

Mass and magnetic dipole moment of negative-parity heavy baryons with spin-3/2

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Abstract. We calculate the mass and residue of the heavy spin-3/2 negative-parity baryons with single heavy bottom or charm quark by use of a two-point correlation function. We use the obtained results to investigate the diagonal radiative transitions among the baryons under consideration. In particular, we compute corresponding transition form factors via light cone QCD sum rules, which are then used to obtain the magnetic dipole moments of the heavy spin-3/2 negative-parity baryons. We remove the pollutions coming from the positive-parity spin-3/2 and positive/negative-parity spin-1/2 baryons by constructing sum rules for different Lorentz structures. We compare the results obtained with the existing theoretical predictions.

1 Introduction

The investigation of electromagnetic properties of hadrons including their electromagnetic form factors and multipole moments is one of the key tools in understanding their internal structure and geometric shapes. Such properties of positive-parity hadrons for both the light and heavy systems have been widely studied theoretically. From the experimental side, the subject of nucleon electromagnetic form factors have been in the focus of much attention for many years although the experimental data via different experiments on some form factors are not in good agreement (see, for instance, [1–13]). We hope we will overcome these deficiencies and experimentally study the electromagnetic parameters of other light and heavy baryons by the developments in the facility of different experiments. However, we have even less theoretical knowledge on the multipole moments of the negative-parity baryons. Hence, both theoretical and experimental studies on the electromagnetic form factors of negative-parity states are welcome as they can help us gain valuable knowledge on the nature of the strong interactions inside these objects. Jefferson Laboratory and Mainz Microtron facility are planing to measure the electromagnetic form factors and multipole moments of the negative-parity baryons [2, 14]. It is desired that the new electron beam facilities would allow to compile a large number of more precise data to study the electro-excitations of the nucleon resonances.

The theoretical studies on the spectroscopic and electromagnetic decays of negative-parity baryons have mainly been devoted to the radial excitations of the nucleons and other light baryons (see, for instance, [15–20] and references therein). In the heavy sector, some spectroscopic properties of negative-parity spin-1/2 and spin-3/2 heavy baryons have been studied in [21–25]. The magnetic moments of negative-parity heavy baryons with $J^P = \frac{1}{2}^-$ are calculated in [26]. The transition magnetic moments between negative-parity heavy spin-1/2 baryons are also investigated in [27, 28]. In the present work, first we study the masses and residues of the heavy spin-3/2 negative-parity baryons using a two-point correlation function in the context of QCD or SVZ sum rules [29, 30]. The obtained results are then used to compute the magnetic dipole moments of the heavy spin-3/2 negative-parity baryons in the frameworks of light cone QCD sum rules (LCSR) by the help of photon distribution amplitudes (DAs). A similar calculations on the spectroscopic and electromagnetic properties of the heavy spin-3/2 positive-parity baryons can be found in [31]. The interpolating current of the baryons under consideration in the present study also couple to the heavy spin-3/2 positive-parity baryons as well as heavy spin-1/2 baryons with both parities. To remove the unwanted contributions coming from these channels, different Lorentz structures entering the calculations as well as an appropriate ordering of the Dirac matrices are used.

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The paper is organized as follows. In the next section, we construct a mass sum rule to evaluate the masses and pole residues of the negative-parity heavy spin-3/2 baryons and numerically analyze the obtained sum rules. The results are compared with the existing theoretical predictions. In sect. 3, we construct the LCSR for the electromagnetic form factors defining the diagonal radiative transitions of the baryons under consideration and compute the corresponding magnetic dipole moments and their numerical values. The last section is dedicated to the concluding remarks.

2 Mass and residue of the negative-parity spin-3/2 heavy baryons

In order to calculate the mass and residue of the negative-parity spin-3/2 baryons with single heavy bottom or charm quark, we start with the following two-point correlation function:

$$T_{\mu\nu} = i \int d^4x e^{ik \cdot x} \langle 0 | \mathcal{T} \{ \eta_\mu^{\mathcal{B}}(x) \bar{\eta}_\nu^{\mathcal{B}}(0) \} | 0 \rangle, \quad (1)$$

where $\eta_\mu^{\mathcal{B}}$ is the interpolating current of \mathcal{B} baryon coupling to both the positive- and negative-parity baryons. To construct the mass sum rules for the baryons under consideration we calculate this correlation function via two different ways: in terms of hadronic parameters and in terms of QCD degrees of freedom by the help of operator product expansion (OPE). The hadronic side of the correlation function is obtained by inserting complete sets of intermediate states with both parities. After performing the four-integral we get

$$T_{\mu\nu}^{HAD} = \frac{\langle 0 | \eta_\mu^{\mathcal{B}} | \mathcal{B}^+(k, s) \rangle \langle \mathcal{B}^+(k, s) | \bar{\eta}_\nu^{\mathcal{B}} | 0 \rangle}{m_{\mathcal{B}^+}^2 - k^2} + \frac{\langle 0 | \eta_\mu^{\mathcal{B}} | \mathcal{B}^-(k, s) \rangle \langle \mathcal{B}^-(k, s) | \bar{\eta}_\nu^{\mathcal{B}} | 0 \rangle}{m_{\mathcal{B}^-}^2 - k^2} + \dots, \quad (2)$$

where \mathcal{B}^+ and \mathcal{B}^- denote the positive- and negative-parity spin-3/2 baryons, respectively, and \dots shows the contributions of the higher states and continuum. To proceed, we need to define the following matrix elements:

$$\begin{aligned} \langle 0 | \eta_\mu^{\mathcal{B}} | \mathcal{B}^+(k, s) \rangle &= \lambda_{\mathcal{B}^+} u_\mu(k, s), \\ \langle 0 | \eta_\mu^{\mathcal{B}} | \mathcal{B}^-(k, s) \rangle &= \lambda_{\mathcal{B}^-} \gamma_5 u_\mu(k, s), \end{aligned} \quad (3)$$

where $u_\mu(k, s)$ is the Rarita-Schwinger spinor for the spin-3/2 particles and $\lambda_{\mathcal{B}^\pm}$ are the residues of the \mathcal{B}^\pm baryons. Here we shall comment that the $\eta_\mu^{\mathcal{B}}$ current not only interacts with the spin-3/2 states, but also with the spin-1/2 states. Hence, we should remove the unwanted pollution coming from the spin-1/2 states. The general form of the matrix element of $\eta_\mu^{\mathcal{B}}$ between the spin-1/2 and vacuum states can be written as

$$\left\langle 0 | \eta_\mu^{\mathcal{B}} | \frac{1}{2}(k) \right\rangle = (A k_\mu + B \gamma_\mu) u(k), \quad (4)$$

where A and B are some constants. By multiplication of both sides of this equation with γ^μ , and by using the condition $\gamma^\mu \eta_\mu^{\mathcal{B}} = 0$ to get rid of transverse spin-1/2 components (for details, see [32]), we get

