



Ion-acoustic shock waves in magnetized pair-ion plasma

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Abstract. A theoretical investigation associated with obliquely propagating ion-acoustic shock waves (IASHWs) in a three-component magnetized plasma having inertialess non-extensive electrons, inertial warm positive, and negative ions has been performed. A Burgers equation is derived by employing the reductive perturbation method. The plasma model supports both positive and negative shock structures. It is found that the positive and negative shock wave potentials increase with the oblique angle (δ) which arises due to the external magnetic field. It is also observed that the magnitude of the amplitude of positive and negative shock waves is not affected by the variation of the ion kinematic viscosity but the steepness of the positive and negative shock waves decreases with ion kinematic viscosity. The implications of our findings in space and laboratory plasmas are briefly discussed.

1 Introduction

The pair-ion (PI) plasma can be observed in astrophysical environments such as upper regions of Titan's atmosphere [1–8], cometary comae [9], (H^+ , O_2^-) and (H^+ , H^-) plasmas in the D- and F-regions of Earth's ionosphere [2–7], and also in the laboratory experiments namely (Ar^+ , F^-) plasma [10], (K^+ , SF_6^-) plasma [11, 12], neutral beam sources [13], plasma processing reactors [14], (Ar^+ , SF_6^-) plasma [15–18], combustion products [19], plasma etching [19], (Xe^+ , F^-) plasma [20], (Ar^+ , O_2^-) plasma, fullerene (C_{60}^+ , C_{60}^-) plasma [21–23], etc. Positive ions are produced by electron impact ionization, and negative ions are produced by attachment of the low-energy electrons. A number of authors studied the nonlinear electrostatic structures in PI plasma [3–8].

Highly energetic particles have been observed in the galaxy clusters [24], the Earth's bow-shock [25], in the upper ionosphere of Mars [26], in the vicinity of the Moon [27], and in the magnetospheres of Jupiter and Saturn [28]. Maxwellian velocity distribution demonstrating the thermally equilibrium state of particles is

not appropriate for explaining the dynamics of these highly energetic particles. Renyi [29] first introduced the non-extensive q -distribution for explaining the dynamics of these highly energetic particles, and further development of q -distribution has been demonstrated by Tsallis [30]. The parameter q in the non-extensive q -distribution describes the deviation of the plasma particles from the thermally equilibrium state. It should be noted that $q = 1$ refers to Maxwellian, and $q < 1$ ($q > 1$) refers to super-extensivity (sub-extensivity). Jannat et al. [7] investigated the ion-acoustic (IA) shock waves (IASHWs) in PI plasma in the presence of non-extensive electrons, and observed that the height of the positive potential decreases (increases) with positive (negative) ion mass. Hussain et al. [31] considered inertial PI and inertialess non-extensive electrons and investigated IASHWs by considering kinematic viscosities of both positive and negative ion species, and observed that the amplitude of the positive IASHWs decreases with q . Tribeche et al. [32] studied IA solitary waves in a two-component plasma, and found that the magnitude of the amplitude of positive and negative solitary structures increases with super-extensive and sub-extensive electrons.

A plasma medium having considerable dissipative properties dictates the formation of shock structures [33–35]. The Landau damping, kinematic viscosity among the plasma species, and the collision between plasma species are the major causes of the dissipation which is mainly responsible for the formation of shock structures in the plasma medium [33–35]. The presence of kinematic viscosity plays a pivotal role in generating nonlinear waves [33–35]. Hafez et al. [33] observed

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that the steepness of the IASHWs decreases with the increase in ion kinematic viscosity but the amplitude of IASHWs remains unchanged. Abdelwahed et al. [34] investigated IASHWs in PI plasma and reported that the kinematic viscosity coefficient of the ion reduces the steepness of the IASHWs.

The external magnetic field is to be considered to change the dynamics of the plasma medium, and associated electrostatic nonlinear structures. Hossen et al. [35] studied the electrostatic shock structures in magnetized dusty plasma, and found that the magnitude of the positive and negative shock profiles increases with the oblique angle (δ) which arises due to the external magnetic field. El-Labany et al. [8] considered a three-component plasma model having inertial PI and inertialess non-extensive electrons, and investigated IASHWs, and found that the amplitude of the positive shock profile decreases with q . To the best knowledge of the authors, no attempt has been made to study the IASHWs in a three-component magnetized plasma by considering kinematic viscosities of both inertial warm positive and negative ion species, and inertialess non-extensive electrons. The aim of the present investigation is, therefore, to derive Burgers' equation and investigate IASHWs in a three-component magnetized PI plasma, and to observe the effects of various plasma parameters on the configuration of IASHWs.

