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Gravitational collapse in generalized teleparallel gravity

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Abstract. The idea of this work is to exhibit the unstable regions of a self-gravitating celestial object which collapses adiabatically with a non-vanishing expansion scalar. We adopted the generalized teleparallel gravity for a plane-symmetric self-gravitating object and matter distribution as dissipative and anisotropic. We analyzed the instability regions of the stellar model by considering a particular model, *i.e.*, $\beta T + \gamma T^n$. We established the basic dynamical equations together with junction conditions and equation of state proposed by Harrison *et al.* The linear perturbation strategy is applied to the metric, matter and $f(T)$ function up to first order. The equations of motion are developed under this perturbative framework to construct collapse equation. We demonstrate the unstable phases at weak field limit and post-Newtonian epochs and deduce a vital role of adiabatic index to explain the stability/instability issues for planar model in $f(T)$ gravity.

1 Introduction

The most recommended and agreeable proposal to the relativistic theory of gravitation is given by Einstein in his theory of general relativity (GR). Nevertheless, at present, there are certain alternative formulations for gravitational theories that aspire to resolve the problems of dark matter (DM) and dark energy (DE), that does not detect adequate justification within the background of GR [1]. Modified gravity [2,3] is one of the two main paths which one can adopt to illustrate the early and late times of the universe characterized as two accelerated phases of expansion, (the other one leads to the notion of DE in the background of (GR)) [4–7]. Also, even if one determines to take the grievous step of modifying gravity, still there would be the question how the gravity theory is modified. Most of the literature works initiate from the usual, curvature-based formulation by using the Einstein-Hilbert action.

In order to formulate a modified gravity with second-order equations in four dimensions, we will begin not from Einstein-Hilbert Lagrangian, but from a teleparallel equivalent to GR (TEGR) [8]. The TEGR is an equivalent concept of classical gravity, in which curvature-less Witzenböck connections are used instead of torsion-less Levi-Civita connections. The four linearly independent vierbeins are served as dynamical objects in this framework [9–12]. The theories, GR and TEGR, are invariant under local Lorentz transformation, but in GR the natural Lorentz scalar acts as geometrical quantity, while in TEGR it depends on the choice of frames [13]. Pereira *et al.* [14] discussed the benefits of TEGR and concluded that it is independent of the universality of free fall and does not need the equivalence principle to describe the gravitational field. Vargas [15] used the teleparallel versions of Landau-Lifshitz energy-momentum and Einstein complexes of gravitational field to obtain the total energy of the universe. He found that the total energy vanishes independently of the pseudo-tensor for the closed universe, along with three dimensionless coupling constants of teleparallel gravity.

The nominal $f(T)$ is the true generalization of TEGR, in which the function of the torsion scalar served as Lagrangian density. Ferraro and Fiorini [16] introduced the $f(T)$ gravity under the Born-Infeld approach which helps to investigate singularity-free solutions as well as to solve particle horizon problem. Jamil *et al.* [17] discussed the Noether symmetries in $f(T)$ theory and discussed its astrophysical implications. Wang [18] explored some static solutions with spherical symmetry in $f(T)$ gravity with the Maxwell term based on conformal Cartesian coordinates. Jawad *et al.* [19] discussed the dynamical instability of shear-free spherical and cylindrically symmetric gravitating objects in the $f(T)$ background. Bamba *et al.* [11] discussed the time-dependent matter instability of star collapse in $f(T)$ and emphasized that the second law of thermodynamics is satisfied in this framework.

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The extra degrees of freedom that exist in $f(T)$ gravity, results in the breaking of local invariance, which is the main feature of this theory. However, the extra degrees of freedom do not unveil up to second-order linear perturbations for the flat Friedmann-Robertson-Walker geometry [20]. Therefore, it is feasible to modify the $f(T)$ theory in order to make it evidently Lorentz invariant which will reduce to $f(T)$ and has generically different dynamics [21, 22]. To be more precise, the aim to unravel the mystery of DM and DE without calling for the new material elements that have not yet been detected by experiments is done by modifying the geometric part [23, 24]. Thus, the $f(T)$ gravity could be a trust worthy approach to deal with the deficiency of GR at high-energy scales [3, 25]. Further, it is demonstrated that inflation and the DE-dominated stage can be realized in the Kaluza-Klein and Randall-Sundrum models, respectively [26]. In this fashion, $f(T)$ theory attained much significance and has been confirmed to exhibit the stimulating cosmological ramifications. Nashed [27] regularized the field equations in $f(T)$ theory with "general tetrad field" and removed the effect of local Lorentz invariance. Most of the research in $f(T)$ gravity uses spherically symmetric background.

Gravitational collapse of self-gravitating objects is highly dissipating process and play dominant role in the formation and evolution of stars, planets etc. The study of gravitational collapse can be done by considering the exterior and interior regions of space time by smooth matching of these regions through some junction conditions. Darmois matching conditions are most appropriate among the junctions conditions presented by Lichnerowicz [28], Darmois [29] and O'Brien and Synge [30]. In this process, the various stages of self-gravitating objects formed which can be analyzed through dynamical equations. The dynamical instability of super massive objects has become the topic of great interest over the past decades. Chandrasekhar [31] was the first one who explored instability regimes of the spherical systems coupled with perfect matter content. Herrera *et al.* [32] investigated stability regimes for non-adiabatic collapsing objects. Due to various physical applications, the study of relativistic anisotropic stars have gained much significance. Herrera and Santos [33] studied the influences of rotation degrees of freedom in a stellar interior and indicated anisotropy in pressure. Chan [34] checked the role of shear viscosity in the evolution of spherical dissipative celestial objects. The role of adiabatic index for non-static axial symmetry with anisotropic matter configuration was explored by [35–37].

In the context of extended gravities, the adiabatic index (Γ_1) bring out the stability ranges which at same time depend on the usual terms of stress energy tensor and extra curvature quantities coming from the extended gravity. In $f(R)$ gravity, the jeans instability for self-gravitating stellar systems through weak field approximation was explored by Capozziello *et al.* [38]. The instability ranges via adiabatic index and homogeneity factors for cylindrical [39, 40], spherically [41–47] and plane [48–50] symmetric collapsing objects was investigated in the background of modified gravity. Sharif and Yousaf [51] discussed the stability of spherical collapsing objects using perturbation on metric and material profiles and checked the influences of adiabatic index in Palatini $f(R)$ gravity. Wu and Yu [52] explored the stability issue by imposing homogenous perturbations to Einstein's static universe under $f(T)$ gravity. Recently, Jawad *et al.* [53, 54] analyzed the instability ranges with both weak field approximation and post-Newtonian (pN) eras with and without shear-free condition. González *et al.* [55] studied three-dimensional hairy black hole with scalar field and a self-interacting potential in TEGR.