$$\left\langle 0 | \eta_\mu^{\mathcal{B}} | \frac{1}{2}^+(k) \right\rangle = B \left(-\frac{4}{m_{\frac{1}{2}^+}} k_\mu + \gamma_\mu \right) u(k), \quad (5)$$

for the positive-parity and

$$\left\langle 0 | \eta_\mu^{\mathcal{B}} | \frac{1}{2}^-(k) \right\rangle = B \gamma_5 \left(-\frac{4}{m_{\frac{1}{2}^-}} k_\mu + \gamma_\mu \right) u(k), \quad (6)$$

for the negative-parity states. From these equations we see that the unwanted contributions coming from the spin-1/2 states are proportional to either k_μ or γ_μ . To remove these contributions, first we order the Dirac matrices as $\gamma_\mu \not{k} \gamma_\nu$ and then set the terms with γ_μ in the beginning and γ_ν at the end and those terms proportional to k_μ and k_ν to zero (for further details about the eliminating the contributions of the spin-1/2 particles, see [33]).

Now, we insert eq. (3) into eq. (2) and use the relation $T_{\mu\nu}^- = -\gamma_5 T_{\mu\nu}^+ \gamma_5$ (see also [16]) and the summation over the spin-3/2 baryons via

$$\sum_s u_\mu(k, s) \bar{u}_\nu(k, s) = -(\not{k} + m_{\mathcal{B}}) \left[g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2k_\mu k_\nu}{3m_{\mathcal{B}}^2} + \frac{k_\mu \gamma_\nu - k_\nu \gamma_\mu}{3m_{\mathcal{B}}} \right]. \quad (7)$$

Table 1. The normalization constant A and the light quark fields q_1 and q_2 for the corresponding baryons.

	A	q_1	q_2
$\Sigma_{b(c)}^{*+(++)}$	$1/\sqrt{3}$	u	u
$\Sigma_{b(c)}^{*0(+)}$	$\sqrt{2/3}$	u	d
$\Sigma_{b(c)}^{*- (0)}$	$1/\sqrt{3}$	d	d
$\Xi_{b(c)}^{*0(+)}$	$\sqrt{2/3}$	s	u
$\Xi_{b(c)}^{*- (0)}$	$\sqrt{2/3}$	s	d
$\Omega_{b(c)}^{*- (0)}$	$1/\sqrt{3}$	s	s

As a result, for the hadronic side of the correlation function in the Borel scheme, we get

$$\widehat{\mathbf{B}}\mathbf{T}_{\mu\nu}^{HAD} = - \left[\lambda_{\mathcal{B}^+}^2 e^{-\frac{m_{\mathcal{B}^+}^2}{M_{\mathcal{B}}^2}} + \lambda_{\mathcal{B}^-}^2 e^{-\frac{m_{\mathcal{B}^-}^2}{M_{\mathcal{B}}^2}} \right] \not{k} g_{\mu\nu} - \left[\lambda_{\mathcal{B}^+}^2 m_{\mathcal{B}^+} e^{-\frac{m_{\mathcal{B}^+}^2}{M_{\mathcal{B}}^2}} - \lambda_{\mathcal{B}^-}^2 m_{\mathcal{B}^-} e^{-\frac{m_{\mathcal{B}^-}^2}{M_{\mathcal{B}}^2}} \right] g_{\mu\nu} + \text{other structures}, \tag{8}$$

where $M_{\mathcal{B}}^2$ is the Borel mass parameter coming from the Borel transformation, which is performed to suppress the contributions of the higher states and continuum.

The OPE side of the aforesaid correlation function is calculated in terms of the QCD degrees of freedom in deep Euclidean region. To this aim, we need the explicit form of the interpolating current $\eta_{\mu}^{\mathcal{B}}$, which is given as (for some details about the baryon currents, see [15, 34–36])

$$\eta_{\mu}^{\mathcal{B}} = A \epsilon_{ijk} \left\{ (q_1^{iT} C \gamma_{\mu} q_2^j) Q^k + (q_2^{iT} C \gamma_{\mu} Q^j) q_1^k + (Q^{iT} C \gamma_{\mu} q_1^j) q_2^k \right\}, \tag{9}$$

where C is the charge conjugation operator; i, j and k are color indices and Q denotes the heavy b or c quark. The normalization constant A and the light quark fields q_1 and q_2 for each heavy baryon is given in table 1.

After inserting the explicit form of the interpolating current into the correlation function in eq. (2) and performing contractions via the Wick’s theorem, we get the OPE side in terms of the heavy and light quarks propagators. For the light quark propagator in coordinate space we use [37]

$$\begin{aligned} S_q(x) = & \frac{i \not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} \left(1 - i \frac{m_q}{4} \not{x} \right) - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left(1 - i \frac{m_q}{6} \not{x} \right) \\ & - i g_s \int_0^1 du \left[\frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(ux) \sigma_{\mu\nu} - \frac{i}{4\pi^2 x^2} u x^{\mu} G_{\mu\nu}(ux) \gamma^{\nu} \right. \\ & \left. - i \frac{m_q}{32\pi^2} G_{\mu\nu} \sigma^{\mu\nu} \left(\ln \left(\frac{-x^2 \Lambda^2}{4} \right) + 2\gamma_E \right) \right], \end{aligned} \tag{10}$$

where $\gamma_E \simeq 0.577$ is the Euler constant and Λ is a scale parameter. The heavy quark propagator in an external field is also taken as

$$S_Q(x) = S_Q^{\text{free}}(x) - i g_s \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \int_0^1 du \left[\frac{\not{k} + m_Q}{2(m_Q^2 - k^2)^2} G^{\mu\nu}(ux) \sigma_{\mu\nu} + \frac{u}{m_Q^2 - k^2} x_{\mu} G^{\mu\nu} \gamma_{\nu} \right], \tag{11}$$

where $S_Q^{\text{free}}(x)$ is the free heavy quark operator in the x -representation and is given by

$$S_Q^{\text{free}}(x) = \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} - i \frac{m_Q^2 \not{x}}{4\pi^2 x^2} K_2(m_Q \sqrt{-x^2}), \tag{12}$$

where K_1 and K_2 are the modified Bessel function of the second kind. By using these propagators in the coordinate space and performing the Fourier and Borel transformations as well as applying the continuum subtraction, after a very lengthy calculations we get

$$\widehat{\mathbf{B}}\mathbf{T}_{\mu\nu}^{\text{OPE}} = \mathbf{T}_1^{\text{OPE}} \not{k} g_{\mu\nu} + \mathbf{T}_2^{\text{OPE}} g_{\mu\nu} + \text{other structures}, \tag{13}$$

where the functions $T_{1,2}^{\text{OPE}}$, for instance, for the Σ_b^{*0} particle, are given as

