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Influence of the gravitational field on a piezothermoelastic rotating medium with G-L theory

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Abstract. In the present paper, two different theories (coupled theory and Green-Lindsay theory with two relaxation times) are applied to study the deformation of a generalized piezothermoelastic rotating medium under the influence of gravity. The normal mode analysis is used to obtain the expressions for the displacement components, the temperature, the stress, the stress components, the electric potential and the electric displacements. Numerical results for the field quantities are given in the physical domain and illustrated graphically. Comparisons are made with the results predicted by coupled and Green-Lindsay theories in the presence and absence of rotation as well as of gravity.

Nomenclature

u_i	mechanical displacement	φ	electric potential
T	absolute temperature	ε_{ij}	strain tensor
σ_{ij}	stress tensor	β_{ij}	thermal elastic coupling tensor
E_i	electric field	D_i	electric displacement
C_{ijkl}	elastic parameters tensor	e_{kij}	piezoelectric moduli
$\in_{ij}$	dielectric moduli	p_i	pyroelectric moduli
ρ	mass density	t_0, t_1	thermal relaxation time parameters
K_{ij}	heat conduction tensor	T_0	reference temperature
C_e	specific heat at constant strain	ε_0, μ_0	electric and magnetic permeability, respectively
α_1, α_3	coefficients of linear thermal expansion	$v_p = \sqrt{C_{11}/\rho}$	longitudinal wave velocity in the medium

1 Introduction

Piezoelectricity is the ability of some materials to generate an electric charge in response to mechanical stress. If the material is not short-circuited, the applied charges induce a voltage across the material. The word piezoelectricity means "electricity by pressure" and derives from the Greek *piezein*, which means to squeeze or press. Piezoelectric substances are commercially produced in single-crystal form as well as ceramics and they belong to the second biggest application of dielectric materials, just after semiconductors [1].

The phenomenon of piezoelectricity was discovered by the Curie brothers: Pierre and Jacques. In 1880, they found that some crystals, when compressed in certain directions, show positive and negative charges on some portions of the surface. These charges are proportional to the pressure and disappear when the pressure ceases. The first paper on piezoelectricity was by Jacques and Pierre Curie [2]. Mason [3] studied piezoelectric crystals and their application to ultrasonics. Redwood [4] studied the transient performance of a piezoelectric transducer. Chen [5] investigated further correspondences between plane piezoelectricity and generalized plane strain in elasticity. Abd-alla *et al.* [6,7] studied the reflection and refraction phenomena in piezoelectric media under initial stresses.

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Mindlin [8] was the first to develop the governing equations of a three-dimensional linear thermo-piezoelectric medium. Nowacki [9] subsequently developed some general theorems and mathematical models of thermo-piezoelectricity which can be viewed as the basis for various numerical methods. Chandrasekharaiah [10] has generalized Mindlin's theory of thermo-piezoelectricity to account for the finite speed of propagation of thermal disturbances. Dunn [11] studied micromechanics models for effective thermal expansion and pyroelectric coefficients of piezoelectric composites. Sharma and Kumar [12] studied plane harmonic waves in piezothermoelastic materials. Youssef and Bassiouny [13] used the state-space approach to study two-temperature generalized thermo-piezoelasticity for the one-dimensional problem. Akbarzadeh *et al.* [14] used coupled theory to investigate the transient thermo-piezoelectric behavior of a one-dimensional (1D) functionally graded piezoelectric medium subjected to a moving heat source. Alshaikh [15] used the mathematical modeling for studying the influence of the initial stresses and relaxation times on the reflection and refraction waves in piezothermoelastic half-space. The generalized theories of thermoelasticity have been developed to overcome the infinite propagation speed of thermal signals predicted by the classical coupled dynamical theory of thermoelasticity by Biot [16]. The subject of generalized thermoelasticity covers a wide range of extensions of the classical theory of thermoelasticity. We recall the two earliest and well-known generalized theories proposed by Lord and Shulman [17] and Green and Lindsay [18]. In the model of [17], the Fourier law of heat conduction is replaced by the Maxwell-Cattaneo law, which introduces one thermal relaxation time parameter in Fourier's law, whereas in the model of Green and Lindsay [18], two relaxation parameters are introduced in the constitutive relations for the stress tensor and the entropy. Othman and Said [19] studied the effect of the mechanical force on generalized thermoelasticity in a fibre-reinforcement by three theories. Chandrasekharaiah and Srikantiah [20] have discussed the plane waves in a homogeneous isotropic unbounded thermoelastic solid rotating with uniform angular velocity in the context of the G-L theory.

Some research in the past has investigated different problems of rotating media. In a paper by Schoenberg and Censor [21], the propagation of plane harmonic waves in a rotating elastic medium without a thermal field has been studied. It was shown there that the rotation causes the elastic medium to be depressive and anisotropic. Othman [22] studied the effect of rotation on plane waves in generalized thermoelasticity with two relaxation times. Sharma and his coworkers [23,24] and Peng and Li [25] discussed the effect of rotation on different types of wave propagating in a thermoelastic medium. Hayat and his coworkers [26–30], Ellahi and Asghar [31] investigated the effect of rotation through different studies.

In the classical theory of elasticity, the gravity effect is generally neglected. The effect of gravity in the problem of propagation of waves in solids, in particular on an elastic globe, was first studied by Bromwich in [32]. Subsequently, an investigation of the effect of gravity was considered by Love in [33] who showed that the velocity of Rayleigh waves is increased to a significant extent by the gravitational field when the wavelengths are large. De and Sengupta in [34, 35] studied the effect of gravity on the surface waves, on the propagation of waves in an elastic layer. Othman *et al.* [36] studied the effect of phase lag and gravity field on a generalized thermoelastic medium in two and three dimensions.

Othman *et al.* [37] investigated the generalized thermoelastic medium with temperature-dependent properties for different theories under the effect of gravity. Othman and Said [38] studied the effect of rotation on the two-dimensional problem of a fibre-reinforced thermoelastic medium with one relaxation time.

The present paper is devoted to the investigations related to the effect of the gravitational field and rotation on a generalized piezothermoelastic medium based on the CT and G-L theories by applying the normal mode analysis. Also, the effects of rotation and the gravitational field on the physical quantities are discussed numerically and illustrated graphically.

2 Formulation of the problem

2.1 Basic equations

The basic equations of generalized hexagonal piezothermoelastic medium for the two-dimensional motion in the x-z plane are the following [12].

– *Strain-displacement relation*:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{1}$$

– *Stress-strain temperature*:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \beta_{ij}\left(1 + t_1\frac{\partial}{\partial t}\right)T\delta_{ij}, \tag{2}$$

where $i, j, k, l = 1, 2, 3$.

