

Tunnelling of vector particles from Lorentzian wormholes in 3+1 dimensions

I. Sakalli^a and A. Ovgun^b

Physics Department, Eastern Mediterranean University, Famagusta, Northern Cyprus, Mersin 10, Turkey

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Abstract. In this article, we consider the Hawking radiation (HR) of vector (massive spin-1) particles from the traversable Lorentzian wormholes (TLWH) in 3+1 dimensions. We start by providing the Proca equations for the TLWH. Using the Hamilton-Jacobi (HJ) ansatz with the WKB approximation in the quantum tunneling method, we obtain the probabilities of the emission/absorption modes. Then, we derive the tunneling rate of the emitted vector particles and manage to read the standard Hawking temperature of the TLWH. The result obtained represents a negative temperature, which is also discussed.

1 Introduction

Since the influential paper of Morris and Thorne [1], one of the most fascinating features of the theory of general relativity is the potential existence of spacetimes with wormholes. It is believed that they are the shortcut between otherwise distant or unconnected regions of the universe. Topologically, the wormhole spacetime is the same as that of black holes (BHs), however its throat, which possesses a minimal surface maintained in the time evolution, allows a traveler to both directions. The throat is held open by the presence of a phantom field [2]. Namely, phantom energy is precisely what is needed to support traversable wormholes. However, this exotic quantity violates the null energy condition, which signals the existence of the dark energy that dominates our Universe [3].

In the mid-1970s, Stephen Hawking looked into whether BHs could radiate thermally according to the quantum mechanics using the Wick rotation method [4–6]. Throughout the space, short-lived “virtual” particles (pair of the real particle and the anti-particle) continually pop into and out of the existence. Hawking realized that if the anti-particle falls into a BH and the real one escapes, then the BH would emit radiation, glowing like dying embers. Heuristically, there exist several derivations of the HR, such as the Damour-Rufini method, the HJ method, and the Parikh-Wilczek tunneling method (PWTM) (see for instance [7–14]). The reader is referred to [15] for the topical review. Meanwhile, it is worth noting that the PWTM is only applicable to a future outer trapping horizon of the wormhole [16]. All these methods can be used to calculate the emission/absorption probabilities of the particles penetrating the particular surface (event horizon) of the BH from the inside to the outside of the horizon, or vice versa, via the following relation

$$\Gamma = e^{-2Im S/\hbar}, \quad (1)$$

where S is the action of the classically forbidden trajectory. Thus, the Hawking temperature is derived from the tunneling rate of the emitted particles (see for example [17–26]).

The remainder of this paper is organized as follows. In sect. 2, we introduce the 3 + 1 dimensional TLWH [27]. Section 3 analyzes the Proca equation for massive vector particles in the past outer trapping horizon [28–30] geometry of the TLWH. We show that the Proca equations amalgamated with the HJ method can be reduced to a single equation, which makes it possible to compute the probabilities of the emission/absorption of the spin-1 particles. Then, we read the tunneling rate of the radiated particles and use it to derive the Hawking temperature of the TLWH. Finally, in sect. 4, the conclusions are summarized and further comments are added.

^a e-mail: izzet.sakalli@emu.edu.tr

^b e-mail: ali.ovgun@emu.edu.tr

2 TLWH in 3 + 1 dimensions

There is an analog of the BHs with the wormhole topology [31]. However, instead of the event horizon, the wormholes must have a throat, which allows the particles to pass through it in both directions. To construct the throat of a wormhole, exotic matter is required. Since the wormholes have two ends, the inside particles can naturally radiate from both ends.

For studying the HR of TLWH, we consider a general spherically symmetric and dynamic wormhole with a past outer trapping horizon. As is shown in [27], this local metric can be expressed in terms of the generalized retarded Eddington-Finkelstein coordinates as

$$ds^2 = -Cdu^2 - 2dudr + r^2 (d\theta^2 + Bd\varphi^2), \quad (2)$$

where $C = 1 - 2M/r$ and $B = \sin^2\theta$. M represents the gravitational energy in space with this symmetry, which is the so-called Misner-Sharp energy [32]. It is defined by $M = \frac{1}{2}r(1 - \partial^a r \partial_a r)$, which becomes $M = \frac{1}{2}r$ on a trapping horizon. Moreover, the retarded coordinates admit that the marginal surfaces in which $C = 0$ (at horizon: $r = r_0$) are the past marginal surfaces [27].

3 HR of vector particles from 3 + 1 dimensional TLWH

We start this section by introducing the Proca equation for a curved spacetime [33,34]:

$$\frac{1}{\sqrt{-g}} \frac{\partial(\sqrt{-g}\psi^{\nu\mu})}{\partial x^\mu} + \frac{m^2}{\hbar^2} \psi^\nu = 0, \quad (3)$$

where the wave function for a 3 + 1 dimension is given by $\psi_\nu = (\psi_0, \psi_1, \psi_2, \psi_3)$. Next, within the framework of the WKB approximation, we substitute the following HJ ansatz into eq. (3):

$$\psi_\nu = (c_0, c_1, c_2, c_3) e^{\frac{i}{\hbar}S(u,r,\theta,\phi)}, \quad (4)$$

where (c_0, c_1, c_2, c_3) denote the arbitrary real constants. The action $S(u, r, \theta, \phi)$ is given by

$$S(u, r, \theta, \phi) = S_0(u, r, \theta, \phi) + \hbar S_1(u, r, \theta, \phi) + \hbar^2 S_2(u, r, \theta, \phi) + \dots \quad (5)$$