$$\begin{aligned}
T_1^{\text{OPE}} = & \int_{m_b^2}^{s_0} ds e^{-\frac{s}{M_B^2}} \left\{ \frac{15m_b^8 s - 4m_b^{10} + 40m_b^4 s^3 - 60m_b^2 s^4 + 9s^5 + 60m_b^4 s^3 \ln[\frac{s}{m_b^2}]}{15 \times 2^8 \pi^4 s^3} \right. \\
& - (\langle u\bar{u} \rangle + \langle d\bar{d} \rangle) \frac{m_b(m_b^2 - s)^2}{24\pi^2 s^2} - \langle g_s^2 GG \rangle \frac{(m_b^6 - 4m_b^2 s^2 + 3s^3)}{2^7 \times 3^2 \pi^4 s^3} - \langle g_s^2 GG \rangle^2 \frac{m_b^2}{2^8 \times 3^4 \pi^4 s^3} \\
& + m_0^2 (\langle u\bar{u} \rangle + \langle d\bar{d} \rangle) \frac{m_b^3}{48\pi^2 s^2} - \langle g_s^2 GG \rangle (\langle u\bar{u} \rangle + \langle d\bar{d} \rangle) \frac{m_b}{2^5 \times 3^2 \pi^2 s^2} \left. \right\} \\
& + e^{-\frac{m_b^2}{M_B^2}} \left\{ \frac{1}{9} \langle u\bar{u} \rangle \langle d\bar{d} \rangle + \langle g_s^2 GG \rangle (\langle u\bar{u} \rangle + \langle d\bar{d} \rangle) \frac{1}{2^5 \times 3^3 \pi^2 m_b} - m_0^2 \langle u\bar{u} \rangle \langle d\bar{d} \rangle \frac{m_b^2 + M_B^2}{18M_B^4} \right. \\
& + \langle g_s^2 GG \rangle^2 \frac{m_b^2 + 2M_B^2}{2^{10} \times 3^4 \pi^4 m_b^2 M_B^2} - m_0^2 \langle d\bar{d} \rangle \langle g_s^2 GG \rangle \frac{m_b^2 - 2M_B^2}{2^7 \times 3^3 \pi^2 m_b M_B^4} \\
& + m_0^2 \langle g_s^2 GG \rangle \langle d\bar{d} \rangle \langle u\bar{u} \rangle \frac{m_b^2(m_b^2 - M_B^2)}{2^4 \times 3^4 M_B^{10}} - \langle d\bar{d} \rangle \langle u\bar{u} \rangle \langle g_s^2 GG \rangle \frac{m_b^2}{2^3 \times 3^4 M_B^6} \\
& \left. - m_0^2 \langle g_s^2 GG \rangle \langle u\bar{u} \rangle \frac{m_b^2 - 2M_B^2}{2^7 \times 3^3 \pi^2 m_b M_B^4} \right\}, \tag{14}
\end{aligned}$$

and

$$\begin{aligned}
T_2^{\text{OPE}} = & \int_{m_b^2}^{s_0} ds e^{-\frac{s}{M_B^2}} \left\{ \frac{(s - m_b^2)(41m_b^4 - sm_b^2 + 2s^2) - 6m_b^4(4s - 3m_b^2) \ln[\frac{s}{m_b^2}]}{2^7 \times 3^2 \pi^4 m_b} \right. \\
& - \frac{(4s + m_b^2)(m_b^2 - s)^4}{2^8 \times 3^2 \pi^4 m_b s^2} - \langle u\bar{u} \rangle \frac{(m_b^2 - s)^2(m_b^4 + 2m_b^2 s - 6s^2)}{72\pi^2 m_b^2 s^2} - \langle d\bar{d} \rangle \frac{(m_b^2 - s)^2(m_b^2 + 2s)}{72\pi^2 s^2} \\
& + \langle g_s^2 GG \rangle \frac{24m_b^4 s - 11m_b^6 - 21m_b^2 s^2 + 8s^3 + 18m_b^2 s^2 \ln[\frac{m_b^2}{s}]}{2^9 \times 3^3 \pi^4 m_b s^2} \\
& + \left(m_0^2 \langle u\bar{u} \rangle + m_0^2 \langle d\bar{d} \rangle \right) \frac{(s^2 + m_b^4)}{96\pi^2 s^2} - \langle g_s^2 GG \rangle^2 \frac{m_b}{2^{10} \times 3^3 \pi^4 s} \left. \right\} \\
& + e^{-\frac{m_b^2}{M_B^2}} \left\{ \frac{1}{3} m_b \langle u\bar{u} \rangle \langle d\bar{d} \rangle - m_0^2 \langle u\bar{u} \rangle \langle d\bar{d} \rangle \frac{m_b^3}{6M_B^4} + \frac{\langle g_s^2 GG \rangle (\langle u\bar{u} \rangle + \langle d\bar{d} \rangle)}{2^4 \times 3^3 \pi^2} \right. \\
& + \langle g_s^2 GG \rangle^2 \frac{m_b^2 - M_B^2}{2^{10} \times 3^4 \pi^4 m_b M_B^2} + m_0^2 \langle g_s^2 GG \rangle \langle u\bar{u} \rangle \langle d\bar{d} \rangle \frac{m_b(m_b^4 - 6m_b^2 M_B^2 + 6M_B^4)}{2^4 \times 3^3 M_B^{10}} \\
& \left. - \langle g_s^2 GG \rangle \langle u\bar{u} \rangle \langle d\bar{d} \rangle \frac{m_b(m_b^2 - 3M_B^2)}{2^3 \times 3^3 M_B^6} - m_0^2 \langle g_s^2 GG \rangle (\langle u\bar{u} \rangle + \langle d\bar{d} \rangle) \frac{m_b^2}{2^6 \times 3^2 \pi^2 M_B^4} \right\}, \tag{15}
\end{aligned}$$

where s_0 is the continuum threshold, and for simplicity, we ignored to present the terms containing the light quark masses and those proportional to m_0^4 . Note that we only ignored to present such terms in the above formulas and we will take into account their contributions in the numerical calculations.

Having calculated both the hadronic and OPE sides of the correlation function, we match the coefficients of the structures $\not{k}g_{\mu\nu}$ and $g_{\mu\nu}$ from these two sides and obtain the mass and residue of the negative-parity heavy spin-3/2 baryons as

$$\begin{aligned}
m_{B^-}^2 &= \frac{\frac{d}{d\frac{1}{M_B^2}} (T_2^{\text{OPE}} - m_{B^+} T_1^{\text{OPE}})}{T_2^{\text{OPE}} - m_{B^+} T_1^{\text{OPE}}}, \\
\lambda_{B^-}^2 &= \frac{T_2^{\text{OPE}} - m_{B^+} T_1^{\text{OPE}}}{m_{B^+} + m_{B^-}} e^{\frac{m_{B^-}^2}{M_B^2}}, \tag{16}
\end{aligned}$$

where $\frac{d}{d\frac{1}{M_B^2}}$ denotes the derivative with respect to $\frac{1}{M_B^2}$.