A lot of work has been carried out related to dynamical instability for spherically symmetric stars. However, the non-spherical objects like cylindrical bodies came into existence by post-shocked clouds on the edge of celestial collapse at stellar background along with galaxy formations. The final fate of non-spherical collapsing of objects furnishes the foundation of some new examples about gravitational collapse. It has been inferred that modified gravity could provide a very effective platform for the description of new solutions with some extra degrees of freedom that were not possible in GR [56–60]. In this context, Yousaf *et al.* [61] have investigated those factors that are responsible for disturbing the stability of regular distribution of compact objects through Γ_1 in modified gravity. Moustakidis [62] performed stability analysis to check the role of Γ_1 in the dynamics of various analytical solutions of compact objects.

Keeping in mind, we continue this analysis of dynamical instability for plane symmetric collapsing objects in $f(T)$ gravity. We explore the dynamical stability epochs at both N and pN epochs by formulating the collapse equation. The paper is lined up in following manner. In sect. 2, we provide the basic equations in $f(T)$ gravity to construct the field equations. Section 3 is devoted to discuss the dynamical equations and matching conditions. In sect. 4, we apply the perturbation scheme on all previous equations in order to evaluate the extended version collapse equation. Section 5 deals with the instability regions under N and pN limits while the last section is devoted to compile our main findings.

2 Structure formation of f(T) gravity

Here, we provide the basic formulation of $f(T)$ gravity with planar geometry as an interior metric. The Lagrangian of teleparallel is dictated by T (T is the torsion scalar) that has been extended to its suitable function to understand the inflation and the late-time acceleration in the universe model. In this scenario, the Einstein-Hilbert action for $f(T)$ gravity is described in following manner [8, 26, 63, 64]:

$$S_{f(T)} = \frac{1}{\kappa^2}\int \mathrm{d}^4x\,(f(T) + \mathcal{L}_M)\,h, \tag{1}$$

where κ, $f(T)$, $\mathcal{L}_M$ are coupling constant, differential function of torsion scalar and matter field Lagrangian respectively while $h=\det(h^{\gamma}_{\delta})$, with h^{γ}_{δ} as a tetrad field which has most significant role in construction of torsion based gravity theory. This on connection with metric tensor gives $g_{\delta\gamma}=\zeta_{ab}h^a_{\delta}h^b_{\gamma}$ with $\zeta_{ab}=\mathrm{diag}(1,-1,-1,-1)$. The indices $(\gamma,\delta,\ldots)$ and $(a,b,\ldots)$ are for the coordinates of tangent space and manifold, respectively. The torsion scalar can be expressed as follows:

$$T=S_{\mu}^{\ \gamma\delta}T^{\mu}_{\ \gamma\delta}, \tag{2}$$

where $T^{\mu}_{\ \gamma\delta}$ is a tensor accepting a relation $T^{\mu}_{\ \gamma\delta}=-T^{\mu}_{\ \delta\gamma}$ and can be represent by means of Weitzenböck connection ($\bar{\Gamma}^{\mu}_{\ \delta\gamma}=h_a^{\ \mu}\partial_{\delta}h^a_{\ \gamma}$) as

$$T^{\mu}_{\ \gamma\delta}=\bar{\Gamma}^{\mu}_{\ \delta\gamma}-\bar{\Gamma}^{\mu}_{\ \gamma\delta}=h_a^{\ \mu}(\partial_{\delta}h^a_{\ \gamma}-\partial_{\gamma}h^a_{\ \delta}), \tag{3}$$

while

$$S_{\mu}^{\ \gamma\delta}=\frac{\delta^{\gamma}_{\mu}}{2}T^{\alpha\delta}_{\ \ \alpha}-\frac{\delta^{\delta}_{\ \mu}}{2}T^{\alpha\gamma}_{\ \ \alpha}+\frac{1}{4}\left(T^{\gamma\delta}_{\mu}+T^{\delta\gamma}_{\mu}-T^{\gamma\delta}_{\mu}\right). \tag{4}$$

The variation in the action given in eq. (1) with respect to tetrad field gives

$$h_a^{\ \mu}S_{\mu}^{\ \gamma\delta}\partial_{\gamma}Tf_{TT}+\frac{f}{4}h_a^{\ \delta}+\frac{f_T}{h}\partial_{\gamma}(hh_a^{\ \mu}S_{\mu}^{\ \gamma\delta})+h_a^{\ \mu}T^{\nu}_{\gamma\mu}S_{\nu}^{\ \delta\gamma}f_T=\frac{\kappa^2}{2}h_a^{\ \mu}T^{\delta(m)}_{\mu}, \tag{5}$$

where $Z_T\equiv\frac{\partial Z}{\partial T}$, $Z_{TT}\equiv\frac{\partial^2 Z}{\partial T^2}$, while $T^{\delta(m)}_{\mu}$ is the standard fluid energy-momentum tensor. We are intrigued to calculate instability epochs of relativistic stellar interiors coupled with the dissipative locally anisotropic fluid configuration with the following energy-momentum tensor:

$$T^{(m)}_{\gamma\delta}=(P_{\perp}+\mu)V_{\delta}V_{\gamma}+(P_z-P_{\perp})\chi_{\gamma}\chi_{\delta}+P_{\perp}g_{\gamma\delta}+q_{\gamma}V_{\delta}+V_{\gamma}q_{\delta}, \tag{6}$$

where μ, $P_{\perp}$, P_z, V_{γ} and q_{γ} are the fluid energy density and different pressures, four-velocity and heat flux, respectively. Also, χ_{γ} is the unit four-vector in the z-direction and all these four-vectors configure a canonical orthonormal tetrad. The effects of covariant formulation of torsion tensor is same as subtraction of torsion scalar from the Ricci invariant. In covariant formalism, $f(T)$ field equations become

$$\Upsilon_{\gamma\delta}f_{TT}+\frac{T}{2}\left(\frac{f}{T}-f_T\right)g_{\gamma\delta}+G_{\gamma\delta}f_T=\kappa^2T^{(m)}_{\gamma\delta}, \tag{7}$$

where $\Upsilon_{\gamma\delta}=S_{\delta\gamma}^{\ \ \mu}\nabla_{\mu}T$, while $G_{\gamma\delta}$ is the Einstein tensor. Equation (7) can be rewritten as follows:

$$G_{\gamma\delta}=\frac{\kappa^2}{f_T}\left(\overset{(T)}{T}_{\gamma\delta}+T^{(m)}_{\gamma\delta}\right), \tag{8}$$

where

$$\overset{(T)}{T}_{\gamma\delta}=\frac{1}{\kappa^2}\left\{-\Upsilon_{\gamma\delta}f_{TT}-\frac{1}{4}(T-\Upsilon f_{TT}+Rf_T)g_{\gamma\delta}\right\}, \tag{9}$$

is a tensor assimilating corrections coming from torsion scalar. One can recover the field equations of GR by substituting $f(T)=T$ in eq. (8).

