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Qualitative structures of electron-acoustic waves in an unmagnetized plasma with q-nonextensive hot electrons

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Abstract. The qualitative structures of electron-acoustic waves are investigated in an unmagnetized plasma containing cold electron fluid, q-nonextensive hot electrons and stationary ions. Applying the phase plane analysis, we present all phase portraits of the dynamical system and corresponding solitary- and periodic-wave solutions. Considering an external periodic perturbation, we study the chaotic structure of the perturbed dynamical system. The non-extensive parameter (q), the ratio between hot electron and cold electron number density at equilibrium (α) and speed of the traveling wave (v) play crucial roles for electron-acoustic solitary, periodic and chaotic structures. The results may have relevance in laboratory plasmas as well as space plasma environments.

1 Introduction

During last few decades, a collection of noticeable research works have been reported on the linear as well as nonlinear propagation of electron-acoustic waves (EAWs) in plasma literature. Fundamentally, electron-acoustic waves are high-frequency electrostatic modes in comparison with the ion plasma frequency [1] which can propagate in unmagnetized plasma [2,3] as well as in magnetized plasma [4–6]. It is important to note that an electrostatic wave in which the inertia is provided by the cold electrons and the restoring force is given by the pressure of the hot electrons. Furthermore, the ions play the role of a neutralizing background and there is no influence of the ion dynamics on the EAWs. It has been noticed that the phase speed of the EAWs lie between the cold- and hot-electron thermal velocities. EA waves are influential for their excitation in space as well as in laboratory plasma, and also vital for their potential importance [7]. The idea of EAWs had been originated by Fried and Gould [8] during their numerical simulations of the linear electrostatic Vlasov dispersion equation in an unmagnetized homogenous plasma. These types of waves can appear in the cusp region of the terrestrial magnetosphere [9,10], the geomagnetic tail [11], and the dayside auroral accelaration region [12]. Dubouloz *et al.* [13,14] studied the electron-acoustic solitons in an unmagnetized and as well as magnetized plasma and reported the fact that the negative-polarity electrostatic solitary potential structures are found by the Viking satellite in the dayside auroral zone. Positive-polarity soliton structures have been studied in the auroral plasma by the FAST and POLAR spacecraft [15]. Singh *et al.* [10] investigated electron-acoustic solitons in four-component plasmas successfully to apply their results to Viking satellite observations in the dayside auroral zone. Lakhina *et al.* [16,17] studied electron-acoustic nonlinear solitary waves in three- and four-component plasmas and applied their results to the magnetosheath plasma and the plasma sheet boundary layer. Anderegg *et al.* [18] observed standing electron-acoustic waves (EAWs) in a pure ion plasma and they showed that at moderate amplitude, EAW-type plasma waves can be excited over a broad range of frequencies. Montgomery *et al.* [19] reported the observation of stimulated electron-acoustic wave scattering (SEAS) and they reinterpreted the previous SRS (stimulated Raman scattering) data from low-density plasmas in terms of SEAS. Kabantsev *et al.* [20] showed that electron-acoustic waves have strong linear Landau damping, and were observed as nonlinear BGK modes in experiments with pure electron plasmas.

It has been observed that systems that present long-range interactions, long-time memory, fractality of the corresponding space-time or phase-space, or intrinsic inhomogeneity are untractable within the conventional Boltzmann-Gibbs (BG) statistics [21,22]. The main cause for this failure is that BG statistics is an additive or extensive formalism.

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In dealing with the statistical properties of systems with long-range correlations, Tsallis [23] consistently extended BG thermodynamics by generalizing the concept of entropy to nonextensive regimes. Nonextensivity means that the entropy of the composition $(A+B)$ of two independent systems A and B is equal to $S_q^{(A+B)} = S_q^{(A)}+S_q^{(B)}+(1-q)S_q^{(A)}S_q^{(B)}$, where the parameter q that underpins the generalized entropy of Tsallis is linked to the underlying dynamics of the system and provides a measure of the degree of its correlation. There is a collection of various physical systems where the Tsallis entropy have been applied are plasma physics [24,25], long-range Hamiltonian systems [26], and gravitational systems [27]. Verheest *et al.* [28] presented successfully that the inclusion of the hot-electron inertia provides compressive type of electron-acoustic solitons. Electron-acoustic waves (EAWs) have been observed in laboratory plasmas consisting of hot and cold electrons [29,30] and in an electron ion plasma consisting of ions hotter than electrons [8]. The nonlinear propagation of EAWs plays a crucial role in laboratory and space plasma environments. Many researchers have payed their contributions on linear [31,32] as well as nonlinear [33–35] features of EAWs in unmagnetized plasma. Ferdousi and Mamun [36] studied electrostatic shock structures in nonextensive plasma with two distinct temperature electrons. El-Wakil *et al.* [37] investigated finite-amplitude electron-acoustic solitary waves in plasma with cold-electron fluid and two different temperature isothermal ions. Their study could be useful to describe the compressive and rarefactive bipolar pulses of the Broadband Electrostatic Noise type emissions in the regions with no electron beams.

