



Fundamental prerequisites for realization of the quantum Zeno effect in the microwave and optical ranges

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Received 4 February 2022 / Accepted 7 August 2022 / Published online 7 September 2022
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Abstract. The main physical premises of the quantum Zeno effect in the problem of stopping the spontaneous radiative decay of excited atoms and nuclei are considered. It is shown for the first time that such an effect can be realized only for low-frequency radiative transitions (not higher than the microwave range), which correspond to the selective interaction of a quantum system with one or several discretely arranged modes of a quantized electromagnetic field in the volume of experimental setups. It is shown that this process occurs at the initial stage of the reversible Rabi precession and is formally unrelated to the true irreversible spontaneous decay. It is also shown that the effect of deceleration of radiative decay in the case of a quasi-continuous distribution of field modes is fundamentally impossible, which explains its absence for quantum sources of visible and shorter-wavelength radiation. In this case, the law of spontaneous radiative decay both at the beginning of the process and after a long time is described by the same standard exponential law. It has been shown for the first time that the quantum Zeno effect and stopping of spontaneous decay are possible in the infrared and visible ranges, provided that there are controlled sources inside the optical microresonators. These results can be used to create high-performance microlasers and to create optical systems for storing and processing information.

1 Introduction

Among the unique effects of modern physics, one of the most surprising is the quantum Zeno effect (QZE), which consists in the possibility of a very significant inhibition of spontaneous decay and related quantum phenomena with frequent nondestructive control of the initial excited state of a quantum system. The QZE was theoretically substantiated in 1977 in [1], and the idea of the possibility of such a process was proposed in [2].

This effect has been confirmed in many experiments on the control of the probability of radiative transitions between nearest atomic levels. In particular, it was observed in the system of energy levels formed during hyperfine splitting of the ground state of Be⁺ ions [3]. The frequency of the transition between these levels corresponded to the microwave range. Similar parameters (frequency 6.8–GHz) corresponded to experiments [4] on the control of the lifetime of excited rubidium atoms that make up a Bose–Einstein condensate.

It is very important that in these and other successful experiments this effect was observed only in systems whose radiative transition frequency corresponded to the microwave range [5–11]. All attempts to realize the QZE in systems with a significantly higher radiation

frequency (IR, visible and UV ranges, X-ray, gamma radiation) have not been successful.

This fact is usually justified by the reason that for higher frequencies the average spontaneous decay time sharply decreases and in experiments it is not possible to make a quick multiple check of the system until the radiative transition has taken place. However, this argument is not correct, for example, in the case of radiative transitions of higher multipolarity. In particular, for excited nuclei, such transitions can be characterized by a very long lifetime, and periodic monitoring of the excited state can be easily realized in a system of sublevels of the excited state. Such experiments correspond, for example, to the standard technique of Mössbauer spectroscopy, which is regularly used in different laboratories of universities and institutes, and such an effect would certainly be noticed.

This method of external influence on decay is fundamentally different from the theoretically predicted [12] and experimentally confirmed [13–15] method of controlling the probability of gamma decay of excited Mössbauer nuclei with a controlled influence on the decay process due to the close (at a distance of several cm) arrangement of a system of similar but unexcited nuclei near the investigated gamma-source.

The formal side of the QZE-based decay control method is based on the typical assumption (e.g., [4]) that the initial stage of the spontaneous decay process is

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characterized by the following expressions for the probability of a quantum system to remain in the initial excited state

$$W(t \ll 1/|B_{eg}|) = 1 - |B_{eg}|^2 t^2 \tag{1}$$

and the probability of transition to the ground state

$$1 - W(t \ll 1/|B_{eg}|) = |B_{eg}|^2 t^2. \tag{2}$$

Here B_{eg} is the coefficient characterizing the efficiency of the process that determines the transition of a given quantum system from the excited state e to the ground state g located below.

If we divide the time interval $[0, t]$ of the initial stage of spontaneous decay into a large number of identical subintervals $\Delta t = t/n$ for checking (inspection) of this state, then the partial probability of the absence of decay in each of them can be described by a similar formula

$$\Delta W = 1 - |B_{eg}|^2 \Delta t^2 \equiv 1 - |B_{eg}|^2 (t/n)^2. \tag{3}$$

The total probability of the system to remain in the initial excited state for time t is determined by the product of partial probabilities (3)

$$W(t = n\Delta t) = \{\Delta W\}^n \equiv \{1 - |B_{eg}|^2 (t/n)^2\}^n. \tag{4}$$

From (4), using the standard calculus formula $(1 - A/N)^N|_{N \rightarrow \infty} \rightarrow e^{-A}$ we can get the expression for the final probability

$$\begin{aligned} W(t) &= \{1 - (|B_{eg}|^2 t^2/n)/n\}^n \\ &= \exp(-|B_{eg}|^2 t^2/n)|_{n \rightarrow \infty} \rightarrow 1. \end{aligned} \tag{5}$$

It can be seen from (5) that in the case of repeated rapid checking of the initial state of the considered quantum system every $\Delta t = t/n$ the spontaneous decay completely stops.

The problem of realization of such a multiple check is a purely technological problem, and the physical substantiation of such a process is related to the validity of the expression used for the law of spontaneous decay (3).

It is obvious that this formula cannot be obtained in the limiting case of small time from the standard full formula for the probability of a quantum system to remain in the initial excited state and its form, which determines the same total probability at the beginning of the decay process

$$W_s(t) = e^{-t/\tau}, \quad W_s(t \rightarrow 0) = 1 - t/\tau. \tag{6}$$

Here τ the quantity is equal to the average lifetime of the excited state of the atom in such a system.

This conclusion follows directly from a simple analysis based on the same technique for dividing the interval

$[0, t]$ into a large number n of small intervals with duration $\Delta t = t/n$, as it was used to obtain (5)

$$W_s(t) = \{1 - t/n\tau\}^n|_{n \rightarrow \infty} = e^{-t/\tau}. \tag{7}$$

The obvious contradiction between the results (5) and (7), referring to the same process, can be resolved with a detailed analysis of the validity of mutually inconsistent formulas (2) and (6).

The traditional assumption about the cause of this contradiction consists in the statement that the nature of the spontaneous decay of atoms and other quantum systems for large and intermediate times corresponds to an exponential law, and for very short times the character of decay is fundamentally different from such a law. It is shown below that the contrasting of these features is incorrect, since they refer to different radiation processes (irreversible spontaneous decay and reversible Rabi oscillation). The main reason for this contradiction is associated with the specifics of the interaction of a particular quantum system (atom, nucleus, molecule) with the surrounding quantized field in a vacuum state at different radiation transition frequencies.