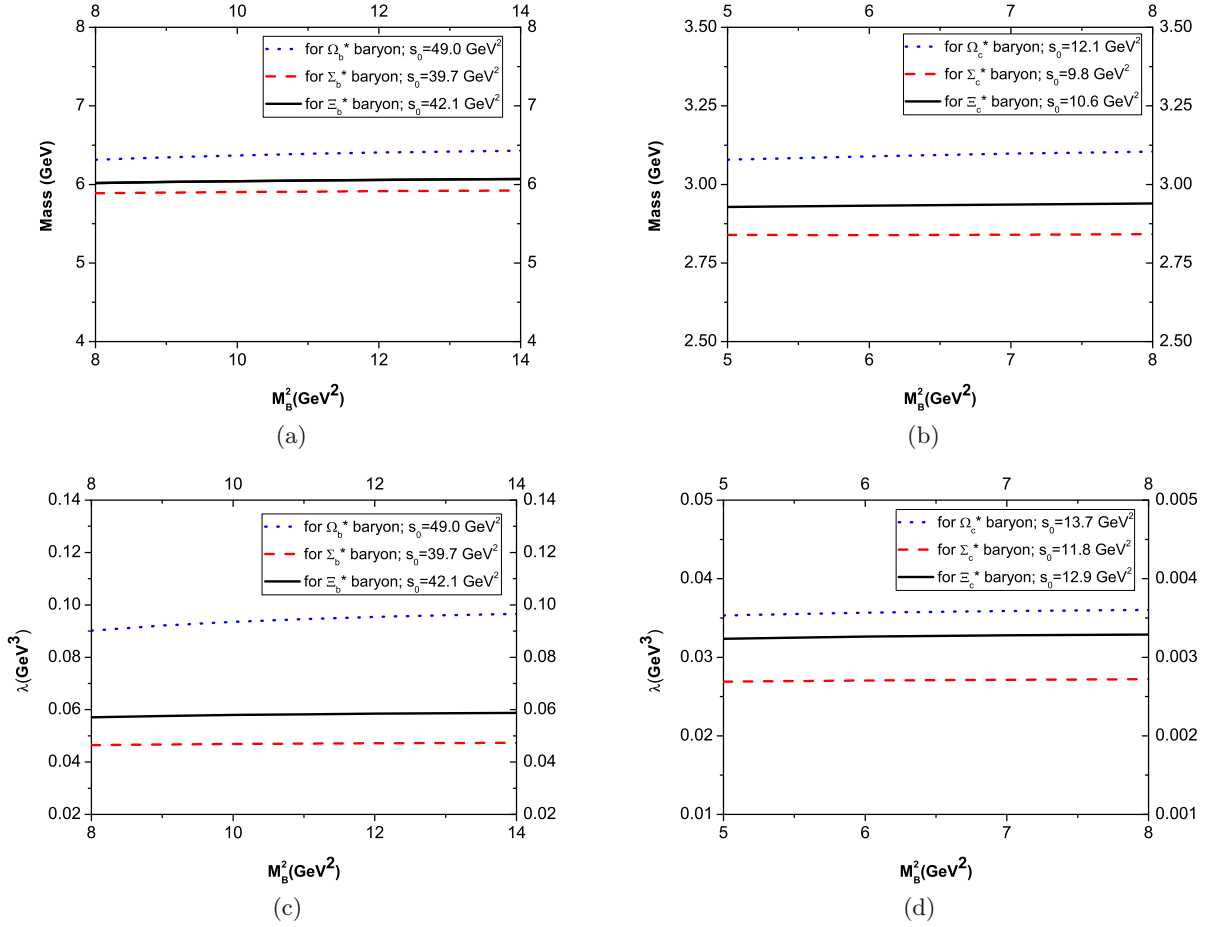


Fig. 1. Variations of the mass and residue of the heavy spin-3/2 negative-parity baryons with respect to M_B^2 at fixed values of s_0 .

To obtain the numerical values of the masses and residues of the negative-parity heavy spin-3/2 baryons we take $\langle \bar{u}u \rangle(1 \text{ GeV}) = \langle \bar{d}d \rangle(1 \text{ GeV}) = -(0.24 \pm 0.01)^3 \text{ GeV}^3$, $\langle \bar{s}s \rangle(1 \text{ GeV}) = 0.8 \langle \bar{u}u \rangle(1 \text{ GeV}) \text{ GeV}^3$, $\langle \frac{\alpha_s G^2}{\pi} \rangle = (0.012 \pm 0.004) \text{ GeV}^4$ and $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$ [38, 39]. Beside these input parameters, we shall also find working regions of the auxiliary parameters s_0 and M_B^2 , such that the physical quantities show weak dependence on these parameters according to the standard criteria of the method used. The continuum threshold is not entirely arbitrary, but it depends on the energy of the first excited state with the same quantum numbers. Hence, according to the standard prescriptions, the value of this threshold is mainly chosen, such that the mass of the first excited state remains above $\sqrt{s_0}$. This may be considered as a weak point of the QCD sum rule approach especially in the case of the negative-parity baryons. Since for many states we know the exact values of the ground state masses experimentally but have not enough information on the energy of the corresponding first excited states. Considering this point, in our calculations, we impose the pole dominance condition and demand that the pole contribution consists the highest possible part of the whole result in each channel. This leads us to take this parameter in the interval $(m_{B^-} + 0.1)^2 \leq s_0 \leq (m_{B^-} + 0.7)^2$. The upper and lower bands on the Borel parameter is determined requiring that not only the contributions of the higher states and continuum are small compared to the pole contributions, but also the perturbative part exceeds the non-perturbative contributions and the series of sum rules converge (for a discussion on the different but equivalent ways of fixing the Borel parameter, see [40]). By these considerations, the following working intervals are found:

$$8 \text{ GeV}^2 \leq M_B^2 \leq 14 \text{ GeV}^2, \quad \text{for } \Omega_b, \Sigma_b \text{ and } \Xi_b,$$

and

$$5 \text{ GeV}^2 \leq M_B^2 \leq 8 \text{ GeV}^2, \quad \text{for } \Omega_c, \Sigma_c \text{ and } \Xi_c. \quad (17)$$

The variations of the mass and residue of the baryons under consideration with respect to M_B^2 at fixed values of s_0 are shown in fig. 1. From this figure, we see that the mass and residue of these baryons show good stabilities with respect to the Borel mass parameter in its working regions. Our numerical calculations show also that the dependence of the results on s_0 is relatively weak at the above-mentioned working region for the continuum threshold.

Table 2. Values of the masses and residues of the heavy spin-3/2 negative-parity baryons with single heavy bottom or charm quark.