It is important to note that the integrability of a system could be destroyed due to the effect of external periodic perturbations occurring in some real physical environments [38–40]. The type of the external periodic perturbation may vary depending upon different physical situations. A significant attention is recently received on the study of nonlinear evolution equations considering an external periodic perturbations as a completely integrable nonlinear wave equations is unable to describe quasi-periodic or chaotic features. But, the presence of an external periodic perturbation to a nonlinear integrable wave equation may lead to quasi-periodic or chaotic motions. Recently, Samanta *et al.* [41] investigated bifurcations nonlinear waves in plasmas using bifurcation theory of planar dynamical systems for the first time in plasma literature. Saha and Chatterjee [42] studied nonlinear electron-acoustic traveling waves in an unmagnetized quantum plasma on the framework of KdV equation employing the bifurcation theory of planar dynamical systems. Saha and Chatterjee [43] reported electron-acoustic blow-up solitary waves and periodic waves in an unmagnetized plasma with kappa distributed hot electrons using bifurcation theory of planar dynamical systems in the framework of KdV equation. Sahu *et al.* [44] investigated the nonlinear wave structures of electron-acoustic waves (EAWs) in an unmagnetized quantum plasma consisting of cold and hot electrons. The authors presented the solitonic, quasi-periodic and periodic pattern of electron-acoustic waves in quantum plasma. Sahu [45] also reported the nonlinear properties of electron-acoustic solitary waves in an unmagnetized plasma consisting of a cold electron fluid, κ distributed hot electrons and stationary ions. Saha *et al.* [46] investigated the dynamic behavior of ion-acoustic waves in electron-positron-ion magnetoplasmas with superthermal electrons and positrons considering external periodic perturbation. Very recently, Saha *et al.* [47] studied bifurcation and quasiperiodic behaviors of ion acoustic waves in electron-ion magnetoplasmas with non-thermal electrons featuring Cairns-Tsallis distribution on the framework of the Kadomtsev-Petviashvili (KP) equation. Zhen *et al.* [48] also studied soliton solution and chaotic motion of the extended ZK equations in a magnetized dusty plasmas with Maxwellian hot and cold ions. But, the chaotic structure of electron-acoustic waves in unmagnitized plasmas has not been reported in the literature to the best of our knowledge.

In the present work, our intention is to study the qualitative structures of electron-acoustic waves in an unmagnetized plasma containing cold electron fluid, q-nonextensive hot electrons and stationary ions. We present all possible phase portraits of the dynamical system and corresponding solitary- and periodic-wave solutions. Considering an external periodic perturbation, we present the chaotic structure of the perturbed dynamical system.

The remaining part of the paper is composed as follows: In sect. 2, we consider model equations. We present phase plane analysis in sect. 3. In sect. 4, we obtain new form of solitary- and periodic-wave solutions. We present the chaotic structure of the perturbed dynamical system in sects. 5 and 6 is kept for conclusions.

2 Model equations

In this work, we consider a homogeneous, unmagnetized plasma consisting of a cold electron fluid, q-nonextensive hot electrons and stationary ions. Nonlinear dynamics of the electron-acoustic waves (EAWs) in this plasma system is described by the following equations:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \alpha\frac{\partial \phi}{\partial x}\,, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + \frac{1}{\alpha}n - \left(1+\frac{1}{\alpha}\right), \tag{3}$$

where n denotes the cold electron number density normalized by n_0, n_e denotes the hot electron number density, u denotes the cold electron velocity normalized by ion fluid speed $C_s = \sqrt{k_B T_h/\alpha m_e}$, where $\alpha = n_{e0}/n_0 > 1$, m_e is the electron mass, T_h is the temperature of hot electron, e is the electron charge, k_B is the Boltzman constant, ϕ is the electrostatic potential normalized by $k_B T_h/e$. The space and time variables are normalized by the hot electron Debye radius $\lambda_h = \sqrt{k_B T_h/4\pi n_{e0} e^2}$ and the inverse of cold electron plasma frequency $\omega_{pc}^{-1} = \sqrt{m_e/4\pi n_0 e^2}$, respectively.

