, despite the change of the initial epoch, need to be comparable to those in [6], provided the time span of the orbital mission remains one year. To the contrary the results for the rotational parameters, which describe phenomena with time scales from 88 days to several years, can significantly change.

This applies especially to the amplitude ε_2 of the librations in longitude due to Jupiter perturbations on Mercury's orbital motion. Accounting for the assumptions made in [5] for the rotational model of Mercury, the determination of ε_2 requires particular care. Indeed, this is a key parameter for drawing information about the size of the possible inner core of Mercury, since its period (*i.e.* Jupiter's orbital period) could be near-resonant with the free librations period, depending on the planet's internal structure [20–22]. The signal due to ε_2 has a periodicity of $\sim 11.86\,\text{yr}$, hence, as already pointed out in [5] and [6], the possibility to determine this parameter is strongly related to the phase of the Jupiter perturbations at the specific epoch of the observations. A delay by one or two years in the beginning of the orbital observations changes significantly the phase of this effect. We will show that, while the time span of the mission in the 2017 scenario was far from optimal, both the 2018 schedules result more suitable for the determination of ε_2.

2 Methods and qualitative discussion

In this section we will briefly recall the methods and the dynamical models, which has been already extensively discussed in [6]. All the simulations of the BepiColombo radio science experiment have been carried out with a novel dedicated software, ORBIT14, developed by the Celestial Mechanics Group of the University of Pisa under an Italian Space Agency contract. The software consists of two main programs (all the details can be found in [6,12]): the *simulator*, which simulates the observables (range and range-rate, accelerometer readings, angular observables from the camera) and generates preliminary orbital elements of the spacecraft planetocentric orbit and of Mercury orbit, and the *differential corrector*, solving for a list of parameters of interest by a global non-linear least squares (LS) fit within an *a priori* constrained multi-arc strategy (details on the *differential correction method* can be found, *e.g.*, in ref. [23], chapt. 5, while a complete description of the constrained multi-arc strategy is presented in [24]).

Concerning the observational model, we can point out that the mercurycentric dynamics of the probe, on one side, and the heliocentric dynamics of Mercury, on the other, take place over the two completely different time scales of 2.3 hours and 88 days, respectively. Thus, we can separate the two dynamics. For the MORE gravimetry and rotation experiments, we are mainly interested in the planetocentric dynamics of the spacecraft.

For the purpose of the rotation experiment, we introduce a rotation matrix $\mathbf{R}$ to convert the spacecraft equations of motion from the body-fixed reference frame, Ψ_{BF}, in which we wrote the gravitational potential of Mercury acting on the probe, to a space-fixed Mercurycentric reference frame, Ψ_{MC}, so that the spacecraft acceleration in the Ψ_{MC} frame can be written as

$$\mathbf{a}_{\text{MC}} = \mathbf{R}^{\text{T}}\,\nabla V\,(\mathbf{R}\,\mathbf{r}_{\text{MC}}), \tag{1}$$

where V is the gravitational potential and $\mathbf{r}_{\text{MC}}$ is the spacecraft position in the Ψ_{MC} frame. For the computation of $\mathbf{R}$ we adopt the semi-empirical model defined in [5]. Referring to that paper for an exhaustive discussion, we recall that the rotation matrix can be decomposed as

$$\mathbf{R} = \mathbf{R}_3(\phi)\,\mathbf{R}_1(\delta_2)\,\mathbf{R}_2(\delta_1), \tag{2}$$

where $\mathbf{R}_i(\alpha)$ is the matrix associated with the rotation by an angle α about the i-th axis ($i = 1, 2, 3$), (δ_1, δ_2) define the space-fixed direction of the rotation axis in the Ψ_{MC} frame and ϕ is the rotation angle around the rotation axis,