The outline of the paper is as follows: The basic equations are displayed in Sect. 2. The Burgers equation has been derived in Sect. 3. Results and discussion are reported in Sect. 4. A brief conclusion is provided in Sect. 5.

2 Governing equations

We consider a magnetized plasma system comprising inertial negatively and positively charged warm ions, and inertialess electrons featuring q -distribution. An external magnetic field \mathbf{B}_0 has been considered in the system directed along the z -axis defining $\mathbf{B}_0 = B_0 \hat{z}$, where B_0 and \hat{z} are the strength of the external magnetic field and unit vector directed along the z -axis, respectively. The dynamics of the magnetized PI plasma system is governed by the following set of equations [36–42]

$$\frac{\partial \tilde{n}_+}{\partial \tilde{t}} + \hat{\nabla} \cdot (\tilde{n}_+ \tilde{u}_+) = 0, \tag{1}$$

$$\begin{aligned} \frac{\partial \tilde{u}_+}{\partial \tilde{t}} + (\tilde{u}_+ \cdot \hat{\nabla}) \tilde{u}_+ = & -\frac{Z_+ e}{m_+} \hat{\nabla} \tilde{\psi} + \frac{Z_+ e B_0}{m_+} (\tilde{u}_+ \times \hat{z}) \\ & - \frac{1}{m_+ n_+} \hat{\nabla} P_+ + \tilde{\eta}_+ \hat{\nabla}^2 \tilde{u}_+, \end{aligned} \tag{2}$$

$$\frac{\partial \tilde{n}_-}{\partial \tilde{t}} + \hat{\nabla} \cdot (\tilde{n}_- \tilde{u}_-) = 0, \tag{3}$$

$$\begin{aligned} \frac{\partial \tilde{u}_-}{\partial \tilde{t}} + (\tilde{u}_- \cdot \hat{\nabla}) \tilde{u}_- = & \frac{Z_- e}{m_-} \hat{\nabla} \tilde{\psi} - \frac{Z_- e B_0}{m_-} (\tilde{u}_- \times \hat{z}) \\ & - \frac{1}{m_- \tilde{n}_-} \hat{\nabla} P_- + \tilde{\eta}_- \hat{\nabla}^2 \tilde{u}_-, \end{aligned} \tag{4}$$

$$\hat{\nabla}^2 \tilde{\psi} = 4\pi e [\tilde{n}_e + Z_- \tilde{n}_- - Z_+ \tilde{n}_+], \tag{5}$$

where \tilde{n}_+ (\tilde{n}_-) is the positive (negative) ion number density, m_+ (m_-) is the positive (negative) ion mass, Z_+ (Z_-) is the charge state of the positive (negative) ion, e being the magnitude of electron charge, \tilde{u}_+ (\tilde{u}_-) is the positive (negative) ion fluid velocity, $\tilde{\eta}_+ = \mu_+/m_+ n_+$ ($\tilde{\eta}_- = \mu_-/m_- n_-$) is the kinematic viscosity of the positive (negative) ion, P_+ (P_-) is the pressure of positive (negative) ion, and $\tilde{\psi}$ represents the electrostatic wave potential. Now, we are introducing normalized variables, namely $n_+ \rightarrow \tilde{n}_+/n_{+0}$, $n_- \rightarrow \tilde{n}_-/n_{-0}$, and $n_e \rightarrow \tilde{n}_e/n_{e0}$, where n_{-0} , n_{+0} , and n_{e0} are the equilibrium number densities of the negative ions, positive ions, and electrons, respectively; $u_+ \rightarrow \tilde{u}_+/C_-$, $u_- \rightarrow \tilde{u}_-/C_-$ [where $C_- = (Z_- k_B T_e/m_-)^{1/2}$, k_B being the Boltzmann constant, and T_e being temperature of the electron]; $\psi \rightarrow \tilde{\psi} e/k_B T_e$; $t \rightarrow \tilde{t}/\omega_{p-}^{-1}$ [where $\omega_{p-}^{-1} = (m_-/4\pi e^2 Z_-^2 n_{-0})^{1/2}$]; $\nabla = \hat{\nabla}/\lambda_D$ [where $\lambda_D = (k_B T_e/4\pi e^2 Z_- n_{-0})^{1/2}$]. The pressure term of the positive and negative ions can be recognized as $P_{\pm} = P_{\pm 0} (N_{\pm}/n_{\pm 0})^\gamma$ with $P_{\pm 0} = n_{\pm 0} k_B T_{\pm}$ being the equilibrium pressure of the positive (for $+0$ sign) and negative (for -0 sign) ions, and T_+ (T_-) being the temperature of warm positive (negative) ion, and $\gamma = (N + 2)/N$ (where N is the degree of freedom and for three-dimensional case $N = 3$, then $\gamma = 5/3$). For simplicity, we have considered ($\tilde{\eta}_+ \approx \tilde{\eta}_- = \eta$), and η is normalized by $\omega_{p-} \lambda_D^2$. The quasi-neutrality condition at equilibrium for our plasma model can be written as $n_{e0} + Z_- n_{-0} \approx Z_+ n_{+0}$. Equations (1)–(5) can be expressed in the normalized form as [7, 8]:

