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Reconstructions of scalar field dark energy models from new holographic dark energy in Galileon universe

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Abstract. Here we briefly discuss the Galileon gravity theory and modified Friedmann equations. By considering new holographic dark energy (NHDE) in the framework of Galileon gravity, we found the energy density, pressure, equation of state and the deceleration parameter in terms of the scale factor. Subsequently, we study the correspondence between the NHDE in the framework of Galileon gravity with other dark energies like quintessence, k-essence, tachyon, dilaton, hessence and DBI-essence dark energies and construct the scalar field and corresponding scalar potentials which describe the dynamics of the scalar fields graphically. All the dark energy models, the scalar field and the potential decrease due to the evolution of the universe.

1 Introduction

The recent cosmological observations have confirmed the existence of an early inflationary epoch and of an accelerated expansion of the present universe. The type Ia Supernovae and Cosmic Microwave Background (CMB) [1,2] observations have shown evidences to support cosmic acceleration. To explain this acceleration, in the context of standard cosmology, we need an anti-gravity fluid with negative pressure, usually dubbed "dark energy" (DE) in the literature. The combined astrophysical observations suggest that the dark energy of the universe occupies about 70% of the total energy of the universe, the contribution of dark matter is $\sim 26\%$, the baryon is 4% and radiation is negligible. Recent WMAP data analysis [3,4] also give us the confirmation of this acceleration. The most simple candidate for dark energy is the vacuum energy or the cosmological constant whose equation of state (EoS) parameter $w = -1$ [5]. Although this model has a good agreement with observational data but it suffers several difficulties such as fine tuning and coincidence problem [2,6]. Further observations show that the EoS of DE w is likely to cross the cosmological constant boundary -1 (or phantom divide), *i.e.*, w is larger than -1 in the recent past and less than -1 today [7, 8]. The well known scalar field model, the quintessence [9,10] with a canonical kinetic term, can only evolve in the region of $w > -1$, whereas the model of phantom with negative kinetic term can always lead to $w < -1$. There are different candidates that obey the property of dark energy given by k-essence [11], dilaton [12], DBI-essence [13,14], hessence [15], tachyon [16], Chaplygin gas [17], holographic dark energy [18–21], etc. A comprehensive review of these DE models is also available in [22,23].

An alternative of dark energy for the accelerated expansion of the universe are the modified theories of gravity. For modified gravity, the effective energy density and pressure coming from the gravity sector which may violate strong energy condition drive the accelerated expansion of the universe [24]. The most well known modified gravity includes DGP brane, $f(R)$ gravity, $f(T)$ gravity, $f(G)$ gravity, Gauss-Bonnet gravity, Horava-Lifshitz gravity, Brans-Dicke gravity, etc. [25–33]. Here we assume one kind of modified gravity such as Galileon gravity [34–39]. The self-interacting solution for Galileon gravity has been constructed in [38]. Recently, Jamil *et al.* [40] and Ranjit et al [41] have discussed the observational constraints of some parameters in Galileon gravity. The cosmological and symmetry issues of Galileons have been investigated in [42]. Now the Brans-Dicke theory is an example of Galileons and it has several interesting cosmological aspects like Brans-Dicke Chameleon Cosmology [43,44]. Also it is possible to derive the Chameleon as an extension of Brans-Dicke using Kaluza-Klein [45].

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Recently, the correspondences between several candidates of dark energy models and modified gravities are very challenging subjects in cosmological phenomena. Till now, several authors [46–50] have discussed the correspondence between different DE models and their cosmological implications. For this purpose, we first briefly discuss the Galileon gravity theory and modified Friedmann equations in sect. 2. Next we discuss the new holographic dark energy (NHDE) in the framework of Galileon gravity. The energy density and pressure of NHDE have been found in terms of the scale factor in Galileon gravity. Subsequently, we study the correspondence between the NHDE with other dark energies like k-essence, tachyon, dilaton, hessence and DBI-essence dark energy in Galileon Universe and construct the scalar field and corresponding scalar potentials which describe the dynamics of the scalar fields graphically. Finally, we give some cosmological implications of the constructed models in sect. 4.

2 Galileon gravity and modified Friedmann equations in short

Recently, a general class of scalar-tensor theories has been explored that includes non-linear derivative interaction terms in the Lagrangian, which can be made not only to account for the current cosmic acceleration, but also to satisfy Solar System and laboratory constraints. That is, the theory leads to Lorentz invariant equations of motion having only a second derivative of the scalar field on a flat background. Such a scalar field is known as "Galileon". The Galileon theory is described by the action [34–39]

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\varphi R-\frac{\omega}{\varphi}\left(\nabla\varphi\right)^2+f(\varphi)\Box\,\varphi\left(\nabla\varphi\right)^2+\mathcal{L}_m\right], \tag{1}$$

where φ is the Galileon field and the coupling $f(\varphi)$ has dimension of length, $\left(\nabla\varphi\right)^2=g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi$, $\Box\,\varphi=g^{\mu\nu}\nabla_\mu\nabla_\nu\varphi$ and $\mathcal{L}_m$ is the matter Lagrangian. Variation with respect to the metric gives Einstein's equations [39]

$$\begin{aligned}G_{\mu\nu}=&\frac{T_{\mu\nu}}{2\varphi}+\frac{1}{\varphi}\left(\nabla_\mu\nabla_\nu\varphi-g_{\mu\nu}\,\Box\,\varphi\right)+\frac{\omega}{\varphi^2}\left[\nabla_\mu\varphi\nabla_\nu\varphi-\frac{1}{2}g_{\mu\nu}\left(\nabla\varphi\right)^2\right]\\&-\frac{1}{\varphi}\left\{\frac{1}{2}g_{\mu\nu}\nabla_\lambda\left[f(\varphi)\left(\nabla\varphi\right)^2\right]\nabla^\lambda\varphi-\nabla_\mu[f(\varphi)\left(\nabla\varphi\right)^2]\nabla_\nu\varphi+f(\varphi)\nabla_\mu\varphi\nabla_\nu\varphi\,\Box\,\varphi\right\}.\end{aligned} \tag{2}$$

Here we consider the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) metric of the universe as follows:

$$\mathrm{d}s^2=-\mathrm{d}t^2+a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2}+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right)\right], \tag{3}$$

where $a(t)$ is the scale factor and k $(=0,\pm1)$ is the curvature scalar. Now in the background of the Galileon gravity, the modified Friedmann equations are (for flat model $k=0$) [39]

$$3H^2=\frac{\rho_D}{2\varphi}-3HI+\frac{\omega}{2}\,I^2+\varphi^2 f(\varphi)\left(3H-\frac{\alpha_1}{2}\,I\right)I^3, \tag{4}$$

$$-3H^2-2\dot{H}=\frac{p_D}{2\varphi}+\dot{I}+I^2+2HI+\frac{\omega}{2}\,I^2+\varphi^2 f(\varphi)\left(\dot{I}+\frac{2+\alpha_1}{2}\,I^2\right)I^2. \tag{5}$$