We use the following nonextensive electron distribution function to model an electron distribution with nonextensive particles:

$$f_e(v) = C_q \left\{1 + (q-1)\left[\frac{m_e v^2}{2T_e} - \frac{e\phi}{T_e}\right]\right\}^{\frac{1}{(q-1)}},$$

where ϕ is the electrostatic potential and the other variables or parameters carry their usual meaning. It is interesting to point out that $f_e(v)$ is the special distribution that maximizes the Tsallis entropy and, thus, conforms to the laws of thermodynamics. In this case, the constant of normalization is given by

$$C_q = n_{e0} \frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q} - \frac{1}{2}\right)} \sqrt{\frac{m_e(1-q)}{2\pi T_e}} \quad \text{for } -1 < q < 1,$$

and

$$C_q = n_{e0} \frac{1+q}{2} \frac{\Gamma\left(\frac{1}{q-1} + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{q-1}\right)} \sqrt{\frac{m_e(q-1)}{2\pi T_e}} \quad \text{for } q > 1.$$

Integrating $f_e(v)$ over all velocity space, one can derive the following nonextensive electron number density as:

$$n_e(\phi) = n_{e0} \left\{1 + (q-1)\frac{e\phi}{T_e}\right\}^{\frac{1}{(q-1)} + \frac{1}{2}}.$$

The normalized number density of q-nonextensive [49] hot electrons is given by

$$n_e = \{1 + (q-1)\phi\}^{\frac{1}{q-1} + \frac{1}{2}}, \tag{4}$$

where, the parameter q indicates the strength of nonextensivity. It is important to note that when $q < -1$, the q-distribution is not normalizable [50]. The strength of nonextensivity, q varies as $-1 < q < 1$. When $q \geq 1$, the distribution function exhibits Maxwell-Boltzmann velocity distribution. In the extensive limiting case $q \to 1$, the electron density, eq. (4) reduces to the well-known Maxwell-Boltzmann distribution.

3 Phase plane analysis

In this section, we transform our model equations into a planar dynamical system and present bifurcations of phase portraits of the system. To do so, we introduce a new variable $\xi = x - vt$, where v is the speed of the electron-acoustic traveling wave. Substituting the new variable ξ into eqs. (1) and (2) and using the initial condition $u = 0$, $n = 1$ and $\phi = 0$, we can express the cold electron number density as

$$n = \frac{v}{\sqrt{v^2 + 2\alpha\phi}}. \tag{5}$$

Substituting eqs. (4), and (5) into eq. (3) and considering the terms involving ϕ up to third degree, we have

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\xi^2} = a\phi + b\phi^2 + c\phi^3, \tag{6}$$

where $a = \frac{(q+1)}{2} - \frac{1}{v^2}$, $b = \frac{(q+1)(3-q)}{8} + \frac{3\alpha}{2v^4}$, and $c = \frac{(q+1)(3-q)(5-3q)}{48} - \frac{5\alpha^2}{2v^6}$.

Then eq. (6) is equivalent to the following planar dynamical system:

$$\begin{cases} \dfrac{\mathrm{d}\phi}{\mathrm{d}\xi} = z, \\ \dfrac{\mathrm{d}z}{\mathrm{d}\xi} = c\phi\left(\phi^2 + \dfrac{b}{c}\phi + \dfrac{a}{c}\right). \end{cases} \tag{7}$$

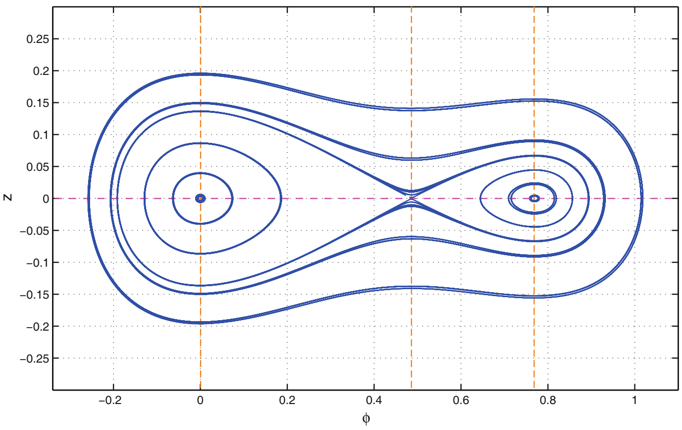

Fig. 1. Phase portrait of eq. (7) for $q = -0.3$, $\alpha = 1.2$ and $v = 1.2$.