– *Gauss equation and electric field relation*:

$$D_{i,i} = 0, \tag{3}$$

$$D_i = e_{ijk}\varepsilon_{jk} + \in_{ij} E_j + p_i\left(1 + t_1\frac{\partial}{\partial t}\right)T, \tag{4}$$

where $E_i = -\varphi_{,i}$.

– *Heat conduction equation*:

$$K_{ij}T_{,ij} = \rho C_e\left(1 + t_0\frac{\partial}{\partial t}\right)\dot{T} + \boldsymbol{T}_0[\boldsymbol{\beta_{ij}}\dot{\boldsymbol{u}}_{\boldsymbol{i,j}} - \boldsymbol{p_i}\dot{\boldsymbol{\varphi}}_{\boldsymbol{,i}}]. \tag{5}$$

– *Equation of motion*:
Since the medium is rotating uniformly with an angular velocity $\boldsymbol{\Omega} = \Omega\mathbf{n}$, where $\mathbf{n}$ is a unit vector representing the direction of the axis of the rotation, the equation of motion in the rotating frame of reference has two additional terms (Schoenberg and Censor [21]): centripetal acceleration, $\boldsymbol{\Omega}\wedge(\boldsymbol{\Omega}\wedge\mathbf{u})$, due to time-varying motion only and Coriolis acceleration, $2\boldsymbol{\Omega}\wedge\dot{\mathbf{u}}$; then the equation of motion in the rotating frame of reference is

$$\sigma_{ij,j} = \rho[\ddot{\boldsymbol{u}}_{\boldsymbol{i}} + \{\boldsymbol{\Omega}\wedge(\boldsymbol{\Omega}\wedge\mathbf{u})\}_{\boldsymbol{i}} + (2\boldsymbol{\Omega}\wedge\dot{\mathbf{u}})_{\boldsymbol{i}}]. \tag{6}$$

The constitutive relation and electric displacement of the hexagonal (6 mm) crystals symmetry given by

$$\sigma_{xx} = C_{11}\varepsilon_{xx} + C_{13}\varepsilon_{zz} - e_{31}E_z - \beta_1\left(1 + t_1\frac{\partial}{\partial t}\right)T, \tag{7}$$

$$\sigma_{zz} = C_{13}\varepsilon_{xx} + C_{33}\varepsilon_{zz} - e_{33}E_z - \beta_3\left(1 + t_1\frac{\partial}{\partial t}\right)T, \tag{8}$$

$$\sigma_{xz} = 2C_{44}\varepsilon_{zx} - e_{15}E_x, \tag{9}$$

$$D_{\mathrm{x}} = e_{15}(u_{,z} + w_{,x}) + \in_{11} E_x, \tag{10}$$

$$D_{\mathrm{z}} = e_{31}u_{,x} + e_{33}w_{,z} + \in_{33} E_z + p_3\left(1 + t_1\frac{\partial}{\partial t}\right)T. \tag{11}$$

2.2 The boundary conditions

1) A mechanical boundary condition:

$$\sigma_{zz}(x,0,t) = -f_1(x,0,t) = -f_1^* e^{ia(x-ct)}, \qquad \sigma_{xz}(x,0,t) = 0, \qquad \frac{\partial\varphi}{\partial z} = 0. \tag{12}$$

2) A thermal boundary condition that the surface of the half-space subjected to thermal shock:

$$T(x,0,t) = f_2(x,0,t) = f_2^* e^{ia(x-ct)}, \tag{13}$$

where $f_1(x,t)$ and $f_2(x,t)$ are arbitrary functions of x, t and $f_1{}^*$, $f_2{}^*$ are constant.

We consider a homogeneous, anisotropic, piezothermoelastic half-space of hexagonal type.

The basic field equations (3), (5), (6) for the temperature change, $T(x,z,t)$, displacement vector, $\boldsymbol{u}(x,z,t) = (u,0,w)$, and electric potential, $\varphi(x,z,t)$, under the effect of rotation and the gravitational field are given by

$$C_{11}u_{,xx} + C_{44}u_{,zz} + (C_{13}+C_{44})w_{,xz} + (e_{31}+e_{15})\varphi_{,xz} - \beta_1\left(1 + t_1\frac{\partial}{\partial t}\right)T_{,x} + \rho g w_{,x} = \rho(\ddot{\boldsymbol{u}} - \boldsymbol{\Omega}^2\boldsymbol{u} + 2\boldsymbol{\Omega}\dot{\boldsymbol{w}}), \tag{14}$$

$$(C_{44}+C_{13})u_{,xz} + C_{44}w_{,xx} + C_{33}w_{,zz} + e_{15}\varphi_{,xx} + e_{33}\varphi_{,zz} - \beta_3\left(1 + t_1\frac{\partial}{\partial t}\right)T_{,z} - \rho g u_{,x} = \rho(\ddot{\boldsymbol{w}} - \boldsymbol{\Omega}^2\boldsymbol{w} - 2\boldsymbol{\Omega}\dot{\boldsymbol{u}}), \tag{15}$$

$$(e_{15}+e_{31})u_{,xz} + e_{15}w_{,xx} + e_{33}w_{,zz} - \in_{11}\varphi_{,xx} - \in_{33}\varphi_{,zz} + p_3\left(1 + t_1\frac{\partial}{\partial t}\right)T_{,z} = 0, \tag{16}$$

$$K_1T_{,xx} + K_3T_{,zz} - \rho C_e\left(1 + t_0\frac{\partial}{\partial t}\right)\dot{T} = \boldsymbol{T}_0[\boldsymbol{\beta_1}\dot{\boldsymbol{u}}_{\boldsymbol{,x}} + \boldsymbol{\beta_3}\dot{\boldsymbol{w}}_{\boldsymbol{,z}} - \boldsymbol{p_3}\dot{\boldsymbol{\varphi}}_{\boldsymbol{,z}}]. \tag{17}$$

