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Segregation of a binary granular mixture in a vibrating sawtooth base container

Shahin Mobarakabadi[1,a], Neda Adrang[2], Mehdi Habibi[3], and Ehsan Nedaaee Oskoee[2,4]

[1] Department of Physics, Islamic Azad University Arak Branch, Arak 38135-567, Iran
[2] Department of Physics, Institute for Advanced Studies in Basic Sciences, Gava Zang, Zanjan 45195-159, Iran
[3] Laboratory of Physics and Physical Chemistry of Food, Department of Agrotechnology and Food Sciences, Wageningen University, Wageningen, The Netherlands
[4] Research Center for Basic Sciences & Modern Technologies (RBST), Institute for Advanced Studies in Basic Sciences (IASBS), Zanjan 45137-66731, Iran

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Abstract. A granular mixture of identical particles of different densities can be segregated when the system is shaken. We present an efficient method of continuously segregating a flow of randomly mixed identical spherical particles of different densities by shaking them in a quasi–two-dimensional container with a sawtooth-shaped base. Using numerical simulation we study the effect of direction of shaking (horizontal/vertical), geometry of the sawtooth, and the friction coefficient between the grains and the container walls on the segregation quality. Finally by performing experiments on the same system we compare our simulation results with the experimental results. The good agreement between our simulation and experiment indicates the validity of our simulation approach and will provide a practical way for granular segregation in industrial applications.

1 Introduction

Granular segregation is ubiquitous in nature, technology, and daily life. Numerous applications of the segregation process in industry motivated many researchers to study the segregation of granular mixtures. In recent years a variety of segregation methods has been proposed for the separation of shaken binary mixtures [1–3]. While many mechanisms have been proposed to describe the granular segregation the underlying physics remains the subject of debate [1, 4]. From the application point of view, continuous separation is a desired feature of a segregation mechanism. For example in many industrial applications a steady input flow of a granular mixture has to be separated in two distinct flows of identical grains. Therefore, segregation methodologies in which a continuous input of mixed grains can be segregated in time would have great potential applications.

Granular flow due to geometrical symmetry breaking of the container is a well-known mechanism for transport and segregation of granular materials [2, 3, 5–14]. In most of the researches a container with sawtooth-shaped walls/base is used to drive the granular gas. Breaking the spatial symmetry by the ratchet structure induces a net drift speed [5–8, 11] which can cause segregation of binary mixtures [2, 3, 9, 14]. Different features of these systems have been studied. For example, Rapaport *et al.* [2] numerically showed that a binary mixture of large and small particles is horizontally drifted in opposite directions while the grains are vertically vibrated in a container with a sawtooth-shaped base. Ratchet-induced transport and horizontal segregation of non-spherical particles is numerically studied by Wambaugh *et al.* [9]. Farkaz *et al.*, proposed a mechanism for segregating identical particles that only differ in stiffness or restitution coefficient [3]. In their study they considered a flow of a binary mixture of spheres falling from a specified distance on an asymmetric tilted sawtooth profile which is shaken vertically. Unlike the other segregation methods in which separation is mainly due to the collective behavior of the particles in their system, a sawtooth-shaped base plays the major role. At low flow rate of particles the interaction between the base and the particles is dominant, which results in a high efficiency of segregation. The efficiency decreases with increasing the particle-particle interactions.

In a different study Ji-Hong *et al.*, numerically investigated the segregation of a horizontally vibrated binary mixture of spheres of different densities in a container with a sawtooth base [14]. They observed different states of separation depending on the frequency and acceleration of the vibration, including vertical and horizontal segregation states and even a mixed state without

[a] e-mail: shahinmobaraki@gmail.com

separation. They concluded that the collective behavior of the particles plays a major role in the segregation process [14]. They also studied the quality of segregation as a function of the shaking frequency and the density ratio. Furthermore, Shi *et al.*, presented results of experiments and numerical simulations on the horizontal transport and segregation of binary granular mixtures with different sizes and/or different densities in a vertically vibrated container [15]. They have shown that, depending on the diameter ratios, difference in densities and vibrating frequencies, the larger particles would migrate to one end of the container. Recently, Bhateja *et al.*, employed the mechanism proposed by some of us [5], for granular segregation in a ratchet corridor [16]. They have found that, the average interfacial pressure gradients acting over long distances causes a slowly axial separation of grains with different sizes [16].