2 Dynamics of the initial stage of the process of interaction of a two-level quantum system with a quantized electromagnetic field

Let us consider the dynamics of the process of interaction of the atom with a quantized electromagnetic field using the well-known Weisskopf–Wigner method (e.g., [12]). Speaking further about the atom, we are keeping in mind that the same calculations apply equally to the nucleus or the molecule.

The Hamilton operator of the complete system (atom and field) has the form

$$\widehat{H}(\vec{r}) = \widehat{H}_A(\vec{r}_A) + \widehat{H}_F(\vec{q}) + \widehat{V}(\vec{r}_A, \vec{q}). \tag{8}$$

In this expression \widehat{H}_A is the Hamilton operator of the atom, \widehat{H}_F is the Hamilton operator of the quantized electromagnetic field surrounding the atom, and $\widehat{V} = \sum_{\beta} \widehat{V}_{\beta}$ is the operator of the interaction of the atom with the modes of this field.

The individual excited and ground states of the atom and the field are characterized, respectively, by the indices a and 0 , β and 0 . The coordinates characterizing the processes related to the atom and field are \vec{r}_A and \vec{q} .

The spontaneous decay process characterizes the dynamics of energy transfer from an initially excited atom to an initially unexcited system of field modes.

The system corresponding to the process of spontaneous decay in free space can be in two fundamentally different (and mutually orthogonal) base states, which are eigenfunctions of the unperturbed (without taking into account the interaction) Hamilton operator

$$\widehat{H}_0(\vec{r}) = \widehat{H}_A(\vec{r}_A) + \widehat{H}_F(\vec{q}).$$

a) The state of the system when the atom is excited and has energy $E_a = \hbar\omega_a$, and all field modes are in vacuum states. In this state, the general coordinate wave function of the entire system (in the cast that the atom does not interact with the field) is the product of the wave function of the atom $\Psi_a(\vec{r}_A)$ and the wave functions of all modes of the field $\varphi_0(\vec{q})$ and has the form

$$\begin{aligned} \Psi_{a0}(\vec{r}) &= \Psi_a(\vec{r}_A)\varphi_{\{0,0,\dots,0\}}(\vec{q}); \\ \varphi_{\{0,0,\dots,0\}}(\vec{q}) &= \prod_{\beta} \varphi_{\{0\beta\}}(\vec{q}). \end{aligned} \tag{9}$$

The subscript in the expression for the wave function of the atom $\Psi_a(\vec{r}_A)$ corresponds to the excited state of the atom with energy $E_a = \hbar\omega_a$. Accordingly, the subscript in the expression for the total wave function of the field $\varphi_{\{0,0,\dots,0\}}(\vec{q})$ means that all mutually independent modes of the field are in the lowest (vacuum) state.

b) One of the large number of possible states of the same system, when after the spontaneous transition the atom is in the ground state with the energy E_g , one of the field modes is excited and has energy $E_{n_{\beta}=1}$, and all other field modes ($\alpha \neq \beta$) are at ground levels (in vacuum states) $E_{n_{\alpha}=0}$. In each of these states, the coordinate wave function of the entire system has the form

$$\begin{aligned} \Psi_{0\beta}(\vec{r}) &= \Psi_0(\vec{r}_A)\varphi_{\{0,\beta,\dots,0\}}(\vec{q}); \\ \varphi_{\{0,\beta,\dots,0\}}(\vec{q}) &= \varphi_{\{1\beta\}}(\vec{q}) \prod_{\alpha \neq \beta} \varphi_{\{0\alpha\}}(\vec{q}). \end{aligned} \tag{10}$$

The subscript in the expression for the wave function of the atom $\Psi_0(\vec{r}_A)$ means that the atom is in the ground state. Accordingly, the subscript in the expression for the total wave function of the field $\varphi_{\{0,\beta,\dots,0\}}(\vec{q})$ means that the field mode with the number β is in the excited state, and other field modes ($\alpha \neq \beta$) are unexcited. It is easy to verify that there are no other possible states of the entire system in the case of only one excitation "wandering" in the system.

For the zero value of the system energy we can take the value corresponding to the presence of the atom and all field modes in the ground state (E_0 and $E_{n_{\beta}=0}$). For this, in further calculations, we use such a scale of energy reference, at which the total energy of the ground state of the entire system is equal to zero

$$E_0 + \sum_{\beta} E_{n_{\beta}=0} = 0. \tag{11}$$

In accordance with the principle of superposition, the general wave function of the entire system $\Psi(\vec{r}, t)$, including the atom and the quantized field, has the form

$$\begin{aligned} \Psi(\vec{r}, t) &= A(t)\Psi_{a0}(\vec{r})e^{-iE_a t/\hbar} + \sum_{\beta} F_{\beta}(t)\Psi_{0\beta}(\vec{r})e^{-iE_{\beta} t/\hbar} \\ &= A(t)\Psi_a(\vec{r}_A)\varphi_{\{0,0,\dots,0\}}(\vec{q})e^{-iE_a t/\hbar} \\ &\quad + \sum_{\beta} F_{\beta}(t)\Psi_0(\vec{r}_A)\varphi_{\{0,\beta,\dots,0\}}(\vec{q})e^{-iE_{\beta} t/\hbar}. \end{aligned} \tag{12}$$

In this expression, the time-dependent coefficients $A(t)$ and $F_{\beta}(t)$ are the amplitudes of the probability that the atom and each mode of the field are in the excited state at the time t . Basing on the principle of superposition and the requirement of completeness of the entire system, these coefficients are related by the normalization condition

$$|A(t)|^2 + \sum_{\beta} |F_{\beta}(t)|^2 = 1. \tag{13}$$

The change in these coefficients with time describes the process of spontaneous decay of the atom and, accordingly, the processes of excitation of each of the field modes. The evolution of the entire system and the dynamics of changes in these coefficients are described by the nonstationary Schrödinger equation

$$i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t} = \{\widehat{H}_A(\vec{r}_A) + \widehat{H}_F(\vec{q}) + \widehat{V}(\vec{r}_A, \vec{q})\}\Psi(\vec{r}, t). \tag{14}$$

Taking into account the explicit form of the results of the individual action of the constituent parts $\widehat{H}_A(\vec{r}_A)$ and the $\widehat{H}_F(\vec{q})$ of the general Hamilton operator on the wave functions of the atom and field modes, which are eigenfunctions of these constituent parts, this equation can be transformed:

$$\begin{aligned} \widehat{H}_A(\vec{r}_A)\Psi_a(\vec{r}_A) &= E_a\Psi_a(\vec{r}_A); \\ \widehat{H}_A(\vec{r}_A)\Psi_0(\vec{r}_A) &= E_0\Psi_0(\vec{r}_A) = 0; \\ \widehat{H}_F(\vec{q})\varphi_{\{0,0,\dots,0\}}(\vec{q}) &= E_0\varphi_{\{0,0,\dots,0\}}(\vec{q}) = 0; \\ \widehat{H}_F(\vec{q})\varphi_{\{0,\beta,\dots,0\}}(\vec{q}) &= E_{\beta}\varphi_{\{0,\beta,\dots,0\}}(\vec{q}). \end{aligned} \tag{15}$$