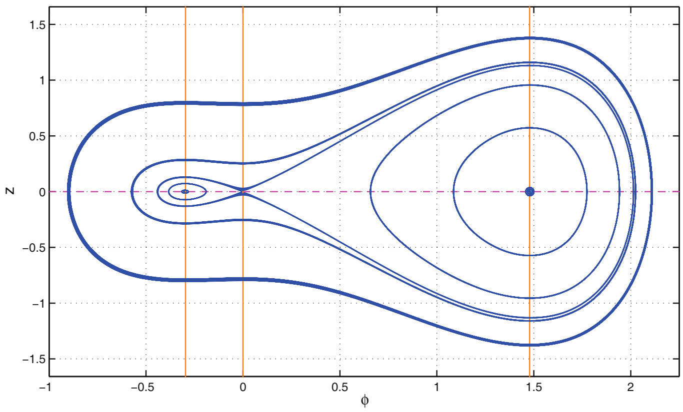

Fig. 2. Phase portrait of eq. (7) for $q = 1.4$, $\alpha = 1.2$ and $v = 1.2$.

The system of equations (7) is a planar Hamiltonian system with Hamiltonian function:

$$H(\phi, z) = \frac{z^2}{2} - a\frac{\phi^2}{2} - b\frac{\phi^3}{3} - c\frac{\phi^4}{4} = h, \text{ say.} \tag{8}$$

The system of equations (7) is a planar dynamical system with parameters q, α and v. It is clear that the phase orbits defined by the vector fields of eq. (7) will determine all traveling-wave solutions of eq. (6). A homoclinic orbit of eq. (7) gives a solitary-wave solution of the system. Similarly, a periodic orbit of eq. (7) gives a periodic traveling-wave solution of the system. In this study, the bifurcation theory of planar dynamical systems plays an important role [51,52].

We study the bifurcation set and phase portraits of the planar Hamiltonian system (7). Clearly, on the (ϕ, z) phase plane, the abscissas of equilibrium points of system (7) are the zeros of $f(\phi) = \phi(\phi^2 + \frac{b}{c}\phi + \frac{a}{c})$. Let $E_i(\phi_i, 0)$ be an equilibrium point of the dynamical system (7) where $f(\phi_i) = 0$. When $b^2 - 4ac > 0$, there exist three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$, where $\phi_0 = 0$, $\phi_1 = \frac{-b+\sqrt{b^2-4ac}}{2c}$ and $\phi_2 = \frac{-b-\sqrt{b^2-4ac}}{2c}$. If $M(\phi_i, 0)$ is the coefficient matrix of the linearized system of the traveling system (7) at an equilibrium point $E_i(\phi_i, 0)$, then we get

$$J = \det M(\phi_i, 0) = -cf'(\phi_i). \tag{9}$$

By the theory of planar dynamical systems [51–53], it is clear that the equilibrium point $E_i(\phi_i, 0)$ of the planar dynamical system (7) is a saddle point when $J < 0$ and the equilibrium point $E_i(\psi_i, 0)$ of the planar dynamical system (7) is a center when $J > 0$.

Applying the systematic analysis of the non-extensive parameter (q), ratio between unperturbed hot electron to cold electron density (α) and the speed of the traveling wave (v), we consider different cases and present all possible phase portraits of the system (7) in figs. 1–3. We mainly consider the effect of the non-extensive parameter q with fixed values of other parameters α and v. It is interesting that we obtain a variety of different phase portraits which represent qualitative different trajectories of the system, where each trajectory represents a solution of the system with respect to a initial condition.