In all the above-mentioned researches, except ref. [3], segregation is performed on a fixed amount of mixed grains in a closed container. On the contrary in our paper we introduce a method of continuously separating a binary mixture of identical spheres of different densities. Although, our system is similar to what proposed by Ji-Hong *et al.* [14], and is composed of a quasi–two-dimensional container with a sawtooth-shaped base; but, we have added some features to enable us to obtain a continuous segregation. This means that a flow of randomly mixed particles continuously enters the container and separates with a constant rate. We perform a molecular dynamic simulation to study the separation mechanism and determine the quality of separation as a function of different parameters such as direction of vibration and geometry of the sawtooth. Finally we experimentally study the separation of a granular mixture in the same system and demonstrate that our numerical results are in a good agreement with those of experiment.

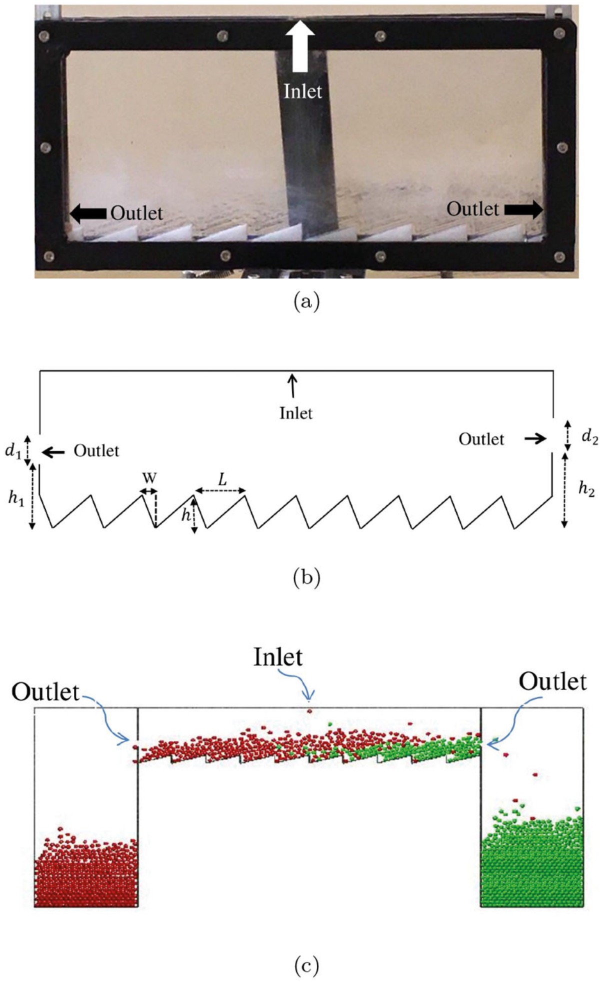

Fig. 1. The Hele-shaw cell. Front view (a) and its schematical representation (b) as well as visualization of the setup used in the simulation (c). Grains enter randomly through an inlet at the top middle of the cell. Separated grains exit from the gates on opposite sides as shown on the image. The geometrical parameters of the setup are as follow: $h = 1.4\,\mathrm{cm}$, $L = 4.3\,\mathrm{cm}$, $d_1 = d_2 = 9\,\mathrm{mm}$, $h_1 = 29\,\mathrm{mm}$, and $h_2 = 38\,\mathrm{mm}$.

2 Simulation methodology

The setup we use for studying the segregation is composed of a vertically mounted Hele-Shaw cell with a ratchet base (fig. 1). The distance between the two walls of the container is slightly larger than the diameter of the beads ($d = 6\,\mathrm{mm}$); so the beads are able to move freely in the container. A binary mixture of spherical beads of identical shape and size with differences in their density and/or friction coefficient is used as a granular mixture. A snapshot of the initial configuration of the particles is presented in fig. 2(a). We vibrate the system either vertically or horizontally and study the segregation mechanism for each case. Two gates are located at the two sides of the chamber to let the particles leaving the container. The gates are closed for about 30 seconds after starting the shaking in order that domain decomposition occur. After that, the two side gates are opened, and let the grains exit from the different sides of the system. Meanwhile, a flow of randomly mixed grains is inserted continuously from the top of the container with a flow rate equal to the exiting rate from both gates. In that way, the number of grains in the container remains constant, and the separation procedure will be carried out continuously. Symmetry breaking by the sawtooth-shaped base is the key parameter for segregation, therefore, we define an asymmetry parameter as $\kappa = W/L$ where, L is the spatial period of the sawtooth and W is shown in fig. 1(b). When $W = 0$ the teeth are right triangles while for $W = L/2$ they are isosceles triangles. Therefore, in our study, κ is a dimensionless parameter that varies from 0 for an asymmetric system (fig. 5(a)) to 1/2 for fully symmetric one (fig. 5(f)). The container consists of 10 identical teeth.