To facilitate the solution, the following non-dimensional quantities are introduced:

$$x' = \frac{\omega^*}{v_p}x, \quad z' = \frac{\omega^*}{v_p}z, \quad u' = \frac{\rho\omega^* v_p}{\beta_1 T_0}u, \quad w' = \frac{\rho\omega^* v_p}{\beta_1 T_0}w, \quad T' = \frac{T}{T_0}, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\beta_1 T_0}, \quad \varphi' = \varepsilon_p\varphi,$$
$$\{t', t_1', t_0'\} = \omega^*\{t, t_1, t_0\}, \quad D_i' = \frac{D_i}{e}, \quad \Omega' = \frac{\Omega}{\omega^*}, \quad g' = \frac{g}{(1/t^2)}, \tag{18}$$

where $\omega^* = \frac{C_e C_{11}}{K_{11}}$, $\varepsilon_p = \frac{\omega^* e_{33}}{v_p \beta_1 T_0}$, $\beta_1 = (C_{11} + C_{12})\alpha_1 + C_{13}\alpha_3$, $\beta_3 = 2C_{13}\alpha_1 + C_{33}\alpha_3$. In terms of the non-dimensional quantities defined in eq. (18), the above governing equations (14)–(17) take the form (dropping the primes over the non-dimensional variables for convenience)

$$\delta_1 u_{,xx} + \delta_2 u_{,zz} + \delta_3 w_{,xz} + \delta_4 \varphi_{,xz} - \left(1 + t_1\frac{\partial}{\partial t}\right)T_{,x} + g w_{,x} = \ddot{\boldsymbol{u}} - \boldsymbol{\Omega}^2\boldsymbol{u} + 2\boldsymbol{\Omega}\dot{\boldsymbol{w}}, \tag{19}$$

$$\delta_3 u_{,xz} + \delta_2 w_{,xx} + \delta_5 w_{,zz} + \delta_6 \varphi_{,xx} + \varphi_{,zz} + \delta_7\left(1 + t_1\frac{\partial}{\partial t}\right)T_{,z} - g u_{,x} = \ddot{\boldsymbol{w}} - \boldsymbol{\Omega}^2\boldsymbol{w} - 2\boldsymbol{\Omega}\dot{\boldsymbol{u}}, \tag{20}$$

$$\delta_8 u_{,xz} + \delta_9 w_{,xx} + \delta_{10} w_{,zz} + \delta_{11}\varphi_{,xx} + \delta_{12}\varphi_{,zz} + \delta_{13}\left(1 + t_1\frac{\partial}{\partial t}\right)T_{,z} = 0, \tag{21}$$

$$\delta_{14} T_{,xx} + \delta_{15} T_{,zz} - \left(1 + t_0\frac{\partial}{\partial t}\right)\dot{T} = [\boldsymbol{\delta}_{16}\dot{\boldsymbol{u}}_{,x} + \boldsymbol{\delta}_{17}\dot{\boldsymbol{w}}_{,z} + \boldsymbol{\delta}_{18}\dot{\boldsymbol{\varphi}}_{,z}]. \tag{22}$$

3 The solution of the problem

In this section, we applied the normal mode analysis, which gives exact solutions without any assumed restrictions on the displacement, stress distributions, and temperature. The solution of the considered physical quantities can be decomposed in terms of the normal mode in the following form:

$$[u, w, \varphi, T](x, z, t) = [u^*, w^*, \varphi^*, T^*](z)e^{ia(x-ct)}, \tag{23}$$

where $D = \frac{\mathrm{d}}{\mathrm{d}z}$, $c = \frac{\omega}{a}$, ω is the complex time constant (frequency), i is the imaginary unit, a is the wave number in the x-direction, u^*, w^*, φ^* and T^* are the amplitudes of the functions u, w, φ, T.

Substituting from eq. (23) in eqs. (19)–(22), we get

$$(\mathrm{D}^2 + A_1)u^* + (A_2\mathrm{D} + A_3)w^* + A_4\mathrm{D}\varphi^* + A_5 T^* = 0, \tag{24}$$
$$(A_6\mathrm{D} + A_7)u^* + (\mathrm{D}^2 + A_8)w^* + (A_9\mathrm{D}^2 + A_{10})\varphi^* + A_{11}\mathrm{D}T^* = 0, \tag{25}$$
$$A_{12}\mathrm{D}u^* + (\mathrm{D}^2 + A_{13})w^* + (A_{14}\mathrm{D}^2 + A_{15})\varphi^* + A_{16}\mathrm{D}T^* = 0, \tag{26}$$
$$A_{17}u^* + A_{18}\mathrm{D}w^* + A_{19}\mathrm{D}\varphi^* + (\mathrm{D}^2 + A_{20})T^* = 0, \tag{27}$$

where A_j, $j = 1$–20 are given in appendix A. Equations (24)–(27) have a non-trivial solution if the determinant coefficients of the physical quantities are equal to zero. Then we get

$$[\mathrm{D}^8 - A\mathrm{D}^6 + B\mathrm{D}^4 - C\mathrm{D}^2 + E]\{u^*(z), w^*(z), \varphi^*(z), T^*(z)\} = 0, \tag{28}$$

where A, B, C, E are given in appendix A. Equation (28) can be factored as

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)(\mathrm{D}^2 - k_3^2)(\mathrm{D}^2 - k_4^2)\{u^*(z), w^*(z), \varphi^*(z), T^*(z)\} = 0. \tag{29}$$

The solution of eq. (29), bound as $z \to \infty$, is given by

$$u^* = \sum_{n=1}^{4} M_n e^{-k_n z}, \tag{30}$$

$$w^* = \sum_{n=1}^{4} H_{1n} M_n e^{-k_n z}, \tag{31}$$

$$\varphi^* = \sum_{n=1}^{4} H_{2n} M_n e^{-k_n z}, \tag{32}$$

$$T^* = \sum_{n=1}^{4} H_{3n} M_n e^{-k_n z}, \tag{33}$$

where $k_n^2(n=1,2,3,4)$ are the roots of the characteristic equation of eq. (29). Taking non-dimension and the normal mode to eqs. (7)–(11), then substituting from eqs. (30)–(33), we obtain

$$\sigma_{xx}^* = \sum_{n=1}^{4} H_{4n} M_n e^{-k_n z}, \tag{34}$$

$$\sigma_{zz}^* = \sum_{n=1}^{4} H_{5n} M_n e^{-k_n z}, \tag{35}$$

$$\sigma_{xz}^* = \sum_{n=1}^{4} H_{6n} M_n e^{-k_n z}, \tag{36}$$

$$D_x^* = \sum_{n=1}^{4} H_{7n} M_n e^{-k_n z}, \tag{37}$$

$$D_z^* = \sum_{n=1}^{4} H_{8n} M_n e^{-k_n z}, \tag{38}$$

where H_{jn}, $j=1-8$, $n=1,2,3,4$ are given in appendix B.