$$\frac{\partial n_+}{\partial t} + \nabla \cdot (n_+ u_+) = 0, \tag{6}$$

$$\begin{aligned} \frac{\partial u_+}{\partial t} + (u_+ \cdot \nabla) u_+ = & -\alpha_1 \nabla \psi + \alpha_1 \Omega_c (u_+ \times \hat{z}) \\ & -\alpha_2 \nabla n_+^{\gamma-1} + \eta \nabla^2 u_+, \end{aligned} \tag{7}$$

$$\frac{\partial n_-}{\partial t} + \nabla \cdot (n_- u_-) = 0, \tag{8}$$

$$\begin{aligned} \frac{\partial u_-}{\partial t} + (u_- \cdot \nabla) u_- = & \nabla \psi - \Omega_c (u_- \times \hat{z}) \\ & -\alpha_3 \nabla n_-^{\gamma-1} + \eta \nabla^2 u_-, \end{aligned} \tag{9}$$

$$\nabla^2 \psi = \mu_e n_e - (1 + \mu_e) n_+ + n_-. \tag{10}$$

Other plasma parameters are defined as $\alpha_1 = Z_+ m_- / Z_- m_+$, $\alpha_2 = \gamma T_+ m_- / (\gamma - 1) Z_- T_e m_+$, $\alpha_3 = \gamma T_- / (\gamma - 1) Z_- T_e$, $\mu_e = n_{e0} / Z_- n_{-0}$, and $\Omega_c = \omega_c / \omega_{p-}$ [where $\omega_c = Z_- e B_0 / m_-$]. Now, the expression for the number

density of electrons following non-extensive q -distribution can be written as [8]

$$n_e = \left[1 + (q - 1)\psi \right]^{\frac{q+1}{2(q-1)}}, \tag{11}$$

where the parameter q represents the non-extensive properties of electrons. We have neglected the effect of the external magnetic field on the non-extensive electron distribution. This is valid due to the fact that the Larmor radii of electrons is so small that as if the electrons are flowing along the magnetic field lines of force. Now, by substituting Eq. (11) into Eq. (10), and expanding up to third order in ψ , we get

$$\nabla^2\psi = \mu_e + n_- - (1 + \mu_e)n_+ + \sigma_1\psi + \sigma_2\psi^2 + \sigma_3\psi^3 + \dots, \tag{12}$$

where

$$\begin{aligned} \sigma_1 &= [\mu_e(q + 1)]/2, & \sigma_2 &= [\mu_e(q + 1)(3 - q)]/8, \\ \sigma_3 &= [\mu_e(q + 1)(3 - q)(5 - 3q)]/48. \end{aligned}$$

We note that the terms containing σ_1 , σ_2 , and σ_3 are the contribution of q -distributed electrons.

3 Derivation of the Burgers' equation

To derive the Burgers' equation for the IASHWs propagating in a magnetized PI plasma, first we introduce the stretched coordinates [35, 43]

$$\xi = \epsilon(l_x x + l_y y + l_z z - v_p t), \tag{13}$$

$$\tau = \epsilon^2 t, \tag{14}$$

where v_p is the phase speed and ϵ is a smallness parameter measuring the weakness of the dissipation ($0 < \epsilon < 1$). The l_x , l_y , and l_z (i.e., $l_x^2 + l_y^2 + l_z^2 = 1$) are the directional cosines of the wave vector k along x , y , and z -axes, respectively. Then, the dependent variables can be expressed in power series of ϵ as [35]

$$n_+ = 1 + \epsilon n_+^{(1)} + \epsilon^2 n_+^{(2)} + \epsilon^3 n_+^{(3)} + \dots, \tag{15}$$

$$n_- = 1 + \epsilon n_-^{(1)} + \epsilon^2 n_-^{(2)} + \epsilon^3 n_-^{(3)} + \dots, \tag{16}$$

$$u_{+x,y} = \epsilon^2 u_{+x,y}^{(1)} + \epsilon^3 u_{+x,y}^{(2)} + \dots, \tag{17}$$

$$u_{-x,y} = \epsilon^2 u_{-x,y}^{(1)} + \epsilon^3 u_{-x,y}^{(2)} + \dots, \tag{18}$$