The equation of motion for the Galileon field is obtained as

$$6(2H^2+\dot{H})-\omega(2\dot{I}+I^2+6HI)-6\varphi^2 f(\varphi)(2H\dot{I}+\dot{H}I+2HI^2+3H^2I)I+4\varphi^2 f(\varphi)\alpha_1 I^2\dot{I}+\varphi^2 f(\varphi)(\alpha_1^2+3\alpha_1+\alpha_2)I^4=0, \tag{6}$$

where $H=\frac{\dot{a}}{a}$, $I=\frac{\dot{\varphi}}{\varphi}$ and $\alpha_n=\frac{\mathrm{d}^n\ln f(\varphi)}{\mathrm{d}\ln\varphi^n}$. Here ρ_D and p_D are, respectively, the density and pressure of dark energy (DE) and we neglect the contributions from matter and radiation in the universe, that is, $\rho_{\mathrm{tot}}=\rho_D$. In addition, we shall assume that the Galileon field φ can be described as a power law of the scale factor, *i.e.*, $\varphi=\varphi_0 a^m$, where φ_0 and m are constants. If $\omega=0$ and $f(\varphi)=\frac{r_c^2}{\varphi^3}$, where r_c is the crossover scale, the above model closely related to the DGP brane world model. Also Kobayashi *et al.* [39] assumed the specific form of $f(\varphi)=\frac{r_c^2}{\varphi^2}$ and give the perturbation equations for Galileon gravity. Since $f(\varphi)$ is the arbitrary function of φ, so we may assume that the power law form of $f(\varphi)=f_0\varphi^n$, where f_0 and n are constants. From this, we get $\alpha_1=n$.

The holographic DE density with the new infrared cut-off is given by $\rho_D=3(\alpha H^2+\beta\dot{H})$ [51,52], where α and β are constants which must satisfy the restrictions imposed by the current observational data. In the Galileon gravity (as well as Brans-Dicke gravity), we assume one candidate of DE like the new holographic dark energy (NHDE) [53–56], in the form

$$\rho_D=3\varphi(\alpha H^2+\beta\dot{H}). \tag{7}$$

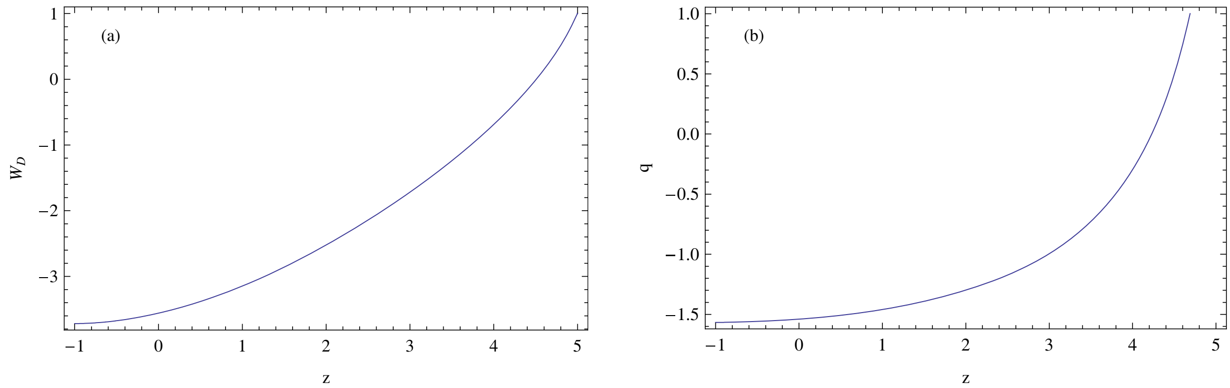

Fig. 1. The variations of the EoS parameter w_D and the deceleration parameter q against redshift z for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$.

From eqs. (4) and (7), we obtain the differential equation of H^2 in the form

$$3\beta\, a\frac{\mathrm{d}H^2}{\mathrm{d}a} + 2(3\alpha + \omega m^2 - 6m - 6)H^2 = 2f_0\varphi_0^{n+2}(mn-6)m^3a^{m(n+2)}H^4. \tag{8}$$

Solving the above equation, we obtain

$$H^2 = \frac{\left(6(\alpha-2) - 3m(4+(2+n)\beta) + 2m^2\omega\right) a^{-\frac{2}{3\beta}\left(-6-6m+3\alpha+m^2\omega\right)}}{\left(6(\alpha-2) - 3m(4+(2+n)\beta) + 2m^2\omega\right) C_1 + 2f_0m^3(mn-6)\varphi_0^{n+2}a^{\frac{1}{3\beta}\left(-6(\alpha-2)+3m(4+(2+n)\beta)-2m^2\omega\right)}}. \tag{9}$$

From (8), we obtain

$$\dot{H} = -\frac{1}{3\beta}(3\alpha + \omega m^2 - 6m - 6)H^2 + \frac{f_0}{3\beta}\,\varphi_0^{n+2}(mn-6)m^3a^{m(n+2)}H^4. \tag{10}$$

The deceleration parameter can be written as

$$q = -1 - \frac{\dot{H}}{H^2} = \frac{1}{3\beta}(3\alpha - 3\beta - \omega m^2 - 6m - 6) - \frac{f_0}{3\beta}\,\varphi_0^{n+2}(mn-6)m^3a^{m(n+2)}H^2. \tag{11}$$

Thus eq. (7) reduces to

$$\rho_D = \varphi_0 a^m\left[(6m + 6 + \omega m^2)H^2 + f_0\,\varphi_0^{n+2}(mn-6)m^3a^{m(n+2)}H^4\right]. \tag{12}$$

From eq. (5), we obtain

$$\begin{aligned} p_D = \frac{1}{3\beta}\,\varphi_0 a^m\Bigg[&\Big\{6(-4+2\alpha-3\beta) + 6m(\alpha - 2(3+\beta)) + 2m^3\omega - m^2(-4(-3+\omega) + 3\beta(2+\omega))\Big\}H^2 \\ &-f_0m^3\varphi_0^{2+n}a^{m(2+n)}\left(-6(2+\alpha) + m(6\beta + n(4+3\beta)) + 2m^2(n-\omega)\right)H^4\Bigg] \end{aligned} \tag{13}$$

In fig. 1, we draw the EoS parameter $w_D = p_D/\rho_D$ and the deceleration parameter q against the redshift $z = \frac{1}{a} - 1$, respectively, for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$. From this figure, we observe that the EoS parameter starts from positive value to -4 at a late stage. Also q shows the signature flips from positive to negative signs, that implies that, after some values of $z \sim 4$, the model generates DE, *i.e.*, the Galileon universe accelerates. Moreover, EoS w_D crosses -1 after $z < 4$, so phantom crossing is possible for this model.