After substituting the general wave function (12) into the nonstationary Schrödinger equation and performing the listed actions, we obtain equation, which describes the related processes of the simultaneous evolution of the coefficients $A(t)$ and $F_{\beta}(t)$:

$$\begin{aligned} i\hbar \frac{\partial A(t)}{\partial t} \Psi_a(\vec{r}_A)\varphi_{\{0,0,\dots,0\}}(\vec{q})e^{-i\omega_a t} \\ + i\hbar \sum_{\beta} \frac{\partial F_{\beta}(t)}{\partial t} \Psi_0(\vec{r}_A)\varphi_{\{0,\beta,\dots,0\}}(\vec{q})e^{-i\omega_{\beta} t} \\ = \widehat{V}(\vec{r}_A, \vec{q})\{A(t)\Psi_a(\vec{r}_A)\varphi_{\{0,0,\dots,0\}}(\vec{q})e^{-i\omega_a t} \\ + \sum_{\beta} F_{\beta}(t)\Psi_0(\vec{r}_A)\varphi_{\{0,\beta,\dots,0\}}(\vec{q})e^{-i\omega_{\beta} t}\} \end{aligned} \tag{16}$$

Equation- (16) can be transformed into a system of simpler coupled algebraic equations. To do this, we can multiply this equation, respectively, once by the eigenfunction $\Psi_a^*(\vec{r}_A)\varphi_{\{0,0,\dots,0\}}^*(\vec{q})$, which corresponds to the excited state of the atom, and then integrate over the entire range of variation of the arguments, and the second time by one of the eigenfunctions $\Psi_0^*(\vec{r}_A)\varphi_{\{0,\beta,\dots,0\}}^*(\vec{q})$ corresponding to the excited state of one of the field modes (mode number β), and also integrate over the same area of arguments change.

Using the orthogonality of the eigenfunctions, we obtain the basic system of coupled equations

$$\frac{\partial A(t)}{\partial t} = \frac{1}{i\hbar} \sum_{\beta} F_{\beta}(t)V_{a\beta}e^{i(\omega_a-\omega_{\beta})t}, \tag{17}$$

$$\frac{\partial F_{\beta}(t)}{\partial t} = \frac{1}{i\hbar} A(t)V_{a\beta}^*e^{i(\omega_{\beta}-\omega_a)t}, \tag{18}$$

which describe the dynamics of a nonstationary change in the amplitudes of the excitation of the atom $A(t)$ and all modes of the field $F_{\beta}(t)$.

In this system, the matrix elements $V_{a\beta}$ of the interaction energy of the atom with the quantized field have the form

$$V_{a\beta} = \iint \Psi_a^*(\vec{r}_A)\varphi_{\{0,0,\dots,0\}}^*(\vec{q}) \widehat{V}(\vec{r}_A, \vec{q})\Psi_0(\vec{r}_A)\varphi_{\{0,\beta,\dots,0\}}(\vec{q})d^3r_A d^3q. \tag{19}$$

Let us analyze these equations using the initial conditions $A(0) = 1, F_{\beta}(0) = 0$ consistent with the conditions for observing of the QZE.

First, we consider the dynamics of changes of the state of the field and atom modes at $t \rightarrow +0$, i.e., at the beginning of the process. From Eq.- (18), we find an expression for the amplitude of excitation of the considered field mode β

$$\begin{aligned} F_{\beta}(t) &= \frac{1}{i\hbar} \int_0^t A(t')V_{a\beta}^*e^{i(\omega_{\beta}-\omega_a)t'} dt' \\ &= \frac{1}{i\hbar} \int_0^t \left\{ A(t')|_0 + \frac{dA(t')}{dt'}|_0 t' + \dots \right\} V_{a\beta}^*e^{i(\omega_{\beta}-\omega_a)t'} dt' \\ &\approx \frac{A(0)}{i\hbar} \int_0^t V_{a\beta}^*e^{i(\omega_{\beta}-\omega_a)t'} dt' \\ &= \frac{A(0)}{i\hbar} V_{a\beta}^* \frac{1 - e^{-i(\omega_{\beta}-\omega_a)t}}{i(\omega_{\beta} - \omega_{\alpha})} \end{aligned} \tag{20}$$

and the probability of this excitation $|F_{\beta}(t)|^2$, which can be written in two alternative forms:

$$|F_{\beta}(t)|^2 \approx |A(0)|^2 \frac{|V_{a\beta}|^2 \sin^2\left(\frac{\omega_{\beta}-\omega_{\alpha}}{2}t\right)}{\left(\frac{\omega_{\beta}-\omega_{\alpha}}{2}\right)^2}$$

$$\approx |A(0)|^2 \frac{|V_{a\beta}|^2}{\hbar^2} t^2, \quad t \ll \frac{\pi}{|\omega_{\beta} - \omega_{\alpha}|}, \tag{21}$$

$$|F_{\beta}(t)|^2 \approx |A(0)|^2 \frac{|V_{a\beta}|^2}{\hbar^2} 2\pi t \delta(|\omega_{\beta} - \omega_{\alpha}|), \quad t \gg \frac{\pi}{|\omega_{\beta} - \omega_{\alpha}|}. \tag{22}$$

The first of them can be used if we take into account only one mode of the electromagnetic field in the case of a large discreteness of the spectrum of modes and the second one in the alternative case of a quasi-continuous spectrum of the field modes.

In the case of interaction of the atom with only one mode of the field, from the conditions

$$|A(t)|^2 + |F_{\beta}(t)|^2 = 1, \quad F_{\gamma \neq \beta}(t) \equiv 0, \tag{23}$$

and formula (21) an expression for the probability of the atom to remain in an excited state for a short time t follows:

$$|A(t)|^2 = 1 - |F_{\beta}(t)|^2 \approx 1 - \frac{|V_{a\beta}|^2}{\hbar^2} t^2. \tag{24}$$

Accordingly, when the condition for the observation of the QZE (inspections of the state of the atom at very small time intervals $\Delta t = t/n$) is realized, we obtain from (24) evidence of the realization of this effect

$$\begin{aligned} |A(t)|^2 &= \left\{ 1 - \frac{1}{n} \frac{|V_{a\beta}|^2}{\hbar^2} (t^2/n) \right\}^n \Big|_{n \rightarrow \infty} \\ &= \exp \left\{ - \frac{|V_{a\beta}|^2}{\hbar^2 n} t^2 \right\} \Big|_{n \rightarrow \infty} = 1. \end{aligned} \tag{25}$$

It can be seen that in such a single-mode regime, formulas (21), (24), and (25) for the corresponding probabilities completely coincide both with the initial formulas (1) and (2) used in the analysis of the quantum Zeno effect, and with the formula (5), demonstrating the QZE.

