Eur. Phys. J. Plus (2016) **131**: 398
DOI 10.1140/epjp/i2016-16398-5

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

Gauss-Bonnet dark energy Chaplygin gas model

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Received: 30 April 2016 / Revised: 1 October 2016
Published online: 14 November 2016 –

Abstract. The correspondence of the Gauss-Bonnet (GB) model and its modified (MGB) model of dark energy with the standard and generalized Chaplygin gas-scalar field models (SCG and GCG) have been studied in a flat universe. The exact solution of potentials and scalar fields, which describe the accelerated expansion of the universe, are reconstructed. According to the evolutionary behavior of the GB and MGB models, the same form of dynamics of scalar field and potential for different SCG and GCG models are derived. By calculating the squared sound speed of the MGB, GB model as well as the SCG, GCG, and investigating the GB-Chaplygin gas from the viewpoint of linear perturbation theory, we find that the best results, which are consistent with the observation, may appear by considering MGB-GCG. Also, we find out some bounds for parameters.

1 Introduction

Astrophysical data, coming from distant Ia supernovae [1–3], Large Scale Structures (LSS) [4,5] and Cosmic Microwave Background (CMB) [6,7], indicate that our universe undergoes an accelerating expansion. This kind of expansion may be due to a mysterious energy component with negative pressure, the so-called dark energy (DE).

However, in the last decades, other models based on modified gravity ($F(R), F(G), F(R,\phi,X), F(T),\ldots$) have been proposed that give another description of the acceleration expansion of the universe. In these models, many authors have showed that all models of DE can be resolved by modifying the curvature term R (Ricci scalar) of the Einstein-Hilbert action with other curvature scalars, such as any scalar function of R, Gauss-Bonnet term (G), torsion (T), scalar-tensor (X,ϕ), etc. (details are in refs. [8–21] and references therein). Moreover, some authors found that the early inflation, the intermediate decelerating expansion and the late time acceleration expansion, could be described together in one model [22].

Lately, among many models of DE, dynamical models, which are considering a time-dependent component of energy density and equation of state, have attracted a great deal of attention. Also, among many dynamical models, the ones that represented a power series of Hubble parameter and its derivative (*i.e.* $\dot{H}, H\dot{H}, H^2,\ldots$) were found of interest [23,24]. Also, the authors of [25–29] have shown that terms of the form H^3, $\dot{H}H^2$ and H^4 can be important for studying the early universe. Hence, it would not be something strange to consider a DE density proportional to the Gauss-Bonnet (GB) term, which is invariant in 4 dimensions. Besides, in geometrical meaning, the GB invariant has a valid dimension of energy density [30]. Also, the authors of [31,32] showed that a unification between early time inflation and late time acceleration in a viable cosmology can be described by coupling between the GB term and a time-varying scalar field [33].

Another successful model of DE is the Chaplygin gas model. The standard Chaplygin gas (SCG) model, first proposed in [34–36], regards a perfect fluid which plays a dual role in the history of the universe: it behaves as dark matter in the first epoch of evolution of the universe and as dark energy at the late time. Unfortunately, this model has some inconsistency with observational data, *e.g.*, SNIa, BAO, CMB [37–39]. So, the Generalized Chaplygin Gas (GCG) [40] and Modified Chaplygin Gas (MCG) models [41–43] have been introduced in order to establish a viable cosmological model. It would be beneficial to study any relationship between the SCG model and its modification, while DE density behaves like a GB invariant term, as mentioned above. In this paper we shall show that it leads to interesting cosmological implications.

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As we will show in this paper, the EoS parameter of the GB DE model on its own does not give rise to a phantom phase of the universe. Besides, in [30], the author shows that the presence of matter drastically converts the Friedmann equation into a nonlinear differential equation, which alters the behavior of the EoS parameter which can lead to $w_\circ \sim -1.17$ and allows for quintom behavior. However, in this paper, we incorporate the GB dark energy density with a SCG component without adding any matter content. In addition, corporation GB or MGB with different CG models (*i.e.* SCG and GCG) would be helpful in obtaining the exact solution for scalar field and potential and would helps us determining some bounds for free parameters of models. So, considering the cosmological solution for different compositions of GB and CG models could show the importance of each. Also, we would succeed in the framework, where $\kappa^2 = 8\pi G = M_p^{-2} = 1$ and in the natural unit where ($\hbar = c = 1$).