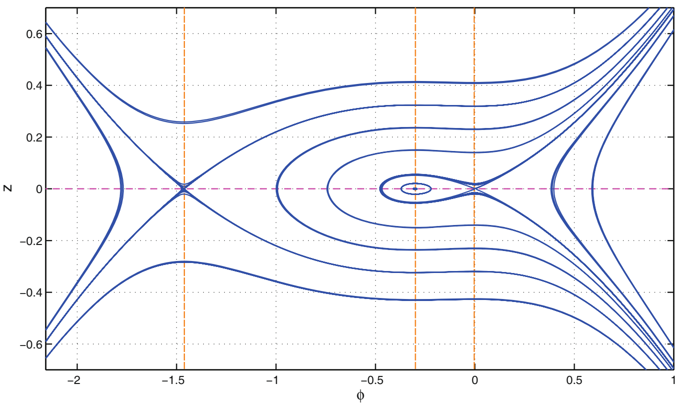

Fig. 3. Phase portrait of eq. (7) for $q = -0.3$, $\alpha = 1.2$ and $v = 2$.

For the phase portrait given by fig. 1, the parameters q, α and v satisfy the relations $c < 0$, $ac > 0$, $bc < 0$, $b^2 - 4ac > 0$ and $2b^2 - 9ac > 0$. In this case, the system (7) has three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$ with $0 < \phi_1 < \phi_2$, where $E_0(\phi_0, 0)$ and $E_2(\phi_2, 0)$ are centers, and $E_1(\phi_1, 0)$ is a saddle point. There is a pair of homoclinic orbits at $E_1(\phi_1, 0)$ enclosing the centers at $E_0(\phi_0, 0)$ and $E_2(\phi_2, 0)$. The phase portrait (see fig. 1) of the system (7) is presented for $q = -0.3$, $\alpha = 1.2$ and $v = 1.2$ considering q in the range $-1 < q < 0$. In this case, the system has compressive and rarefactive solitary-wave solutions corresponding to the pair of homoclinic orbits (see fig. 1) at $E_1(\phi_1, 0)$ and two families of periodic-wave solutions corresponding to two families of periodic orbits about the equilibrium points at $E_0(\phi_0, 0)$ and $E_2(\phi_2, 0)$. If we increase value of q and consider it in the range $0 < q < 1$ with fixed values of other parameters ($\alpha = 1.2$ and $v = 1.2$), then one also can obtain the similar phase portrait as fig. 1.

For the phase portrait given by fig. 2, the parameters q, α and v are connected by the relations $c < 0$, $ac < 0$, $bc < 0$, and $b^2 - 4ac > 0$. Then the system (7) has three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$ with $\phi_1 < 0 < \phi_2$, where $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$ are centers, and $E_0(\phi_0, 0)$ is a saddle point. There is a pair of homoclinic orbits at $E_0(\phi_0, 0)$ enclosing the centers at $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$. The phase portrait (see fig. 2) of the system (7) is shown considering $q = 1.4$ in the range $q > 1$ with same values of α and v as fig. 1. In this case, the system has compressive and rarefactive solitary-wave solutions corresponding to the pair of homoclinic orbits (see fig. 2) at $E_0(\phi_0, 0)$ and two families of periodic-wave solutions corresponding to two families of periodic orbits about the equilibrium points at $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$.

For the phase portrait given by fig. 3, the parameters q, α and v are connected by the relations $c > 0$, $ac > 0$, $bc > 0$, $b^2 - 4ac > 0$ and $2b^2 - 9ac > 0$. Then the system (7) has three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ at and $E_2(\phi_2, 0)$ with $\phi_2 < \phi_1 < 0$, where $E_0(\phi_0, 0)$ and $E_2(\phi_2, 0)$ are saddle points, and $E_1(\phi_1, 0)$ is a center. There is a homoclinic orbit at $E_0(\phi_0, 0)$ enclosing the center at $E_1(\phi_1, 0)$ (see fig. 3). The phase portrait (see fig. 3) of the system (7) is presented for $q = -0.3$, $\alpha = 1.2$ and $v = 2$ considering q in the range $-1 < q < 0$. In this case, the system has solitary-wave solution corresponding to the homoclinic orbit (see fig. 3) at $E_2(\phi_2, 0)$ and a family of periodic-wave solutions corresponding to the family of periodic orbits about the equilibrium point at $E_0(\phi_0, 0)$.