$$u_{+z} = \epsilon u_{+z}^{(1)} + \epsilon^2 u_{+z}^{(2)} + \dots, \tag{19}$$

$$u_{-z} = \epsilon u_{-z}^{(1)} + \epsilon^2 u_{-z}^{(2)} + \dots, \tag{20}$$

$$\psi = \epsilon\psi^{(1)} + \epsilon^2\psi^{(2)} + \dots. \tag{21}$$

Now, by substituting Eqs. (13)–(21) into Eqs. (6)–(9), and (12), and collecting the terms containing ϵ , the first-

order equations reduce to

$$n_+^{(1)} = \frac{3\alpha_1 l_z^2}{3v_p^2 - 2\alpha_2 l_z^2} \psi^{(1)}, \tag{22}$$

$$u_{+z}^{(1)} = \frac{3v_p \alpha_1 l_z}{3v_p^2 - 2\alpha_2 l_z^2} \psi^{(1)}, \tag{23}$$

$$n_-^{(1)} = -\frac{3l_z^2}{3v_p^2 - 2\alpha_3 l_z^2} \psi^{(1)}, \tag{24}$$

$$u_{-z}^{(1)} = -\frac{3v_p l_z}{3v_p^2 - 2\alpha_3 l_z^2} \psi^{(1)}. \tag{25}$$

Now, the phase speed of IASHWs can be written as

$$v_p \equiv v_{p+} = l_z \sqrt{\frac{-a_1 + \sqrt{a_1^2 - 36\sigma_1 a_2}}{18\sigma_1}}, \tag{26}$$

$$v_p \equiv v_{p-} = l_z \sqrt{\frac{-a_1 - \sqrt{a_1^2 - 36\sigma_1 a_2}}{18\sigma_1}}, \tag{27}$$

where $a_1 = -9 - 6\alpha_2\sigma_1 - 6\alpha_3\sigma_1 - 9\alpha_1\mu_e - 9\alpha_1$ and $a_2 = 6\alpha_2 + 4\alpha_2\alpha_3\sigma_1 + 6\alpha_1\alpha_3\mu_e + 6\alpha_1\alpha_3$. The x and y -components of the first-order momentum equations can be manifested as

$$u_{+x}^{(1)} = -\frac{3l_y v_p^2}{\Omega_c(3v_p^2 - 2\alpha_2 l_z^2)} \frac{\partial\psi^{(1)}}{\partial\xi}, \tag{28}$$

$$u_{+y}^{(1)} = \frac{3l_x v_p^2}{\Omega_c(3v_p^2 - 2\alpha_2 l_z^2)} \frac{\partial\psi^{(1)}}{\partial\xi}, \tag{29}$$

$$u_{-x}^{(1)} = -\frac{3l_y v_p^2}{\Omega_c(3v_p^2 - 2\alpha_3 l_z^2)} \frac{\partial\psi^{(1)}}{\partial\xi}, \tag{30}$$

$$u_{-y}^{(1)} = \frac{3l_x v_p^2}{\Omega_c(3v_p^2 - 2\alpha_3 l_z^2)} \frac{\partial\psi^{(1)}}{\partial\xi}. \tag{31}$$

Now, by taking the next higher-order terms, the equation of continuity, momentum equation, and Poisson's equation can be written as

$$\begin{aligned} \frac{\partial n_+^{(1)}}{\partial\tau} - v_p \frac{\partial n_+^{(2)}}{\partial\xi} + l_x \frac{\partial u_{+x}^{(1)}}{\partial\xi} + l_y \frac{\partial u_{+y}^{(1)}}{\partial\xi} \\ + l_z \frac{\partial u_{+z}^{(2)}}{\partial\xi} + l_z \frac{\partial}{\partial\xi} (n_+^{(1)} u_{+z}^{(1)}) = 0, \end{aligned} \tag{32}$$

$$\begin{aligned} \frac{\partial u_{+z}^{(1)}}{\partial\tau} - v_p \frac{\partial u_{+z}^{(2)}}{\partial\xi} + l_z u_{+z}^{(1)} \frac{\partial u_{+z}^{(1)}}{\partial\xi} + \alpha_1 l_z \frac{\partial\psi^{(2)}}{\partial\xi} \\ + \alpha_2 l_z \frac{\partial}{\partial\xi} \left[\frac{2}{3} n_+^{(2)} - \frac{1}{9} (n_+^{(1)})^2 \right] - \eta \frac{\partial^2 u_{+z}^{(1)}}{\partial\xi^2} = 0, \end{aligned} \tag{33}$$