3 Correspondence between NHDE in Galileon gravity and other dark energies

In the following subsections, we consider several types of dark energy models namely, quintessence, k-essence, tachyon, dilaton, hessence and DBI-essence. Then we investigate the correspondence between NHDE in Galileon gravity model and the above-mentioned candidates of DE. After that we shall try to investigate the nature of potentials as well as dynamics of scalar fields of DE candidates in Galileon gravity model.

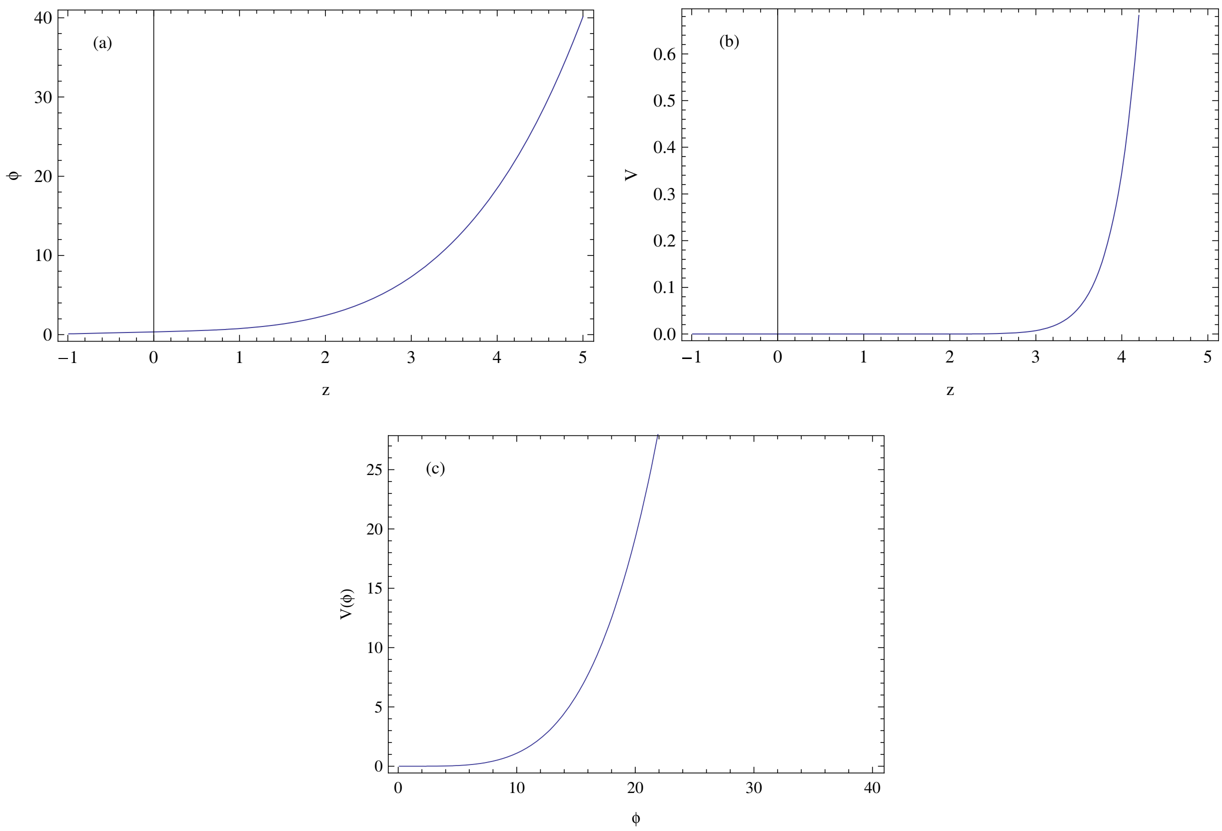

Fig. 2. Panels (a) and (b) show the variations of ϕ and V against z and panel (c) shows the variation of $V(\phi)$ against ϕ for the quintessence model for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$.

3.1 Quintessence model

Quintessence leads to the accelerated expansion of the universe with the help of scalar field (time-dependent and homogeneous) minimally coupled to gravity through a particular potential. Its scalar field applies the slow-roll condition on the potential which leads to a kinetic energy of the field less than the potential energy. The quintessence energy density and pressure with EoS parameter have the following form [9,10]:

$$\rho_Q = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{14}$$

$$\rho_Q = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{15}$$

The EoS for the quintessence scalar field is given by

$$w_Q = \frac{\dot{\phi}^2 + 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{16}$$

Now we obtain the potential energy

$$V = \frac{1}{2}(\rho_Q - p_Q) \tag{17}$$

and the scalar field as in the form

$$\phi(a) = \phi(a_0) + \int_{a_0}^{a} \frac{1}{Ha}\ \sqrt{\rho_Q + p_Q}\ \mathrm{d}a. \tag{18}$$

Now we take the correspondence between the quintessence scalar field model and new holographic dark energy model in our considered Galileon universe. For this purpose, we must equate their energy densities and pressures, *i.e.*, $\rho_Q = \rho_D$ and $p_Q = p_D$. Here ρ_D, p_D are given in eqs. (12) and (13), where H^2 is given in eq. (9). Now, we put their expressions in eqs. (17) and (18) and we may obtain the expressions of V and ϕ in terms of a, which are very complicated. Now we draw the variations of ϕ and V in terms of the redshift z in figs. 2(a) and (b) for $m = 2$, $n = -2$,

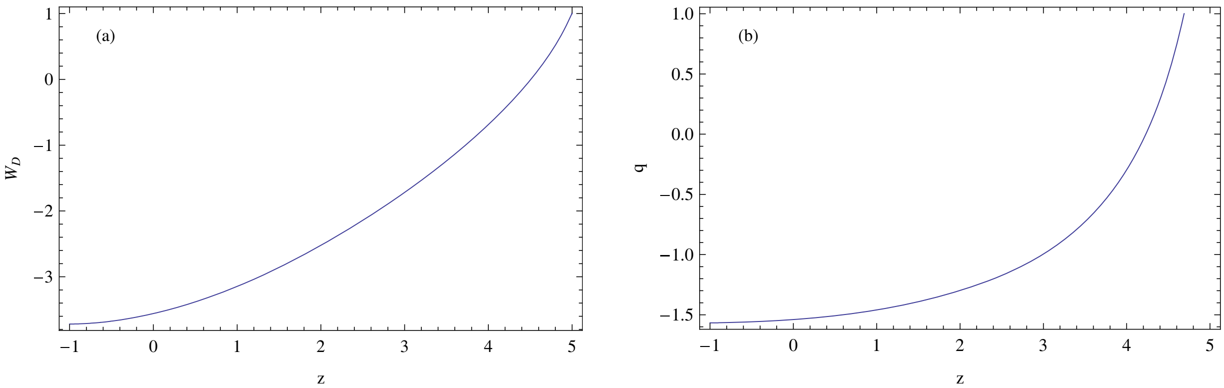

Fig. 3. Panels (a) and (b) show the variations of ϕ and V against z and panel (c) shows the variation of $V(\phi)$ against ϕ for the k-essence model for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$.