By applying the boundary conditions (12) and (13) to determine the coefficients $M_n(n=1,2,3,4)$, we obtain

$$\sum_{n=1}^{4} H_{5n} M_n = -f_1^*, \tag{39}$$

$$\sum_{n=1}^{4} H_{6n} M_n = 0, \tag{40}$$

$$\sum_{n=1}^{4} k_n H_{2n} M_n = 0, \tag{41}$$

$$\sum_{n=1}^{4} H_{3n} M_n = f_2^*. \tag{42}$$

Let us solve the above system of equations (39)–(42) of $M_n(n=1,2,3,4)$, by using the inverse of matrix method as follows:

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} H_{51} & H_{52} & H_{53} & H_{54} \\ H_{61} & H_{62} & H_{63} & H_{64} \\ k_1 H_{21} & k_2 H_{22} & k_3 H_{23} & k_4 H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \end{pmatrix}^{-1} \begin{pmatrix} -f_1^* \\ 0 \\ 0 \\ f_2^* \end{pmatrix}. \tag{43}$$

4 Numerical results and discussion

The material chosen for the purpose of numerical calculations is cadmium selenide (CdSe) having hexagonal symmetry (6 mm class)

$$C_{11} = 7.41\times10^{10}\,\mathrm{N\,m^{-2}},\quad C_{12} = 4.52\times10^{10}\,\mathrm{N\,m^{-2}},\quad C_{13} = 3.93\times10^{10}\,\mathrm{N\,m^{-2}},$$
$$C_{33} = 8.36\times10^{10}\,\mathrm{N\,m^{-2}},\quad C_{44} = 1.32\times10^{10}\,\mathrm{N\,m^{-2}},\quad T_0 = 298\,\mathrm{K},\quad \rho = 5504\,\mathrm{Kg\,m^{-3}},$$
$$e_{13} = -0.160\,\mathrm{C\,m^{-2}},\quad e_{33} = 0.347\,\mathrm{C\,m^{-2}},\quad e_{15} = -0.138\,\mathrm{C\,m^{-2}},\quad \beta_1 = 0.621\times10^{6}\,\mathrm{N\,K^{-1}\cdot m^{-2}},$$
$$\beta_3 = 0.551\times10^{6}\,\mathrm{N\,K^{-1}\cdot m^{-2}},\quad p_3 = -2.94\times10^{-6}\,\mathrm{C\,K^{-1}\cdot m^{-2}},\quad K_1 = K_3 = 9\,\mathrm{W\,m^{-1}\cdot K^{-1}},$$
$$\in_{11} = 8.26\times10^{-11}\,\mathrm{C^2\,N^{-1}\cdot m^{-2}},\quad \in_{33} = 9.03\times10^{-11}\,\mathrm{C^2\,N^{-1}\cdot m^{-2}},\quad C_e = 260\,\mathrm{J\cdot Kg^{-1}K^{-1}},\quad \mu_0 = 4\pi\times10^{-7}.$$

The numerical technique, outlined above, was used for the distribution of the real part of the displacement component u, the temperature T, the stress components σ_{zz} and σ_{xz}, the electric potential φ and the electric displacement D_z, for the problem. The results are shown in figs. 1–12. The graphs show the four curves predicted by two different theories of thermoelasticity (CT and G-L). In these figures, the solid lines represent the solution in the generalized

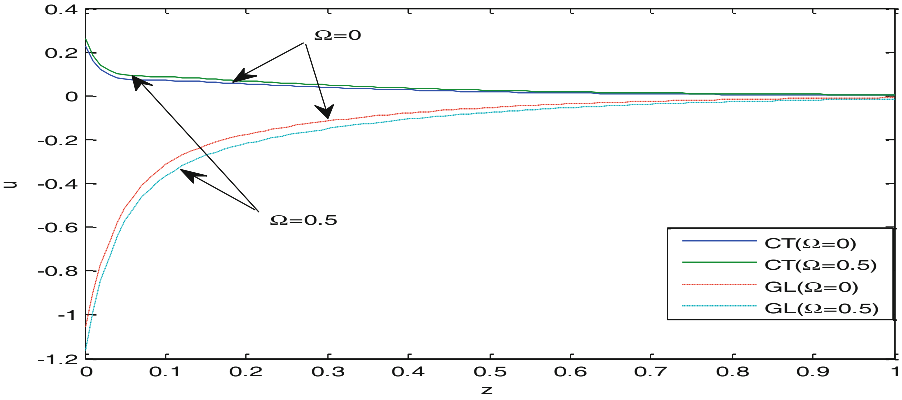

Fig. 1. Horizontal displacement distribution u in the absence and in the presence of rotation.

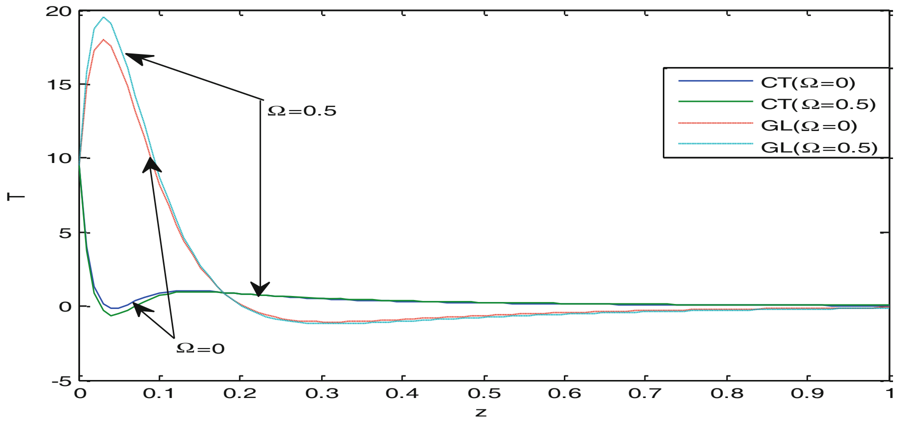

Fig. 2. Temperature distribution T in the absence and in the presence of rotation.

CT theory and the dashed lines represent the solution using the G-L theory; here all the variables are taken in non-dimensional form. Comparisons were carried out for $x = 1.9$, $t = 1.6$, $f_1^* = 10$, $f_2^* = 10$.

Figures. 1–6 show the comparison between the considered variables in the absence and in the presence of rotation (*i.e.* $\Omega = 0,\ 0.5$) at $g = 9.8$. Figure 1 shows that the distribution of the horizontal displacement u, in the context of the CT theory for $\Omega = 0,\ 0.5$, decreases and then converges to zero. However, in the context of G-L theory, the values of u increase and then converge to zero for $\Omega = 0,\ 0.5$. It is observed that the horizontal displacement u, in the context of the CT for $\Omega = 0,\ 0.5$, is higher than that in the context of the G-L for $\Omega = 0,\ 0.5$. It is clear that rotation acts to decrease the values of u. Figure 2 shows that the distribution of the temperature T, in the context of the CT theory for $\Omega = 0,\ 0.5$, decreases, then increases and converges to zero. However, in the context of G-L theory for $\Omega = 0,\ 0.5$, it increases to the maximum value, then decreases and converges to zero. Figure 3 shows the distribution of the stress components σ_{zz}, in the context of the (CT and GL) theories for $\Omega = 0,\ 0.5$. It is clear that rotation acts to decrease the values of σ_{zz}. Figure 4 shows that the distribution of the stress component σ_{xz}, in the context of the (CT and G-L) theories, begins from zero and satisfies the boundary conditions at $z = 0$. In the context of the CT theory, the values of σ_{xz} increase and then decrease for $\Omega = 0,\ 0.5$. However, in the context of the G-L theory, the values of σ_{xz} decrease in the range $0 \le z \le 0.1$, then increase for $\Omega = 0,\ 0.5$. It is observed that the values of the stress components σ_{xz}, in the context of the CT theory, are higher than those in the context of the G-L theory for $\Omega = 0,\ 0.5$. Figure 5 shows that the distribution of the electric potential φ, in the context of the (CT and GL) theories for $\Omega = 0,\ 0.5$, decreases, then increases and converges to zero at $z \ge 1$. Figure 6 shows that the distribution of the electric displacement D_z, in the context the (CT and G-L) theories, always begins from positive values for $\Omega = 0,\ 0.5$. It shows that the values of D_z, in the context of the (CT and G-L) theories, decrease then increase and finally converge to zero for $\Omega = 0,\ 0.5$. It is clear that rotation acts to decrease the values of D_z.