Mass and residue	Present work	[21]	[23]	[24, 25]
$m_{\Omega_b^*}$ (GeV)	6.40 ± 0.26	6.26 ± 0.15	6.304	6.330
$m_{\Omega_c^*}$ (GeV)	3.08 ± 0.12	2.98 ± 0.16	2.986	2.998
$m_{\Sigma_b^*}$ (GeV)	5.91 ± 0.22	6.00 ± 0.18	6.101	6.076
$m_{\Sigma_c^*}$ (GeV)	2.84 ± 0.11	2.74 ± 0.20	2.763	2.761
$m_{\Xi_b^*}$ (GeV)	6.06 ± 0.24	6.14 ± 0.16	6.194	6.212
$m_{\Xi_c^*}$ (GeV)	2.93 ± 0.11	2.86 ± 0.17	2.871	2.900
$\lambda_{\Omega_b^*}$ (GeV ³)	0.094 ± 0.028	0.095 ± 0.019	–	–
$\lambda_{\Omega_c^*}$ (GeV ³)	0.036 ± 0.011	0.072 ± 0.013	–	–
$\lambda_{\Sigma_b^*}$ (GeV ³)	0.048 ± 0.014	0.047 ± 0.012	–	–
$\lambda_{\Sigma_c^*}$ (GeV ³)	0.027 ± 0.009	0.037 ± 0.009	–	–
$\lambda_{\Xi_b^*}$ (GeV ³)	0.058 ± 0.017	0.054 ± 0.013	–	–
$\lambda_{\Xi_c^*}$ (GeV ³)	0.033 ± 0.010	0.045 ± 0.009	–	–

We depict the numerical results of the masses and residues of the negative-parity heavy spin-3/2 baryons in table 2. The errors in the presented results are due to the uncertainties in determination of the working regions for the auxiliary parameters as well as the errors of other input parameters. With a quick glance at this table we read that the values of the masses of the heavy spin-3/2 negative-parity baryons obtained in the present work are overall close to the predictions of [21, 23–25]. For the residue of b-baryons, our results are very close to those of [21], but for the residues of c-baryons our results considerably small compared to the predictions of [21]. The results of the present work for the masses and residues will be used in determination of the magnetic dipole moments of the negative-parity heavy spin-3/2 baryons in the following section.

3 Magnetic dipole moment of the negative-parity spin-3/2 heavy baryons

In order to obtain the LCSR for the magnetic dipole moment of the negative-parity heavy spin-3/2 baryons we choose the following two-point correlation function in the presence of a background photon field:

$$\Pi_{\mu\nu} = i \int d^4x e^{ip \cdot x} \langle \gamma(q) | \mathcal{T} (\eta_\mu^{\mathcal{B}}(x) \bar{\eta}_\nu^{\mathcal{B}}(0)) | 0 \rangle, \quad (18)$$

where $\gamma(q)$ means the electromagnetic field. Let us note that we use the background electromagnetic plane wave field and the photon DAs instead of the electromagnetic current in accordance with the light cone QCD sum rule approach and the point that at $q^2 = 0$ the em-current gives the charge only. In terms of twist expansion the local em-current is twist-2 but the photon DAs contain also the higher twists. We will also calculate this correlation function once in terms of hadronic parameters called the physical or hadronic side, and the second in terms of QCD parameters called the QCD or OPE side. By matching these two sides through a dispersion relation in the Borel scheme one can calculate the magnetic dipole moment of the baryons under consideration in terms of other parameters entering the calculations.

3.1 Hadronic side

For the hadronic side of the calculation one inserts complete sets of states between the interpolating currents in eq. (18) with quantum numbers of heavy baryons. Integration over four- x give

$$\begin{aligned}
 \Pi_{\mu\nu}^{HAD} = & \frac{\langle 0 | \eta_{\mu}^{\mathcal{B}} | \mathcal{B}^+(p_2) \rangle \langle \mathcal{B}^+(p_2) \gamma(q) | \mathcal{B}^+(p_1) \rangle \langle \mathcal{B}^+(p_1) | \bar{\eta}_{\nu}^{\mathcal{B}} | 0 \rangle}{(m_{\mathcal{B}^+}^2 - p_1^2)(m_{\mathcal{B}^+}^2 - p_2^2)} \\
 & + \frac{\langle 0 | \eta_{\mu}^{\mathcal{B}} | \mathcal{B}^-(p_2) \rangle \langle \mathcal{B}^-(p_2) \gamma(q) | \mathcal{B}^+(p_1) \rangle \langle \mathcal{B}^+(p_1) | \bar{\eta}_{\nu}^{\mathcal{B}} | 0 \rangle}{(m_{\mathcal{B}^+}^2 - p_1^2)(m_{\mathcal{B}^-}^2 - p_2^2)} \\
 & + \frac{\langle 0 | \eta_{\mu}^{\mathcal{B}} | \mathcal{B}^+(p_2) \rangle \langle \mathcal{B}^+(p_2) \gamma(q) | \mathcal{B}^-(p_1) \rangle \langle \mathcal{B}^-(p_1) | \bar{\eta}_{\nu}^{\mathcal{B}} | 0 \rangle}{(m_{\mathcal{B}^-}^2 - p_1^2)(m_{\mathcal{B}^+}^2 - p_2^2)} \\
 & + \frac{\langle 0 | \eta_{\mu}^{\mathcal{B}} | \mathcal{B}^-(p_2) \rangle \langle \mathcal{B}^-(p_2) \gamma(q) | \mathcal{B}^-(p_1) \rangle \langle \mathcal{B}^-(p_1) | \bar{\eta}_{\nu}^{\mathcal{B}} | 0 \rangle}{(m_{\mathcal{B}^-}^2 - p_1^2)(m_{\mathcal{B}^-}^2 - p_2^2)} \\
 & + \dots,
 \end{aligned} \tag{19}$$

where $p_1 = p + q$, $p_2 = p$ and \dots stand for the contributions coming from the higher states and continuum. To proceed, beside the matrix elements defined in the previous section in terms of residues, we need the following matrix elements parametrized in terms of electromagnetic form factors (see also [33]):