$$\begin{aligned} \frac{\partial n_-^{(1)}}{\partial\tau} - v_p \frac{\partial n_-^{(2)}}{\partial\xi} + l_x \frac{\partial u_{-x}^{(1)}}{\partial\xi} + l_y \frac{\partial u_{-y}^{(1)}}{\partial\xi} \\ + l_z \frac{\partial u_{-z}^{(2)}}{\partial\xi} + l_z \frac{\partial}{\partial\xi} (n_-^{(1)} u_{-z}^{(1)}) = 0, \end{aligned} \tag{34}$$

$$\frac{\partial u_{-z}^{(1)}}{\partial \tau} - v_p \frac{\partial u_{-z}^{(2)}}{\partial \xi} + l_z u_{-z}^{(1)} \frac{\partial u_{-z}^{(1)}}{\partial \xi} - l_z \frac{\partial \psi^{(2)}}{\partial \xi} + \alpha_3 l_z \frac{\partial}{\partial \xi} \left[\frac{2}{3} n_-^{(2)} - \frac{1}{9} (n_-^{(1)})^2 \right] - \eta \frac{\partial^2 u_{-z}^{(1)}}{\partial \xi^2} = 0, \tag{35}$$

$$\sigma_1 \psi^{(2)} + \sigma_2 [\psi^{(1)}]^2 + n_-^{(2)} - (\mu_e + 1) n_+^{(2)} = 0. \tag{36}$$

Finally, the next higher-order terms of Eqs. (6)–(9), and (12), with the help of Eqs. (22)–(36), can provide the Burgers equation as

$$\frac{\partial \Psi}{\partial \tau} + A \Psi \frac{\partial \Psi}{\partial \xi} = C \frac{\partial^2 \Psi}{\partial \xi^2}, \tag{37}$$

where $\Psi = \psi^{(1)}$ is used for simplicity. In Eq. (37), the nonlinear coefficient A and dissipative coefficient C are given by

$$A = \frac{81\alpha_1^2 v_p^2 s_1^3 l_z^4 + F_1}{18v_p s_1 l_z^2 s_2^3 + F_2}, \quad \text{and} \quad C = \frac{\eta}{2}, \tag{38}$$

where

$$\begin{aligned} F_1 &= 81\mu_e \alpha_1^2 v_p^2 s_1^3 l_z^4 - 81v_p^2 s_1^3 l_z^4 + 2\mu_e \alpha_2 \alpha_1^2 s_1^3 l_z^6 \\ &\quad + 2\alpha_2 \alpha_1^2 s_1^3 l_z^6 + 2\alpha_3 s_1^3 l_z^6 - 2\sigma_2 s_1^3 s_2^3, \\ F_2 &= 18\alpha_1 v_p s_2 l_z^2 s_1^3 + 18\alpha_1 \mu_e v_p s_2 l_z^2 s_1^3, \\ s_1 &= 3v_p^2 - 2\alpha_3 l_z^2, \quad s_2 = 3v_p^2 - 2\alpha_2 l_z^2. \end{aligned}$$

Now, we look for stationary shock wave solution of this Burgers' equation by considering $\zeta = \xi - U_0 \tau'$ and $\tau = \tau'$ (where U_0 is the speed of the shock waves in the reference frame). These allow us to write the stationary shock wave solution as [35, 44, 45]

$$\Psi = \Psi_m \left[1 - \tanh \left(\frac{\zeta}{\Delta} \right) \right], \tag{39}$$

where the amplitude Ψ_m and width Δ are given by

$$\Psi_m = \frac{U_0}{A}, \quad \text{and} \quad \Delta = \frac{2C}{U_0}. \tag{40}$$

It is clear from Eqs. (39) and (40) that the IASHWs exist, which are formed due to the balance between nonlinearity and dissipation, because $C > 0$ and the IASHWs with $\Psi > 0$ ($\Psi < 0$) exist if $A > 0$ ($A < 0$) because $U_0 > 0$.

4 Results and discussion

The balance between nonlinearity and dissipation leads to generate IASHWs in a three-component magnetized PI plasma. We have numerically analyzed the variation of A with μ_e in the left panel of Fig. 1, and it is obvious

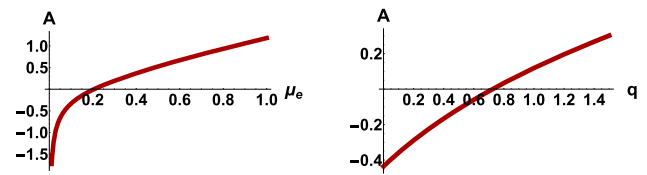


Fig. 1 The variation of nonlinear coefficient A with μ_e when $q = 1.2$ (left panel), and the variation of nonlinear coefficient A with q when $\mu_e = 0.3$ (right panel). Other plasma parameters are $\alpha_1 = 1.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\delta = 30^\circ$, and $v_p \equiv v_{p+}$