Figures 7–12 show the comparisons between the considered variables in the absence and in the presence of a magnetic field (*i.e.* $g = 0,\ 9.8$) at $\Omega = 0.5$.

Figure 7 shows that the distribution of the horizontal displacement u, in the context of CT theory, increases with the increase of gravity. However, in the context of the G-L theory, gravity has a decreasing effect on u. Figure 8 displays the distribution of temperature T. In the context of the CT theory, gravity has a decreasing effect on T, while in the

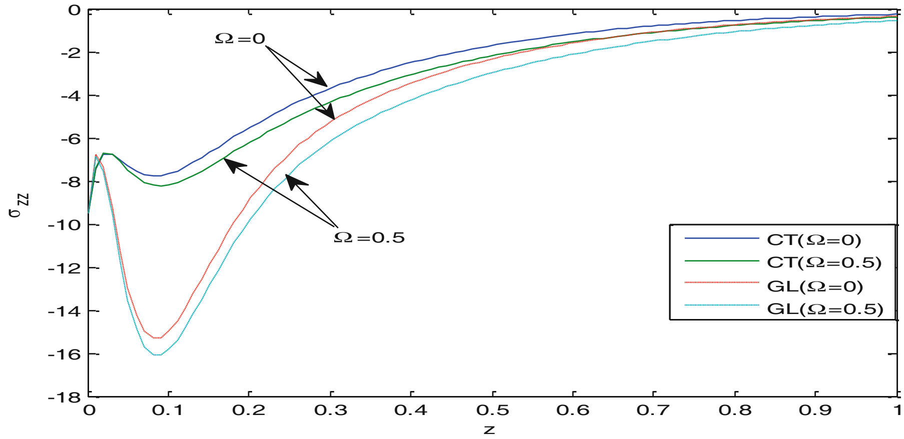

Fig. 3. Distribution of stress component σ_{zz} in the absence and in the presence of rotation.

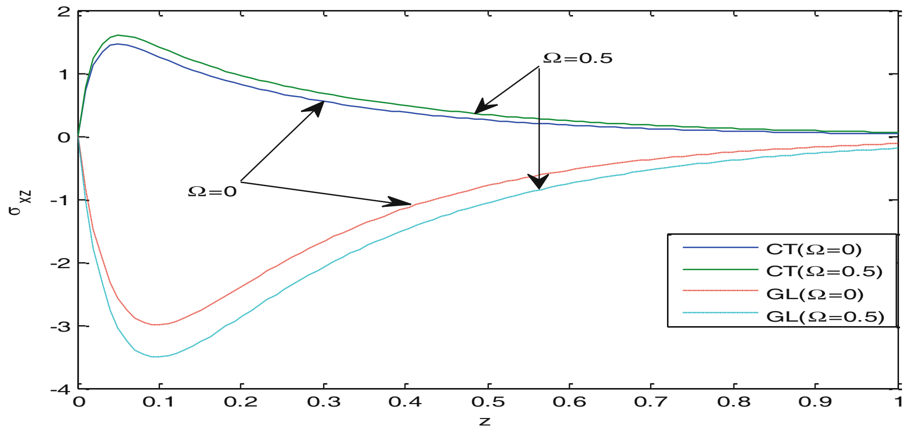

Fig. 4. Distribution of stress component σ_{xz} in the absence and in the presence of rotation.

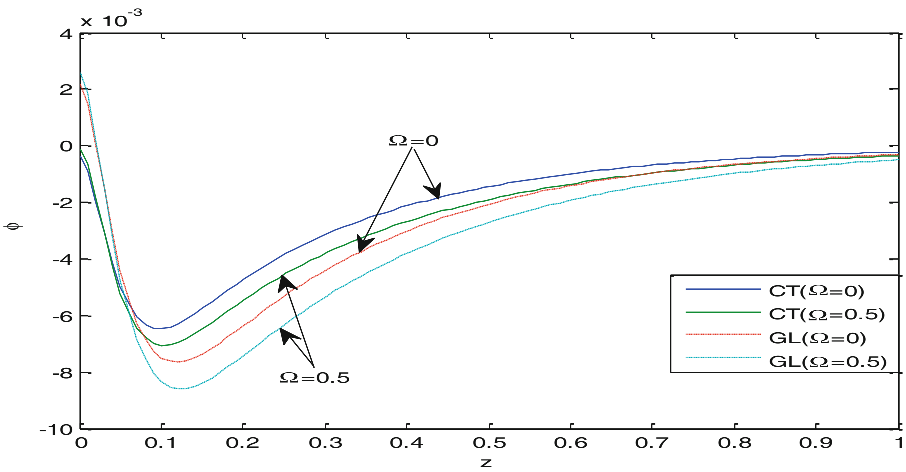

Fig. 5. Distribution of electric potential ϕ in the absence and in the presence of rotation.

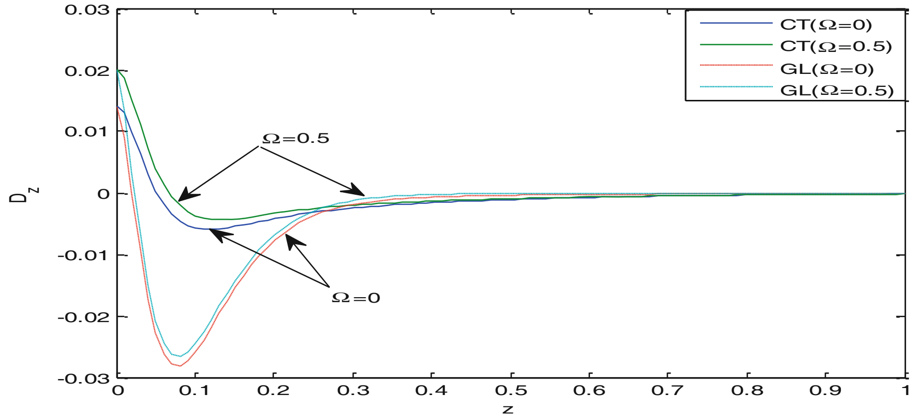

Fig. 6. Distribution of electric displacement component D_z in the absence and in the presence of rotation.