$$\begin{aligned}
 \langle \mathcal{B}^+(p_2) \gamma(q) | \mathcal{B}^+(p_1) \rangle &= \varepsilon_{\rho} \bar{u}_{\alpha}(p_2) \left\{ -g^{\alpha\beta} \left[\gamma^{\rho}(f_1 + f_2) + \frac{(p_1 + p_2)^{\rho}}{2m_{\mathcal{B}^+}} f_2 + q^{\rho} f_3 \right] \right. \\
 &\quad \left. - \frac{q^{\alpha} q^{\beta}}{2m_{\mathcal{B}^+}^2} \left[\gamma^{\rho}(g_1 + g_2) + \frac{(p_1 + p_2)^{\rho}}{2m_{\mathcal{B}^+}} g_2 + q^{\rho} g_3 \right] \right\} u_{\beta}(p_1), \\
 \langle \mathcal{B}^-(p_2) \gamma(q) | \mathcal{B}^-(p_1) \rangle &= \varepsilon_{\rho} \bar{u}_{\alpha}(p_2) \left\{ -g^{\alpha\beta} \left[\gamma^{\rho}(f_1^* + f_2^*) + \frac{(p_1 + p_2)^{\rho}}{2m_{\mathcal{B}^-}} f_2^* + q^{\rho} f_3^* \right] \right. \\
 &\quad \left. - \frac{q^{\alpha} q^{\beta}}{2m_{\mathcal{B}^-}^2} \left[\gamma^{\rho}(g_1^* + g_2^*) + \frac{(p_1 + p_2)^{\rho}}{2m_{\mathcal{B}^-}} g_2^* + q^{\rho} g_3^* \right] \right\} u_{\beta}(p_1), \\
 \langle \mathcal{B}^+(p_2) \gamma(q) | \mathcal{B}^-(p_1) \rangle &= \varepsilon_{\rho} \bar{u}_{\alpha}(p_2) \left\{ -g^{\alpha\beta} \left[\gamma^{\rho}(f_1^T + f_2^T) + \frac{(p_1 + p_2)^{\rho}}{m_{\mathcal{B}^+} + m_{\mathcal{B}^-}} f_2^T + q^{\rho} f_3^T \right] \right. \\
 &\quad \left. - \frac{q^{\alpha} q^{\beta}}{m_{\mathcal{B}^+}^2 + m_{\mathcal{B}^-}^2} \left[\gamma^{\rho}(g_1^T + g_2^T) + \frac{(p_1 + p_2)^{\rho}}{m_{\mathcal{B}^+} + m_{\mathcal{B}^-}} g_2^T + q^{\rho} g_3^T \right] \right\} \gamma_5 u_{\beta}(p_1), \\
 \langle \mathcal{B}^-(p_2) \gamma(q) | \mathcal{B}^+(p_1) \rangle &= -\varepsilon_{\rho} \bar{u}_{\alpha}(p_2) \gamma_5 \left\{ -g^{\alpha\beta} \left[\gamma^{\rho}(f_1^{T*} + f_2^{T*}) + \frac{(p_1 + p_2)^{\rho}}{m_{\mathcal{B}^+} + m_{\mathcal{B}^-}} f_2^{T*} + q^{\rho} f_3^{T*} \right] \right. \\
 &\quad \left. - \frac{q^{\alpha} q^{\beta}}{m_{\mathcal{B}^+}^2 + m_{\mathcal{B}^-}^2} \left[\gamma^{\rho}(g_1^{T*} + g_2^{T*}) + \frac{(p_1 + p_2)^{\rho}}{m_{\mathcal{B}^+} + m_{\mathcal{B}^-}} g_2^{T*} + q^{\rho} g_3^{T*} \right] \right\} u_{\beta}(p_1),
 \end{aligned} \tag{20}$$

where $f_i, f_i^*, f_i^T, f_i^{T*}, g_i, g_i^*, g_i^T$ and g_i^{T*} are electromagnetic form factors, whose values at $q^2 = 0$ are needed in determination of the multipole moments. In the above equation, ε_{ρ} is the four-polarization vector of the electromagnetic field.

Here also to remove the pollution from the spin-1/2 baryons a process similar to that of the previous section is followed and the ordering $\gamma_{\mu} \not{\not{q}} \not{p} \gamma_{\nu}$ is applied. To complete the task, the terms containing the γ_{μ} at the beginning, γ_{ν} at the end and those proportional to $p_{1\mu}$ and $p_{2\nu}$ are set to zero.

Substituting eqs. (3) and (20) in eq. (19) and using eq. (7), the final form of the hadronic side of the correlation function in Borel scheme is obtained as

$$\begin{aligned}
\widehat{\mathbf{B}}\Pi_{\mu\nu}^{\text{HAD}} = & \left[\lambda_{\mathcal{B}^-} \lambda_{\mathcal{B}^+} m_{\mathcal{B}^-} (f_1^T + f_2^T) e^{-\frac{m_{\mathcal{B}^-}^2}{M_1^2} - \frac{m_{\mathcal{B}^+}^2}{M_2^2}} + \lambda_{\mathcal{B}^-} \lambda_{\mathcal{B}^+} m_{\mathcal{B}^+} (f_1^{T^*} + f_2^{T^*}) e^{-\frac{m_{\mathcal{B}^+}^2}{M_1^2} - \frac{m_{\mathcal{B}^-}^2}{M_2^2}} \right. \\
& - \lambda_{\mathcal{B}^-}^2 m_{\mathcal{B}^-} (f_1^* + f_2^*) e^{-\frac{m_{\mathcal{B}^-}^2}{M_1^2} - \frac{m_{\mathcal{B}^-}^2}{M_2^2}} - \lambda_{\mathcal{B}^+}^2 m_{\mathcal{B}^+} (f_1 + f_2) e^{-\frac{m_{\mathcal{B}^+}^2}{M_1^2} - \frac{m_{\mathcal{B}^+}^2}{M_2^2}} \left. \right] g_{\mu\nu} \not{\epsilon} \not{\epsilon} \\
& + \left[\lambda_{\mathcal{B}^-}^2 (m_{\mathcal{B}^-}^2 - m_{\mathcal{B}^+}^2) (f_1^* + f_2^*) e^{-\frac{m_{\mathcal{B}^-}^2}{M_1^2} - \frac{m_{\mathcal{B}^-}^2}{M_2^2}} + \lambda_{\mathcal{B}^-} \lambda_{\mathcal{B}^+} m_{\mathcal{B}^-} (m_{\mathcal{B}^-} + m_{\mathcal{B}^+}) (f_1^T + f_2^T) \right. \\
& \times e^{-\frac{m_{\mathcal{B}^-}^2}{M_1^2} - \frac{m_{\mathcal{B}^+}^2}{M_2^2}} - \lambda_{\mathcal{B}^-} \lambda_{\mathcal{B}^+} m_{\mathcal{B}^+} (m_{\mathcal{B}^+} + m_{\mathcal{B}^-}) (f_1^{T^*} + f_2^{T^*}) e^{-\frac{m_{\mathcal{B}^+}^2}{M_1^2} - \frac{m_{\mathcal{B}^-}^2}{M_2^2}} \left. \right] g_{\mu\nu} \not{\epsilon} \\
& - \left[\lambda_{\mathcal{B}^-} \lambda_{\mathcal{B}^+} (f_1^T + f_2^T) e^{-\frac{m_{\mathcal{B}^-}^2}{M_1^2} - \frac{m_{\mathcal{B}^+}^2}{M_2^2}} - \lambda_{\mathcal{B}^-} \lambda_{\mathcal{B}^+} (f_1^{T^*} + f_2^{T^*}) e^{-\frac{m_{\mathcal{B}^+}^2}{M_1^2} - \frac{m_{\mathcal{B}^-}^2}{M_2^2}} \right. \\
& + \lambda_{\mathcal{B}^+}^2 (f_1 + f_2) e^{-\frac{m_{\mathcal{B}^+}^2}{M_1^2} - \frac{m_{\mathcal{B}^+}^2}{M_2^2}} - \lambda_{\mathcal{B}^-}^2 (f_1^* + f_2^*) e^{-\frac{m_{\mathcal{B}^-}^2}{M_1^2} - \frac{m_{\mathcal{B}^-}^2}{M_2^2}} \left. \right] g_{\mu\nu} \not{\epsilon} \not{\epsilon} \\
& - \left[\lambda_{\mathcal{B}^-} \lambda_{\mathcal{B}^+} (m_{\mathcal{B}^-} + m_{\mathcal{B}^+}) (f_1^{T^*} + f_2^{T^*}) e^{-\frac{m_{\mathcal{B}^+}^2}{M_1^2} - \frac{m_{\mathcal{B}^-}^2}{M_2^2}} + \lambda_{\mathcal{B}^-} \lambda_{\mathcal{B}^+} (m_{\mathcal{B}^-} + m_{\mathcal{B}^+}) \right. \\
& \times (f_1^T + f_2^T) e^{-\frac{m_{\mathcal{B}^-}^2}{M_1^2} - \frac{m_{\mathcal{B}^+}^2}{M_2^2}} \left. \right] g_{\mu\nu} \not{\epsilon} \not{\epsilon} \\
& + \text{other structures,} \tag{21}
\end{aligned}$$