We perform a molecular dynamics simulation using a simplified version of the Hertzain model [17, 18]. In our simulation, the normal force between bead i and bead j is given by

$$\mathbf{F}_{n_{i,j}} = \left(\frac{\delta_{i,j}}{d}\right)^{1/2} \left(k_n \delta_{i,j} \mathbf{n}_{i,j} - \gamma_n m_{eff} \mathbf{v}_{n_{i,j}}\right), \qquad (1)$$

Table 1. Numeric values of the simulation parameters. m indicates the mass of the grains, R is the radius, E is Young's modulus, and ν is Poisson's ratio.

Material	m (g)	R (mm)	E (Pa)	ν
Steel	0.872	3	2.16×10^{11}	0.276
Plastic	0.093	3	2×10^{9}	0.24

Table 2. Friction coefficients used in the simulations. The first letter of the subscripts refers to dynamic (d) *versus* static (s), the second letter is the particle material, s for steel and p for plastic and the third letter is particle material or wall (w). The friction coefficients are determined by matching the simulation results to the experimental data.

μ_{sss}	μ_{spp}	μ_{ssp}	μ_{dsw}	μ_{dpw}	μ_{ssw}	μ_{spw}
0.14	0.17	0.18	0.5	0.4	0.8	0.94

where $\mathbf{v}_{n_{i,j}}$ is the normal component of the relative velocity between the two beads, evaluated at the contact point, $\mathbf{n}_{i,j}$ is the unit vector corresponding to the distance $r_{i,j}$ between the two beads centers, d is the particle diameter, m_{eff} is the effective mass $(m_i m_j/(m_i + m_j))$, and $\delta_{i,j} = d - r_{i,j}$ is the normal compression at contact. In our simulation the density of heavy particles is set to $7.714\,\mathrm{g/cm^3}$ (density of steel) and for the light particles we consider the density of plastic ($0.822\,\mathrm{g/cm^3}$). The elastic constant is numerically calculated based on the following relation: $k_n = \frac{4E'\sqrt{R_{eff}}}{3}$, where $\frac{1}{E'} = \frac{1-\nu_i^2}{E_i} + \frac{1-\nu_j^2}{E_j}$, $\frac{1}{R_{eff}} = \frac{1}{R_i} + \frac{1}{R_j}$ [18]. Here E is the Young modulus and ν is the Poisson ratio (the corresponding values are listed in table 1). The normal viscoelastic constant is $\gamma_n = -2\ln e\sqrt{\frac{k_n m_{eff}}{(\ln e)^2+\pi^2}}$, where e is the coefficient of restitution. The same equations are used to model the interaction between the grains and the channel walls, the corresponding coefficients have a subscript "w". In table 2 the numerical values for friction coefficients used in the simulations are shown. The mass of the container is set to infinity, consequentially $m_{eff} = m$.

Friction usually plays an important role in the collective behavior in granular materials. Indeed, the difference between friction coefficients of distinct grains is a key factor in segregation. In order to model the shear force between the two grains due to friction, we use a force law proposed by Haff and Werner [18, 19];

$$\mathbf{F}_{t_{i,j}} = -\,\mathrm{sign}(\mathbf{v}_{t_{i,j}}) \cdot \min(\gamma_t|\mathbf{v}_{t_{i,j}}|, \mu_d|\mathbf{F}_{n_{i,j}}|), \qquad (2)$$

where $\mathbf{v}_{t_{i,j}}$ is the relative velocity of spheres at the point of contact, $\gamma_t = 2g/s$ is the tangential viscoelastic constant, and μ_d is the relative friction coefficient. The first argument of the minimum function is the linear shear damping tangential force which grows linearly with $\mathbf{v}_{t_{i,j}}$ while the second one is Coulomb's friction law. Here, the shear force is either smaller than Coulomb's friction law for small relative velocities and large normal force, or equal to Coulomb's friction for the opposite case. Therefore, the shear force is in agreement with Coulomb's friction low in general.