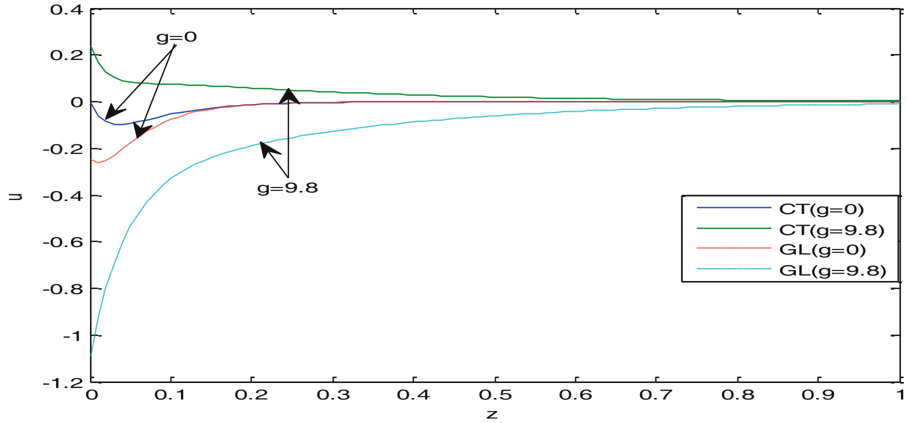

Fig. 7. Horizontal displacement distribution u in the absence and in the presence of gravity.

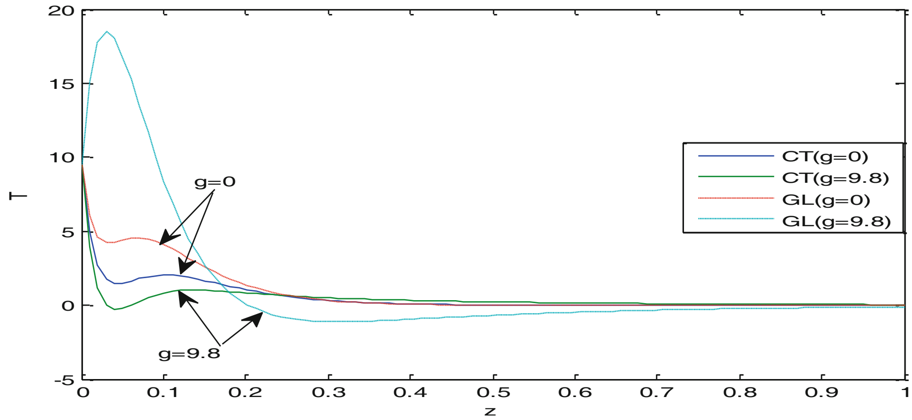

Fig. 8. Temperature distribution T in the absence and in the presence of gravity.

context of G-L theory, it has an increasing effect. Figure 9 shows that the distribution of the stress component σ_{zz}, in the context of the (CT and G-L) theories, always begins a negative value for $g = 0$, 9.8. In the context of the (CT and G-L) theories gravity has a decreasing effect on σ_{zz}. Figure 10 shows that the distribution of the stress component σ_{xz}, in the context of the (CT and G-L) theories, begins from zero and satisfies the boundary conditions at $z = 0$. In the context of the CT theory, gravity has an increasing effect, while in the context of the G-L theory, it has a decreasing effect on σ_{xz}. Figure 11 shows that gravity has a decreasing effect on the variation of the electric potential φ, in the context of the (CT and G-L) theories. Figure 12 shows that gravity has an increasing effect on the distribution of the electric displacement D_z, in the context of the (CT and G-L) theories.

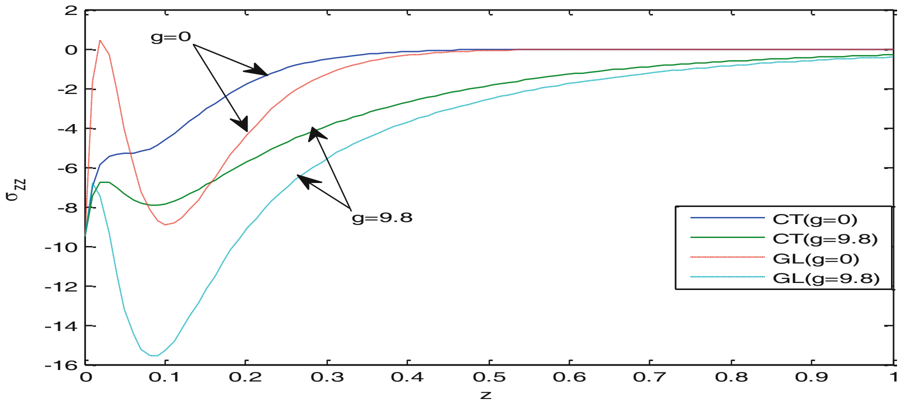

Fig. 9. Distribution of stress component σ_{zz} in the absence and in the presence of gravity.

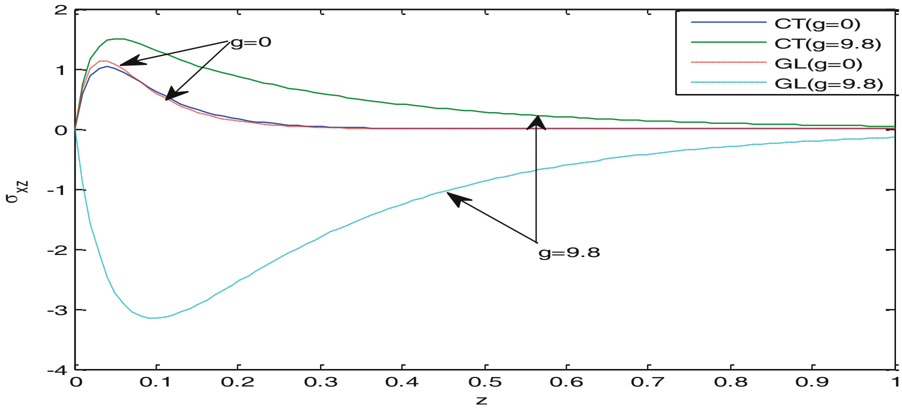

Fig. 10. Distribution of stress component σ_{xz} in the absence and in the presence of gravity.