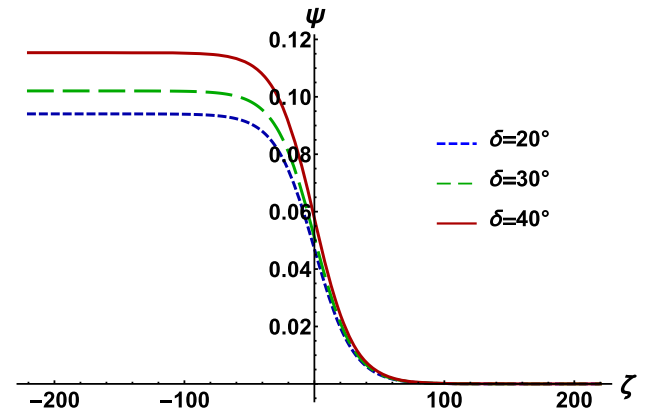


Fig. 2 The variation of Ψ with ζ for different values of δ under the consideration $\mu_e > \mu_{ec}$. Other plasma parameters are $\alpha_1 = 1.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\eta = 0.3$, $\mu_e = 0.3$, $q = 1.2$, $U_0 = 0.01$, and $v_p \equiv v_{p+}$

from this figure that (a) A can be negative, zero, and positive depending on the values of μ_e ; (b) the value of μ_e for which A becomes zero is known as critical value of μ_e (i.e., μ_{ec}), and the μ_{ec} for our present analysis is almost 0.2; and (c) the parametric regimes for the formation of positive (i.e., $\psi > 0$) and negative (i.e., $\psi < 0$) potential shock structures can be found corresponding to $A > 0$ and $A < 0$. The right panel of Fig. 1 describes the variation of A with q when other plasma parameters are constant and in this case, A becomes zero for the critical value of q (i.e., $q = q_c \simeq 0.7$). The positive (negative) potential can exist for $q > 0.7$ ($q < 0.7$) [Figures are not included].

Figures 2 and 3 display the variation of the positive potential shock structure under the consideration $\mu_e > \mu_{ec}$ and negative potential shock structure under the consideration $\mu_e < \mu_{ec}$ with the oblique angle (δ), respectively. It is clear from these figures that (a) the magnitude of the amplitude of positive and negative potential structures increases with an increase in the value of the δ , and this result agrees with the result of Hossen et al. [35]; (b) the magnitude of the negative potential is always greater than the positive potential for same plasma parameters. So, the oblique angle enhances the amplitude of the potential profiles.

Figures 4 and 5 illustrate the effects of the ion kinematic viscosity on the positive (under the consideration $\mu_e > \mu_{ec}$) and negative (under the consideration

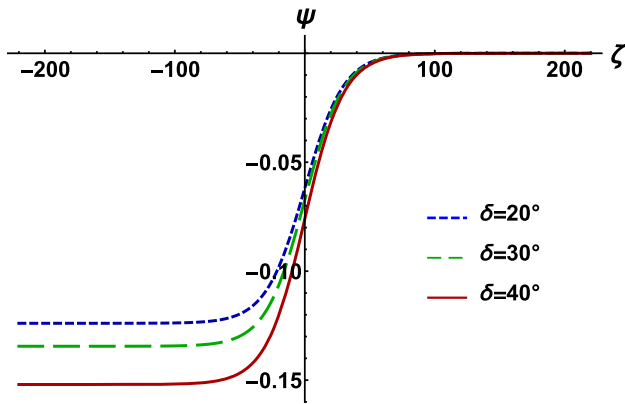


Fig. 3 The variation of Ψ with ζ for different values of δ under the consideration $\mu_e < \mu_{ec}$. Other plasma parameters are $\alpha_1 = 1.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\eta = 0.3$, $\mu_e = 0.15$, $q = 1.2$, $U_0 = 0.01$, and $v_p \equiv v_{p+}$