where M_1^2 , M_2^2 are the Borel mass parameters in the initial and final channels, respectively. We are interested in calculation of the magnetic dipole moment of only the negative-parity baryons which is given as $\mu = 3(f_1^* + f_2^*)$ in the unit of their natural magneton, *i.e.* $e\hbar/(2m_{\mathcal{B}c})$. So we have diagonal transitions for this case and the initial and final baryon masses are the same. Hence, in the calculations, we take $M_1^2 = M_2^2 = M_{\mathcal{B}}^2 = 2M^2$.

3.2 OPE Side

In order to obtain the OPE side of the correlation function, we insert the interpolating current of the heavy spin-3/2 baryons into eq. (18). After performing contractions of all quark pairs using the Wick's theorem, we get

$$\begin{aligned}
\Pi_{\mu\nu}^{\text{OPE}} = & -iA^2 \epsilon_{abc} \epsilon_{a'b'c'} \int d^4x e^{ipx} \langle \gamma(q) \left\{ S_Q^{ca'} \gamma_\nu S_{q_2}^{tbb'} \gamma_\mu S_{q_1}^{ac'} \right. \\
& + S_Q^{cb'} \gamma_\nu S_{q_1}^{taa'} \gamma_\mu S_{q_2}^{bc'} + S_{q_2}^{ca'} \gamma_\nu S_{q_1}^{tbb'} \gamma_\mu S_Q^{ac'} + S_{q_2}^{cb'} \gamma_\nu S_Q^{taa'} \gamma_\mu S_{q_1}^{bc'} \\
& + S_{q_1}^{cb'} \gamma_\nu S_{q_2}^{taa'} \gamma_\mu S_Q^{bc'} + S_{q_1}^{ca'} \gamma_\nu S_Q^{tbb'} \gamma_\mu S_{q_2}^{ac'} + Tr \left(\gamma_\mu S_{q_1}^{ab'} \gamma_\nu S_{q_2}^{tba'} \right) S_Q^{cc'} \\
& \left. + Tr \left(\gamma_\mu S_Q^{ab'} \gamma_\nu S_{q_1}^{tba'} \right) S_{q_2}^{cc'} + Tr \left(\gamma_\mu S_{q_2}^{ab'} \gamma_\nu S_Q^{tba'} \right) S_{q_1}^{cc'} \right\} | 0 \rangle, \tag{22}
\end{aligned}$$

where $S' = CS^TC$; and S_Q and S_q are the heavy and light quark propagators, respectively. The correlation function in OPE side contains three different contributions:

- perturbative contributions;
- mixed contributions where the photon is radiated from the short distances and at least one of the quarks interacts with the QCD vacuum and makes a condensate;
- non-perturbative contributions where the photon is radiated at long distances.

To proceed in calculations of these contributions, we use the heavy and light quark propagators again in coordinate space. We also need some matrix elements defined in terms of the photon DAs [41]:

$$\begin{aligned}
\langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle &= -ie_q \bar{q} q (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left(\chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right) \\
&\quad - \frac{i}{2(qx)} e_q \langle \bar{q} q \rangle \left[x_\nu \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u), \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle &= e_q f_{3\gamma} \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u), \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -\frac{1}{4} e_q f_{3\gamma} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u), \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q} q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i), \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu} i\gamma_5(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q} q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \tilde{\mathcal{S}}(\alpha_i), \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i), \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i\gamma_\alpha q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i), \\
\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q} q \rangle \left\{ \left[\left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\nu} - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) \right) q_\beta \right. \right. \\
&\quad - \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\nu} - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha \\
&\quad - \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\mu} - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta \\
&\quad + \left. \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\mu} - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) \right) q_\alpha \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_1(\alpha_i) \\
&\quad + \left[\left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) q_\nu \right. \\
&\quad - \left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) q_\mu \\
&\quad - \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) q_\nu \\
&\quad + \left. \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) q_\mu \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_2(\alpha_i) \\
&\quad + \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_3(\alpha_i) \\
&\quad \left. + \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_4(\alpha_i) \right\}, \tag{23}
\end{aligned}$$

where $\varphi_\gamma(u)$ is the leading twist-2 photon DAs, $\psi^v(u)$, $\psi^a(u)$, $\mathcal{A}(\alpha_i)$ and $\mathcal{V}(\alpha_i)$ are twist 3 and $h_\gamma(u)$, $\mathbb{A}(u)$ and \mathcal{T}_i ($i = 1, 2, 3, 4$) are twist 4 photon DAs [41]. In the above equations χ is the magnetic susceptibility of the light quarks. The measure $\int \mathcal{D}\alpha_i$ is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g) \tag{24}$$

and the photon DAs are given as [41]