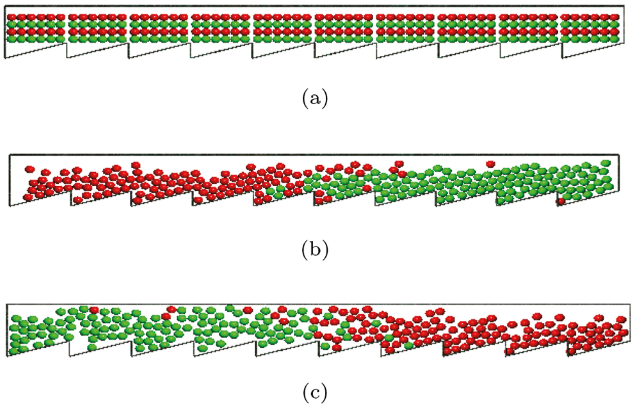

Fig. 2. (a) Initial position of the grains in the simulation box, light particles are presented in red while the heavy ones are green. Separation caused by the horizontal vibration (b), and by the vertical one (c) for the following parameters: $f = 15\,\mathrm{Hz}$, $x_0 = 0.5\,\mathrm{cm}$, $\kappa = 0$.

The tangential force exerted by the cell bottom and walls on a grain is evaluated by a model proposed by Kondic [20]. This model distinguishes between sliding and rolling contacts by assuming the contact is a rolling contact at the first step. Therefore, the acceleration of the sphere is taken in the frame of the laboratory and can be computed as $\mathbf{a} = 2/7\mathbf{a}_s$ in which $\mathbf{a}_s$ is the acceleration of the surface. This could be the final step if the no-sliding condition, $|\mathbf{F}_t| \leqslant \mu_s|\mathbf{F}_n|$ is satisfied (here, μ_s is the static coefficient of friction). Otherwise, the tangential force has to be evaluated using the dynamic friction coefficient μ_d:

$$\mathbf{F}_t = -\mu_d|\mathbf{F}_n|\frac{\mathbf{v}_w}{|\mathbf{v}_w|}\,, \qquad (3)$$

where $\mathbf{v}_w$ is the relative velocity of the contact point respect to the wall.

The shaking of the container is modeled by a sinusoidal excitation $a_i(t) = a_0\cos(\omega t)$, where i indicates on the vibration direction, ω is the oscillation frequency and a_0 is its amplitude. The code is written in C++ using a 5th-order predictor-corrector algorithm for numerical integration of the equations of motion. The time step increment (Δt) is set to 10^{-5} s in all simulations. In our simulation we use a restitution coefficient of 0.7 and 0.87 for steel and plastic, respectively.

Shaking the container causes random mixing of the grains at the early stages of the simulation, then continuing the procedure results in segregation of the particles at later time, as it has been shown in figs. 2(b) and (c) for horizontal and vertical vibration, respectively.

In order to quantify the separation process, we define a separation factor (q), which is an averaged ratio of difference between the two kinds of grains separated during the process, normalized by the total number of grains:

$$q = \frac{|N_h - N_l|_{right} + |N_h - N_l|_{left}}{|N_h + N_l|_{right} + |N_h + N_l|_{left}} \times 100\%. \qquad (4)$$

Here we divide the container into left and right sections which are represented by "left" and "right" indices, and

N_h and N_l are the number of heavy and light particles, respectively. Since the initial number of heavy and light particles are roughly equal, in case of complete separation of the particles to the left and right sides the quality factor would be 100%.

3 Results and discussion

3.1 Separation mechanism

The separation mechanism is a combination of two distinct mechanisms: separation due to difference in density (*i.e.*, gravity separation), and a granular convection directed by the asymmetric base of the container. At the early stages of the process, the light particles are pushed upwards by the heavy ones and mainly occupy the upper layers. By continuing the process, the heavy grains collide with the sawtooth walls. The asymmetry of the teeth, causes an asymmetric velocity distribution on the particles. For example, for $\kappa = 0$ as is schematically presented in fig. 2(b), and for horizontal shaking, in the first half period when the container is moving to the left, the force imposed by the left side of a sawtooth has both horizontal and vertical components, whereas, in the second half period the imposed force by the right side has just the horizontal component. This difference between imposed forces causes a net drift of heavy grains to the right direction, which in turn, imposes a granular convection in the whole system. Convection moves the heavy grains to the right and the light ones to the left, resulting in the separation of particles. The granular convection, however, could not continue by continuing the shaking, because the light grains could not push the heavy ones upward. Therefore, the system is locked in the separation phase and the global motion in grains can not occur anymore.