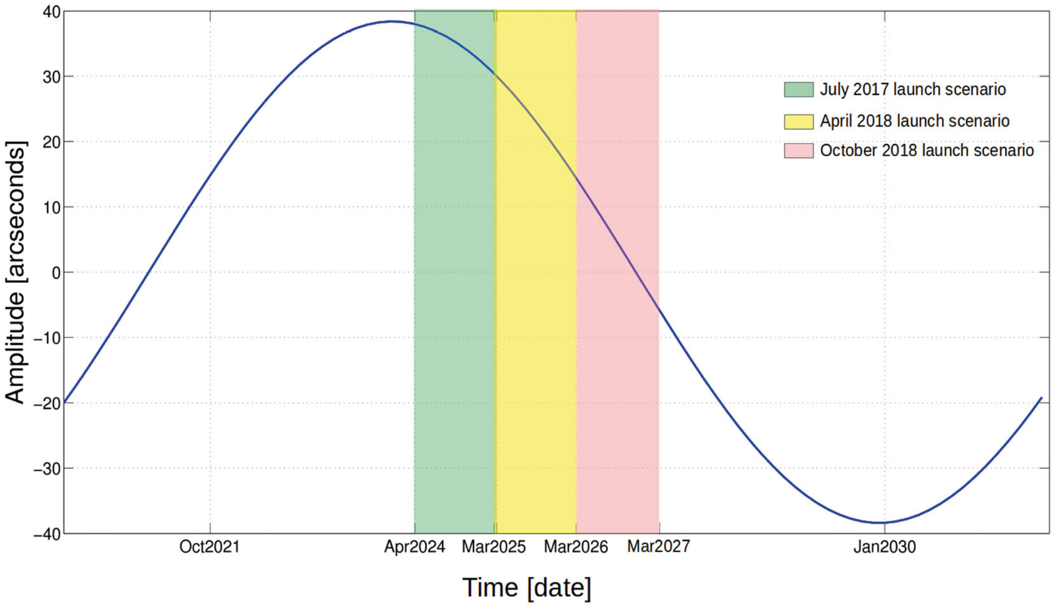

Fig. 1. Evolution in time of the contribution to the rotation angle due to ε_2: comparison between the July 2017 launch scenario (green - left box), the April 2018 launch (yellow - central box) and the October 2018 launch (pink - right box).

assuming the unit vector along the longest axis of the equator of Mercury (minimum momentum of inertia) as the rotational reference meridian. The fundamental aspects to describe the rotational state of Mercury in the adopted semi-empirical model are the following (see [25]): I) we assume the *Cassini state theory*, defining the obliquity η with respect to the orbit normal as $\cos\eta = \cos\delta_2 \cos\delta_1$, set constant over the mission time span; II) we include in the description two librations in longitude terms, the amplitude ε_1 of 88 days forced librations and the amplitude ε_2 of the Jupiter-forced librations.

We can have some *a priori* expectations on the effect of whatever change of the orbital phase initial date, provided that the mission time span remains unchanged, *i.e.* one year. From item I), since the obliquity η is variable over secular time scales, we do not expect that a change of few years in the initial date can affect in a significant way the accuracy in determining δ_1 and δ_2. The librations in longitude at 88 days arise as a consequence of the peculiar 3:2 resonant state of Mercury. Due to the 88 days periodicity, averaging over one year time span (*i.e.* about 4 cycles of libration), a relevant variation in the accuracy for the amplitude ε_1 is not likely at all. To the contrary, due to the 11.86 yr periodicity of the libration term ε_2 forced by Jupiter, a change of one or two years in the initial date can produce a relevant effect. From the analytical formula adopted for the rotation angle $\phi(t)$ given in [6], eq. (6), the contribution from the long-period librations about the 3:2 resonance results:

$$\phi_{\varepsilon_2}(t) = \varepsilon_2 \cos[w_j\,(t - t_p) + \varphi_j], \tag{3}$$

where the values of ε_2 and w_j are given in [22] at the reference epoch J2000 for the mass distribution asymmetry $(B - A)/C_m = 2.18 \times 10^{-4}$ taken from [26] (consistent with the results from MESSENGER [27]). In the software we assumed the time of perihelion t_p as the reference epoch, thus the phase φ_j, which is model-dependent as well, has been deduced conveniently from the values given in [22]. The evolution in time of ϕ_{ε_2} from the beginning of 2020 to the end of 2031 is shown in fig. 1. Three temporal windows have been highlighted for comparison: from April 2024 to April 2025 for the 2017 launch scenario (green - left box), from March 2025 to March 2026 for the April 2018 launch scenario (yellow - central box) and from March 2026 to March 2027 for the October 2018 launch scenatio (pink - right box), respectively.