The outline of this paper is as follows: In the next section, we introduce the GB dark energy and calculate the deceleration and EoS parameters. Then, in subsects. 2.1 and 2.2 we investigate corporation GB with SCG and GCG, in turn, and then scalar field and scalar potential are obtained by exact solution. In sect. 3, the same procedure is followed for the MGB energy density. In sect. 4, we investigate the adiabatic sound speed, v^2, which is one of the critical physical quantities in the theory of linear perturbation. In sect. 5, we discuss on the behavior of scalar field, scalar potential and deceleration parameter *versus* x for GB and MGB models and we gain some bounds for free parameters of models. Finally, we summarize our results in sect. 6.

2 Gauss-Bonnet dark energy in a flat universe

The energy density of GB DE is given by

$$\rho_D = \alpha \mathcal{G}, \tag{1}$$

where α is a positive dimensionless parameter [30]. the Gauss-Bonnet invariant, $\mathcal{G}$, is topological invariant in four dimensions and may lead to some interesting cosmological effects in higher-dimensional brane-world (for a review, see [44]). It is defined as

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\eta\gamma}R^{\mu\nu\eta\gamma}, \tag{2}$$

where R, $R_{\mu\nu}$ and $R_{\mu\nu\eta\gamma}$ are scalar curvature, Ricci curvature tensor and Riemann curvature tensor, respectively. In a spatially flat FRW universe,

$$\mathrm{d}^2 s = -\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right], \tag{3}$$

eq. (1) takes the form

$$\rho_D = 24\alpha H^2\left(H^2 + \dot{H}\right). \tag{4}$$

By using the energy density, ρ_D, without any matter component, the Friedmann equation in flat universe in reduced Planck mass units ($8\pi G = \hbar = c = 1$) is

$$H^2 = \frac{1}{3}\rho_D = 8\alpha H^2\left(H^2 + \dot{H}\right). \tag{5}$$

Defining the e-folding x with the definition $x = \ln a = -\ln(1+z)$, where z is the redshift parameter, and using $\mathrm{d}/\mathrm{d}(x) = \frac{1}{H}\mathrm{d}/\mathrm{d}(t)$, we get the following differential equation:

$$H^2 + \frac{1}{2}\frac{\mathrm{d}H^2}{\mathrm{d}x} - \frac{1}{8\alpha} = 0, \tag{6}$$

which immediately gives the solution

$$H(x) = \sqrt{\frac{1}{8\alpha}(1 + \xi e^{-2x})}. \tag{7}$$

The parameter ξ is a constant of integration which is obtained by $\xi = 8\alpha H_0^2 - 1$. Also, it gives $\alpha = (1+\xi)/(8H_0^2)$. Using the continuity equation,

$$\dot{\rho_D} + 3H\left(1 + w_D\right)\rho_D = 0, \tag{8}$$

and eqs. (4) and (5), the equation-of-state (EoS) parameter yields

$$w_D = -1 - \frac{\dot{\rho_D}}{3H\rho_D} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -1 - \frac{2}{3}\left(\frac{1}{8\alpha H^2} - 1\right). \tag{9}$$

It is more preferable to write the above equation in terms of e-folding, x. Hence, by using eq. (7), the EoS parameter can be rewritten as

$$w_D = -1 + \frac{2}{3}\left(\frac{\xi e^{-2x}}{1+\xi e^{-2x}}\right). \tag{10}$$

We see that the constant ξ plays a crucial role in the behavior of the EoS parameter. For $\xi = 0$ (*i.e.* $8\alpha H_0^2 = 1$), the EoS parameter for the ΛCDM model ($w_\Lambda = -1$) is retrieved. For $\xi > 0$ the expanding universe accelerates in quintessence phase ($-1 < w_D < -1/3$). Using eqs. (5) and (7), the deceleration parameter is calculated as

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{1}{8\alpha H^2} = -\frac{1}{1+\xi e^{-2x}}. \tag{11}$$

Since α and H_0^2 are positive parameters, ξ must always be greater than -1. Therefore, the deceleration parameter is always negative except for $-1 < \xi < 0$. In this way, the universe which is characterized by the GB dark-energy model could not exhibit a transition from deceleration to acceleration phase for $\xi \geq 0$, against what we expect from observations.