A fundamentally different result corresponds to the case of interaction of the atom with an ensemble of modes with close frequencies.

To analyze this case, it is convenient to find the desired probability $|A(t)|^2$ of the atom to remain in an excited state using the normalization condition (13) and replacing the summation over modes in formula (17) by integration over the frequencies of these modes, taking into account the standard expression for the spectral density of modes of the electromagnetic field in the volume V_0 :

$$\rho(\omega_{\beta}) \equiv \frac{dN}{d\omega_{\beta}} = \frac{V_0 \omega_{\beta}^2}{\pi^2 c^3}. \tag{26}$$

When these relations are used, the expression for the probability of the atom to remain in the excited state a short time after the formation of such a state takes a fundamentally different form

$$\begin{aligned}
 |A(t)|^2 &= 1 - \sum_{\beta} |F_{\beta}(t)|^2 \\
 &\approx 1 - |A(0)|^2 \frac{1}{\hbar^2} \int_0^{\infty} \frac{dN}{d\omega_{\beta}} |V_{a\beta}|^2 2\pi t \delta(|\omega_{\beta} - \omega_{\alpha}|) d\omega_{\beta} \\
 &= 1 - |A(0)|^2 \frac{2V_0\omega_{\alpha}^2}{\pi c^3 \hbar^2} |V_{a\beta}|^2 t. \tag{27}
 \end{aligned}$$

If we use the same method, which was used to obtain (5), dividing the interval $[0, t]$ into a large number n of very small subintervals with duration t/n , then on the basis of dependence (27) we can get the standard formula describing the process of spontaneous decay

$$\begin{aligned}
 |A(t)|^2 &= \left\{ 1 - \frac{1}{n} \left(\frac{2V_0\omega_{\alpha}^2}{\pi c^3 \hbar^2} |V_{a\beta}|^2 t \right) \right\}^n \Big|_{n \rightarrow \infty} = e^{-t/\tau}, \\
 \tau &= \frac{\pi c^3 \hbar^2}{2|V_{a\beta}|^2 V_0 \omega_{\alpha}^2}, \tag{28}
 \end{aligned}$$

which has the structure, that is fully consistent with formula (6) and completely excludes the QZE. This formula can be modified taking into account that in the dipole approximation, the interaction energy of the atom with a specific mode of a quantized electromagnetic field, which is in a vacuum state, is determined by the standard expression

$$V = -\vec{d}\vec{E}_{\beta} = -(\vec{d}\vec{e}_k)E_{\beta}. \tag{29}$$

Here $\vec{d} = e \vec{r}$ is the dipole moment of the atom, \vec{e}_k is the unit polarization vector of the mode. After finding the matrix elements and the formal procedure for spatial averaging of the scalar product $(\vec{d}\vec{e}_k)$, we find

$$\langle |V_{a\beta}|^2 \rangle = \langle |\vec{d}_{a\beta}\vec{e}_k|^2 \rangle \langle |E_{\beta}|^2 \rangle. \tag{30}$$

The value

$$\langle |E_{\beta}|^2 \rangle = \frac{2\pi\hbar\omega_{\beta}}{V_0} \tag{31}$$

can be found if we take into account that the total energy of the mode in the ground (vacuum) state is determined by the product of the average energy density of the electromagnetic field in the vacuum state $\hbar\omega_{\beta}/2$ by the volume of the mode V_0 . After spatial averaging of the scalar product $(\vec{d}\vec{e}_k)$ in (30), we find

$$\langle |V_{a\beta}|^2 \rangle = \frac{2\pi\hbar\omega_{\beta}|\vec{d}_{a\beta}|^2}{3V_0}. \tag{32}$$

Taking this result into account, expression (28) for the average lifetime of an excited state of the atom τ

in such a multimode system takes the final form

$$\tau = \frac{3c^3\hbar}{4|\vec{d}_{a\beta}|^2\omega_{\alpha}^3}. \tag{33}$$

It is very important that in such a multimode interaction regime, expression (33) (and expressions (28) $W(t) = |A(t)|^2 = e^{-t/\tau}$ for the probability of an excited state of the atom) does not depend on the magnitude of the quantization volume V_0 and does not depend on periodic inspection of the state of an excited atom, which is the standard method for the QZE formation.

It can be seen from this result that at the interaction of the atom with the field in the multimode regime the QZE is completely absent! For a more detailed conclusion about the reasons for the fundamentally different mechanisms of spontaneous decay in single- and multimode regimes, we consider the same problem without being limited by the requirement of a short time after the formation of an excited state of the atom.

3 The full time dynamics of the process of interaction of a two-level quantum system with a quantized electromagnetic field

An analysis of the complete, without time limitation, dynamics of the interaction of a quantized electromagnetic field with a two-level quantum system corresponds to the general solution of the system of Eqs.- (17) and (18). This solution is found using the method of direct and inverse integral Laplace transforms, which make it possible to transform these differential equations for the probability amplitudes $A(t)$, $F_{\beta}(t)$ into a system of algebraic equations for the images of these amplitudes.

For such a transformation, first we multiply each of these equations by a value e^{-pt} , and then we integrate new equations with respect to the variable p in the range from 0 to ∞ .

We can introduce image functions

$$A_p = \int_0^{\infty} A(t)e^{-i\omega_{\alpha}t}e^{-pt}dt, \quad F_{\beta p} = \int_0^{\infty} F_{\beta}(t)e^{-i\omega_{\beta}t}e^{-pt}dt \tag{34}$$

on combinations of initial functions $A(t)e^{-i\omega_{\alpha}t}$ and $F_{\beta}(t)e^{-i\omega_{\beta}t}$. After integration by parts, the system of these equations takes the form

$$(i\omega_{\alpha} + p)A_p = A(0) + \frac{1}{i\hbar} \sum_{\beta} F_{\beta p} V_{a\beta}, \tag{35}$$

$$(i\omega_{\beta} + p)F_{\beta p} = F_{\beta}(0) + \frac{1}{i\hbar} A_p V_{a\beta}^*. \tag{36}$$