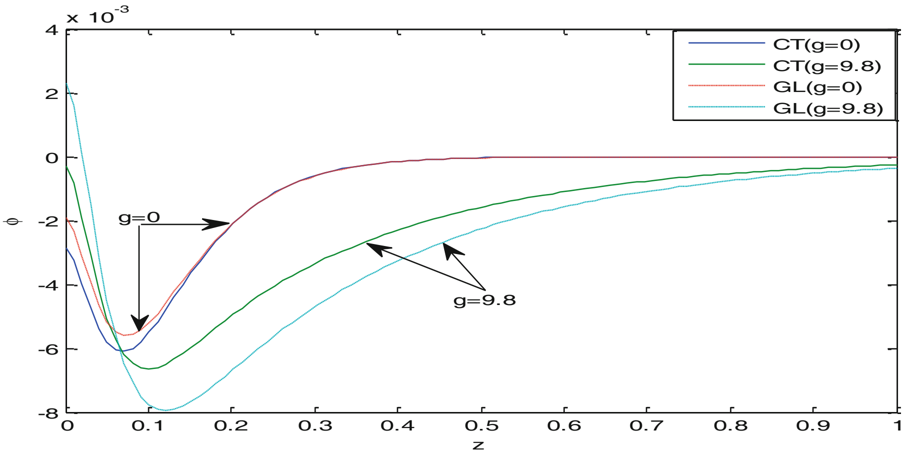

Fig. 11. Distribution of electric potential ϕ in the absence and in the presence of gravity.

5 Conclusions

From comparing the figures, which were obtained for the three thermoelastic theories, important phenomena are observed.

1) The values of all physical quantities converge to zero with increasing distance z, and all functions are continuous.
2) All the physical quantities satisfy the boundary conditions.
3) The phenomenon of finite speed of propagation is manifested in all figures.
4) The analytical solutions based upon the normal mode analysis of the thermoelastic problem in solids have been developed and used.

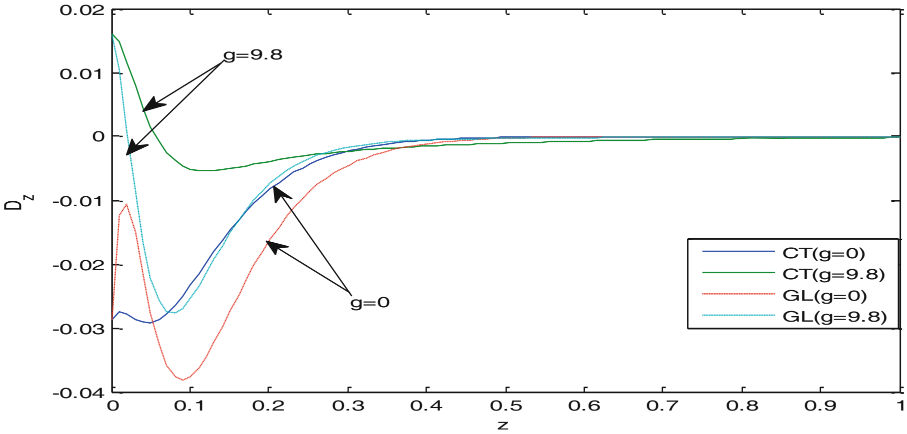

Fig. 12. Distribution of electric displacement component D_z in the absence and in the presence of gravity.

5) The comparisons of two different theories of thermoelasticity, (CT and G-L) theories are carried out.
6) The gravity and rotation have an obvious effect on the magnitude of the real part of the physical quantities.

Appendix A.

$$\delta_1=\frac{C_{11}}{\rho v_p^2},\quad \delta_2=\frac{C_{44}}{\rho v_p^2},\quad \delta_3=\frac{C_{13}+C_{44}}{\rho v_p^2},\quad \delta_4=\frac{(e_{31}+e_{15})}{e_{33}},\quad \delta_5=\frac{C_{33}}{\rho v_p^2},\quad \delta_6=\frac{e_{15}}{e_{33}}, \delta_7=-\frac{\beta_3}{\beta_1},$$
$$\delta_8=\frac{(e_{15}+e_{31})}{\rho v_p^2},\quad \delta_9=\frac{e_{15}}{\rho v_p^2},\quad \delta_{10}=\frac{e_{33}}{\rho v_p^2},\quad \delta_{11}=-\frac{\in_{11}}{e_{33}},\quad \delta_{12}=-\frac{\in_{33}}{e_{33}},\quad \delta_{13}=\frac{p_3}{\beta_1},$$
$$\delta_{14}=\frac{K_1\omega^*}{\rho C_e v_p^2},\quad \delta_{15}=\frac{K_3\omega^*}{\rho C_e v_p^2},\quad \delta_{16}=\frac{\beta_1^2 T_0}{\rho^2 C_e v_p^2},\quad \delta_{17}=\frac{\beta_1\beta_3 T_0}{\rho^2 C_e v_p^2},\quad \delta_{18}=-\frac{p_3\beta_1 T_0}{\rho C_e e_{33}}.$$
$$A_1=-\frac{(a^2\delta_1-a^2c^2-\Omega^2)}{\delta_2},\quad A_2=\frac{ia\delta_3}{\delta_2},\quad A_3=\frac{iag+2iac\Omega}{\delta_2},\quad A_4=\frac{ia\delta_4}{\delta_2},\quad A_5=-\frac{ia(1-iact_1)}{\delta_2},$$
$$A_6=\frac{ia\delta_3}{\delta_5},\quad A_7=-\frac{(iag+2iac\Omega)}{\delta_5},\quad A_8=-\frac{(a^2\delta_2-a^2c^2-\Omega^2)}{\delta_5},\quad A_9=\frac{1}{\delta_5},\quad A_{10}=-\frac{a^2\delta_6}{\delta_5},$$
$$A_{11}=\frac{\delta_7(1-iact_1)}{\delta_5},\quad A_{12}=\frac{ia\delta_8}{\delta_{10}},\quad A_{13}=-\frac{a^2\delta_9}{\delta_{10}},\quad A_{14}=\frac{\delta_{12}}{\delta_{10}},\quad A_{15}=-\frac{a^2\delta_{11}}{\delta_{10}},\quad A_{16}=\frac{\delta_{13}(1-iact_1)}{\delta_{10}},$$
$$A_{17}=-\frac{a^2c\delta_{16}}{\delta_{15}},\quad A_{18}=\frac{iac\delta_{17}}{\delta_{15}},\quad A_{19}=\frac{iac\delta_{18}}{\delta_{15}},\quad A_{20}=-\frac{(a^2\delta_{14}-iac-a^2c^2t_0)}{\delta_{15}}.$$
$$\begin{aligned}A=&-\left(\frac{1}{A_{14}-A_9}\right)(A_{14}A_{20}+A_{15}-A_{16}A_{19}+A_8A_{14}-A_9A_{13}-A_9A_{20}+A_9A_{16}A_{18}-A_{10}\\&+A_{11}A_{19}-A_{11}A_{14}A_{18}+A_1A_{14}-A_1A_9-A_2A_6A_{14}+A_2A_9A_{12}+A_4A_6-A_4A_{12})\\B=&\left(\frac{1}{A_{14}-A_9}\right)(A_{15}A_{20}+A_8A_{14}A_{20}+A_8A_{15}-A_8A_{16}A_{19}-A_9A_{13}A_{20}-A_{10}A_{13}-A_{10}A_{20}\\&+A_{10}A_{16}A_{18}+A_{11}A_{13}A_{19}-A_{11}A_{15}A_{18}+A_1A_{14}A_{20}+A_1A_{15}-A_1A_{16}A_{19}+A_1A_8A_{14}\\&-A_1A_9A_{13}-A_1A_9A_{20}+A_1A_9A_{16}A_{18}-A_1A_{10}+A_1A_{11}A_{19}-A_1A_{11}A_{14}A_{18}-A_2A_6A_{14}A_{20}\\&-A_2A_6A_{15}+A_2A_6A_{16}A_{19}+A_2A_9A_{12}A_{20}-A_2A_9A_{16}A_{17}+A_2A_{10}A_{12}-A_2A_{11}A_{12}A_{19}\\&+A_2A_{11}A_{14}A_{17}-A_3A_7A_{14}+A_4A_6A_{13}+A_4A_6A_{20}-A_4A_6A_{16}A_{18}-A_4A_{12}A_{20}+A_4A_{16}A_{17}\\&-A_4A_8A_{12}+A_4A_{11}A_{12}A_{18}-A_4A_{11}A_{17}-A_5A_6A_{19}+A_5A_6A_{14}A_{18}+A_5A_{12}A_{19}-A_5A_{14}A_{17}\\&-A_5A_9A_{12}A_{18}+A_5A_9A_{17})\end{aligned}$$