To substantiate the above conjecture, we examined the vertical vibration of the Hele-Shaw cell. In the case of vertical shaking and $\kappa = 0$, the left side of a sawtooth does not interact with any grain, therefore, does not impose any momentum on the grains. In this case, heavy grains tend to move rightward, and the separation should occur in the opposite direction, as is shown in fig. 2(c). In order to quantize this we define Δx_{cm}, the difference between the center of mass of two distinct grains as follows:

$$\Delta x_{cm} = x_{cm,l} - x_{cm,h}, \qquad (5)$$

where $x_{cm,l}$ and $x_{cm,h}$ are the center of mass of light and heavy grains, respectively.

Figure 3 is a plot of Δx_{cm} as a function of time for both vertical and horizontal vibration. In both cases, the relative center of mass is zero at the beginning of the simulation, indicating a uniform mixing of both grains. Δx_{cm} starts to grow relatively fast with time and after a few seconds the separation becomes complete. This is clear from fig. 3 by crossing from a growing to an approximately saturate regime in the Δx_{cm} curve. Note that in all simulations in this section, gates are closed and we study the domain decomposition phase of the system. The difference between the sign of Δx_{cm} at any time during the simulation is indicated by the formation of clusters of heavy grains in the opposite sides of the container during horizontal and vertical shaking. The same is true for the cluster of light grains, therefore, it is easy to observe that the mechanism of separation in vertical and horizontal shaking is different.

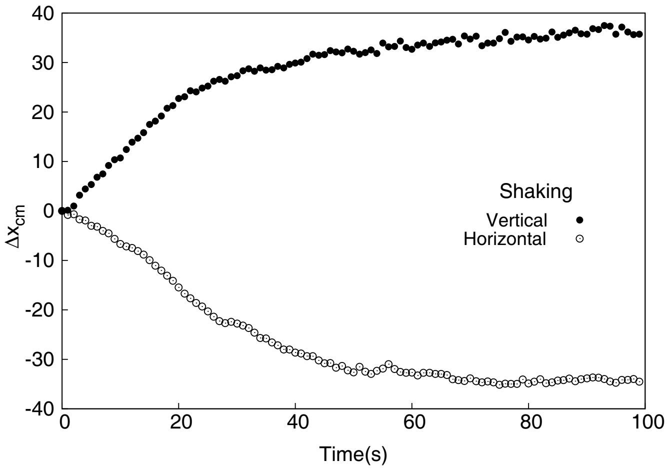

Fig. 3. Difference between the center of mass of the heavy and light grains as a function of time for the following parameters: $f = 15\,\mathrm{Hz}$, $x_0 = 0.5\,\mathrm{cm}$, $\kappa = 0$.

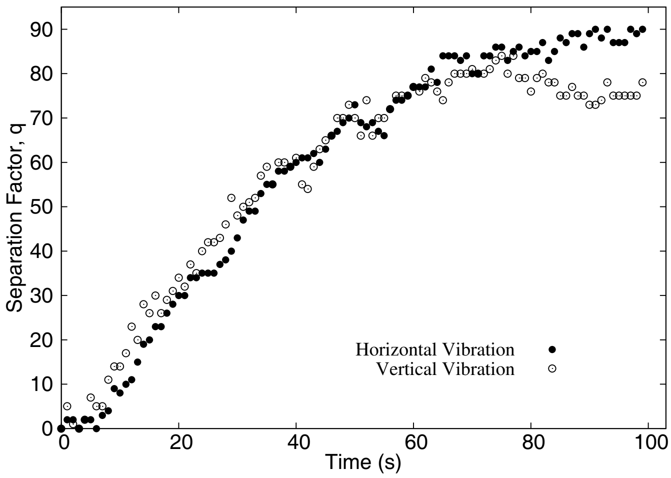

Fig. 4. Separation factor, q, as a function of time for both horizontal and vertical shaking for the following parameters: $f = 15\,\mathrm{Hz}$, $x_0 = 0.5\,\mathrm{cm}$, $\kappa = 0$.

In order to study the role of vertical and horizontal shaking in more details, we divide the container into two equal parts and compute the separation factor as the averaged ratio of the difference of two kinds of grains in the left (right) parts. Results are shown in fig. 4

At the beginning of the simulation, since grains are mixed, $q = 0$. The separation quality increases with time, indicating that domains of identical grains are formed. It finally reaches a constant value that indicates the system has arrived at a stationary state. The growth of q in time is roughly the same for both vertical and horizontal shaking.