$\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$. From these figures, we see that ϕ and V are both decreasing with z and ultimately approach zero in the late stage of the evolution of the Galileon universe. Moreover fig. 2(c) shows the variation of $V(\phi)$ with ϕ and we see that $V(\phi)$ increases with ϕ.

3.2 k-essence model

In the kinetically driven scalar field theory, we have a *non-canonical* kinetic energy term with *no* potential. Scalars, modelling this theory, are popularly known as *k-essence*. Tt was used for the first time for kinetically driven inflation. Later on, it was used as a source of dark energy. Motivated by the Born-Infeld action of String Theory [57], it was used as a source to explain the mechanism for producing the late time acceleration of the universe. For the k-essence model the expressions of energy density and pressure are in the following forms [58]:

$$\rho_k = V(\phi)[-X + 3X^2] \tag{19}$$

and

$$p_k = V(\phi)[-X + X^2], \tag{20}$$

where $X = (1/2)\dot{\phi}^2$ for homogeneous scalar field ϕ and $V(\phi)$ is the potential. Therefore, from eqs. (19) and (20), the EoS parameter for the k-essence scalar field is given as

$$w_k = p_k/\rho_k = \frac{(X-1)}{(3X-1)}. \tag{21}$$

If the effective energy density and pressure governed by the Galileon gravity correspond to the energy density and pressure of the k-essence field, we may obtain

$$V(\phi) = \frac{(\rho_k - 3p_k)^2}{2(\rho_k - p_k)}. \tag{22}$$

Using the relation $\dot{\phi}^2 = 2X$ and the above expression, the evolutionary form of the k-essence scalar field is obtained as

$$\phi(a) = \phi(a_0) + \int_{a_0}^{a} \frac{\sqrt{2}}{Ha} \sqrt{\frac{\rho_k - p_k}{\rho_k - 3p_k}}\, \mathrm{d}a. \tag{23}$$

Now we take the correspondence between the k-essence model and the new holographic dark energy model in our considered Galileon universe. For this purpose, we must equate the energy densities and pressures of them, *i.e.*, $\rho_k = \rho_D$ and $p_k = p_D$. Here ρ_D, p_D are given in eqs. (12) and (13), where H^2 is given in eq. (9). Now, we put their expressions in eqs. (22) and (23) and we may obtain the expressions of V and ϕ in terms of a, which are very complicated. Now we draw the variations of ϕ and V in terms of redshift z in figs. 3(a) and (b) for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$. From these figures, we see that ϕ and V are both decreasing with z and ultimately approach zero in the late stage of the evolution of the Galileon universe. Moreover, fig. 3(c) shows the variation of $V(\phi)$ with ϕ and we see that $V(\phi)$ increases with ϕ.

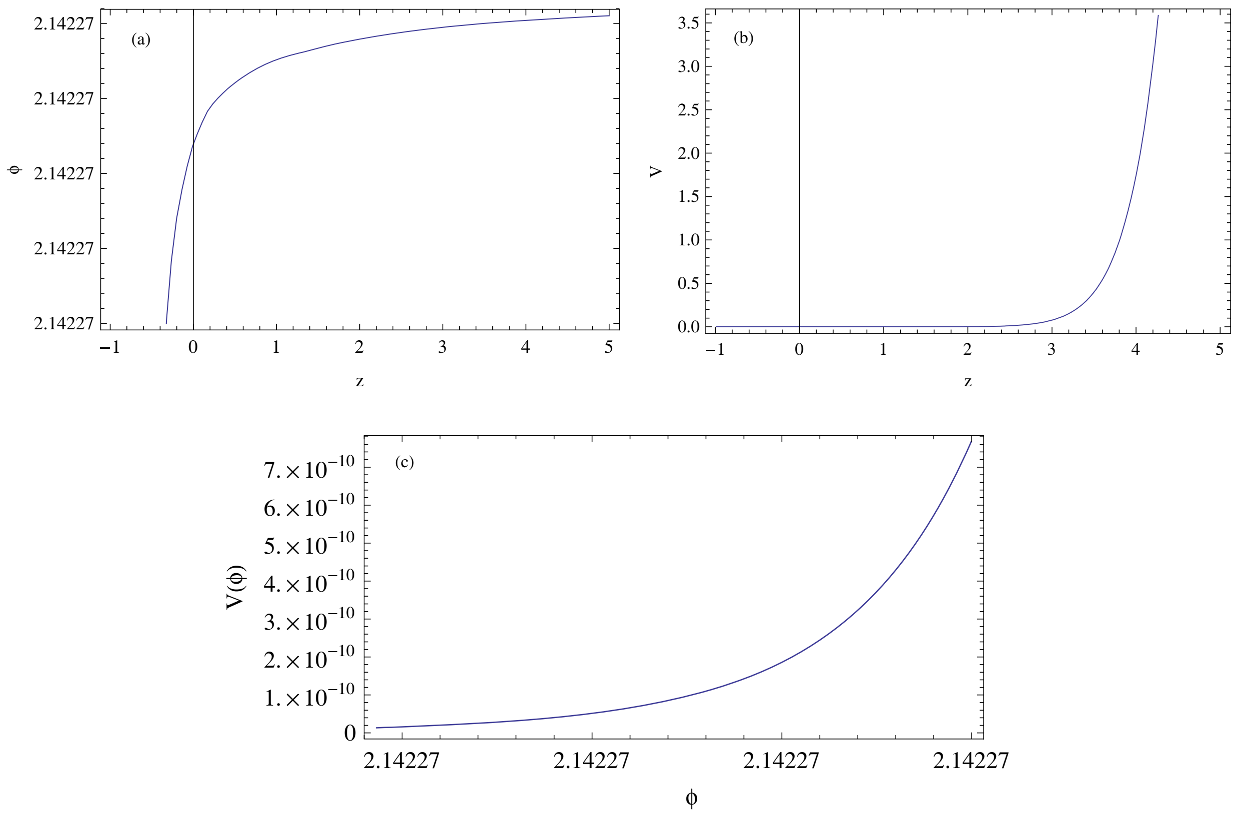

Fig. 4. Panels (a) and(b) show the variations of ϕ and V against z and panel (c) shows the variation of $V(\phi)$ against ϕ for the tachyon model for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$.