For a qualitative analysis, the signal in the three temporal windows can be roughly approximated as an almost linear variation over one year, with a trend of ~ 8 arcsec/yr for the July 2017 schedule, ~ 16 arcsec/yr for the April 2018 one and ~ 20 arcsec/yr for the October one. If the signal trend is too small, as it happens if the mission orbital phase is close to a stationary point (*e.g.* during the year 2023), the contribution of the ϕ_{ε_2} term to the rotation angle ϕ could be envisaged as an almost constant term. Thus its effect would be aliased with that of the harmonic coefficient S_{22}. Indeed, the ratio of the two harmonic coefficients S_{22}/C_{22} represents (the tangent of) the angle between the axis of minimum momentum of inertia and the zero meridian, assumed to be constant. The result is a near one correlation between S_{22} and ε_2: it would not be possible to solve for both the parameters in the LS fit unless attaining inaccurate results.

Table 2. MORE rotational parameters: δ_1, δ_2 (both in arcmin); ε_1, ε_2 (both in arcsec). Comparison between previous 2017 launch scenario and the 2018 scenarios (launch in April or October 2018). Results for the normalized spherical harmonics coefficient $\bar{S}_{22}$ and its correlation $\rho_{(S_{22},\cdot)}$ with the rotational parameters is also shown.

	July 2017 launch				April 2018 launch				October 2018 launch			
Parameter	Formal	True	T/F	$\rho_{(S_{22},\cdot)}$	Formal	True	T/F	$\rho_{(S_{22},\cdot)}$	Formal	True	T/F	$\rho_{(S_{22},\cdot)}$
δ_1 [am]	0.0008	0.0011	1.4	< 0.1	0.0007	0.0009	1.2	< 0.1	0.0004	0.0008	2.0	< 0.1
δ_2 [am]	0.0005	0.0013	2.6	< 0.1	0.0005	0.0009	1.7	< 0.1	0.0006	0.0009	1.5	< 0.1
ε_1 [as]	0.047	0.11	2.3	< 0.1	0.044	0.08	1.7	< 0.1	0.056	0.10	1.8	< 0.1
ε_2 [as]	0.57	0.69	1.2	0.80	0.29	0.44	1.5	0.43	0.25	0.50	2.0	0.12
$\bar{S}_{22}$	7.9e-11	7.9e-11	1.0	–	5.0e-11	9.9e-11	2.0	–	4.4e-11	9.7e-11	2.2	–

With the July 2017 schedule, we found a correlation $\rho(S_{22}, \varepsilon_2) = 0.97$ if only tracking observables are adopted for the fit (test case II in [6]), leading to a highly downgraded solution for both S_{22} and ε_2. Adding the camera observations, the result was significantly improved: assuming a Gaussian error $\sigma_{opt} = 2.5$ arcsec for the angular observables (test case I), the correlation is $\rho(S_{22}, \varepsilon_2) = 0.80$, while adopting $\sigma_{opt} = 5$ arcsec (test case III) the result is $\rho(S_{22}, \varepsilon_2) = 0.91$. These results were encouraging, but still an aliasing between S_{22} and ε_2 was present, resulting, even in the most favourable test case I, in an increase of the degree variance for harmonic degree $\ell = 2$ of Mercury's gravity field, as it clearly appears from fig. 9 in [6]. Moreover, the assumption of $\sigma_{opt} = 2.5$ arcsec for camera observables has not yet been confirmed by the BepiColombo project.

Adopting the April 2018 mission scenario, it turns out from fig. 1 that the trend of the ε_2 signal is twice the value of the 2017 scenario, thus the correlation with S_{22} is expected to be lower. The trend of the signal is even more steep assuming the October 2018 scenario, which seems particularly favourable for the determination of ε_2. These points will be addressed in sect. 3.