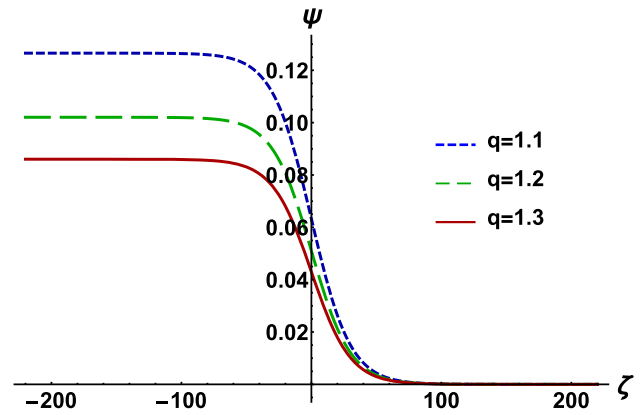


Fig. 6 The variation of Ψ with ζ for different values of q under the consideration $\mu_e > \mu_{ec}$. Other plasma parameters are $\alpha_1 = 1.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\delta = 30^\circ$, $\eta = 0.3$, $\mu_e = 0.3$, $U_0 = 0.01$, and $v_p \equiv v_{p+}$

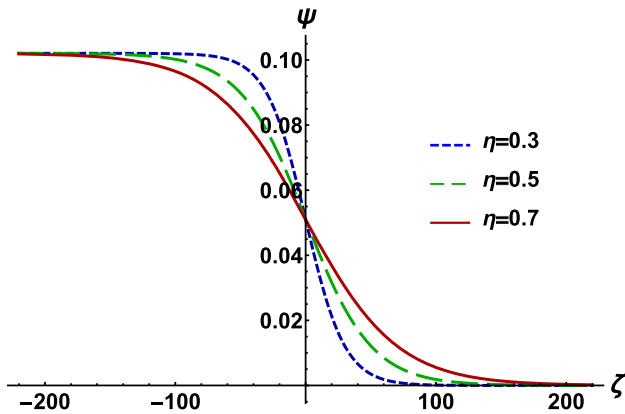


Fig. 4 The variation of Ψ with ζ for different values of η under the consideration $\mu_e > \mu_{ec}$. Other plasma parameters are $\alpha_1 = 1.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\delta = 30^\circ$, $\mu_e = 0.3$, $q = 1.2$, $U_0 = 0.01$, and $v_p \equiv v_{p+}$

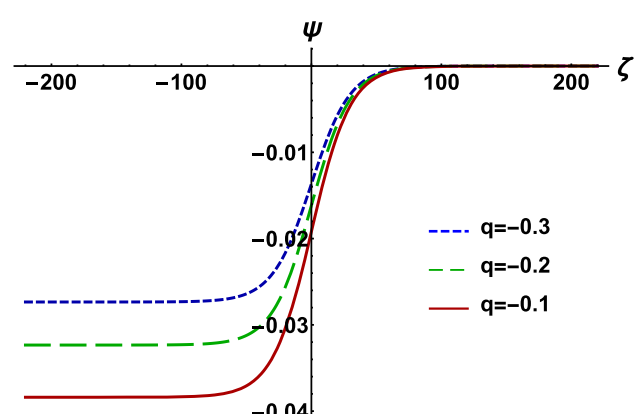


Fig. 7 The variation of Ψ with ζ for different values of q under the consideration $\mu_e > \mu_{ec}$. Other plasma parameters are $\alpha_1 = 1.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\delta = 30^\circ$, $\eta = 0.3$, $\mu_e = 0.3$, $U_0 = 0.01$, and $v_p \equiv v_{p+}$

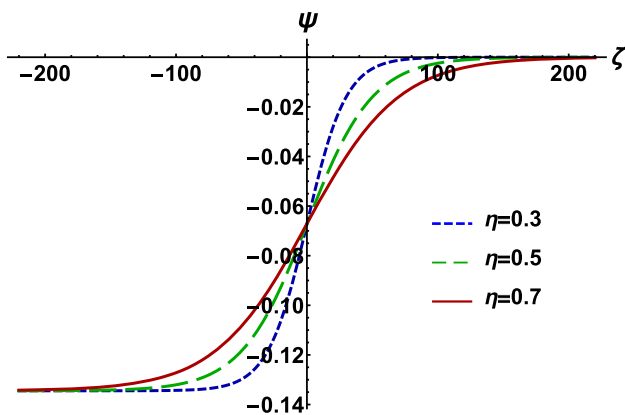


Fig. 5 The variation of Ψ with ζ for different values of η under the consideration $\mu_e < \mu_{ec}$. Other plasma parameters are $\alpha_1 = 1.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\delta = 30^\circ$, $\mu_e = 0.15$, $q = 1.2$, $U_0 = 0.01$, and $v_p \equiv v_{p+}$

$\mu_e < \mu_{ec}$) shock profiles. It is really interesting that the magnitude of the amplitude of positive and negative shock profiles is not affected by the variation of the ion kinematic viscosity but the steepness of the shock profile decreases with ion kinematic viscosity, and this result agrees with the previous work of Refs. [33,34].