Since metric (2) is symmetric, we have the Killing vectors ∂_θ and ∂_ϕ . So, one can apply the separation of variables method to the action $S_0(u, r, \theta, \phi)$:

$$S_0 = Eu - W(r) - j\theta - k\phi, \quad (6)$$

where E and (j, k) are energy and real angular constants, respectively. After inserting eqs. (4), (5) and (6) into eq. (3), we obtain a matrix equation $\Delta(c_0, c_1, c_2, c_3)^T = 0$ (to the leading order in \hbar), which has the following non-zero components:

$$\begin{aligned} \Delta_{11} &= 2B [\partial_r W(r)]^2 r^2, \\ \Delta_{12} &= \Delta_{21} = 2m^2 r^2 B + 2B \partial_r W(r) E r^2 + 2B j^2 + 2k^2, \\ \Delta_{13} &= -\frac{2\Delta_{31}}{r^2} = -2B j \partial_r W(r), \\ \Delta_{14} &= \frac{\Delta_{41}}{B r^2} = -2k \partial_r W(r), \\ \Delta_{22} &= -2BC m^2 r^2 + 2E^2 r^2 B - 2j^2 BC - 2k^2 C, \\ \Delta_{23} &= \frac{-2\Delta_{32}}{r^2} = 2j BC \partial_r W(r) + 2E j B, \\ \Delta_{24} &= \frac{\Delta_{42}}{B r^2} = 2k C \partial_r W(r) + 2k E, \\ \Delta_{33} &= m^2 r^2 B + 2B E r^2 \partial_r W(r) + r^2 BC [\partial_r W(r)]^2 + k^2, \\ \Delta_{34} &= \frac{-\Delta_{43}}{2B} = -k j, \\ \Delta_{44} &= -2r^2 BC [\partial_r W(r)]^2 - 4B E r^2 \partial_r W(r) - 2B(m^2 r^2 + j^2). \end{aligned} \quad (7)$$

A non-trivial solution is conditional on the termination of the determinant of the Δ -matrix ($\det \Delta = 0$). Hence, we get

$$\det \Delta = 64Bm^2r^2 \left\{ \frac{1}{2}r^2BC [\partial_r W(r)]^2 + BEr^2\partial_r W(r) + \frac{B}{2} (m^2r^2 + j^2) + \frac{k^2}{2} \right\}^3 = 0. \tag{8}$$

Solving eq. (8) for $W(r)$ yields

$$W_{\pm}(r) = \int \left(\frac{-E}{C} \pm \sqrt{\frac{E^2}{C^2} - \frac{m^2}{C} - \frac{j^2}{CB^2r^2} - \frac{k^2}{Cr^2}} \right) dr. \tag{9}$$

In the vicinity of the horizon ($r \rightarrow r_0$), the above intergral takes the following form:

$$W_{\pm}(r) \simeq \int \left(\frac{-E}{C} \pm \frac{E}{C} \right) dr. \tag{10}$$

According to eq. (1), the probabilities of emitted/absorbed particles depend on the imaginary contribution of the action. Since $C = 0$ on the horizon, eq. (10) has a pole. Therefore, the associated contribution is obtained by deforming the contour of integration in the upper r half-plane. In short, at the horizon, eq. (10) becomes

$$W_{\pm} = i\pi \left(\frac{-E}{2\kappa|_H} \pm \frac{E}{2\kappa|_H} \right). \tag{11}$$

Whence,

$$Im S = Im W_{\pm}, \tag{12}$$

where $\kappa|_H = \partial_r C/2$ is the surface gravity at the horizon. It should be noted that since the throat is an outer trapping horizon, the $\kappa|_H$ is positive quantity [27,35]. If we set the probability of absorption to 100% (*i.e.*, $\Gamma_{\text{absorption}} \approx e^{-2Im W} \approx 1$) so that we consider W_+ for the ingoing particles, and consequently W_- stands for the outgoing particles, we can compute the tunnelling rate [12,33,21–23] of the vector particles as

$$\Gamma = \frac{\Gamma_{\text{emission}}}{\Gamma_{\text{absorption}}} = \Gamma_{\text{emission}} \approx e^{-2Im W_-} = e^{\frac{2\pi E}{\kappa|_H}}. \tag{13}$$

Comparing eq. (13) with the Boltzmann factor $\Gamma \approx e^{-\beta E}$ (β is the inverse temperature), we then have

$$T|_H = -\frac{\kappa|_H}{2\pi}, \tag{14}$$

where $T|_H$ is the Hawking temperature of the TLWH. But $T|_H$ is negative, as formerly stated by [27,35]. The main reason of this negativeness is the phantom energy [27,36], which is located at the throat of wormhole. Furthermore, because of the phantom energy, the ordinary matter can travel backward in time [35].

4 Conclusion

In summary, we have calculated the HR of the massive vector particles from the TLWH in 3 + 1 dimensions. To this end, we have used the Proca equation. The probabilities of the vector particles crossing the trapped horizon of the TLWH have been obtained by applying the HJ method. The tunneling rate of the vector particles has been obtained, and comparing it with the Boltzmann factor the Hawking temperature of the TLWH has been obtained. Although the computed temperature is negative, our result is consistent with the results of [27,35]. Remarkably, we infer from the the negative $T|_H$ that past the outer trapping horizon of the TLWH radiates thermal phantom energy. On the other hand, it is a fact that phantom energy radiation must decrease both the size of the throat of the wormhole and its entropy. However, this does not constitute a problem. Because the total entropy of the universe always increases, and consequently it prevents violation of the second law of thermodynamics [37].

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