It is noted that fig. 1 is presented for $q = -0.3$, $\alpha = 1.2$ and $v = 1.2$. Then considering same values of α and v we obtain fig. 2 increasing the value of q in the range $q > 1$. Again considering the same values of q and α as fig. 1, we obtain fig. 3 taking $v = 2$. It is seen that for different values of nonextensive parameter q with fixed values α and v, and for different values of v with fixed values of q and α play crucial role to obtain qualitatively different phase portraits of the system. Thus, the dynamical behavior of the system is affected by the nonextensive parameter q and v with fixed value of α. We could in fact vary α with fixed values of q and v, but this does not give us any significant different qualitative results.

4 Solitary- and periodic-wave solutions

In this section, we will present explicit electron-acoustic solitary wave solutions and periodic wave solutions with the help of the dynamical system (7) and the Hamiltonian function (8). It is well-known that if a phase portrait of a dynamical system has a homoclinic orbit at an equilibrium point of the system, then the system has solitary wave solution corresponding to the homoclinic orbit at that point. If a phase portrait of a dynamical system has a family of periodic orbits about an equilibrium point of the system, then the system has a family of periodic wave solutions corresponding to the family of periodic orbits about that point. It should be noted that $\mathrm{sn}(\Omega\xi, k)$ is the Jacobian elliptic function [54] with the modulo k.

i) When the parameters q, α and v satisfy the relations $c < 0$, $ac > 0$, $bc < 0$, $b^2 - 4ac > 0$ and $2b^2 - 9ac > 0$, then the phase portrait of the dynamical system (7) has a family of periodic orbits about $E_0(\phi_0, 0)$ (see fig. 1) and the corresponding family of electron-acoustic periodic-wave solutions is given by

$$\phi(\xi) = \frac{(\gamma_1 - \delta_1)\beta_1 \operatorname{sn}^2(\Omega\xi, k) - \gamma_1(\beta_1 - \delta_1)}{(\gamma_1 - \delta_1)\operatorname{sn}(\Omega\xi, k) - (\beta_1 - \delta_1)}, \tag{10}$$

with $\Omega = \sqrt{-\frac{c}{8}(\beta_1 - \delta_1)(\alpha_1 - \gamma_1)}$, $k = \sqrt{\frac{(\alpha_1-\beta_1)(\gamma_1-\delta_1)}{(\alpha_1-\gamma_1)(\beta_1-\delta_1)}}$, where $\alpha_1, \beta_1, \gamma_1$ and δ_1 are roots of the equation $h + \frac{c}{4}\phi^4 + \frac{b}{3}\phi^3 + \frac{a}{2}\phi^2 = 0$, $h \in (0, h_2)$.

ii) When the parameters q, α and v satisfy the relations $c < 0$, $ac < 0$, $bc < 0$, and $b^2 - 4ac > 0$, then the phase portrait of the dynamical system (7) has a pair of homoclinic orbits at $E_0(\phi_0, 0)$ (see fig. 2) and the corresponding compressive and rarefactive electron-acoustic solitary-wave solutions are given by

$$\phi(\xi) = \pm \frac{1}{\sqrt{2\left(1 - \frac{b^2}{9ac}\right) \sin\left(2\sqrt{\frac{a}{c}}\xi\right) + \frac{b}{6a}}}\,. \tag{11}$$

iii) When the parameters q, α and v satisfy the relations $c > 0$, $ac > 0$, $bc > 0$, $b^2 - 4ac > 0$ and $2b^2 - 9ac > 0$, then the phase portrait of the dynamical system (7) has a homoclinic orbit at $E_0(\phi_0, 0)$ (see fig. 3) and the corresponding electron-acoustic solitary-wave solution is given by

$$\phi(\xi) = \frac{\left(\frac{4b}{c} + 6A\sqrt{\frac{2a}{c}}\right)}{3(A^2 - 1)}\,, \tag{12}$$

where $A = \exp(\sqrt{a}\xi) - \frac{b}{3}\sqrt{\frac{2}{ac}}$.