2.1 Gauss-Bonnet standard Chaplygin gas

The SCG is a perfect fluid with an equation of state as

$$p_{\mathrm{SCG}} = -\frac{A}{\rho}, \tag{12}$$

where p, ρ and A are pressure, energy density and a positive constant, respectively. By substituting eq. (12) into the continuity equation (8), the energy density is immediately solved,

$$\rho_{\mathrm{SCG}} = \sqrt{A + Be^{-6x}}, \tag{13}$$

where B is an integration constant [45]. Using the standard scalar field DE model in which the energy density and pressure are defined as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \sqrt{A + Be^{-6x}}, \tag{14}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = \frac{-A}{\sqrt{A + Be^{-6x}}}, \tag{15}$$

and equating $p_{\mathrm{SCG}} = p_\phi$ and $\rho_{\mathrm{SCG}} = \rho_\phi$, the scalar potential and the kinetic energy term of the SCG model are given as

$$V(\phi) = \frac{2A + Be^{-6x}}{2\sqrt{A + Be^{-6x}}} \tag{16}$$

$$\dot{\phi}^2 = \frac{Be^{-6x}}{\sqrt{A + Be^{-6x}}}. \tag{17}$$

Also, the EOS parameter becomes

$$w_{\mathrm{SCG}} = \frac{p}{\rho} = -\frac{A}{A + Be^{-6x}}. \tag{18}$$

Equating the energy densities (*i.e.*, $\rho_{\mathrm{SCG}} = \rho_D$) and EoS parameters (*i.e.*, $w_{\mathrm{SCG}} = w_D$), after using the Friedmann equation (5), constants A and B are immediately given by

$$A = \frac{3}{(8\alpha)^2}\left[(2 + \xi e^{-2x})^2 - 1\right], \tag{19}$$

$$B = e^{6x}\left[\left(\frac{3}{8\alpha}(1 + \xi e^{-2x})\right)^2 - A\right], \tag{20}$$

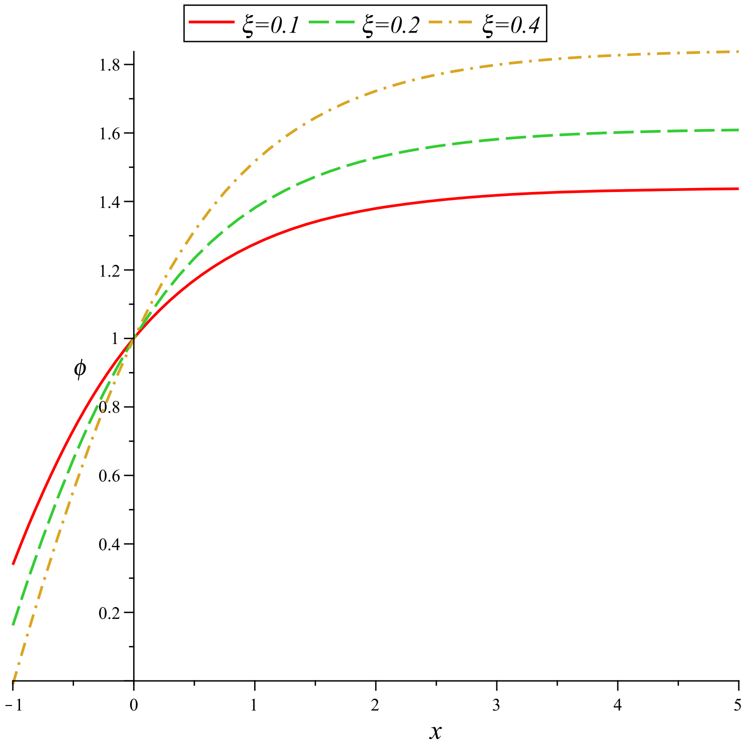

Fig. 1. behavior of normalized scalar field *versus* e-folding x for some values of $\xi \geq 0$.

and hence the scalar potential and kinetic energy term are rewritten as

$$V(x) = \frac{1}{8\alpha}\left(3 + 2\xi e^{-2x}\right) = \frac{H_0^2}{1+\xi}\left(3 + 2\xi e^{-2x}\right), \tag{21}$$

$$\dot{\phi} = \frac{1}{2}\sqrt{\frac{\xi e^{-2x}}{\alpha}}\,. \tag{22}$$

By inserting $\phi' = \dot{\phi}/H$, where prime means derivative with respect to $x = \ln a$, the differential equation (22) gives the normalized scalar field ($\phi = 1$ at present, $x = 0$) in terms of x as

$$\phi = 1 - \frac{\sqrt{2}}{2}\ln\left(\frac{1 + 2\xi e^{-2x} + 2\sqrt{\xi e^{-2x}(1 + \xi e^{-2x})}}{1 + 2\xi + 2\sqrt{\xi(1+\xi)}}\right). \tag{23}$$