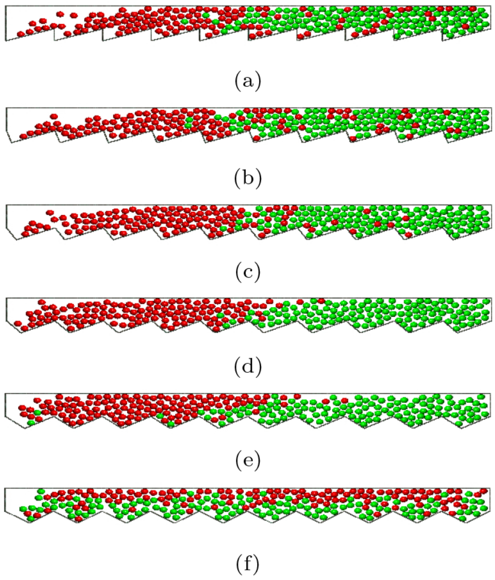

Fig. 5. Snapshots of segregation in containers with different asymmetry ratio. κ is equal to 0, 0.1, 0.2, 0.3, 0.4 and 0.5 from (a) to (f). Simulations are performed for horizontal shaking with frequency of 10 Hz and amplitues of 1 cm.

However, the final value of q at the stationary state is slightly larger for the horizontal shaking. It indicates that horizontal shaking provide a relatively higher separation efficiency with respect to the vertical shaking for the same geometrical parameters.

3.2 Role of the geometry of the sawtooth

Drift and granular convection are results of the symmetry breaking in the system. To study this effect in depth, we set up some simulations for systems with different asymmetric parameter, κ, as schematically represented in fig. 5. The quality of separation increases slightly by increasing the asymmetric factor as one can get from fig. 5. However, the separation suddenly vanishes for the symmetric system, (*i.e.*, $\kappa = 0.5$), which indicates the role of symmetry breaking in the separation process.

This is quantitatively described in fig. 6. The inset of fig. 6 is the separation factor, q, as a function of time for different asymmetry parameter, κ. The main plot represents the time average of the separation quality ($\langle q \rangle_t$) as a function of κ. $\langle q \rangle_t$ slightly increases by increasing κ. This is due to the facilitating effect inherent in the granular convection mechanism; increasing k and changing the slopes of the sawthooth structure facilitates convection of grains and results in a more effective separation. The separation process, however, will be halted if the asymmetry of the sawtooth vanishes.

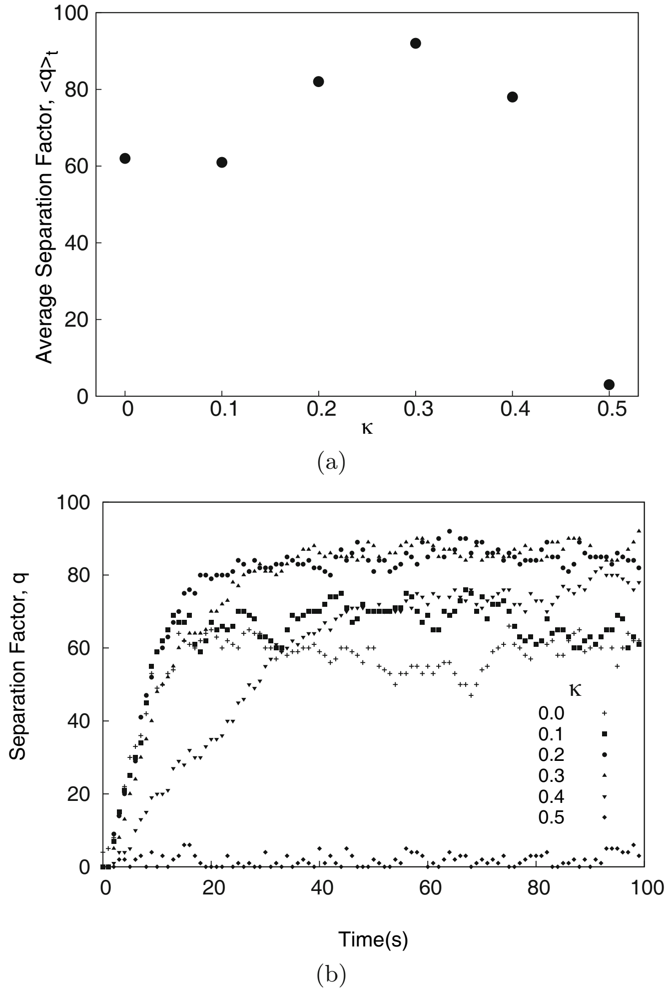

Fig. 6. (a) Average separation factor ($\langle q \rangle_t$) as a function of the asymmetry factor (κ). (b) Time evaluation of the separation factor for different κ.