$$\begin{aligned}
\varphi_\gamma(u) &= 6u\bar{u} \left(1 + \varphi_2(\mu) C_2^{\frac{3}{2}}(u - \bar{u}) \right), \\
\psi^v(u) &= 3(3(2u-1)^2 - 1) + \frac{3}{64} (15w_\gamma^V - 5w_\gamma^A) (3 - 30(2u-1)^2 + 35(2u-1)^4), \\
\psi^a(u) &= (1 - (2u-1)^2) (5(2u-1)^2 - 1) \frac{5}{2} \left(1 + \frac{9}{16} w_\gamma^V - \frac{3}{16} w_\gamma^A \right), \\
\mathcal{A}(\alpha_i) &= 360\alpha_q\alpha_{\bar{q}}\alpha_g^2 \left(1 + w_\gamma^A \frac{1}{2}(7\alpha_g - 3) \right), \\
\mathcal{V}(\alpha_i) &= 540w_\gamma^V (\alpha_q - \alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2, \\
h_\gamma(u) &= -10(1 + 2\kappa^+) C_2^{\frac{1}{2}}(u - \bar{u}), \\
\mathbb{A}(u) &= 40u^2\bar{u}^2 (3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2) [u\bar{u}(2 + 13u\bar{u}) \\
&\quad + 2u^3(10 - 15u + 6u^2)\ln(u) + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})], \\
\mathcal{T}_1(\alpha_i) &= -120(3\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_2(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q) ((\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)), \\
\mathcal{T}_3(\alpha_i) &= -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_4(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q) ((\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)), \\
\mathcal{S}(\alpha_i) &= 30\alpha_g^2\{(\kappa + \kappa^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
&\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}, \\
\tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2\{(\kappa - \kappa^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
&\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}, \tag{25}
\end{aligned}$$

where $\varphi_2(1 \text{ GeV}) = 0$, $w_\gamma^V = 3.8 \pm 1.8$, $w_\gamma^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$ and $\zeta_2^+ = 0$ [41].

After lengthy calculations (for details see, for instance, [20,31]), the OPE side of the correlation function is obtained in terms of the selected structures as

$$\widehat{\mathbf{B}}\Pi_{\mu\nu}^{\text{OPE}} = \Pi_1^{\text{OPE}} g_{\mu\nu} \not{\epsilon} \not{\not{q}} + \Pi_2^{\text{OPE}} g_{\mu\nu} \not{\not{q}} + \Pi_3^{\text{OPE}} g_{\mu\nu} \not{\not{q}} \not{\not{q}} + \Pi_4^{\text{OPE}} g_{\mu\nu} \not{\not{q}} \not{\not{q}} + \text{other structures}, \tag{26}$$

where $\Pi_{1,2,3,4}^{\text{OPE}}$ are very lengthy functions, hence we do not present their explicit expressions here.

Having obtained both the hadronic and OPE sides of the correlation function in Borel scheme it is the time for equating the two sides in order to obtain LCSR for the magnetic moment of the spin-3/2⁻ baryons with single heavy bottom or charm quark. Before doing this we should remind that the magnetic dipole moment is defined in terms of form factors, in the unit of the natural magneton, *i.e.* $e\hbar/(2m_B c)$, as $\mu = 3(f_1^* + f_2^*)$ at $q^2 = 0$, where the factor 3 is due the fact that in the non-relativistic limit the interaction Hamiltonian with magnetic field is given as $\mu B = 3(f_1^* + f_2^*)B$. After replacement $f_1^* + f_2^* \rightarrow \mu/3$ in the final expression of the physical side in eq. (21) and equating the obtained result to the OPE side in eq. (26), we get the following expression for the magnetic dipole moment of the baryons under consideration:

$$\mu = 3 \frac{\Pi_2^{\text{OPE}} - (m_{B^+} + m_{B^-})\Pi_1^{\text{OPE}} + m_{B^+}(m_{B^+} + m_{B^-})\Pi_3^{\text{OPE}} - m_{B^+}\Pi_4^{\text{OPE}}}{2\lambda_{B^-}^2 m_{B^-} (m_{B^+} + m_{B^-})} e^{\frac{m_{B^-}^2}{M^2}}. \tag{27}$$

The numerical values for the magnetic dipole moments of the heavy spin-3/2 negative-parity baryons in units of nuclear magneton are presented in table 3. The errors in numerical values in the presented results again belong to the uncertainties in calculations of the working regions for the Borel mass parameter M^2 and the continuum threshold s_0 ,

Table 3. Values of the magnetic moment of the heavy spin-3/2 negative-parity baryons with single heavy bottom or charm quark in units of nuclear magneton μ_N . The signs +, - and 0 on baryons show their charge.

Heavy baryons with $J^P = \frac{3}{2}^-$	μ
Ω_b^{*-}	-1.37 ± 0.41
Ω_c^{*0}	-3.46 ± 1.04
Σ_b^{*0}	0.55 ± 0.17
Σ_c^{*+}	2.10 ± 0.59
Σ_b^{*+}	2.24 ± 0.67
Σ_c^{*++}	7.73 ± 2.31
Σ_b^{*-}	-1.53 ± 0.46
Σ_c^{*0}	-4.02 ± 1.16
Ξ_b^{*0}	0.57 ± 0.17
Ξ_c^{*+}	2.29 ± 0.68
Ξ_b^{*-}	-1.32 ± 0.39
Ξ_c^{*0}	-2.95 ± 0.83

those uncertainties coming from the parameters entering the photon DAs as well as the uncertainties of other input parameters. Quantitatively, in average, 47%, 14% and 39% of the uncertainties belong to the variations of s_0 , M^2 and DAs together with other inputs, respectively. When we compare these results with the magnetic dipole moments of the positive-parity spin-3/2 heavy baryons [31], we see that the magnetic dipole moments of the negative-parity heavy baryons are compatible with those of the positive-parity baryons except for the Ω_c^{*0} , Σ_c^{*0} , Σ_c^{*++} and Ξ_c^{*0} baryons which there exist considerable differences in the values. Hence the naive expectation, relation between the magnetic moments and masses of the positive- and negative-parity baryons, *i.e.*,

$$\mu_n = \frac{m_p}{m_n} \mu_p, \quad (28)$$

where n and p stand for the negative- and positive-parity heavy baryons, respectively, holds for all b-baryons and some of c-baryons, but is considerably violated for the c-baryons mentioned above. This violation can be attributed to the fact that in our calculations we take into account also the contributions of the positive-to-positive, positive-to-negative and negative-to-positive transitions that affect the c-baryons more compared to the b-baryons. The sign of the magnetic dipole moments of the heavy spin-3/2 baryons with both parities are the same. Our results may be checked via other non-perturbative approaches as well as by future experiments.

4 Conclusion

We calculated the masses and residues of the negative-parity heavy spin-3/2 baryons with single heavy b- or c-quark in the framework of QCD sum rules and compared the results with the existing predictions in the literature. We used the values obtained to calculate the electromagnetic form factors and finally the values of the magnetic dipole moments of the considered baryons in the context of the light cone QCD sum rules using the photon DAs. Our results may be checked via different non-perturbative methods. Checking our predictions by future experiments can be very useful for understanding the internal structure as well as the geometric shape of the negative-parity heavy spin-3/2 baryons.

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