It is easy to see that from eq. (7), at present, we must have $1 + \xi \geq 0$ and from (23), it must be required that $\xi(1+\xi) \geq 0$. Therefore in this model, we must have $\xi \geq 0$. As is shown in fig. 1, the normalized scalar field grows up to a saturated value at late time in such a way that this value exceeds for larger values of ξ. Also, eq. (21) shows that the universe goes to a stable equilibrium at infinity where $V(\infty) = 3H_0^2/(1+\xi)$ and from (23), the scalar field reaches to $\phi(\infty) = 1 + (\sqrt{2}/2)\ln(1 + 2\xi + 2\sqrt{\xi(1+\xi)})$.

2.2 Gauss-Bonnet generalized Chaplygin gas

The equation of state of the generalized Chaplygin gas (GCG) is defined as [46]

$$p = -\frac{A}{\rho^{\delta-1}}\,, \tag{24}$$

where A is a constant and $1 \leq \delta \leq 2$. For $\delta = 2$, it reaches to the SCG model. The energy density, similar to the previous case, is given by

$$\rho_{\rm GCG} = \left(A + Be^{(-3\delta x)}\right)^{\frac{1}{\delta}}, \tag{25}$$

and the scalar field model gives energy density and pressure of GCG as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \left(A + Be^{-3\delta x}\right)^{\frac{1}{\delta}}, \tag{26}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -A\left(A + Be^{-3\delta x}\right)^{-\frac{\delta-1}{\delta}}. \tag{27}$$

After further calculation, the three quantities, scalar potential, kinetic term and EoS parameter, are given by

$$V(x) = \frac{2A + Be^{-3\delta x}}{2(A + Be^{-3\delta x})^{\frac{\delta-1}{\delta}}}, \tag{28}$$

$$\dot{\phi}^2 = \frac{Be^{-3\delta x}}{(A + Be^{-3\delta x})^{\frac{\delta-1}{\delta}}}, \tag{29}$$

$$w_{\mathrm{GCG}} = \frac{p}{\rho} = -\frac{A}{A + Be^{-3\delta x}}. \tag{30}$$

In the same way as previously, the constants A and B are reconstructed as

$$A = \frac{3 + \xi e^{-2x}}{(8\alpha)^\delta}\left[3\left(1 + \xi e^{(-2x)}\right)\right]^{\delta-1}, \tag{31}$$

$$B = e^{3\delta x}\left[\left(\frac{3}{8\alpha}(1 + \xi e^{-2x})\right)^\delta - A\right], \tag{32}$$

and the potential and dynamics of GB-GCG can be written as

$$V(x) = \frac{3 + 2\xi e^{-2x}}{8\alpha}, \tag{33}$$

$$\dot{\phi} = \frac{1}{2}\sqrt{\frac{\xi e^{-2x}}{\alpha}}. \tag{34}$$

As is seen, the potential and dynamics of GB-GCG are not a function of parameter δ and they are exactly as in the previous case (see eqs. (21) and (22)). Therefore the reconstructed scalar potential and scalar field are obtained by eqs. (21) and (23). It is worthwhile to mention that both models that we have studied, bring an essential problem. Despite observational predictions, the phase transition between deceleration to acceleration expansion did not happen in the GB-DE model. Therefore we will study the MGB model, which may solve this problem.

3 Modified Gauss Bonnet dark energy

The energy density MGB is defined as

$$\rho_D = 3H^2(\gamma H^2 + \lambda\dot{H}), \tag{35}$$

where γ and λ are dimensionless constants [30]. The Friedmann equation in dark-energy–dominated flat universe gives

$$\gamma H^2 + \frac{1}{2}\lambda\left(\frac{\mathrm{d}H^2}{\mathrm{d}x}\right) - 1 = 0, \tag{36}$$

and the Hubble parameter is given by

$$H(x) = \sqrt{\frac{1}{\gamma}\left(1 + \eta e^{-\frac{2\gamma x}{\lambda}}\right)}, \tag{37}$$

where η is an integration constant which is obtained by $\eta = \gamma H_0^2 - 1$. The EoS parameter becomes

$$w_D = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -1 + \frac{2\gamma}{3\lambda}\left(\frac{\eta e^{-\frac{2\gamma x}{\lambda}}}{1 + \eta e^{-\frac{2\gamma x}{\lambda}}}\right) \tag{38}$$

and the deceleration parameter is obtained as follows:

$$q = -1 - \frac{\dot{H}}{H^2} = -1 + \frac{\gamma}{\lambda}\left(\frac{\eta e^{-\frac{2\gamma x}{\lambda}}}{1 + \eta e^{-\frac{2\gamma x}{\lambda}}}\right). \tag{39}$$