3 Simulation scenario and results

The main assumptions on the nominal simulation scenario have been outlined in [6], the only difference consisting in the epoch adopted for the beginning of scientific operations, assuming in any case a nominal 365 days long mission. The MPO orbit is polar, with a period of about 2.3 hours and an altitude of perihermion and apohermion of 480 and 1500 km, respectively. We included the angular observables from the camera accounting for a Gaussian error of 2.5 arcsec. The list of solve for parameters in the global LS fit, as extensively discussed in [6], is summarized as follows:

- state vector (position and velocity) of the planetocentric orbit of MPO at the central epoch of each observed arc;
- normalized harmonic coefficients of the expansion of Mercury gravity field, $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$, up to degree ℓ and order m 25 (see, *e.g.*, [23], chapt. 13);
- the elastic constant to account for tidal deformations of Mercury due to the Sun, which is Love number k_2 [28];
- obliquity of Mercury rotational axis with respect to the orbit normal, parameterized through the angles δ_1 and δ_2, the amplitude ε_1 of the 88-days forced librations and the amplitude ε_2 of the Jupiter-forced librations;
- on-board accelerometer calibration constants ($\mathcal{C}^1$ spline model, six parmeters per arc [29]);
- the geodetic coordinates of reference points observed by the on-board high resolution camera, namely latitude and longitude of each reference point on the Mercury surface.

This list includes a total of about 8000 parameters solved simultaneously in the LS fit within a constrained multi-arc strategy [24].

The results for the rotational parameters achieved in both the 2018 scenarios are shown on the right part of table 2, while on the left side the corresponding results from [6] (July 2017 launch scenario) are displayed for comparison. The results for the $\bar{S}_{22}$ harmonic coefficient of Mercury are also included and the correlation $\rho_{S_{22}}$ of each rotational parameter with this coefficient is highlighted. For each parameter, the following meaningful quantities are shown: the formal uncertainty, computed from the diagonal of the covariance matrix in the LS fit; the "true error", defined as the difference between the parameter "ground-truth" value, adopted in the simulation, and the nominal value estimated through the differential corrections process; the true-to-formal (T/F) error ratio. Each quantity has been obtained as rms value over a number of runs, obtained by changing the seed of the random numbers generator; it turns out that ~ 10 runs are adequately representative to quantify systematic errors.

A delicate issue of simulations is to assess the reliability of the formal uncertainties as a measure of the accuracy of the results. Indeed, caution has to be paid: signatures from systematic effects due, for example, to the on-board accelerometer are unavoidably present, and they are ignored by the covariance analysis. This could lead to biased estimated values for the parameters, possibly with a misleading estimated accuracy. Thus, first of all, we introduced

Table 3. Sensitivity study of the formal uncertainty of libration amplitude ε_2 assuming: (i) a 5 arcsec Gaussian error in the optical observations, (ii) removing at all the optical observations from the fit. Comparison between the addressed launch scenarios. Values are in arcsec.

	July 2017	April 2018	October 2018
5 arcsec error	0.98	0.51	0.49
no camera	1.7	0.89	0.94

in the simulation stage an error model for the generation of the accelerometer readings, containing both random and systematic components as extensively discussed in [6]. The critical parameter is the T/F ratio, that is the comparison between the "true error" measuring the propagation of the systematics and the formally estimated standard deviation. If the T/F ratio for a given parameter is close to unity, according to Gaussian statistics, then the covariance analysis is appropriate to assess the accuracy. Conversely, a significantly higher-than-one value points out that some systematics in the measurements can degrade the quality of the results.