3.3 Tachyon model

The present subsection aims to investigate the conditions under which there is a correspondence between Galileon gravity and the tachyonic field in the flat FRW Universe. That is, to determine an appropriate potential for tachyonic field which makes the two dark energies to coincide with each other. The energy density and pressure for the tachyonic field are [16]

$$\rho_T = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \tag{24}$$

and

$$p_T = -V(\phi)\sqrt{1-\dot{\phi}^2}, \tag{25}$$

for which the EoS reads

$$w_T = \frac{p_T}{\rho_T} = \dot{\phi}^2 - 1, \tag{26}$$

where ϕ is the tachyonic field and $V(\phi)$ is the tachyonic potential. Now we obtain

$$V(\phi) = \sqrt{-\rho_T p_T} \tag{27}$$

and

$$\phi(a) = \phi(a_0) + \int_{a_0}^{a} \frac{1}{Ha}\sqrt{\frac{\rho_T + p_T}{\rho_T}}\,\mathrm{d}a. \tag{28}$$

Now we take correspondence between the tachyonic field model and new holographic dark energy model in our considered Galileon universe. For this purpose, we must equate their energy densities and pressures, *i.e.*, $\rho_T = \rho_D$ and $p_T = p_D$. Here ρ_D, p_D are given in eqs. (12) and (13), where H^2 is given in eq. (9). Now we put their expressions in eqs. (27) and (28) and we may obtain the expressions of V and ϕ in terms of a, which are very complicated. Now we draw the variations of ϕ and V in terms of the redshift z in figs. 4(a) and (b) for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$. From these figures, we see that ϕ and V are both decreasing with z and ultimately approach zero in late stage of the evolution of the Galileon universe. Moreover, fig. 4(c) shows the variation of $V(\phi)$ with ϕ and we see that $V(\phi)$ increases with ϕ.

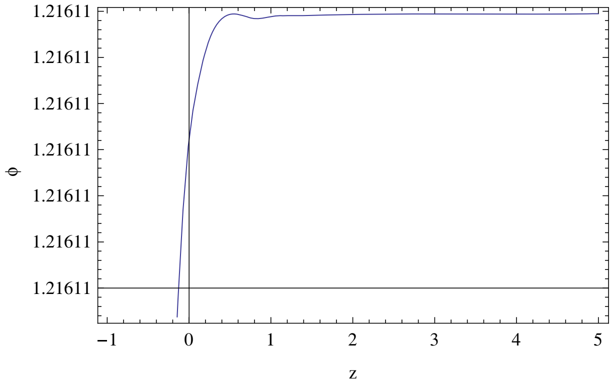

Fig. 5. The variation of ϕ against z for the dilaton model for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$, $c = 104, 4\lambda = 3$.

3.4 Dilaton model

We consider here the dilaton dark energy which is basically motivated from the low-energy limit of the string theory. The dilaton leads to an initial inflationary phase followed by a kinetic-energy–dominated phase. In the Einstein frame, the coefficient of the kinematic term of the dilaton can be negative to behave as a phantom-type scalar field. The energy density and pressure of the dilaton scalar field ϕ are given by [12]

$$\rho_d = -X + 3ce^{\lambda\phi}X^2 \tag{29}$$

and

$$p_d = -X + ce^{\lambda\phi}X^2, \tag{30}$$

where c and λ are positive constants and $\dot{\phi}^2 = 2X$. Consequently, the EoS parameter for the dilaton scalar field can be written as

$$w_d = \frac{p_d}{\rho_d} = \frac{-1 + ce^{\lambda\phi}X}{-1 + 3ce^{\lambda\phi}X}. \tag{31}$$

The dilaton scalar field can be obtained and we get

$$\phi(a) = \frac{2}{\lambda}\ln\left[e^{\frac{\lambda}{2}\phi(a_0)} + \frac{\lambda}{\sqrt{2c}}\int_{a_0}^{a}\frac{1}{aH}\sqrt{\frac{\rho_d - p_d}{\rho_d - 3p_d}}\,\mathrm{d}a\right]. \tag{32}$$

Now we take the correspondence between the dilaton field model and new holographic dark energy model in our considered Galileon universe. For this purpose, we must equate their energy densities and pressures, *i.e.*, $\rho_d = \rho_D$ and $p_d = p_D$. Here ρ_D, p_D are given in eqs. (12) and (13), where H^2 is given in eq. (9). Now we put their expressions in eq. (32) and we may obtain the expression of ϕ in terms of a, which are very complicated. Now we draw the variation of ϕ in terms of the redshift z in fig. 5 for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$, $c = 10$, $\lambda = 3$. From the figure, we see that ϕ is decreasing with z and ultimately approaches zero in the late stage of the evolution of the Galileon universe.

3.5 Hessence model

In 2005, Wei *et al.* [15] proposed a novel non-canonical complex scalar field named as "hessence", which plays the role of quintom dark energy. In the hessence model, the so-called internal motion $\dot{\theta}$ (where θ is the internal degree of freedom) of hessence plays a phantom-like role and in this case, the phantom divide transitions are also possible. The Lagrangian density of the hessence field ϕ is given by [15]

$$\mathcal{L}_h = \frac{1}{2}[(\partial_\mu\phi)^2 - \phi^2(\partial_\mu\theta)^2] - V(\phi). \tag{33}$$

The energy density and pressure for the hessence model are given by

$$\rho_h = \frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) + V(\phi) \tag{34}$$