For positive values of γ and λ, from eq. (37), it is easy to see that η must be always greater than -1 and, from eq. (39), a transition from deceleration to acceleration is expected provided that $\eta \geq 0$. Section 5 is devoted to detailed discussion.

3.1 Modified Gauss-Bonnet and SCG

Now we want to investigate the correspondence between the MGB-SCG models and reconstruct the potential and dynamics of the scalar field. As before, equating energy densities (*i.e.*, eqs. (13) and (35)) and EoS parameters (*i.e.*, eqs. (18) and (38)), yields

$$A = \frac{9}{\gamma^2}\left(1+\eta e^{-\frac{2\gamma x}{\lambda}}\right)\left[1+\left(1-\frac{2\gamma}{3\lambda}\right)\eta e^{-\frac{2\gamma x}{\lambda}}\right], \tag{40}$$

$$B = e^{6x}\left[\left(\frac{3}{\gamma}\left(1+\eta e^{-\frac{2\gamma x}{\lambda}}\right)\right)^2 - A\right]. \tag{41}$$

By substituting A and B in eqs. (16) and (17), we find

$$V(x) = \frac{3}{\gamma}\left[1+\left(1-\frac{\gamma}{3\lambda}\right)\eta e^{-\frac{2\gamma x}{\lambda}}\right], \tag{42}$$

$$\dot{\phi} = \sqrt{\frac{2\eta}{\lambda}e^{-\frac{2\gamma x}{\lambda}}}, \tag{43}$$

which immediately gives the normalized scalar field as

$$\phi = 1 - \frac{\sqrt{2}}{2}\sqrt{\frac{\gamma}{\lambda}}\ln\left(\frac{1+2\eta e^{-\frac{2\gamma x}{\lambda}} + 2\sqrt{\eta e^{-\frac{2\gamma x}{\lambda}}(1+\eta e^{-\frac{2\gamma x}{\lambda}})}}{1+2\eta+2\sqrt{\eta(1+\eta)}}\right). \tag{44}$$

The behavior of the scalar field in this model is the similar to that of the GB-DE model, as discussed in sect. 2.1.

3.2 Modified Gauss-Bonnet and GCG

As previously stated, the constants A and B are

$$A = \left(\frac{3}{\gamma}\right)^{\delta}\left(1+\eta e^{-\frac{2\gamma x}{\lambda}}\right)^{\delta-1}\left[1+\left(1-\frac{2\gamma}{3\lambda}\right)\eta e^{-\frac{2\gamma x}{\lambda}}\right], \tag{45}$$

$$B = e^{3\delta x}\left[\left(\frac{3}{\gamma}\left(1+\eta e^{-\frac{2\gamma x}{\lambda}}\right)\right)^{\delta} - A\right] \tag{46}$$

and the potential and dynamics of MGB-GCG are given by

$$V(\phi) = \frac{3}{\gamma}\left[1+\left(1-\frac{\gamma}{3\lambda}\right)\eta e^{-\frac{2\gamma x}{\lambda}}\right] \tag{47}$$

$$\dot{\phi} = \sqrt{\frac{2\eta}{\lambda}e^{-\frac{2\gamma x}{\lambda}}}, \tag{48}$$

which are exactly similar to (42) and (43) in the previous model. Therefore the behavior of the normalized scalar field and potential is the same as in the MGB-SCG model.