In this system of the algebraic equations values $A(0) = 1$ and $F_\beta(0) = 0$ correspond to the initial excited state of the atom and the initial unexcited state of an arbitrary field mode. From this system, we can find the explicit form of the image function for the probability amplitude of the excited state of the atom

$$A_p = \frac{A(0)}{i\omega_a + p + \sum_\beta \frac{|V_{a\beta}|^2}{\hbar^2(i\omega_\beta + p)}}, \tag{37}$$

and then, using the inverse Laplace transform (34), we can find a solution for $A(t)$:

$$\begin{aligned} A(t) &= \frac{1}{2\pi i} e^{i\omega_a t} \int_{\delta-i\infty}^{\delta+i\infty} A_p e^{pt} dp \\ &= \frac{A(0)}{2\pi i} \int_{\delta-i\infty}^{\delta+i\infty} \frac{e^{(i\omega_a + p)t} dp}{i\omega_a + p + \sum_\beta \frac{|V_{a\beta}/\hbar|^2}{(i\omega_\beta + p)}}. \end{aligned} \tag{38}$$

After changing the variable $p = i(\omega - \omega_a)$ in the integrand, this formula takes the final form

$$A(t) = \frac{A(0)}{2\pi i} \int_{-\infty-i\delta}^{\infty-i\delta} \frac{e^{i\omega t} d\omega}{\omega - \sum_\beta \frac{|V_{a\beta}/\hbar|^2}{(\omega_\beta - \omega_a + \omega)}} \tag{39}$$

and corresponds to the general solution of the problem of finding the time-dependent characteristics of spontaneous decay for an arbitrary form of the distribution of quantized modes of the electromagnetic field. Specific cases of calculating (39) require to detail the explicit form of the spectrum of the field modes with which the atom interacts.

3.1 The full time dynamics of a two-level system in its interaction with one field mode

As it follows from the general solution of (39), the amplitude and probability of spontaneous decay substantially depend on the value of the sum $\sum_\beta |V_{a\beta}|^2 / [\hbar^2(\omega_\beta - \omega_a + \omega)]$ in the denominator of (39), which takes into account the interaction of each field mode with the considered atom. Let us consider the case when the absolute value of the intermode interval $\delta\omega = \omega_\beta - \omega_a$ in a particular electrodynamic system is so large that only one (resonance) term makes a significant contribution to this sum. All other (nonresonant) terms of the sum are small, and the following condition is satisfied:

$$\frac{|V_{a,\beta\pm\Delta\beta}|^2}{|\omega_{\beta\pm\Delta\beta} - \omega_a + \omega|} \ll \frac{|V_{a\beta}|^2}{|\omega_\beta - \omega_a + \omega|}. \tag{40}$$

It should be noted that the limitation of the number of modes influencing the process of interaction of the

atom with a field to only one mode is based not only on the fact that the denominators of the corresponding nonresonant terms of sum (40) decrease with the increase of the intermode distance. Obviously, such a decrease (according to the law $1/|\omega_{\beta\pm\Delta\beta} - \omega_a + \omega|$) by itself cannot ensure the convergence of the entire sum and, in the absence of other convergence mechanisms, would lead to a logarithmic discrepancy. The very rapid decrease in the nonresonant terms of this sum is associated, first of all, with a very rapid decrease in the corresponding matrix elements $|V_{a\beta}|$ for the nonresonant modes with increasing detuning from the optimal mode with $\Delta\beta = 0$. In a symbolic form, such a limitation of the number of field modes essential for the process of interaction with the atom is shown in Fig.- 1a.

Under condition (40), from the general solution (39) we obtain

$$\begin{aligned} A(t) &= \frac{A(0)}{2\pi i} \int_{-\infty-i\delta}^{\infty-i\delta} \frac{e^{i\omega t} d\omega}{\omega - \Omega_0^2 / (\delta\omega + \omega)}, \quad \delta\omega \\ &= \omega_\beta - \omega_a, \quad \Omega_0 = |V_{a\beta}| / \hbar. \end{aligned} \tag{41}$$

This expression can be easily converted to a more convenient form

$$A(t) = \frac{A(0)}{2\pi i} \int_{-\infty-i\delta}^{\infty-i\delta} \frac{(\delta\omega + \omega) e^{i\omega t} d\omega}{(\omega - \omega_1)(\omega - \omega_2)}, \tag{42}$$

where two characteristic frequencies

$$\omega_{1,2} = -\frac{\delta\omega}{2} \pm \Omega, \quad \Omega = \sqrt{\left(\frac{\delta\omega}{2}\right)^2 + \Omega_0^2} \tag{43}$$

are the poles of the integrand, which lie on the real axis of the complex frequency plane ω .

The explicit form of the solution for the probability amplitude $A(t)$ (42) can be easily found by the method of complex integration by the method of residues at these poles

$$A(t) = A(0) \left\{ \cos \Omega t + i \frac{\delta\omega}{2\Omega} \sin \Omega t \right\} e^{-i \frac{\delta\omega}{2} t}. \tag{44}$$

The expression for the probability of finding the atom in an excited state at any time $t \geq 0$ in the case of interaction with only one field mode is found from (44) and has the form

$$W(t) = |A(t)|^2 = 1 - \frac{\Omega_0^2}{\Omega^2} \sin^2 \Omega t. \tag{45}$$

In this case, the probability of excitation of the resonant mode is described by the formula

$$|F_\beta(t)|^2 = \frac{\Omega_0^2}{\Omega^2} \sin^2 \Omega t. \tag{46}$$

Fig. 1 Resonance interaction of the atom A with one (a) or two (b) discretely located modes of the electromagnetic field F . The frequency distribution function on the left corresponds to the spectrum of a particular discrete mode

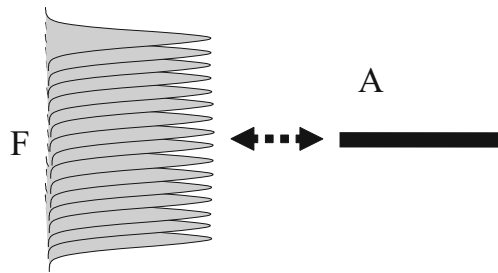
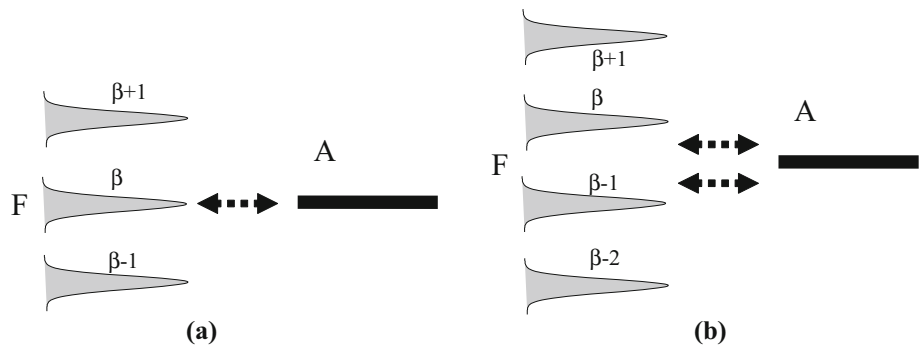


Fig. 2 The scheme of interaction of the atom A with a quasi-continuous ensemble of modes of an electromagnetic field F in a vacuum state

It can be seen from these results that the process of interaction of the atom with a resonant mode corresponds to a continuous oscillatory process with periodic excitation of the atom (transition from the ground state to the excited state) and subsequent reverse transition from the excited to the ground state. This process corresponds to a periodic exchange of energy between the atom and the field mode (Rabi precession).