As it can be seen from table 2, the achievable accuracies for the rotational parameters in both the 2018 launch scenarios are consistent with the discussion made in the previous section. For the parameters δ_1, δ_2 and ε_1 there is only a slight improvement in terms of formal uncertainty, while the T/F ratio decreases significantly, indicating a general improvement in the robustness of the solution. Concerning ε_2, in the April 2018 launch scenario the formal accuracy improves by almost a factor 2 and the correlation $\rho(S_{22}, \varepsilon_2)$ decreases by the same factor, as suggested by fig. 1. As a consequence, an improvement in the determination of $\bar{S}_{22}$ can be achieved as well. In the case of the October 2018 scenario, comparing the slope of the curve in fig. 1 with the case of April 2018, we expect a slight further improvement by a factor 1.2 in the determination of ε_2, which is indeed found in the formal accuracy of ε_2, together with a comparable improvement in the determination of $\bar{S}_{22}$. In terms of correlation between the two paramters we found, instead, a significant improvement with respect to the previous scenarios. Nevertheless, by looking at fig. 1 it is clear that the latest scenario is the most favourable for the determination of ε_2, hence any additional decrease in $\rho(S_{22}, \varepsilon_2)$ would not lead to a further improvement in terms of formal accuracies of the two parameters themselves but only in terms of robustness of the global solution. In conclusion, from fig. 1 we can envisage that only an extension of the orbital phase of the mission to more than one year would increase the accuracy in determining the amplitude ε_2.

Referring to both the 2018 launch scenarios, within the assumption of a 2.5 arcsec Gaussian error for the angular observables, we can expect to achieve an accuracy of less then 1/2 arcsec on ε_2. This result is very encouraging. Indeed, due to the possible resonance, the value of the amplitude ε_2 itself is extremely sensitive to the size of the planet's inner core, which is described by the parameter $(B - A)/C_m$. For instance, the amplitude of the forced librations at Jupiter period is expected to be at the level of 1.4 arcsec, as shown in [21], but it could be significantly enhanced, at the level of tens of arcsec, if it experiences a near-resonance condition with the free librations of the internal core of the planet. Conversely, the frequency of the free librations strongly depends on the radius of the internal core, hence an accurate measurement of ε_2 provides a strong constraint to the internal structure of Mercury. We remark that these results have been obtained under the assumption of the specific model discussed in [21]. Different assumptions on the planet's interior, as detailed in [22], could lead to different resonant terms.

Relaxing the requirement for the camera Gaussian error, the results are nevertheless promising. In table 3 we show the results in terms of formal accuracies for ε_2 in the three launch scenarios, accounting for two different cases: i) a camera Gaussian error of 5 arcsec instead of 2.5 arcsec, ii) no optical observations at all in addition to tracking measurements. As it can be seen, in case i) the formal accuracies are almost twice the corrisponding values shown in table 2, thus an accuracy at the level of 1/2 arcsec in determining ε_2 can still be achieved for both the 2018 scenarios. Even in the worst case of not including optical observations in the fit (case ii), the ε_2 accuracy would nevertheless remain a little below the arcsec level. Thus, contrary to the 2017 launch schedule, the MORE rotation experiment becomes, with the new launch date, more robust with respect to possible issues concerning optical observations.

4 Conclusions

In this paper we discussed the accuracy in the determination of the amplitude ε_2 of the Jupiter-forced librations in longitude, to be achieved with the BepiColombo MORE rotation experiment. This is an important parameter to constrain the internal structure of the planet Mercury. The authors updated the results published in [6] taking into account the new 2018 date for the launch: both the April 2018 launch option and the novel schedule of October 2018 launch have been addressed. The two schedules result in a delay of almost one year and two years, respectively, in the spacecraft orbit insertion around Mercury with respect to the 2017 schedule. We have shown that, while the gravity field coefficients, the obliquity angles and the amplitude of the 88-days librations in longitude are not significantly affected by this time shift, an improvement of at least a factor 2 can be achieved in the determination of the formal accuracy of the amplitude of Jupiter-forced librations in longitude. Also the harmonic coefficient S_{22} is significantly improved.

The stability in the determination of the ε_2 parameter is strongly affected by the phase of the periodic signal at the observation times and by the duration of the mission. Due to the 11.86 yr periodicity, the 2018 mission scenario provides for a favourable geometrical configuration of the planets Jupiter and Mercury, which entails a factor 2 improvement in the accuracy with respect to the 2017 scenario. With the combined use of tracking and optical observations, we expect to determine the amplitude ε_2 with an accuracy of some tenths of arcsec. Finally, a further relevant improvement could be achieved by an extension of the time span of the orbital mission.

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