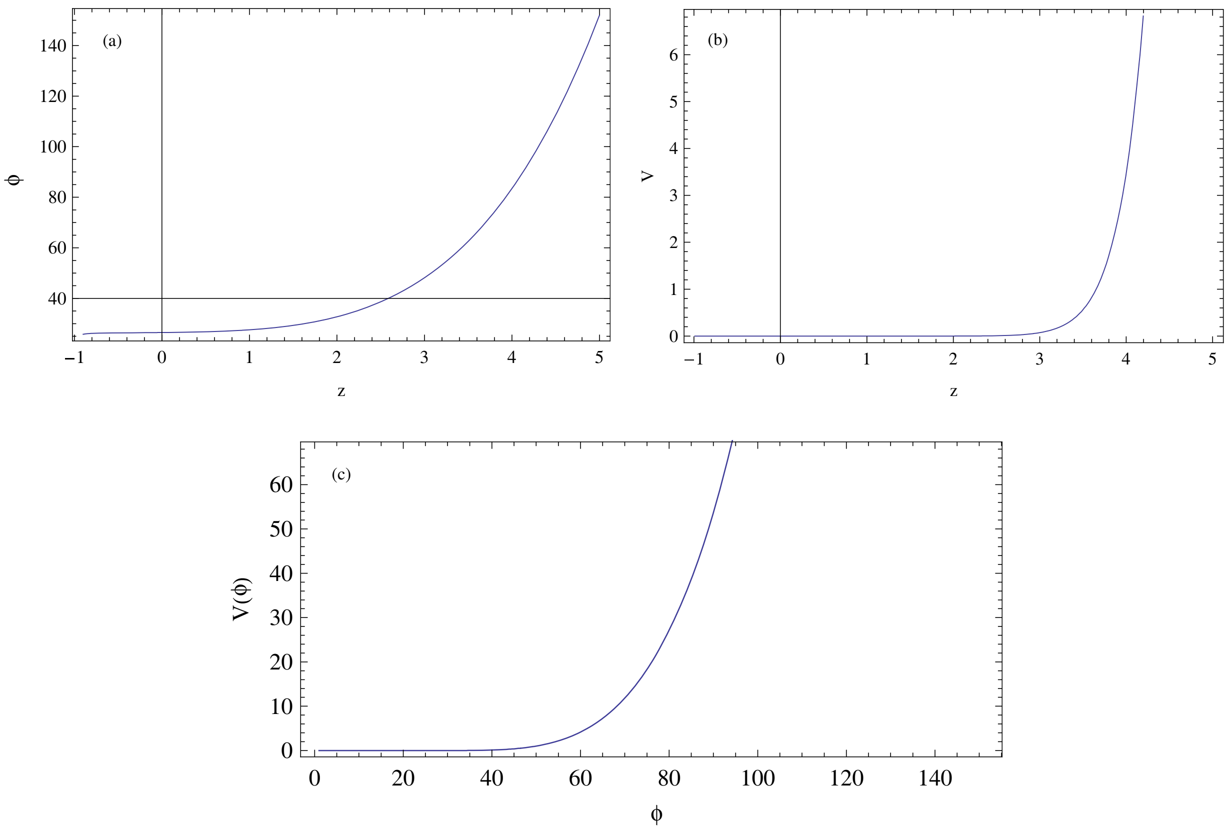

Fig. 6. Panels (a) and (b) show the variations of ϕ and V against z and panel (c) shows the variation of $V(\phi)$ against ϕ for the hessence model for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$, $Q = 0.5$.

and

$$p_h = \frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) - V(\phi), \tag{35}$$

with

$$Q = a^3\phi^2\dot{\theta} = \text{const.} \tag{36}$$

The corresponding equation of state parameter for hessence DE is given by

$$w_h = \frac{p_h}{\rho_h} = \frac{(\dot{\phi}^2 - \phi^2\dot{\theta}^2) - 2V(\phi)}{(\dot{\phi}^2 - \phi^2\dot{\theta}^2) + 2V(\phi)}. \tag{37}$$

We get the expression of potential as

$$V = \frac{1}{2}(\rho_h - p_h). \tag{38}$$

Also the scalar field ϕ can be found from the following first-order non-linear ordinary differential equation:

$$a^2H^2\left(\frac{\mathrm{d}\phi}{\mathrm{d}a}\right)^2 - \frac{Q^2}{a^6\phi^2} = \rho_h + p_h. \tag{39}$$

Now we take correspondence between the hessence model and the new holographic dark energy model in our considered Galileon universe. For this purpose, we must equate their energy densities and pressures, *i.e.*, $\rho_h = \rho_D$ and $p_h = p_D$. Here ρ_D, p_D are given in eqs. (12) and (13), where H^2 is given in eq. (9). Now we put their expressions in eqs. (38) and (39) and we may obtain the expressions of V and ϕ in terms of a, which are very complicated. Now we draw the variations of ϕ and V in terms of the redshift z in figs. 6(a) and (b) for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$, $Q = 0.5$. From these figures, we see that ϕ and V are both decreasing with z and ultimately approach zero in the late stage of the evolution of the Galileon universe. Moreover, fig. 6(c) shows the variation of $V(\phi)$ with ϕ and we see that $V(\phi)$ increases with ϕ.

3.6 DBI-essence model

In recent years, the inflation driven by the open string sector through dynamical Dp-branes is well explored. This is the so-called DBI (Dirac-Born-Infield) inflation, which represents a special class of k-inflation models. Here we consider the dark energy scalar field which is a Dirac-Born-Infeld (DBI) scalar field. In this case, the action of the field can be written as [13,14]

$$S_{\mathrm{dbi}} = -\int \mathrm{d}^4 x a^3(t) \left[T(\phi)\sqrt{1-\frac{\dot{\phi}^2}{T(\phi)}} + V(\phi) - T(\phi) \right], \tag{40}$$

where $T(\phi)$ is the warped brane tension and $V(\phi)$ is the DBI potential. From the above expression, the corresponding energy density and pressure of the scalar field become

$$\rho_{\mathrm{dbi}} = (\gamma - 1)T(\phi) + V(\phi) \tag{41}$$

and

$$p_{\mathrm{dbi}} = \frac{\gamma-1}{\gamma} T(\phi) - V(\phi), \tag{42}$$

where γ is reminiscent of the usual relativistic Lorentz factor and is given by

$$\gamma = \left(1 - \frac{\dot{\phi}^2}{T(\phi)}\right)^{-\frac{1}{2}}. \tag{43}$$

Thus the equation of state for DBI-essence is given by

$$w_{\mathrm{dbi}} = \frac{(\gamma-1)T(\phi) - \gamma V(\phi)}{\gamma((\gamma-1)T(\phi) + V(\phi))}. \tag{44}$$

Now we consider here two particular cases $\gamma = \text{const.}$ and $\gamma \neq \text{const.}$ [50].

Case I. $\gamma = \text{const.}$ In this case, we have $T(\phi) = \mu\dot{\phi}^2$ ($\mu > 1$), where $\gamma = \sqrt{\frac{\mu}{\mu-1}}$. In this case the expressions for ϕ, $T(\phi)$ and $V(\phi)$ are given by

$$\phi(a) = \phi(a_0) + \int_{a_0}^{a} \frac{1}{aH\sqrt{\gamma}} (\rho_{\mathrm{dbi}} - p_{\mathrm{dbi}})^{\frac{1}{2}} \; \mathrm{d}a, \tag{45}$$

$$T = \sqrt{\mu(\mu-1)}\,(\rho_{\mathrm{dbi}} - p_{\mathrm{dbi}}) \tag{46}$$

and

$$V = \frac{(\rho_{\mathrm{dbi}} - \gamma p_{\mathrm{dbi}})}{1+\gamma}. \tag{47}$$

Case II. $\gamma \neq \text{const.}$ In this case let us assume $\gamma = \dot{\phi}^s$. So we have $T(\phi) = \frac{\dot{\phi}^{2s+2}}{\dot{\phi}^{2s}-1}$. In this case the expressions for ϕ, $T(\phi)$ and $V(\phi)$ are given by

$$\phi(a) = \phi(a_0) + \int_{a_0}^{a} \frac{1}{Ha} (\rho_{\mathrm{dbi}} - p_{\mathrm{dbi}})^{\frac{1}{s+2}} \; \mathrm{d}a, \tag{48}$$

$$T = \frac{(\rho_{\mathrm{dbi}} - p_{\mathrm{dbi}})^{\frac{2s+2}{s+2}}}{(\rho_{\mathrm{dbi}} - p_{\mathrm{dbi}})^{\frac{2s}{s+2}} - 1} \tag{49}$$

and

$$V = -p_{\mathrm{dbi}} + \frac{(\rho_{\mathrm{dbi}} + p_{\mathrm{dbi}})}{1 + (\rho_{\mathrm{dbi}} - p_{\mathrm{dbi}})^{\frac{s}{s+2}}}. \tag{50}$$

Now we take the correspondence between the DBI-essence model and the new holographic dark energy model in our considered Galileon universe. For this purpose, we must equate their energy densities and pressures, *i.e.*, $\rho_{\mathrm{dbi}} = \rho_D$ and $p_{\mathrm{dbi}} = p_D$. Here ρ_D, p_D are given in eqs. (12) and (13), where H^2 is given in eq. (9). Now we put their expressions in eqs. (45)–(50) and we may obtain the expressions of ϕ, T and V in terms of a for cases I and II, which are very complicated. Now we draw the variations of ϕ, T and V in terms of redshift z in fig. 7 (for Case I) and fig. 8 (for Case II), respectively, for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$. Moreover, in Case I, we have assumed $\gamma = 2$ and, in Case II, we have assumed $s = 2$. From these figures, we see that ϕ, T and V are decreasing with z and ultimately approach zero in the late stage of the evolution of the Galileon universe. Also the variations of $T(\phi)$ and $V(\phi)$ are drawn in figs. 7(d) and (e) (for Case I) and figs. 8(d) and (e) (for Case II), respectively. From the figures, we see that $T(\phi)$ and $V(\phi)$ increase with ϕ for both cases.