5 Chaotic structure in the perturbed system

In this section, we will present the chaotic structure of the following perturbed system:

$$\begin{cases} \dfrac{\mathrm{d}\phi}{\mathrm{d}\xi} = z, \\[2ex] \dfrac{\mathrm{d}z}{\mathrm{d}\xi} = a\phi + b\phi^2 + c\phi^3 + f_0\cos(\omega\xi), \end{cases} \tag{13}$$

where $f_0\cos(\omega\xi)$ is an external periodic perturbation, f_0 is strength of the external perturbation and ω is the frequency. The difference between the system (7) and the system (13) is that only external periodic perturbation is added with the system (7). The system (13) depends on five independent parameters q, α, v, f_0 and ω. An investigation of such a system for complete range of parametric space or the influence of each parameter is complicated and difficult. To simplify the analysis, all parameters are kept as constants except q to be changed. In order to explore the possible chaotic structure of the perturbed system (13), we consider special values of the parameter q with fixed values of α, v, f_0 and ω in three possible regimes $-1 < q < 0$, $0 < q < 1$ and $q > 1$. We could in fact vary any of the other parameters, but this does not give us any significant different qualitative results.

In fig. 4, we have presented phase portrait and variation of z with ξ of the perturbed system (13) for $q = -0.3$, $\alpha = 1.2$, $v = 1.2$, $f_0 = 1.42$ and $\omega = 2.6$. In fig. 5, we have presented phase portrait and variation of z with ξ of the perturbed system (13) for $q = 0.2$, and other parameters are same as fig. 4. In fig. 6, we have presented phase portrait and variation of z with ξ of the perturbed system (13) for $q = 1.42$, and other parameters are same as fig. 4. It is clear that the perturbed system (13) shows chaotic structure of electron-acoustic waves for some special values of q in the different ranges $-1 < q < 0$, $0 < q < 1$ and $q > 1$ with fixed values of other parameters α, v, f_0 and ω. Furthermore, the developed chaotic structures of electron-acoustic occurs (see figs. 4–6) and the solutions ignore the periodic and quasiperiodic structures and represent random sequences of uncorrelated oscillations. It is easily seen that the chaotic structures of electron-acoustic waves are visible in the system (13) for some special values of q in different regimes.

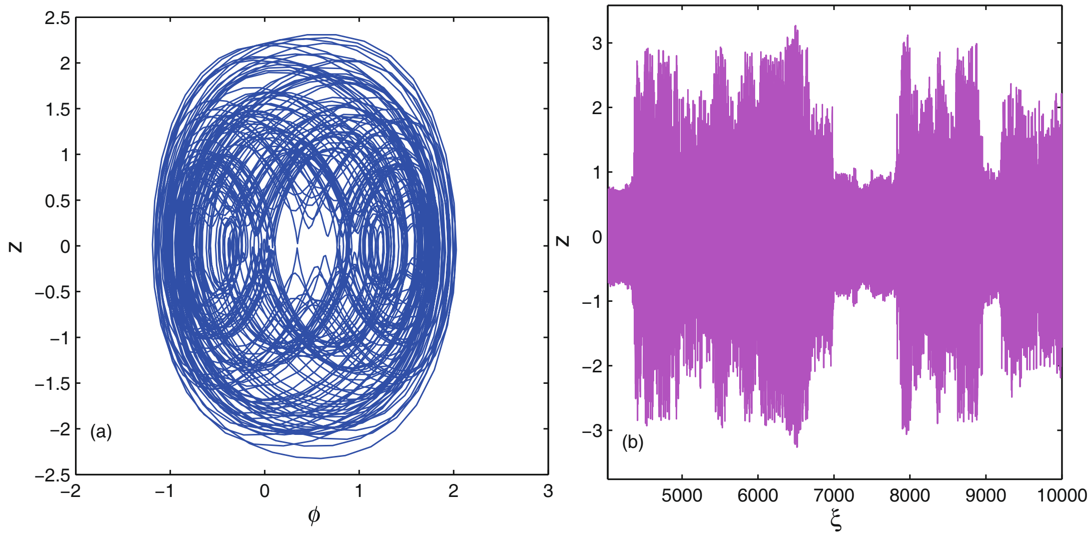

Fig. 4. Phase portrait and variation of z with ξ of the perturbed system (13) for $q = -0.3$, $\alpha = 1.2$, $v = 1.2$, $f_0 = 1.42$ and $\omega = 2.6$.