The $f(T)$ gravity has unlike features in comparison with other modified gravity theories, for instance, the second-order equations of motion are utilized in $f(T)$ gravity rather than fourth-order ones as in metric $f(R)$ gravity. It is demonstrated that additional interaction between two bodies may be involved due to extra propagating degrees of freedom. Some techniques must be imposed to minimize the non-linear effects or interactions of these extra propagating modes of the theory to be compatible with observational data. The FLRW solution is stable in $f(T)$ gravity then it would be the exact theory depicting the nature of our universe. But one can demonstrate the stability issue after knowing the behavior of all degrees of freedom of $f(T)$. In the scenario of non-linear massive gravity, the non-linear instability emerges due to the hidden degrees of freedom and consequently one cannot demonstrate the problem of stability in $f(T)$ gravity theory. Since the real universe is surely not isotropic and homogeneous completely so the stability problem has great significance in cosmology. However, to perform a complete non-linear analysis in $f(T)$ is quiet difficult. Therefore, it stays a hot topic to explore the effects of these degrees of freedom in an anisotropic model of cosmology with linear or radial perturbations. A detailed report has been presented by Cai *et al.* [65] reviewing different torsional formulations, from teleparallel to metric-affine gauge and Einstein-Cartan theories, resulting in extending torsional gravity in the framework of $f(T)$ gravity. They also presented some astrophysical and cosmological applications for solutions arising in $f(T)$ gravity.

The interior geometry is taken to be non-static plane symmetric whose line element is given as [66]

$$\mathrm{d}s_-^2 = -A^2(t,z)\mathrm{d}t^2 + B^2(t,z)(\mathrm{d}x^2 + \mathrm{d}y^2) + C^2(t,z)\mathrm{d}z^2. \tag{10}$$

This system is configured with co-moving reference frame due to which the fluid four-vectors are defined as $V_\gamma = A^{-1}\delta_0^\gamma$, $\chi_\gamma = C^{-1}\delta_3^\gamma$, $q_\gamma = qC^{-1}\delta_3^\gamma$ satisfying

$$V^\gamma V_\gamma = -1, \qquad \chi^\gamma \chi_\gamma = 1, \qquad \chi^\gamma V_\gamma = 0 = V^\gamma q_\gamma.$$

Keeping in view the possible extension of $f(T)$ theory, we assume diagonal tetrad and its inverse for the metric (10) as follows:

$$h_\gamma^a = (-A, B, B, C), \qquad h_a^\gamma = (-A^{-1}, B^{-1}, B^{-1}, C^{-1}) \tag{11}$$

In this context, the torsion scalar takes the form

$$T = 2\left(\frac{\dot{B}}{AB}\right)^2 + \frac{1}{ABC}\left(\frac{-2A'B'}{C} + \frac{3\dot{B}\dot{C}}{A}\right) - \frac{C'}{C}\left\{\frac{1}{A^2}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{1}{C^2}\left(\frac{A'}{A} + \frac{B'}{B}\right)\right\}, \tag{12}$$

where prime and over dot represent $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial t}$ operators, respectively. The non-vanishing components of $f(T)$ field equations are

$$G_{00} = \frac{\kappa^2}{f_T}\left[\mu A^2 - \frac{A^2}{\kappa^2}\left\{\frac{Tf_T - f}{2} + \frac{1}{2C^2}\left(\frac{C'}{C} + \frac{B'}{B}\right)f_T'\right\}\right], \tag{13}$$

$$G_{11} = \frac{\kappa^2}{f_T}\left[P_\perp B^2 + \frac{B^2}{\kappa^2}\left\{\frac{Tf_T - f}{2} - \frac{1}{2}\left(\frac{1}{A^2}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\dot{f}_T - \frac{1}{C^2}\left(\frac{A'}{A} + \frac{B'}{B}\right)f_T'\right)\right\}\right], \tag{14}$$

$$G_{33} = \frac{\kappa^2}{f_T}\left[P_z C^2 + \frac{C^2}{\kappa^2}\left\{\frac{Tf_T - f}{2} - \frac{1}{2}\frac{\dot{B}}{A^2 B}\dot{f}_T\right\}\right], \tag{15}$$

$$G_{03} = \frac{\kappa^2}{f_T}\left[\frac{qC}{A} - \frac{1}{2\kappa^2}\frac{\dot{B}}{B}f_T'\right], \tag{16}$$

$$G_{30} = \frac{\kappa^2}{f_T}\left[\frac{qC}{A} - \frac{1}{2\kappa^2}\left(\frac{C'}{C} + \frac{B'}{B}\right)\dot{f}_T\right], \tag{17}$$

where the components of $G_{\gamma\delta}$ are given in [66].

3 f(T) dynamical equations

The contracted form of Bianchi identities are

$$\left|T^{\gamma\delta} + \overset{(T)}{T^{\gamma\delta}}\right|_{;\delta} V_\gamma = 0, \qquad \left|T^{\gamma\delta} + \overset{(T)}{T^{\gamma\delta}}\right|_{;\delta} \chi_\gamma = 0,$$

with $\chi_\gamma = (0, B, 0, 0)$ yielding

$$\dot{\mu} + (\mu + P_z)\frac{\dot{C}}{C} + 2(\mu + P_\perp)\frac{\dot{B}}{B} + \frac{Aq'}{C} + 2\frac{qA}{C}\left(\frac{A'}{A} + \frac{B'}{B}\right) + \frac{D_0}{\kappa^2} = 0, \tag{18}$$

$$\dot{q} + (\mu + P_z)\frac{A'}{A} + 2(P_z - P_\perp)\frac{AB'}{CB} + \frac{A}{C}P_z' + 2q\frac{B\dot{C}}{BC} + \frac{D_1}{\kappa^2} = 0, \tag{19}$$

where D_0 and D_1 are $f(T)$ corrections given in appendix A. The mass function from Taub formalism takes the form [67]

$$m(t,z) = \frac{B}{2}\left(\frac{\dot{B}^2}{A^2} - \frac{\acute{B}^2}{C^2}\right). \tag{20}$$

In order to couple different geometrical structure of spacetime across the boundary, different junction conditions are proposed which includes the matching criteria proposed by Darmois in 1927. We determine these junction conditions for plane-symmetric interior geometry with a suitable exterior spacetime outside the hypersurface Φ. The exterior geometry is given as [68]

$$\mathrm{d}s_+^2 = \left(\frac{2M(\nu)}{Z}\right)\mathrm{d}\nu^2 - 2\mathrm{d}Z\,\mathrm{d}\nu + Z^2\left(\mathrm{d}X^2 + \mathrm{d}Y^2\right), \tag{21}$$

where M is a cylindrical mass and ν is the retarded time. The continuity of both first and second fundamental form over Φ yield

$$M \overset{\Phi}{=} m(t,z), \qquad P_z \overset{\Phi}{=} -\frac{\overset{(T)}{T_{33}}}{C^2} - \frac{\overset{(T)}{T_{03}}}{AC} + q. \tag{22}$$