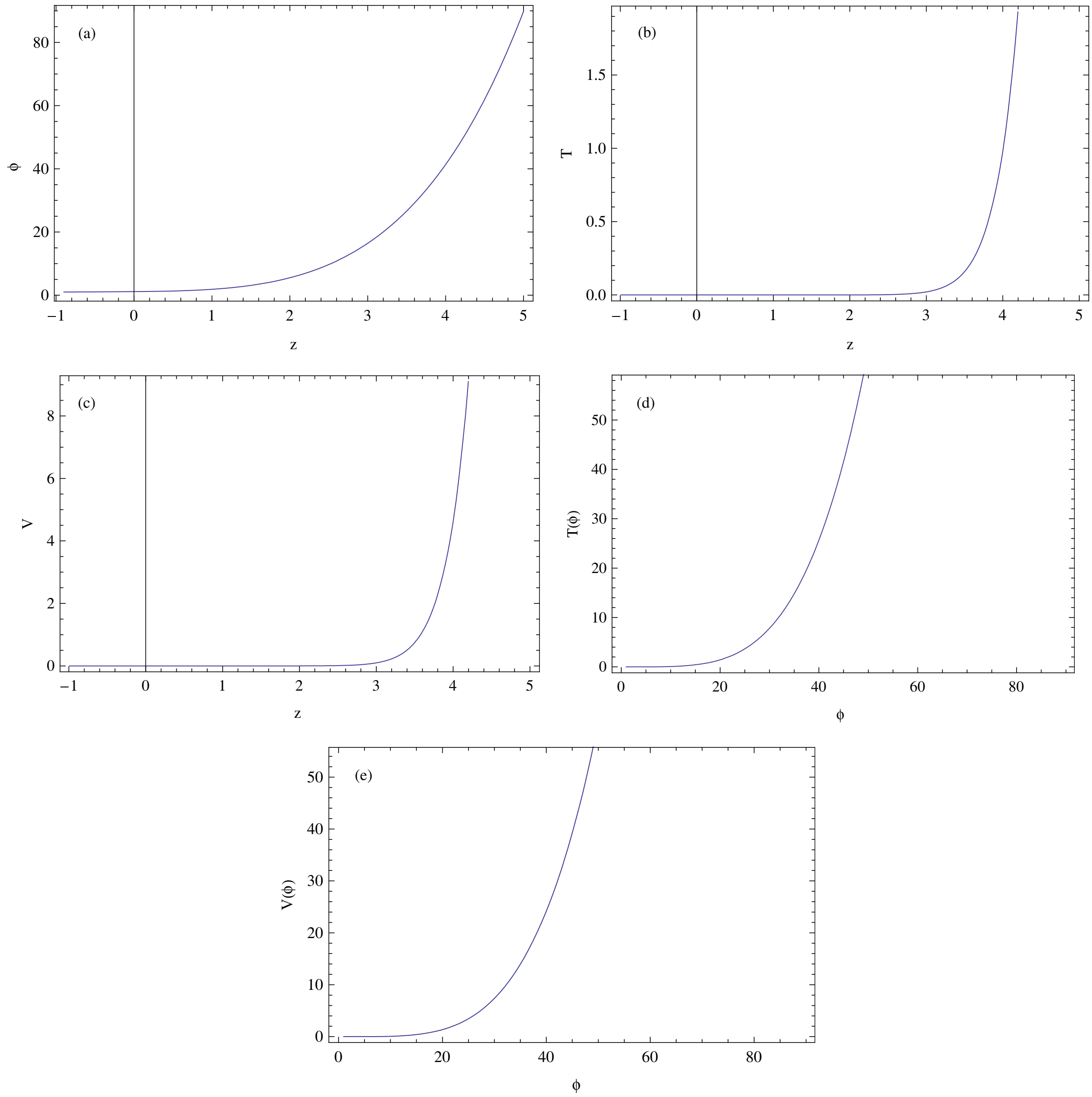

Fig. 7. Panels (a), (b) and (c) show the variations of ϕ, T and V against z and panels (d) and (e) show the variations of $T(\phi)$ and $V(\phi)$ against ϕ for the DBI-essence (Case I) model for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$, $\gamma = 2$.

4 Discussions and concluding remarks

Here we briefly discuss the Galileon gravity theory and the corresponding modified Friedmann equations. By considering the new holographic dark energy (NHDE) in the framework of Galileon gravity, we have found the energy density, pressure, equation of state and the deceleration parameter in terms of the scale factor. Here, we have assumed that the Galileon field φ can be described as a power law of the scale factor, *i.e.*, $\varphi = \varphi_0 a^m$, where φ_0 and m are constants and the power law form of $f(\varphi) = f_0\varphi^n$, where f_0 and n are constants. In fig. 1, we draw the EoS parameter $w_D = p_D/\rho_D$ and the deceleration parameter q against the redshift $z = \frac{1}{a} - 1$, respectively. From this figure, we observe that the EoS parameter starts from positive value to -4 at a late stage. Moreover, q shows the signature flips from positive to negative signs, which implies that, after some values of $z \sim 4$, the model generates DE *i.e.*, the Galileon universe accelerates. Moreover, EoS w_D crosses -1 after $z < 4$, so phantom crossing is possible for this model. In all the figures in this work, we have assumed the values of the parameters $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$.