The exchange frequency $\Omega = \sqrt{(\delta\omega/2)^2 + \Omega_0^2}$ and the amplitude of oscillations of the probability of an excited state of the atom depends both on the energy of interaction of the atom with the mode $|V_{a\beta}| = \Omega_0 \hbar$ and on the spectral detuning $\delta\omega$ from the resonance. In the case of exact resonance $\delta\omega = 0$ we have

$$|A(t)|^2 = \cos^2 \Omega_0 t, \quad |F_\beta(t)|^2 = \sin^2 \Omega_0 t. \quad (47)$$

The beginning of the process of such interaction is characterized by the expression

$$|A(t \ll 1/\Omega)|^2 \approx 1 - \Omega_0^2 t^2 = 1 - \frac{|V_{a\beta}|^2}{\hbar^2} t^2 = 1 - \frac{2\pi \hbar \omega_\beta |\vec{d}_{a\beta}|^2}{V_0} t^2, \quad (48)$$

which completely coincides with both formula (24) and the original formula (1) for the initial stage of the change in the state of the atom.

3.2 The full time dynamics of the interaction of a two-level system with two modes

Let us consider the case of interaction of the atom with two nearest in frequency field modes numbered β and $\beta + 1$ (Fig.- 1b). We assume that only modes with frequencies ω_β and $\omega_{\beta+1}$, for which the conditions of resonant interaction with the atom are satisfied

$$|\omega_a - \omega_\beta|, |\omega_a - \omega_{\beta+1}| \ll |\omega_a - \omega_{\beta-1}|, |\omega_a - \omega_{\beta+2}|, |\omega_a - \omega_{\beta+3}|, \dots \quad (49)$$

make a significant contribution to the sum, which is in the denominator of the expression for the general solution (39).

To simplify the calculation, we assume that the frequencies ω_β and $\omega_{\beta+1}$ are located symmetrically with respect to the frequency of the resonant transition in the atom

$$\omega_{\beta+1, \beta} = \omega_a \pm \delta\omega/2. \quad (50)$$

In a symbolic form, such conditions are shown in Fig.- 1b.

In addition, we assume that the interaction of these modes with the atom is characterized by the same matrix elements of the interaction energy and, accordingly, the same value of the effective interaction frequency

$$|V_{a\beta}| = |V_{a, \beta+1}| = \hbar \Omega_0 \quad (51)$$

Under these conditions, from the general solution (39), we find

$$A(t) = \frac{A(0)}{2\pi i} \int_{-\infty - i\delta}^{\infty - i\delta} \frac{(\omega^2 - (\delta\omega/2)^2) e^{i\omega t} d\omega}{\omega(\omega - \Omega_1)(\omega + \Omega_1)}, \quad (52)$$

where

$$\Omega_1 = \sqrt{2\Omega_0^2 + (\delta\omega/2)^2}. \quad (53)$$

The integrand in formula (52) has three poles at the points

$$\omega_1 = 0, \quad \omega_{1,2} = \pm\Omega_1 \tag{54}$$

on the real axis of the plane of complex frequency ω . Taking it into account, the probability amplitude $A(t)$ and total probability $|A(t)|^2$ of finding the atom in an excited state at moment of time $t \geq 0$ in the case of interacting with only two modes can be easily found by the complex integration by the method of residues

$$A(t) = A(0) \left\{ 1 - 4 \left(\frac{\Omega_0}{\Omega_1} \right)^2 \sin^2 \frac{\Omega_1}{2} t \right\}, \quad W(t) = |A(t)|^2. \tag{55}$$

This regime corresponds to the same Rabi precession. The beginning of the process of such interaction is characterized by the expression

$$|A(t \ll 1/\Omega_1)|^2 \approx 1 - 2\Omega_0^2 t^2 = 1 - 2 \frac{|V_{a\beta}|^2}{\hbar^2} t^2 = 1 - \frac{4\pi\hbar\omega_\beta |\vec{d}_{a\beta}|^2}{V_0} t^2, \tag{56}$$

which is similar in structure to the original formula (1) for the initial stage of the change in the state of the atom.

Thus, the process of interaction of the atom with one and two resonant modes is periodic (oscillating) and it has no damping. A qualitatively similar situation is in the case when the atom interacts with a small number of field modes, the frequencies of which are located asymmetrically with respect to the transition frequency of the atom. In this case, the law of energy exchange between the atom and the field modes differs from the simple harmonic law, which is typical for the interaction with one or two symmetrically located modes, but the main feature of the process remains unchanged. The exchange process is periodic, it has no damping, and at the beginning of system evolution it allows the existence of the QZE.

4 The full time dynamics of the interaction of the atom with a quasi-continuous distribution of electromagnetic field modes

Let us consider an alternative situation, when the field modes are located so densely that the resonance approximation based on the selective interaction of the atom with one or several separate modes can't be used. The analysis of such a problem has to be done basing on the general solution (39), in which the contribution of all field modes is determined by the sum

$$S = \sum_{\beta} |V_{a\beta}|^2 / [\hbar^2(\omega_\beta - \omega_a + \omega)], \tag{57}$$

which is in the denominator of formula (39) and takes into account the connection between the atom and the ensemble of modes. To calculate this sum, we need to make two steps. On the first step, we need to make the procedure of averaging of the mutual orientation of different field modes with respect to the matrix element (30) (the mutual orientation of the dipole moment and polarization vectors of the field modes in the case of dipole transitions), which is responsible for a specific transition in the atom. On the second one, we need to integrate the quantity $|V_{a\beta}|^2/\hbar^2(\omega_\beta - \omega_a + \omega)$ using the spectral density of the modes of the quantized field $\rho(\omega_\beta)$ (26). In the case of dipole transitions with such a replacement sum (57) corresponds to the integral

$$S = \int_0^\infty \rho(\omega_\beta) \frac{\langle |V_{a\beta}|^2 \rangle d\omega_\beta}{\hbar^2(\omega_\beta - \omega_a + \omega)} = \frac{2}{3\pi\hbar c^3} \int_0^\infty \frac{|\vec{d}_{a\beta}|^2 \omega_\beta^3 d\omega_\beta}{(\omega_\beta - \omega_a + \omega)}. \tag{58}$$

The convergence of the integral in expression (58) is ensured by the fact that the matrix element of the dipole moment of the atom $\vec{d}_{eg}(\omega_\beta)$, which determines the explicit form of the matrix element of the interaction energy of a particular mode with the atom (19), decreases exponentially with an increase of the frequency difference $|\omega_\beta - \omega_a| \rightarrow \infty$ due to the presence of a rapidly changing phase in the integrand. This decrease at $\omega_\beta \rightarrow \infty$ is much faster than the growth determined by the function ω_β^3 in the numerator of the integrand.