3.3 Effect of friction on the separation process

In general friction can play important roles in the segregation mechanism in granular systems [21] by dissipating energy in the frictional contacts between the grains and the grains and the container walls. In order to check how friction between the grains and the container walls can affect the segregation mechanism in our system we changed the friction coefficient between side walls and the grains in the simulation. Results are shown in fig. 7. The segregation quality q is approximately constant for a wide range of friction coefficients, indicating the irrelevance of the side wall friction in the separation process. The separation process in our system is a dynamical mechanism and the dissipation is compensated by the energy injection through shaking. Therefore, the side wall friction dose not play an important role, as is clear from fig. 7.

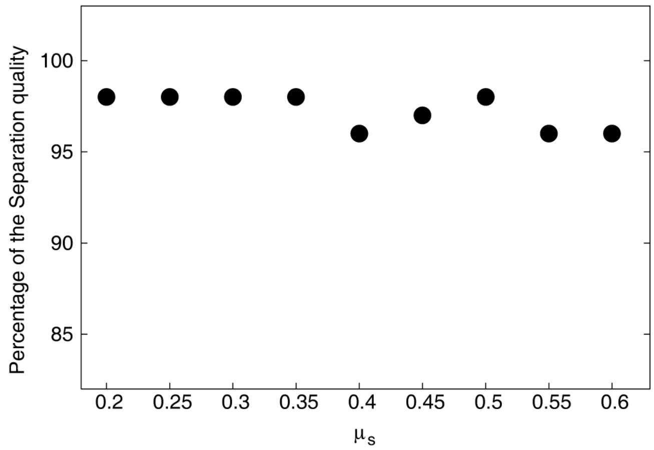

Fig. 7. Separation factor, q, as a function of the friction coefficient between the grains and the side walls of the Hele-Shaw cell.

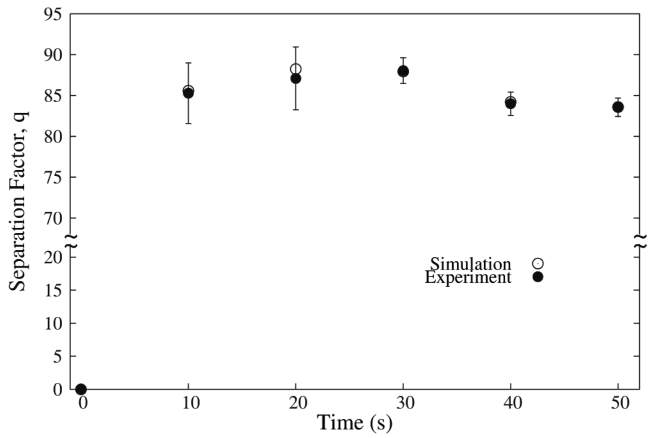

Fig. 8. Separation factor, q, as a function of time, measured in both simulation and experiment. q starts from zero at the beginning of the experiment (simulation), then grows quickly and reaches its maximum value immediately. The error bars are the standard deviation over 5 cycles. In both simulation and experiment the oscillations are performed in the horizontal direction with an amplitude of 1 cm and a driving frequency of 5.7 Hz.

4 Comparing the simulation with the experiment

Finally, we compare our simulation results with results from experiments in a similar system. The experimental setup is shown in fig. 1(a). In the experiments a granular mixture of steel and plastic particles with density of $7.714\,\text{g/cm}^3$ and $0.822\,\text{g/cm}^3$ and diameter of 3 mm is used. We run the simulation for density, elastic modulus and Poisson ratio of steel and plastic as shown in table 1. The experimental setup is the same as the setup used for the simulation. The asymmetry factor κ is set to zero in the experiment. The side walls of the container are separated by a distance slightly larger than the grains diameter preserving the quasi–two-dimensional nature of the system. The side walls and the sawtooth shaped floor of the Hele-Shaw cell are made from plexiglass. The Hele-Shaw cell is mounted vertically and shaken horizontally using an off-center pulley which is driven by an AC motor, as is shown in fig. 1(a). The oscillations are in horizontal direction with an amplitude of 1 cm and a driving frequency of 5.7 Hz. A randomly mixed blend of two kinds of spherical grains are initiated at the beginning of the experiment. The standard deviation over 5 cycles of experiments provides the error bars in our experiments.