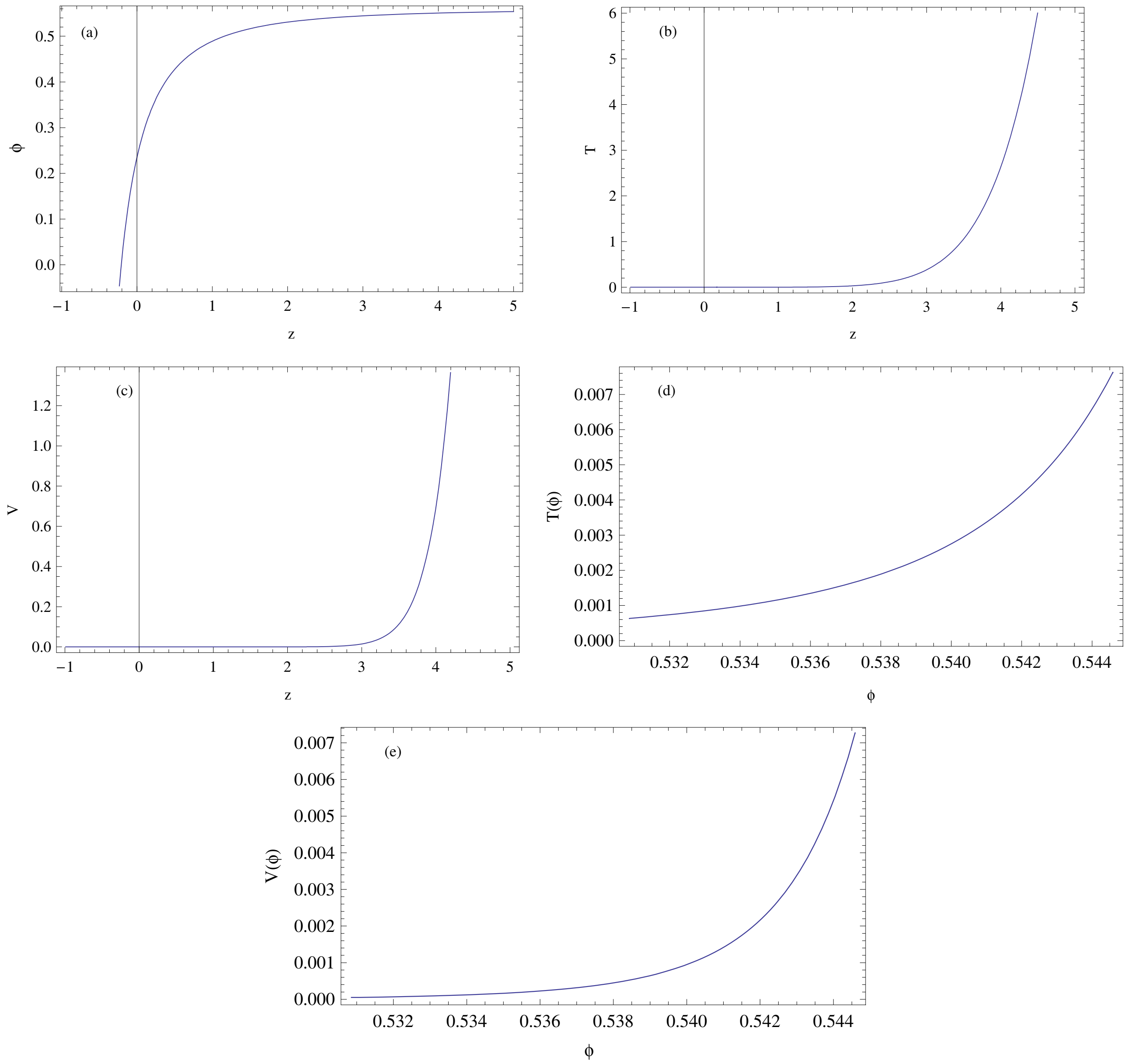

Fig. 8. Panels (a), (b) and (c) show the variations of ϕ, T and V against z and panels (d) and (e) show the variations of $T(\phi)$ and $V(\phi)$ against ϕ for the DBI-essence (Case II) model for $m = 2$, $n = -2$, $\alpha = 2$, $\beta = 3$, $\omega = 5$, $C_1 = 1$, $f_0 = 1.2$, $\phi_0 = 1.5$, $s = 2$.

Subsequently, we have studied the correspondence between the NHDE in the framework of Galileon gravity with other scalar field dark energies like quintessence, k-essence, tachyon, dilaton, hessence and DBI-essence and constructed the scalar field and corresponding scalar potentials which describe the dynamics of the scalar fields graphically. The physical interpretations of all the dark energy models have been discussed separately.

i) Quintessence: We have studied the correspondence between the quintessence scalar field model and the new holographic dark energy model in our considered Galileon universe. For this situation, we have drawn the variations of ϕ and V in terms of redshift z in figs. 2(a) and (b), from which we see that ϕ and V are both decreasing with z and ultimately approach zero in the late stage of the evolution of the Galileon universe. Moreover, fig. 2(c) shows the variation of $V(\phi)$ with ϕ and we see that $V(\phi)$ increases with ϕ.

ii) k-essence: For the correspondence between the k-essence model and the new holographic dark energy model in our considered Galileon universe, we have drawn the variations of ϕ and V in terms of the redshift z in figs. 3(a) and (b). From these figures, we see that ϕ and V are both decreasing with z and ultimately approach zero in the late stage of the evolution of the Galileon universe. Moreover, fig. 3(c) shows the variation of $V(\phi)$ with ϕ and we see that $V(\phi)$ increases with ϕ.

iii) Tachyon: For the tachyon model, we have drawn the variations of ϕ and V in terms of the redshift z in figs. 4(a) and (b) and we see that ϕ and V are both decreasing with z and ultimately approach zero in the late stage of the evolution of the Galileon universe. Moreover, fig. 4(c) shows the variation of $V(\phi)$ with ϕ and we see that $V(\phi)$ increases with ϕ.
iv) Dilaton: For the dilaton model, the variation of ϕ in terms of the redshift z has been drawn in fig. 5 for $c = 10$, $\lambda = 3$. From the figure we see that ϕ is decreasing with z and ultimately approaches zero in the late stage of the evolution.
v) Hessence: The variations of ϕ and V in terms of the redshift z has been drawn in figs. 6(a) and (b) in the hessence model for $Q = 0.5$. From these figures, we see that ϕ and V are both decreasing with z and approach zero in the late stage of the universe. Moreover, fig. 6(c) shows the variation of $V(\phi)$ with ϕ and we see that $V(\phi)$ increases with ϕ.
vi) DBI-essence: For the correspondence between the DBI-essence model and the new holographic dark energy model in our considered Galileon universe, the variations of ϕ, T and V in terms of the redshift z are drawn in figs. 7(a), (b) and (c) (for Case I) and figs. 8(a), (b) and (c) (for Case II), respectively. In Case I, we have assumed $\gamma = 2$ and, in Case II, we have assumed $s = 2$. From these figures, we see that ϕ, T and V are decreasing with z and tend to zero. Moreover, the variations of $T(\phi)$ and $V(\phi)$ are drawn in figs. 7(d) and (e) (for Case I) and figs. 8(d) and (e) (for Case II), respectively. From the figures, we see that $T(\phi)$ and $V(\phi)$ increase with ϕ for both cases.

From the above analysis, we may conclude that in all dark energy models, the scalar field and potential decrease due to the evolution of the universe. In future work, it will be interesting to construct the potentials for various dark energy models in other versions of modified gravity theories.

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