4 Implementations of perturbation theory

In this section, we discuss the gravitational collapse by taking into account a model of particular form [69]

$$f(T) = \beta T + \gamma T^n, \quad n > 1. \tag{23}$$

We used the linear perturbation strategy [32, 34] to explore $f(T)$ plane symmetric collapse equation compiled with dissipative anisotropic matter configuration. We used the effects of perturbation parameter ε up to first order which help us to examine the N and pN eras of dynamical instability. Initially, we take static state of the system which then enters into non-static phase upon evolution with similar dependence of time as follows:

$$X(t,r) = X_0(z) + \varepsilon\omega(t)x(z), \qquad Y(t,r) = Y_0(z) + \varepsilon\hat{y}(t,z), \tag{24}$$

here X and Y indicate ansatz for perturbation on metric coefficients and matter variables, respectively. Using this approach, eqs. (13)-(17) under hydrostatic background give

$$\begin{aligned} G_{00}^{(S)} &= \frac{A_0^2\kappa^2}{(\beta+\gamma nT_0^{n-1})}\left[\mu_0 - \frac{1}{\kappa^2}\left\{\frac{\gamma(n-1)T_0^n}{2} + \frac{1}{C_0^2}\right.\right. \\ &\times \left.\left.\left(\frac{C_0'}{C_0} + \frac{B_0'}{B_0}\right)\left(\gamma(n-1)nT_0^{n-2}T_o'\right)\right\}\right], \end{aligned} \tag{25}$$

$$\begin{aligned} G_{11}^{(S)} &= \frac{B_0^2\kappa^2}{(\beta+\gamma nT_0^{n-1})}\left[P\perp_0 + \frac{1}{\kappa^2}\left\{\frac{\gamma(n-1)T_0^n}{2} - \frac{1}{2C_0^2}\left(\frac{A_0'}{A_0} + \frac{B_0'}{B_0}\right)\right.\right. \\ &\times \left.\left.\left(\gamma(n-1)nT_0^{n-2}T_o'\right)\right\}\right], \end{aligned} \tag{26}$$

$$G_{33}^{(S)} = \frac{C_0^2\kappa^2}{(\beta+\gamma nT_0^{n-1})}\left[Pz_0 + \frac{1}{\kappa^2}\left\{\frac{\gamma(n-1)T_0^n}{2}\right\}\right], \tag{27}$$

while their non-static perturbed distributions are

$$\begin{aligned} \hat{G}_{00} = \frac{A_0^2\kappa^2}{(\beta+\gamma nT_0^{n-1})}&\left[\bar{\mu} + \omega\mu_0\left(\frac{2a}{A_0} - \frac{\gamma(n-1)eT_0^{n-2}}{(\beta+\gamma nT_0^{n-1})}\right)\right. \\ &- \frac{\omega}{\kappa^2}\left\{\frac{\gamma n(n-1)eT_0^{n-1}}{2} + \frac{1}{C_0^2}\left(\frac{C_0'}{C_0} + \frac{B_0'}{B_0}\right)\gamma n(n-1)\left(e'T_0^{n-2} + e(n-2)T_0^{n-3}T_0'\right)\right. \\ &+ \frac{(\gamma n(n-1)T_0^{n-2}T_0')}{C_0^2}\left(\frac{c'}{C_0} - \frac{3cC_0'}{C_0^2} + \frac{b'}{B_0}, -\frac{bB_0'}{B_0^2} - \frac{2cB_0'}{B_0C_0} + \left(\frac{C_0'}{C_0} + \frac{B_0'}{B_0}\right)\right. \\ &\times \left.\left.\left.\left(\frac{\gamma(n-1)eT_0^{n-2}}{(\beta+\gamma nT_0^{n-1})} + \frac{2a}{A_0}\right)\right) + \gamma(n-1)T_0^n\left(\frac{\gamma(n-1)eT_0^{n-2}}{2(\beta+\gamma nT_0^{n-1})} + \frac{a}{A_0}\right)\right\}\right], \end{aligned} \tag{28}$$

$$\hat{G}_{11} = \frac{B_0^2\kappa^2}{(\beta+\gamma n T_0^{n-1})}\left[\bar{P\perp} + \omega P\perp_0\left(\frac{\gamma e n(n-1)T_0^{n-2}}{(\beta+\gamma n T_0^{n-1})} + \frac{2b}{B_0}\right) + \frac{\omega}{\kappa^2}\right.$$
$$\times\left\{\left(\frac{\gamma e n(n-1)T_0^{n-2}}{(\beta+\gamma n T_0^{n-1})} + \frac{2b}{B_0} + \frac{ne}{T_0}\right)\frac{\gamma(n-1)T_0^n}{2} + \frac{1}{2C_0^2}\left(\frac{A_0'}{A_0} + \frac{B_0'}{B_0}\right)\right.$$
$$\times\left\{\gamma n(n-1)(e'T_0^{n-2} + e(n-2)T_0^{n-3}T_0') + \left(\frac{2b}{B_0} - \frac{2c}{C_0} + \frac{e\gamma n(n-1)T_0^{n-2}}{(\beta+\gamma n T_0^{n-1})}\right)\right.$$
$$\left.\left.\left.\times\ (\gamma n(n-1)T_0^{n-2}T_0')\right\} - \frac{1}{2C_0^2}\left(\frac{A_0'a}{A_0^2} - \frac{a'}{A_0} + \frac{B_0'b}{B_0^2} - \frac{b'}{B_0}\right)(\gamma n(n-1)T_0^{n-2}T_0')\right\}\right], \tag{29}$$

$$\hat{G}_{03} = \frac{\kappa^2}{(\beta+\gamma n T_0^{n-1})}\left[\frac{\bar{q}C_0}{A_0} - \frac{1}{\kappa^2}\left(\frac{\dot{\omega}\gamma n(n-1)T_0^{n-2}T_0'}{B_0}\right)\right], \tag{30}$$

$$\hat{G}_{30} = \frac{\kappa^2}{(\beta+\gamma n T_0^{n-1})}\left[\frac{\bar{q}C_0}{A_0} - \frac{1}{2\kappa^2}\left(\frac{C_0'}{C_0} + \frac{B_0'}{B_0}\right)\dot{\omega}e\gamma n(n-1)T_0^{n-2}\right], \tag{31}$$

$$\hat{G}_{33} = \frac{C_0^2\kappa^2}{(\beta+\gamma n T_0^{n-1})}\left[\bar{P}_z + \omega P z_0\left(\frac{\gamma n e(n-1)T_0^{n-2}}{(\beta+\gamma n T_0^{n-1})} + \frac{2\omega c}{C_0}\right) + \frac{\omega}{\kappa^2}\right.$$
$$\left.\times\left\{\frac{\gamma(n-1)T_0^n}{2}\left(\frac{\gamma n(n-1)eT_0^{n-2}}{(\beta+\gamma n T_0^{n-1})} + \frac{\omega c}{C_0}\right) + \frac{\gamma n(n-1)eT_0^{n-1}}{2}\right\}\right]. \tag{32}$$