4 Adiabatic sound speed

Investigation of the squared speed of sound, v^2, could help us determine the growth of the perturbation in linear theory [47]. The sign of v_s^2 plays a crucial role in determining the stability of the background evolution. A positive sign of v^2 shows the periodic propagating mode for density perturbation and, probably, represents a stable universe against perturbations. A negative sign shows an exponentially growing/decaying mode in density perturbation, and can show signs of instability for a given model. The squared speed of sound is defined as [47]

$$v^2 = \frac{\mathrm{d}P}{\mathrm{d}\rho} = \frac{\dot{P}}{\dot{\rho}}. \tag{49}$$

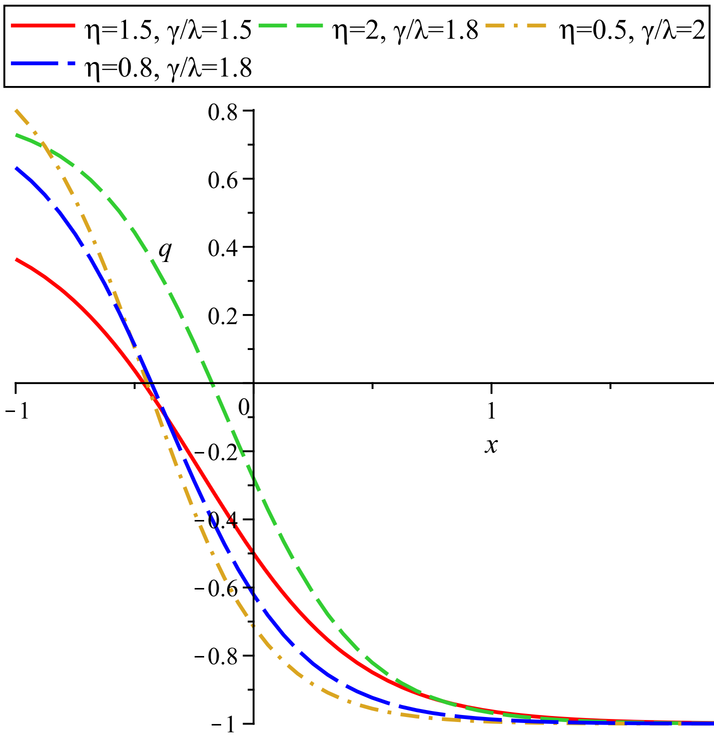

Fig. 2. The behavior of the deceleration parameter q *versus* e-folding x for varying η and γ/λ. The transition from deceleration to acceleration happens around $x \sim 0.5$.

In a dark-energy–dominated flat universe, it can be written as

$$v^2 = -1 - \frac{1}{3}\left(\frac{\ddot{H}}{\dot{H}H}\right) \tag{50}$$

and it immediately gives a constant squared speed of sound for GB-DE as $v^2 = -1/3$. Therefore it may reveal an instability against the density perturbation in the GB-DE model. For MGB-DE, eq.(50) gives

$$v^2 = -1 + \frac{2\gamma}{3\lambda}\,. \tag{51}$$

It shows that v^2 can be positive provided that $\gamma/\lambda > 3/2$. Thus a stable DE-dominated universe may be achieved in this model. In the next section we will improve this bound for γ/λ in a proper way.

5 Discussion

We shall focus on the MGB-DE model. At first, we start with eq. (39) and plot the deceleration parameter with respect to x in fig. 2. This figure shows that the deceleration parameter goes from deceleration ($q > 0$) to acceleration ($q < 0$) at some point in the past. The parameters η and γ/λ play a crucial role for this point. As η or γ/λ adopt bigger values, the transition point approaches to the present time. By choosing the best values of q_0 (~ -0.6) and the inflection point as ($x \simeq -0.5$), which has been recently parameterized [48–50], we obtain some bounds for η and γ/λ as follows:

$$0 < \eta < 2.5, \quad 1.5 \leq \frac{\gamma}{\lambda} \leq 3. \tag{52}$$

Using eq. (42) for the MGB model, we plot $\tilde{V(\phi)} = \gamma V(\phi)$ *versus* x for different values of γ/λ and $\eta = 1.5$ in fig. 3. This figure shows that, as time goes by, $\tilde{V(\phi)}$ decreases to small values and the potential reaches to a constant at infinity. In addition, by increasing the ratio of γ/λ, the tracking potential adopts bigger values in the future.