Figure 8 represents the experimental and simulation results for the separation factor as a function of time. It takes time for the grains in the Hele-Shaw cell to start a convection flow from the horizontal shaking. Without such a flow, the chance of escaping for the grains from the container through the side gates is very low and separation does not occur at the early stages of the process. After the convection flow is established, the grains start to leave from the gates on both sides of the container and q grows in time. However, the separation factor reaches to its final value immediately and remains roughly constant in time which means that the system reaches a steady state, in which the segregation continues in time at a constant rate.

5 Conclusion

Using molecular dynamics simulation we have studied the segregation of shaken binary mixtures of identical spheres with different density in a vertically mounted Hele-Shaw cell with a sawtooth-shaped base. We have found that vertical and horizontal shaking provide different segregation mechanisms. Under the conditions that we studied, it appears that horizontal shaking often provides a slightly larger steady-state value of q with respect to the vertical shaking for the same geometrical parameters. Therefore, horizontal shaking is relatively efficient with respect to the vertical one. Our computational results show that the friction between the container walls and the grains has no effect on the segregation mechanism but the geometry of the sawtooth base will influence the segregation efficiency. Moreover, performing experiments on a similar system enables us to validate some of our computational results with the experimental data, which show very good agreement. We believe our results are of industrial importance and our simulation provides insight for designing efficient segregation methods especially from a practical point of view.

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Author contribution statement

ShM designed the research. NA performed the experiments under the supervision of ENO and MH. ShM performed the simulations and prepared the simulation figures. NA and ENO prepared the experimental figures. All authors contributed to the analysis and interpretation of the results and writing the manuscript.

References

1. Matthias Schröter, Stephan Ulrich, Jennifer Kreft, Jack B. Swift, Harry L. Swinney, Phys. Rev. E **74**, 011307 (2006).
2. D.C. Rapaport, Phys. Rev. E **64**, 061304 (2001).
3. Zn Farkas, Ferenc Szalai, Dietrich E. Wolf, Tams Vicsek, Phys. Rev. E **65**, 022301 (2002).
4. Matthias Schröter, Karen E. Daniels, arXiv:1206.4101.
5. Shahin Mobarakabadi, Ehsan Nedaaee Oskoee, Matthias Schröter, Mehdi Habibi, Phys. Rev. E **88**, 042201 (2013).
6. I. Derényi, P. Tegzes, T. Vicsek, Chaos **8**, 657 (1998).
7. Z. Farkas, P. Tegzes, A. Vukics, T. Vicsek, Phys. Rev. E **60**, 7022 (1999).
8. M. Levanon, D.C. Rapaport, Phys. Rev. E **64**, 011304 (2001).
9. J.F. Wambaugh, C. Reichhardt, C.J. Olson, Phys. Rev. E **65**, 031308 (2002).
10. A.J. Bae, W.A.M. Morgado, J.J.P. Veerman, G.L. Vasconcelos, Physica A **342**, 22 (2004).
11. Michael Heckel, Patric Müller, Thorsten Pöschel, Jason A.C. Gallas, Phys. Rev. E **86**, 061310 (2012).
12. KuanHua Chen, Y.C. Chou, Kiwing To, Phys. Rev. E **87**, 012711 (2013).
13. Fariba Mortazavi, Mehdi Habibi, Ehsan Nedaaee Oskoee, Eur. Phys. J. E **39**, 93 (2016).
14. Xia Ji-Hong, You Yu-Wei, Wang Pan-Pan, Wang Wei-Lu, Liu Chang-Song, Chin. Phys. B **19**, 056404 (2010).
15. Xiaodong Shi, Guoqing Miao, Hua Zhang, Phys. Rev. E **80**, 061306 (2009).
16. Ashish Bhateja, Ishan Sharma, Jayant K. Singh, Phys. Rev. Fluids **2**, 052301(R) (2017).
17. Leonardo E. Silbert, Deniz Erta, Gary S. Grest, Thomas C. Halsey, Dov Levine, Steven J. Plimpton, Phys. Rev. E **64**, 051302 (2001).
18. Thorsten Pöschel, T. Schwager, *Computational Granular Dynamics, Models and Algorithms* (Springer Science & Business Media, 2005).
19. P.K. Haff, B.T. Werner, Powder Technol. **48**, 239 (1986).
20. Ljubinko Kondic, Phys. Rev. E **60**, 751 (1999).
21. Hamed Maleki, Fatemeh Ebrahimi, Ehsan Nedaaee Oskoee, J. Stat. Mech.: Theor. Exp. **2008**, P04026 (2008).