The static and non-static oscillatory components of mass function are

$$m_0 = \frac{-B_0'B_0}{2C_0^2}, \qquad \hat{m} = \frac{\omega}{2C_0^2}\left[B_0\left(\frac{cB_0'}{c_0} - b'\right) - bB_0'\right]. \tag{33}$$

The static form of second dynamical equation from contracted form of Bianchi identities is

$$P_{z0}'\frac{A_0}{C_0} + (\mu_0 + P_{z0})\frac{A_0'}{A_0} + 2(P_{z0} - P_{\perp 0})\frac{A_0B_0'}{C_0B_0} + \frac{D_1^{(S)}}{\kappa^2} = 0, \tag{34}$$

where

$$D_1^{(S)} = \frac{1}{C_0^2}\left\{\frac{\gamma(n-1)T_0^n}{2}\right\}_{,3} + \left[\frac{A_0'}{2A_0C_0^4}\left(\frac{B_0'}{B_0} - \frac{C_0'}{C_0}\right) + \left(\frac{B_0'}{B_0C_0^2}\right)^2\right](\gamma n(n-1)T_0^{n-2}T_0'). \tag{35}$$

Equation (27) and first of eq. (33) provides

$$\frac{A_0'}{A_0} = \frac{B_0'^4(\beta+\gamma n T_0^{n-1}) - 4\kappa^2 m_0^2 C_0^6[P_{z0} + \frac{1}{\kappa^2}\{\frac{\gamma(n-1)T_0^n}{2}\}]}{4m_0C_0^2B_0'^2(\beta+\gamma n T_0^{n-1})}. \tag{36}$$

Using the above equation in static form of second dynamical equation, we get

$$P_{z0}' = -\frac{C_0}{A_0}\left[(\mu_0 + P_{z0})\frac{B_0'(\beta+\gamma n T_0^{n-1}) - 4\kappa^2 m_0^2 C_0^6[P_{z0} + \frac{1}{\kappa^2}\{\frac{\gamma(n-1)T_0^n}{2}\}]}{4m_0C_0^2B_0'^2(\beta+\gamma n T_0^{n-1})} + 2(P_{z0} - P_{\perp 0})\frac{A_0B_0'}{C_0B_0} + \frac{D_1^{(s)}}{\kappa^2}\right]. \tag{37}$$

The perturbed part of first dynamical equation from of Bianchi identities is

$$\dot{\bar{\mu}} + (\mu_0 + P_{z0})\frac{\dot{\omega}c}{C_0} + 2(\mu_0 + P_{\perp 0})\frac{\dot{\omega}b}{B_0} + \frac{A_0\bar{q}'}{C_0} + \frac{2A_0\bar{q}}{C_0}\left(\frac{A_0'}{A_0} + \frac{B_0'}{B_0}\right) + \frac{D_{0p}\dot{\omega}}{\kappa^2} = 0. \tag{38}$$

Similarly, by making use of eq. (30), we obtain the value of $\bar{q}$ as $\bar{q} = \gamma\dot{\omega}$ with

$$\gamma = \frac{A_0(\beta+\gamma n T_0^{n-1})}{\kappa^2 C_0}\left[2\left(\frac{-b'}{B_0} + \frac{A_0'b}{A_0B_0} + \frac{B_0'c}{B_0C_0}\right) + \frac{1}{2(\beta+\gamma n T_0^{n-1})}\left(\frac{C_0'}{C_0} + \frac{B_0'}{B_0}\right)\gamma n(n-1)eT_0^{n-2}\right]. \tag{39}$$

Equation (33) with field equations leads to

$$\frac{C_0'}{C_0} = \frac{(B_0'^2 + 2B_0''B_0)(\beta+\gamma n T_0^{n-1}) - B_0'B_0 n\gamma(n-1)T_0^{n-2}T_0' - \kappa^2 B_0^3 B_0'\{\mu_0 + \frac{1}{\kappa^2}\frac{\gamma(n-1)T_0^n}{2}\}}{2B_0^2 m_0(n\gamma(n-1)T_0^{n-2}T_0' + 2B_0B_0'(\beta+\gamma n T_0^{n-1}))}. \tag{40}$$

Applying value of $\bar{q}$ in eq. (38), we get $\dot{\bar{\mu}} = -\chi\dot{\omega}$ where

$$\chi = (\mu_0 + P_{z0})\frac{c}{C_0} + 2(\mu_0 + P_{\perp 0})\frac{b}{B_0} + \frac{A_0}{C_0}\gamma' + \left(\frac{A_0'}{A_0} + \frac{B_0'}{B_0}\right)\gamma + \frac{D_{0P}}{\kappa^2}\,. \tag{41}$$

Integrating $\dot{\bar{\mu}} = -\chi\dot{\omega}$ with respect to time, we get

$$\bar{\mu} = -\left[(\mu_0 + P_{z0})\frac{c}{C_0} + 2(\mu_0 + P_{\perp 0})\frac{b}{B_0} + \frac{A_0}{C_0}\gamma' + \left(\frac{A_0'}{A_0} + \frac{B_0'}{B_0}\right)\gamma + \frac{D_{0P}}{\kappa^2}\right]\omega. \tag{42}$$

The perturbed form of second Bianchi identity is evaluated as

$$\begin{aligned}&\dot{\bar{q}} + \left[\omega\left(\frac{a}{A_0} - \frac{c}{C_0}\right)P_{z0}' + \bar{P}_z'\right]\frac{A_0}{C_0} + \omega(\mu_0 + P_{z0})\left(\frac{a'}{A_0} - \frac{aA_0'}{A_0^2}\right) + (\bar{\mu} + \bar{P}_z)\frac{A_0'}{A_0}\\ &- 2\omega(P_{z0} - P_{\perp 0})\left\{\frac{A_0B_0'}{C_0B_0}\left(\frac{b}{B_0} + \frac{c}{C_0}\right) - \frac{1}{B_0C_0}(A_0b' + aB_0')\right\} + 2(\bar{P}_z - \bar{P}_\perp)\frac{A_0B_0'}{B_0C_0} + \frac{D_1P}{\kappa^2} = 0,\end{aligned} \tag{43}$$

while D_{0p} and D_{1p} are mentioned in appendix A. The use of perturbation scheme on matching conditions give rise to

$$P_{z0} = -\frac{\gamma(n-1)T_0^n}{2\kappa^2}\,, \tag{44}$$

$$\bar{P}_z = \frac{\gamma n(n-1)\omega e T_0^{n-1}}{2\kappa^2} + \frac{2\omega c\gamma(n-1)T_0^n}{\kappa^2 C_0} + \frac{\dot{\omega b}}{A_0C_0B_0\kappa^2}(\beta + \gamma n(n-1)T_0^{n-2}T_0'). \tag{45}$$