The presence of such an exponential decrease is the part of the standard procedure for obtaining the dipole approximation in the theory of the interaction of an electromagnetic field with atoms. In this case we can neglect the change of the phase of the electromagnetic field $\vec{E}_\beta \sim e^{-i\vec{k}_\beta \vec{r}}$ while calculating the matrix element (19) of the energy of interaction of the field with the atom

$$V_{a\beta} = \int \Psi_a^*(\vec{r}) \left(\vec{E}_\beta \vec{p} \right) \Psi_\beta(\vec{r}) d^3r \sim \int \Psi_a^*(\vec{r}) e^{-i\vec{k}_\beta \vec{r}} (\vec{e}_\beta \vec{p}) \Psi_\beta(\vec{r}) d^3r \sim \int \Psi_a^*(\vec{r}) e^{-i\vec{k}_\beta \vec{r}_a} (\vec{d}_{a\beta} \vec{e}_\beta) \Psi_\beta(\vec{r}) d^3r = e^{-i\vec{k}_\beta \vec{r}_a} (\vec{d}_{a\beta} \vec{e}_\beta), \tag{59}$$

in the volume of atom and replace it by 1. It is obvious that it is possible to do so only in the case when the field frequency satisfies the condition

$$|\vec{k}_\beta(\vec{r} - \vec{r}_a)|_{\max} = \omega_\beta |(\vec{r} - \vec{r}_a) \vec{k}_\beta / ck_\beta|_{\max} \leq \omega_\beta R_a / c \ll 1. \tag{60}$$

This condition can be satisfied when the field frequency ω_β , which is the integration variable in (58), is

limited by the maximum value $|\omega_\beta| \ll c/R_a$, where R_a is the effective size of the atom. The same situation is for transitions of arbitrary multipolarity.

The opposite case $|\omega_\beta| \gg c/R_a$ corresponds to the formal expansion of the integration limits $|\omega_\beta| \rightarrow \infty$ in the process of calculating the complex integral in expression (58). It is obvious that in this case the phase factor becomes a rapidly oscillating function of frequency, leading to an exponential decrease of the interaction energy. Based on these conditions, the limits of integration in (58) can be extended to the interval $[-\infty, \infty]$.

From the analysis of the formula (39), it can be seen that the value $A(t)$ is determined by the poles of the integrand at one or several values of the complex frequency $\omega = \omega' + i\omega''$. The positions of these poles, which determine the temporal change of the amplitude of the probability of finding the atom in an excited state, are characterized by small values of ω' in comparison with the frequency ω_a , which determines the temporal change of the wave function itself. This circumstance follows directly from (39). In mathematical physics, it is shown that the function $(\omega_\beta - \omega_a + \omega)^{-1}$, which is part of the integrand in (39), in the case of small values of ω' corresponds to δ -function and can be represented in the form

$$\frac{1}{(\omega_\beta - \omega_a + \omega' + i\omega'')} = \pi i \delta(\omega_\beta - \omega_a + \omega') + \widehat{P} \frac{1}{(\omega_\beta - \omega_a + \omega')}. \quad (61)$$

Here \widehat{P} is the operator of the Cauchy principal value.

Using this replacement in formula (58), we find

$$\begin{aligned} S &= i \frac{\Gamma_0}{2} + \Delta\omega_0; \\ \Gamma_0 &\equiv \frac{1}{\tau} = \frac{4(\omega_a - \omega')^3}{3\hbar c^3} |\vec{d}_{a\beta}(\omega_a - \omega')|^2 \approx \frac{4\omega_a^3}{3\hbar c^3} |\vec{d}_{a\beta}(\omega_a)|^2, \\ \Delta\omega_0 &= \widehat{P} \int_0^\infty \frac{2\omega_\beta^3 |\vec{d}_{a\beta}(\omega_\beta)|^2 d\omega_\beta}{3\pi\hbar c^3 (\omega_\beta - \omega_a + \omega)}. \end{aligned} \quad (62)$$

In this relation, the quantity $\Delta\omega_0$ determines the Lamb shift of the excited level of the atom.

Substituting (62) into the general solution for $A(t)$ (39), we find the final formula for the probability amplitude and total probability of the atom to remain in an excited state when it interacts with a quasi-continuous continuum of electromagnetic modes

$$\begin{aligned} A(t) &= \frac{A(0)}{2\pi i} \int_{-\infty - i\delta}^{\infty - i\delta} \frac{e^{i\omega t} d\omega}{\omega - \Delta\omega_0 - i\Gamma_0/2} \\ &= A(0) e^{-\frac{\Gamma_0}{2}t + i\Delta\omega_0 t}, \quad |A(t)|^2 = e^{-t/\tau}. \end{aligned} \quad (63)$$

The resulting expression (62) for the average lifetime τ of the excited state of the atom and the law of irreversible spontaneous decay (63) in such a multi-mode system are identical to expressions (28) and (33) obtained on the basis of the analysis of the decay process at the initial moment of time in the same multi-mode system. This result leads to the conclusion that it is impossible to realize the QZE in a system with multimode interaction of the atom with an electromagnetic field in the ground (vacuum) state.

5 On the possibility of realizing the quantum Zeno effect in IR and visible ranges

The analysis performed and the results obtained allow us to make a conclusion about the conditions for the realization of the QZE. It is shown that such an effect can be realized only under the condition that the considered quantum system (atom, nucleus, or other two-level quantum object) interacts with single mode or several discrete modes of the electromagnetic field in the ground (vacuum) state. In such systems, if we do not take into account the possible additional influence of other factors on these modes, the interaction process does not correspond to a unidirectional monotonic spontaneous decay, but has the form of a pendulum process with a periodic phased energy exchange between a quantum object and specific resonant discrete modes. Such dynamics of states of a quantum particle and field corresponds to the standard Rabi precession. This process is analogous to the well-known phenomenon of periodic reversible energy transfer between coupled oscillators.