$$
\begin{aligned}
C = &-\left(\frac{1}{A_{14}-A_9}\right)(A_8A_{15}A_{20} - A_{10}A_{13}A_{20} + A_1A_{15}A_{20} + A_1A_8A_{14}A_{20} + A_1A_8A_{15} - A_1A_8A_{16}A_{19}\\
&- A_1A_9A_{13}A_{20} - A_1A_{10}A_{13} - A_1A_{10}A_{20} + A_1A_{10}A_{16}A_{18} + A_1A_{11}A_{13}A_{19} - A_1A_{11}A_{15}A_{18}\\
&- A_2A_6A_{15}A_{20} + A_2A_{10}A_{12}A_{20} - A_2A_{10}A_{16}A_{17} + A_2A_{11}A_{15}A_{17} - A_3A_7A_{14}A_{20} - A_3A_7A_{15}\\
&+ A_3A_7A_{16}A_{19} + A_4A_6A_{13}A_{20} - A_4A_8A_{12}A_{20} + A_4A_8A_{16}A_{17} - A_4A_{11}A_{13}A_{17} - A_5A_6A_{13}A_{19}\\
&+ A_5A_6A_{15}A_{18} - A_5A_{15}A_{17} + A_5A_8A_{12}A_{19} - A_5A_8A_{14}A_{17} + A_5A_9A_{13}A_{17} - A_5A_{10}A_{12}A_{18}\\
&+ A_5A_{10}A_{17})\\
E = &\left(\frac{1}{A_{14}-A_9}\right)(A_1A_8A_{15}A_{20} - A_1A_{10}A_{13}A_{20} - A_3A_7A_{15}A_{20} - A_5A_8A_{15}A_{17} + A_5A_{10}A_{13}A_{17})
\end{aligned}
$$

Appendix B.

$$
\begin{aligned}
&H_{1n} = -\frac{s_{1n}}{s_{2n}}, H_{2n} = -\frac{(q_{1n}+q_{2n}H_{1n})}{q_{3n}}, H_{3n} = -\frac{1}{A_5}[(k_n^2+A_1) + (-A_2k_n + A_3)H_{1n} - A_4k_nH_{2n}],\\
&H_{4n} = r_1 - l_1k_nH_{1n} - l_2k_nH_{2n} + r_2H_{3n}, \qquad H_{5n} = r_3 - \delta_5k_nH_{1n} - k_nH_{2n} + r_4H_{3n},\\
&H_{6n} = -\delta_2k_n + r_5H_{1n} + r_6H_{2n}, \qquad H_{7n} = -l_3k_n + r_7H_{1n} + r_8H_{2n},\\
&H_{8n} = r_9 - l_6k_nH_{1n} - l_7k_nH_{2n} + r_{10}H_{3n}, \qquad n = 1,2,3,4.\\
&q_{1n} = A_{11}k_n^3 + (A_1A_{11} - A_5A_6)k_n + A_5A_7, \qquad q_{2n} = (-A_2A_{11} + A_5)k_n^2 + A_3A_{11}k_n + A_5A_8,\\
&q_{3n} = (-A_4A_{11} + A_5A_9)k_n^2 + A_5A_{10}, \qquad q_{4n} = A_{16}k_n^3 + (A_1A_{16} - A_5A_{12})k_n,\\
&q_{5n} = (A_5 - A_2A_{16})k_n^2 + A_3A_{16}k_n + A_5A_{13}, \qquad q_{6n} = (A_5A_{14} - A_4A_{16})k_n^2 + A_5A_{15},\\
&s_{1n} = q_{1n}q_{6n} - q_{3n}q_{4n}, \quad s_{2n} = q_{2n}q_{6n} - q_{3n}q_{5n}, \quad l_1 = \frac{C_{13}}{\rho v_p^2}, \quad l_2 = \frac{e_{31}}{e_{33}}, \quad l_3 = \frac{e_{15}\beta_1T_0}{e\rho v_p^2},\\
&l_4 = -\frac{\in_{11}\beta_1T_0}{e_{33}e}, \quad l_5 = \frac{e_{31}\beta_1T_0}{e\rho v_p^2}, \quad l_6 = \frac{e_{33}\beta_1T_0}{e\rho v_p^2}, \quad l_7 = -\frac{\in_{33}\beta_1T_0}{e_{33}e}, \quad l_8 = \frac{P_3T_0}{e},\\
&\{r_1,r_3,r_5,r_6,r_7,r_8,r_9\} = ia\{\delta_1,l_1,\delta_2,\delta_6,l_3,l_4,l_5\}, \quad r_2 = -(1-iact_1), \quad r_4 = \delta_7(1-iact_1),\\
&r_{10} = l_8(1-iact_1).
\end{aligned}
$$

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