Using the above values in eq. (29), we obtain

$$\chi_1\ddot{\omega} + \chi_2\dot{\omega} + \chi_3\omega = 0, \tag{46}$$

where χ_i are supposed to be positive and are given in appendix A. The solution of the above equation is

$$\omega(t) = -\exp(\xi_\Phi t), \quad \text{where } \xi_\Phi = \frac{-\chi_2 + \sqrt{\chi_2^2 - 4\chi_1\chi_3}}{2\chi_1}\,. \tag{47}$$

5 Stability analysis

In this section, we construct the collapse equation in order to study the dynamical instability at N and pN regimes with dissipative anisotropic fluid. The Harrison-Wheeler equation of state [70] is used for this purpose which is the relation between perturbed pressure components and system's energy density given as

$$\bar{P}_i = \Gamma_1\frac{P_{i0}}{\mu_0 + P_{i0}}\bar{\mu}, \tag{48}$$

where parameter Γ_1 spotlight the fluid stiffness which is treated as a constant term. By making use of eq. (42) in the above, it follows that:

$$\bar{P}_z = -\Gamma_1\frac{P_{z0}\chi\omega}{\mu_0 + P_{z0}}\,, \qquad \bar{P}_\perp = -\Gamma_1\frac{P_{\perp 0}\chi\omega}{\mu_0 + P_{\perp 0}}\,. \tag{49}$$

On substituting all these values in equation in the perturbed form of second dynamical equation, we formulate the required equation as

$$\begin{aligned}&\gamma\xi^2 + \left(\frac{a}{A_0} - \frac{c}{C_0}\right)\frac{P_{z0}'A_0}{C_0} - \Gamma_1\left[\left\{\left(\frac{P_{z0}'}{\mu_0 + P_{z0}} - \frac{\mu_0'P_{z0}}{(\mu_0 + P_{z0})^2} - \frac{P_{z0}'P_{z0}}{(\mu_0 + P_{z0})^2}\right)\chi\right.\right.\\ &+ \left.\left.\frac{P_{z0}\chi'}{(\mu_0 + P_{z0})}\right\}\frac{A_0}{C_0} + \frac{P_{z0}\chi}{(\mu_0 + P_{z0})}\frac{A_0'}{A_0} - \chi\left(\frac{P_{\perp 0}}{(\mu_0 + P_{\perp 0})} - \frac{P_{z0}}{(\mu_0 + P_{z0})}\right)\frac{A_0B_0'}{C_0B_0}\right]\\ &+ \frac{1}{A_0}(\mu_0 + P_{z0})\left(a' - \frac{aA_0'}{A_0}\right) - \chi\frac{A_0'}{A_0} - 2(P_{z0} - P_{\perp 0})\left\{\frac{A_0B_0'}{B_0C_0}\left(\frac{b}{B_0} + \frac{c}{C_0}\right)\right.\\ &- \left.\frac{1}{B_0C_0}(A_0b' + aB_0')\right\} - \frac{1}{\exp(\xi_\Phi t)}\frac{D_{1P}}{\kappa^2} = 0.\end{aligned} \tag{50}$$

This is the collapse equation for plane symmetric geometry within the frame of $f(T)$ gravity.

It has been seen from the literature [71–73] that there exists instability analysis that demonstrated instability regions independent of adiabatic index (the index which measures the fluid stiffness). In Newtonian regime, we have considered a flat background metric, thereby providing weak-field approximations, while in post-Newtonian limit, we have continued our systematic analysis with some known static profile of the interior metric. We examine the unstable regions at both N and pN regimes and found corresponding instability constraints that depends on the value of adiabatic index, thereby suggesting the significance of adiabatic index. In the following subsections, we shall evaluate instability regions with N and pN limits.

N approximation

Here, we discuss the instability of non-static plane symmetric system at N era within the framework of $f(T)$ gravity. For this purpose, we consider $A_0 = 1 = B_0$ as well as $\mu_0 \gg P_{i0}$. We use this expression in collapse equation and result follows by

$$\Gamma_1 P_{z0}\chi\frac{1}{C_0} + \gamma_N\xi_N^2 + \left(\frac{a}{C_0} - \frac{c}{C_0^2}\right)P'_{z0} + \mu_0 a' - 2(P_{\perp 0} - P_{z0})\frac{b'}{C_0} - \frac{1}{\exp(\xi_\Phi t)}\frac{D_{1P}^N}{\kappa^2} = 0, \tag{51}$$

which further can written for unstable region as

$$\Gamma_1 < \frac{|\gamma_N\xi_N^2| + \Pi - \mu_0 a' + 2(P_{\perp 0} - P_{z0})\frac{b'}{C_0} + \frac{1}{\exp(\xi_\Phi t)}\frac{D_{1P}^N}{\kappa^2}}{\Delta}. \tag{52}$$

The values of γ_N, Π, Δ and $D_{1P}^{(N)}$ are given in appendix A. Here, D_{1P}^N represents those N approximation terms which left behind after applying above constraints. This equation shows that Γ_1 depends on torsion as well as structural quantities like energy density and anisotropic pressure. Here, we have assumed that every term in the extended version of collapse equation is positive. The system turns to be unstable under the influence of this inequality. So, we conclude the following possibilities:

i) $[|\gamma_N\xi_N^2| + \Pi - \mu_0 a' + 2(P_{\perp 0} - P_{z0})\frac{b'}{C_0} + \frac{1}{\exp(\xi_\Phi t)}\frac{D_{1P}^N}{\kappa^2}] > \Delta,$

ii) $[|\gamma_N\xi_N^2| + \Pi - \mu_0 a' + 2(P_{\perp 0} - P_{z0})\frac{b'}{C_0} + \frac{1}{\exp(\xi_\Phi t)}\frac{D_{1P}^N}{\kappa^2}] = \Delta,$

iii) $[|\gamma_N\xi_N^2| + \Pi - \mu_0 a' + 2(P_{\perp 0} - P_{z0})\frac{b'}{C_0} + \frac{1}{\exp(\xi_\Phi t)}\frac{D_{1P}^N}{\kappa^2}] < \Delta.$

The first possibility indicates that the system will be in stable state as the effects of the modified gravity forces are more than the term Δ, *i.e.*, the stability constraint is $\Gamma_1 > 1$. As it is shown form ii) that the stable forces are furnished so the system is in state of stable equilibrium. The system will be unstable for iii) and unstable range of planar compact object is $0 < \Gamma_1 < 1$.