It should be noted that in experiments on the realization of the QZE the conditions were just like that. For example, in [6] it was a relatively slow periodic transition of a supercooled Bose–Einstein condensate, consisting of rubidium atoms, from an excited state to a ground state. In this case, the QZE was realized at the time when this condensate was in an excited state and was irradiated by a controlling electromagnetic radiation.

In such a system, frequent inspection of the state of an excited quantum object violates the optimal phase states, which determine the current direction of the process (from the atom to the field), and after that the process of energy transfer begins from the initial moment.

Basing on the expression for the intermode frequency interval

$$\frac{d\omega_\beta}{dN} \equiv \frac{1}{\rho(\omega_\beta)} = \frac{\pi^2 c^3}{V_0 \omega_\beta^2}, \quad (64)$$

it can be concluded that such conditions are realized for relatively low frequencies in a small quantization volume V_0 of the electromagnetic field.

For example, in microwave resonator with a volume of $V_0 = 10^3 \text{ cm}^3$ at a wavelength $\lambda_\beta = 5\text{-cm}$ ($\omega_\beta \approx 3.5 \cdot 10^{10} \text{ Hz}$), the interval between modes is equal to $d\omega/dN \approx 3 \cdot 10^8 \text{ Hz}$.

On the other hand, it should be taken into account that the spectral width of the mode in a microwave resonator with a Q-factor is equal to $\Delta\omega_Q \approx \omega_\beta/Q$. In a typical microwave resonator, this Q-factor is equal to the value $Q = 10^3 - 10^4$ that corresponds to the spectral width of the mode $\Delta\omega_Q \approx 3.5 \cdot (10^6 - 10^7) \text{ Hz}$. It can be seen that in this case the spectral width of the mode is 10–100 times less than the intermode distance, which makes it possible to ensure mode selection and interaction of the atom with only one mode.

It is interesting to note that with a similar inspection of the state not of the atom, but of an excited mode of the field (in fact, of another oscillator), the complete inhibition of the opposite process is possible: the excitation of the atom due to the energy of the field!

A fundamentally different situation corresponds to the visible and shorter wavelengths.

In “usual” macroscopic optical resonator with the same volume V_0 (for example, in the form of a rectangular parallelepiped with mirror surfaces) at a wavelength $\lambda = 0.5 \text{ micron}$ ($\omega \approx 2 \cdot 10^{15} \text{ Hz}$), the intermode distance is equal to $d\omega/dN \approx 0.1\text{-Hz}$. With a typical Q-factor for the best optical resonators $Q = 10^6 - 10^7$, the spectral width of the mode in such a resonator $\Delta\omega_Q \approx 2 \cdot (10^8 - 10^9) \text{ Hz}$ is many orders of magnitude greater than the intermode distance, which makes the process of selective interaction of the atom with a separate mode impossible! In ordinary (with a lower effective Q-factor) optical systems, the spectral width of the modes is even larger. The same conclusion about the impossibility of selective interaction in the visible range directly follows from the general formula for the ratio of the mode width to the interval between modes

$$\left| \frac{\Delta\omega_Q}{d\omega/dN} \right| = \rho(\omega_\beta)\Delta\omega_Q \approx \frac{V_0\omega_\beta^3}{\pi^2 c^3 Q} \sim \omega_\beta^3 \quad (65)$$

It follows from this formula that for the realization of the QZE, the condition $|\Delta\omega_Q/(d\omega/dN)| \ll 1$ in the visible range can be realized only in hypothetical optical three-dimensional nanocavities with linear dimensions $V_0^{1/3} \leq 10^{-3} \text{ cm}$, which should be analogous to microwave resonators. The possibility of creating of such nanocavities is rather questionable. In addition, it should be taken into account that as the size of the resonator decreases, its Q-factor, as a rule, rapidly decreases, which leads to an additional increase of the spectral width of the modes. Basing on this consideration, it can be concluded that the quantum Zeno effect can be realized only in the frequency range not exceeding the microwave and, possibly, the frequencies of the near-IR range.

In shorter wavelength ranges (ultraviolet, X-ray, gamma radiation), this condition is fundamentally unrealizable, which directly follows from (65). In these cases, the process of interaction of the atom with the

field corresponds to irreversible spontaneous decay and the QZE is impossible.

The analysis carried out makes it possible to determine the conditions under which the prerequisites for the realization of the Zeno effect will be fulfilled for optical radiation. To fulfill these conditions, it is necessary to significantly increase the distance between the modes of the electromagnetic field and at the same time sharply increase the quality factor of these modes. The successes of modern microphysics make it possible to fulfill these conditions. Such requirements are met, for example, by optical dielectric spherical microresonators, which have a very small size (radius $R \approx 25 \text{ microns}$) and a very high quality factor of $Q \approx 10^9$ [16–20].

In such a microresonator, the interval between neighboring (nearest) modes for the visible range with a wavelength of 0.5 microns is 10^9-Hz , and the spectral width of each mode does not exceed 10^6-Hz , which provides the necessary interaction selectivity and fully satisfies the conditions for observing the Zeno effect.

6 Conclusion

The above features of the interaction of quantum systems with an electromagnetic field clearly demonstrate the ambiguous result of the influence of the standard method of stimulating the Zeno effect (frequent checking of the state of the system) on the possibility of its implementation. Such a method turns out to be effective in the case of the interaction of a quantum system with only one or several discrete modes of the electromagnetic field and is completely ineffective in the case of simultaneous interaction of this system with a quasi-continuous ensemble of field modes. The paper shows that such a significant difference in the process of interaction of field modes with a quantum system is associated with the important fact that the process of interaction with one or several discrete modes does not lead to irreversible spontaneous decay, but corresponds to a periodic energy exchange between the field and the quantum system. It is obvious that the direction of this process depends on the phase relations, which change completely with frequent checking of the state of the system, and this process stops the transfer of energy from the excited system to the field. It is obvious that a similar stopping process should occur with frequent inspection of the resonant mode, although this has not yet been observed. In contrast, the process of interaction of a system with a very large ensemble of modes cannot invert the direction of energy exchange for all modes.

Simple estimates based on this idea show that the QZE can be realized using electrodynamic systems with a very rarefied mode spectrum (in particular, using microwave resonator or optical resonator with a very small distance between mirrors).

It should also be noted that interference processes associated with multimode interaction take place not

only in the analysis of QZE, but also in other cases (in particular, in the analysis of the formation of coherent correlated states, which are associated with the Schrödinger–Robertson uncertainty relation and the generation of large energy fluctuations [21–27]). These processes will be considered in detail in future works.

Author contributions

All authors contributed equally to the paper.

Data Availability Statement This manuscript has associated data in a data repository. [Authors' comment: Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study. This article is purely theoretical and it has no additional data in a data repository. All the data that is used in the article is given in the article itself and there is no additional data.]

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