As it can be seen, the potential describes a tracker solution. According to the quintessential tracker solution, our universe undergoes a phase from $w = 0$ to $w = -1$ and the effective EoS is $w_{\rm eff} = -0.75$ [51]. The huge advantage

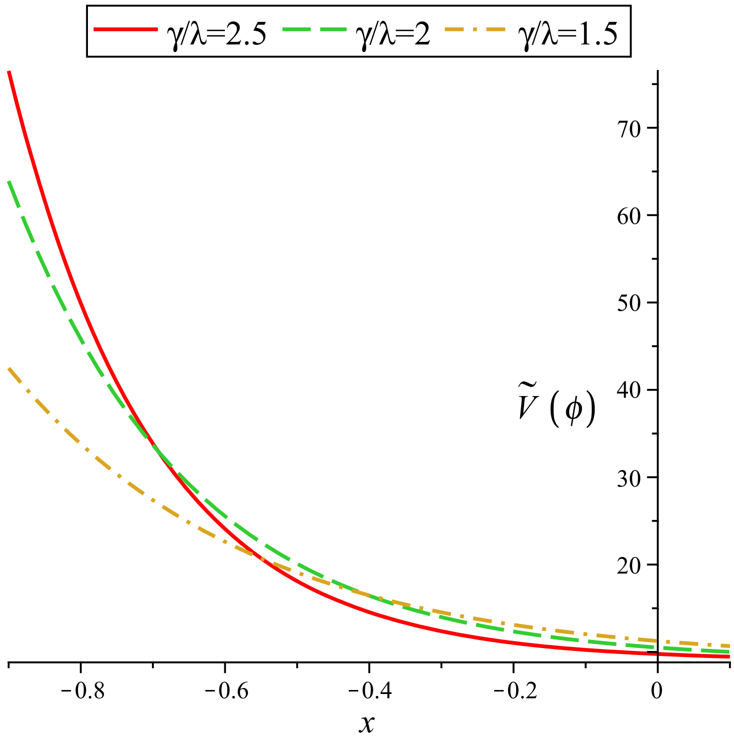

Fig. 3. The behavior of $V\tilde{(}\phi)$ *versus* e-folding x for varying γ/λ and $\eta = 1.5$.

of the tracker solution is that it allows the quintessence model to be insensitive to initial conditions [52]. So we use this feature in order to improve the obtained bounds of parameters. In this way, eqs. (38) and (47), for the matter-dominated universe ($w = 0$), lead to $V(\phi) = 3/(2\gamma - 3\lambda)$. On the other hand, the quantity $V(\phi)$ for the quintessence barrier ($w = -1/3$) reaches to $V(\phi) = 2/(\gamma - \lambda)$, so that the value $\gamma/\lambda = 1$ is illegal. It is also consistent with what we got from the investigation of the deceleration parameter. Finally, the potential might give a tracking solution provided that $1.5 < \gamma/\lambda \leq 3$.

6 Conclusion

In this paper, the reconstruction of GB-DE and some variety of the Chaplygin gas have been studied. We obtained exact solutions for the reconstructed scalar field and its potential in each model (GB-SCH, GB-MCG, MGB-SCG and MGB-MCG). According to cosmological predictions and historical evolutions, some models should be rejected (*i.e.*, models combined with GB-DE) and other models, which have been combined with MGB-DE, can be allowed to express the evolution of the universe. The equation-of-state and deceleration parameters for both GB and MGB models were calculated. In the GB-DE model, the deceleration parameter was always negative, except for $-1 < \xi < 0$. This fact shows that a transition from a decelerated to an accelerated expansion could not have happened in the past, which is contrary to the facts of cosmology. Also, it was easily shown that the EoS parameter in the GB-DE model could not ever reach the phantom phase (*i.e.* $w_D < -1$). We showed that for $\xi = 0$ (*i.e.* $8\alpha H_0^2 = 1$), the EoS parameter for the ΛCDM model was retrieved. Investigation on the squared speed of sound, revealed an instability of the model against density perturbation in the GB-DE model.

In the MGB-DE model, we found that the transition from deceleration to acceleration is allowed only for a limited range of values of η and γ/λ. Choosing the best values for the deceleration parameter at present and deflection point, according to observations, some bounds of $0 < \eta < 2.5$ and $1.5 \leq \gamma/\lambda \leq 3$ were obtained. We showed that by redefining $\gamma V(\phi) = V\tilde{(}\phi)$, the scalar potential decreased to smaller values and reaches to a saturated constant at late time. Our investigation on $V(\phi)$ for the two phases, matter-dominated and quintessence ones, showed that γ/λ could not take the two values 1 and 3/2.

It will be interesting to find the constraints of these models against the data of cosmological observations and structure formation. We hope to discuss these issues in the future.

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