pN approximation

Here, we take, $A_0 = 1 - \phi$, $B_0 = 1 + \phi$ upto $O(\phi)$ where $\phi = \frac{m_0}{r}$ in pN-limits. By substituting these limits the collapse equation yield the following unstable ranges for planar compact model:

$$\Gamma_1 < 1 \quad \text{iff } X_1 \to X_2, \tag{53}$$

$$\begin{aligned} X_1 = {} & \gamma_{pN}\xi_{pN}^2 + \left\{(az + am_0 - cz + cm_0)P'_{z0}\left(\frac{z - m_0}{z^2}\right)\right\} - \Pi_1\left(\chi + (\mu_0 + P_{z_0})\left(\frac{z - m_0}{z}\right)\right) \\ & + \left(\frac{a'z + a'm_0}{z}\right)(\mu_0 + P_{z0}) - 2(P_{z0} - P_{\perp 0})\left(\frac{z - 2m_0}{z}\right)\left\{\left(\frac{bB'_0}{B_0^2} + \frac{cB'_0}{B_0}\frac{z - m_0}{z}\right)\right. \\ & \left. - \left(\frac{aB'_0}{B_0} + \frac{(z - m_0)b'}{zB_0}\right)\right\} - \frac{1}{\exp(\xi_\Phi t)}\frac{D_{1P}^p N}{\kappa^2}, \end{aligned} \tag{54}$$

$$\begin{aligned} X_2 = {} & \left[\left\{\left(\frac{P'_{z0}}{(\mu_0 + P_{z0})} - \frac{\mu'_0 P_{z0}}{(\mu_0 + P_{z0})^2} - \frac{P'_{z0}P_{z0}}{(\mu_0 + P_{z0})^2}\right)\chi + \frac{P_{z0}\chi'\Pi_1}{(\mu_0 + P_{z0})}\right\}\right. \\ & \left. \times\left(\frac{Z - 2m_0}{z}\right) + \frac{P_{z0}\chi}{\mu_0 + P_{z0}} + \frac{2\chi B'_0}{B_0}\left(\frac{p_{\perp 0}}{\mu_0 + P_{\perp 0}} - \frac{P_z 0}{(mu_0 + P_{z0})}\left(\frac{z - 2m_0}{z}\right)\right)\right], \end{aligned}$$

where the values of γ_{pN}, Π_1 and $D_{1P}^{(pN)}$ are given in appendix A. Similar to N approximation, we study the instability ranges at pN approximation. We conclude that the instability range for our celestial model via adiabatic index is $0 < \Gamma_1 < 1$.

6 Concluding remarks

The compact object collapses if the equilibrium between outwardly drawn pressure and inward gravitational pull on star is disturbed. In extended gravity, the dynamical instabilities depend on both usual terms and dark-source terms determined by Γ_1. In this paper, we have investigated the stable and unstable regions of plane-symmetric collapsing model under gravitational field of $f(T)$ gravity. We deal with such interior system which is filled with anisotropic dissipative fluid whose modified field equations are viewed by using diagonal tetrad. In this manner, we explored the dynamical equations using the contracted form of Bianchi identities. We have taken the interior metric as plane symmetry and joined it with suitable exterior geometry to detect some particular constraints over the boundary. This shows that masses of both geometries are not the same over the hypersurface. Also, non-static component of pressure turns out to be non-zero because of dark source terms.

We have used a particular model of $f(T)$ gravity to analyze the effects of this theory under linear perturbations. We have applied the perturbation strategy on all functions to examine the dynamics of collapse with the evolution of time. We build up the collapse equation through perturbed form of second dynamical equation. We found that Γ_1 plays a vital role in stability analysis of self-gravitating objects. We analyzed the stable/unstable regions via Γ_1 after inducing weak-field and pN limits on the collapse equation. It has been observed through these approximations that Γ_1 depends upon various physical and geometrical variables. We explored that the system would be unstable until it satisfies the inequality (52) while its violation move it towards the stable eras during the evolution of plane-symmetric object. Chandrasekhar [31] in his pioneer work found the numerical value to adiabatic index to be $\frac{4}{3}$ for spherical star model, but we have explored that the instability range of adiabatic index should be between 0 and 1 for plane symmetric object within the background of $f(T)$ gravity as described in (52). For different models of $f(T)$ other than the one used in this paper, the qualitative outcomes remains unchanged. The fact is that all instability range rely on the metric, matter and torsion dependent terms. Almost all work in $f(T)$ gravity is carried out for an anisotropic fluid configuration. In this paper, to obtain the numerical results of stability same as Chandrasekhar [31], we consider three different possibilities. The comparison of the stability condition of this paper with the classical results have key physical interpretations. We have observed this regimes to be of great significance for the astrophysical tests of stellar objects in background of $f(T)$ gravity.

From a very genuinely observational stand point, this analysis could be related to the potential evident characteristics of $f(T)$ black holes. Particularly, the possibility to inquire $f(T)$ with $f(R)$ black holes could be a powerful tool to distinguish the torsional (TEGR) and curvature (GR) formulation of gravity theories. The discovery of compact objects such as pulsars and quasars, etc., developed huge concern of researchers in the field of relativity, and its applications in astrophysics and cosmology. The current instability analysis would be helpful to investigate the gravitational behavior of compact bodies. The obtained limits can show any specific astrophysical region and describe the possible importance of the expanding and expansion-free scenarios. The key role in our dynamical analysis is played by the adiabatic index which measures the rigidity in the matter configurations. It is also established that the stability conditions have the influence of extra curvature ingredients of modified gravity for any self-gravitating object. Capozziello *et al.* [74] explored that the criteria for Jeans stability is distinct in $f(R)$ gravity as compared to GR due to the change in different physical parameters in the gravity model under discussion. Ahmed *et al.* [75] discussed the accretion process for heteroclinic and cyclic flows near a black hole in the context of $f(T)$ gravity and found that these flows can be assorted according to the equation of state and the black hole characteristics. They also compared their results with those obtained in $f(R)$ gravity. Here, we have examined the influence of modified terms on the instability ranges of self-gravitating object along with different physical parameters.

Appendix A.