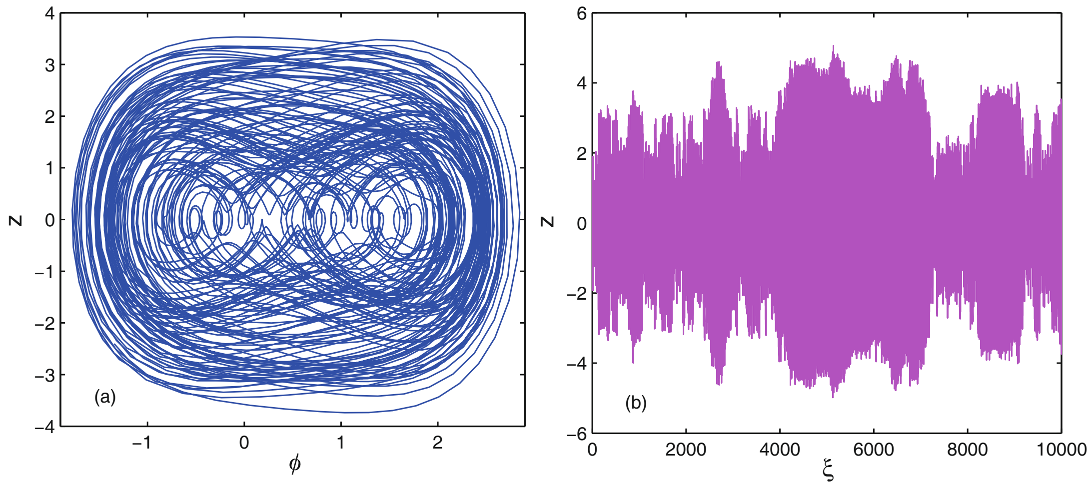

Fig. 5. Phase portrait and variation of z with ξ of the perturbed system (13) for $q = 0.2$, and other parameters are same as fig. 4.

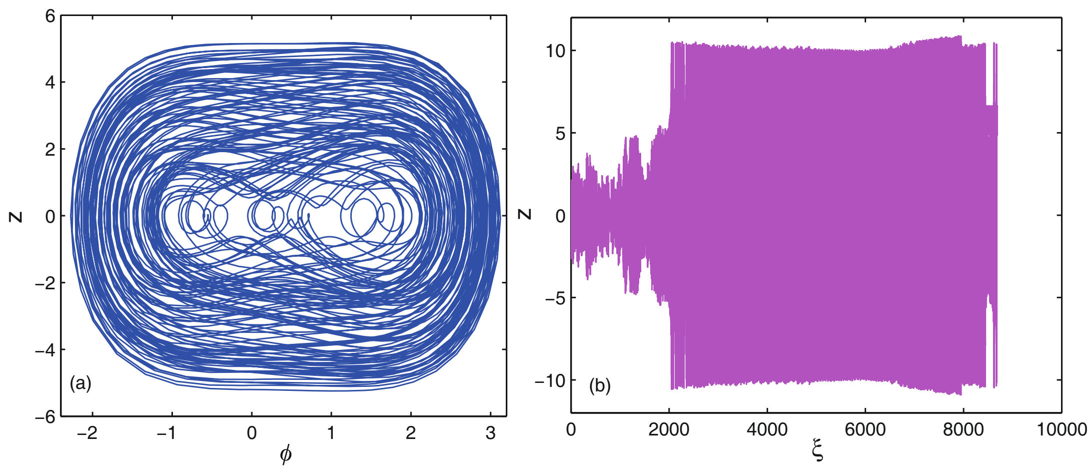

Fig. 6. Phase portrait and variation of z with ξ of the perturbed system (13) for $q = 1.42$, and other parameters are same as fig. 4.

6 Conclusions

We have addressed the qualitative structures of electron-acoustic waves in an unmagnetized plasma whose constituents are cold electron fluid, q-nonextensive hot electrons and stationary ions. With the help of phase plane analysis, we have observed that our model has electron-acoustic solitary-wave solutions (compressive and rarefactive) and periodic wave solutions in terms of Jacobian elliptic function. We have presented three new analytical forms for electron-acoustic wave solutions depending on the nonextensive parameter (q), ratio between unperturbed hot-electron to cold-electron density (α) and speed of the traveling wave (v). Considering an external periodic perturbation, the chaotic structure of electron-acoustic waves has been presented. Depending upon different regimes of the nonextensive parameter q, we have shown the effect of q on the chaotic structures of electron-acoustic waves. It is found that the perturbed system has chaotic structures for some special values of q in the different ranges $-1 < q < 0$, $0 < q < 1$, and $q > 1$ with fixed values of other parameters α, v, f_0 and ω. The results of this study may be helpful to understand the qualitative structures of nonlinear electron-acoustic waves in unmagnetized plasmas with high energetic nonextensive hot electrons.

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