The dark portions of eqs. (18) and (19) are

$$D_0 = \frac{-1}{A^2}\left\{\frac{Tf_T - f}{2} + \frac{1}{2C^2}\left(\frac{C'}{C} + \frac{B'}{B}\right)f'_T\right\}_{,0} + \left[\frac{1}{A^2BC}\left\{\frac{A'}{AC}\left(\frac{BC'}{C} + B'\right) - \frac{\dot{B}\dot{C}}{A^2}\left(\frac{\dot{B}}{B} + \frac{3\dot{C}}{2}\right) + \frac{B'}{C}\left(\frac{C'}{C} + \frac{B'}{B}\right)\right\}\right]\dot{f}_T + \left\{\frac{\dot{B}}{A^2BC^2}\left(\frac{2A'}{A} - \frac{C'}{C}\right)\right\}f'_T + \frac{\dot{C}}{A^2C}\left(\frac{Tf_T - f}{2}\right) + \left\{\frac{\dot{B}}{A^2BC^2}f'_T\right\}_{,3},$$

$$D_1=\left\{\frac{-\dot{A}}{A^3C^2}\left(\frac{C'}{C}+\frac{B'}{B}\right)-\frac{\dot{C}}{2A^2C^3}\left(\frac{C'}{C}-\frac{B'}{B}\right)-\frac{\dot{B}}{A^2BC^2}\left(\frac{A'}{A}-\frac{4B'}{B}\right)\right\}\dot{f}_T$$
$$+\frac{1}{2A^2C^2}\left\{\left(\frac{C'}{C}+\frac{B'}{B}\right)\dot{f}_T\right\}_{,0}+\frac{1}{C^2}\left\{\frac{Tf_T-f}{2}-\frac{\dot{B}}{2A^2B}\right\}_{,3}$$
$$+\left\{\frac{A'}{2AC^4}\left(\frac{B'}{B}-\frac{C'}{C}\right)+\left(\frac{B'}{BC^2}\right)^2+\frac{\dot{B}}{A^2BC^2}\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}+\frac{2\dot{C}}{C}\right)\right\}f'_T.$$

The perturbed parts of eq. (41) and (43) are

$$D_{0p}=-\frac{1}{A_0^2}\left[\frac{\gamma(n-1)neT_0^{n-1}}{2}+\frac{1}{C_0^2}\left\{\left(\frac{c'}{C_0}-3\frac{cC'_0}{C_0^2}+\frac{b'}{B_0}-\frac{bB'_0}{B_0^2}-2\frac{cB'_0}{B_0C_0}\right)\right.\right.$$
$$\times(\gamma n(n-1)T_0^{n-2}T'_0)+\left(\frac{C'_0}{C_0}+\frac{B'_0}{B_0}\right)(\gamma n(n-1)(e'T_0^{n-2}+e(n-2)T_0^{n-3}T'_0)$$
$$\left.\left.-\ \frac{2a}{A_0}(\gamma n(n-1)T_0^{n-2}T'_0)\right)-\frac{2a}{A_0}\frac{\gamma(n-1)T_0^n}{2}\right\}\Bigg]+\left[\frac{1}{A_0^2B_0C_0}\left(\frac{A'_0}{A_0C_0}\left(\frac{C'_0B_0}{C_0}+B'_0\right)\right.\right.$$
$$\left.\left.+\ \frac{B'_0}{C_0}\left(\frac{C'_0}{C_0}+1B_0\right)\right)e\gamma n(n-1)T_0^{n-2}\right]+\frac{b}{A_0^2B_0C_0^2}\left(\frac{2A'_0}{A_0}-\frac{C'_0}{C_0}\right)(\gamma n(n-1)T_0^{n-2}T'_0)$$
$$+\ \frac{c}{A_0^2C_0}\frac{\gamma(n-1)T_0^n}{2}+\left\{\frac{b}{A_0^2B_0C_0^2}\gamma n(n-1)T_0^{n-2}T'_0\right\}_{,3},$$

$$D_{1p}=\frac{1}{C_0^2}\left[\left\{\frac{\gamma n(n-1)\omega eT_0^{n-1}}{2}-\frac{\dot{\omega b}}{2A_0^2B_0}\right\}_{,3}-\frac{2\omega c}{C_0}\left\{\frac{\gamma(n-1)T_0^n}{2}\right\}_{,3}\right]$$
$$+\ \frac{1}{2A_0^2C_0^2}\left\{\left(\frac{C'_0}{C_0}+\frac{B'_0}{B_0}\right)\dot{\omega}e\gamma n(n-1)T_0^{n-2}\right\}_{,0}+\left[\left(\frac{A'_0}{2A_0C_0^4}\right.\right.$$
$$\left.\times\left(\frac{B'_0}{B_0}-\frac{C'_0}{C_0}\right)+\left(\frac{B'_0}{B_0C_0^2}\right)^2\right)(\omega n(n-1)\gamma(e'T_0^{n-2}+e(n-2)T_0^{n-3}T'_0))$$
$$\left.-\left\{\frac{A'_0\omega}{2A_0C_0^4}\left(\frac{4c}{C_0}+\frac{a}{A_0}-\frac{a'}{A'_0}\right)\left(\frac{B'_0}{B_0}-\frac{C'_0}{C_0}\right)+2\left(\frac{B'_0}{B_0C_0}\right)^2\left(\frac{2\omega c}{C_0}+\frac{\omega b}{B_0}+\frac{\omega b'}{B'_0}\right)\right\}\right]$$
$$\times(\gamma n(n-1)T_0^{n-2}T'_0).$$

The values of different parameters used in this paper is given below

$$\Pi=-\left(\frac{a}{C_0}-\frac{c}{C_0^2}\right)P'_{z0},\qquad \Delta=\frac{\chi P_{z0}}{C_0},\qquad \gamma_N=\frac{(\beta+\gamma nT_0^{n-1})}{\kappa^2C_0}\left[-2b'+\frac{1}{2(\beta+\gamma nT_0^{n-1})}\right.$$
$$\left.\times\ \frac{C'_0e\gamma n(n-1)T_0^{n-2}}{C_0}\right],$$

where

$$\chi_1=\frac{bC_0^2}{B_0A_0^2},\qquad \chi_2=\left[\frac{bC_0(\beta+\gamma n(n-1)T_0^{n-2}T'_0)}{A_0B_0(\beta+\gamma nT_0^{n-1})}\right],$$
$$\chi_3=-\left[\frac{2B'_0}{B_0^2}\left(b'-\frac{bB'_0}{B_0}\right)+\frac{2A'_0}{A_0B_0}\left(\frac{a'B'_0}{A'_0}+b'-\frac{aB'_0}{A_0}-\frac{bB'_0}{B_0}\right)-\frac{\gamma n(n-1)C_0^2eT_0^{n-1}}{(\beta+\gamma nT_0^{n-1})}\right].$$

Also,

$$\Delta_1=\left[\left\{\left(\frac{P'_{z0}}{(\mu_0+P_{z0})}-\frac{\mu'_0P_{z0}}{(\mu_0+P_{z0})^2}-\frac{P'_{z0}P_{z0}}{(\mu_0+P_{z0})^2}\right)\chi+\frac{P_{z0}\chi'\Pi_1}{(\mu_0+P_{z0})}\right\}\right.$$
$$\left.\times\left(\frac{Z-2m_0}{z}\right)+\frac{P_{z0}\chi}{\mu_0+P_{z0}}+\frac{2\chi B'_0}{B_0}\left(\frac{p_{\perp 0}}{\mu_0+P_{\perp 0}}-\frac{P_z0}{(mu_0+P_{z0})}\left(\frac{z-2m_0}{z}\right)\right)\right].$$

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