The effects of the sub-extensive electrons (i.e., $q > 1$) on the positive potential profile can be seen in Fig. 6 under the consideration $\mu_e > \mu_{ec}$. The height of the positive potential decreases with q , and this result is a good agreement with the result of El-Labany et al. [8] and Hussain et al. [31]. Figures 7 and 8 illustrate the role of super-extensive electrons (i.e., $q < 1$) on the formation of the negative potential under the consideration $\mu_e > \mu_{ec}$, and this is really interesting that the existence of the super-extensive electron produces negative potential, and the magnitude of the amplitude of negative potential increases with q . So, the orientation of the potential profiles (positive and negative) has been organized by the sign of q under the consideration $\mu_e > \mu_{ec}$.

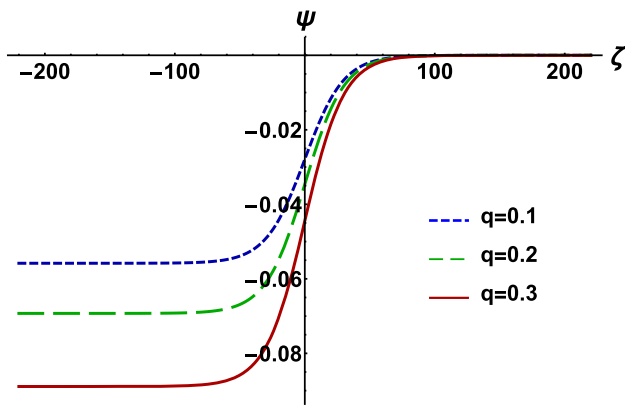


Fig. 8 The variation of Ψ with ζ for different values of q under the consideration $\mu_e > \mu_{ec}$. Other plasma parameters are $\alpha_1 = 1.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\delta = 30^\circ$, $\eta = 0.3$, $\mu_e = 0.3$, $U_0 = 0.01$, and $v_p \equiv v_{p+}$

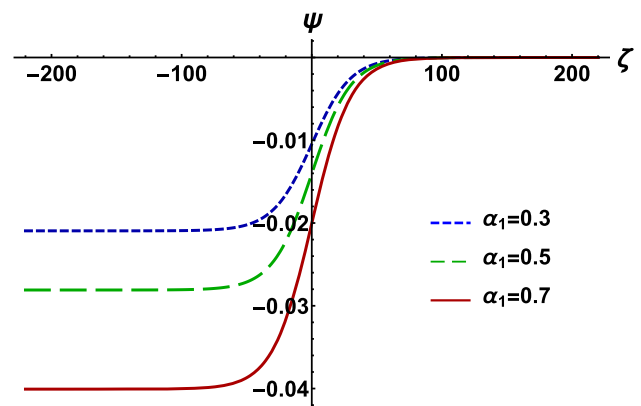


Fig. 10 The variation of Ψ with ζ for different values of α_1 under the consideration $\mu_e > \mu_{ec}$. Other plasma parameters are $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\delta = 30^\circ$, $\eta = 0.3$, $\mu_e = 0.3$, $q = 1.2$, $U_0 = 0.01$, and $v_p \equiv v_{p+}$

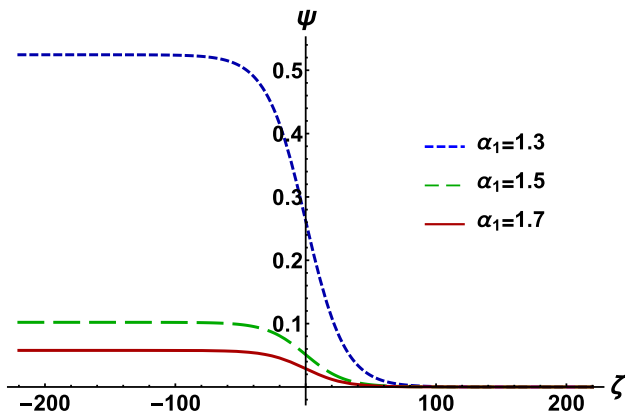


Fig. 9 The variation of Ψ with ζ for different values of α_1 under the consideration $\mu_e > \mu_{ec}$. Other plasma parameters are $\alpha_2 = 0.2$, $\alpha_3 = 0.02$, $\delta = 30^\circ$, $\eta = 0.3$, $\mu_e = 0.3$, $q = 1.2$, $U_0 = 0.01$, and $v_p \equiv v_{p+}$

It can be seen from the literature that the PI plasma system can support these conditions: $m_- > m_+$ (i.e., $H^+ - O_2^-$ [2–7], $Ar^+ - SF_6^-$ [15–18], and $Xe^+ - SF_6^-$ [15–18]), $m_- = m_+$ (i.e., $H^+ - H^-$ [2–7] and $C_{60}^+ - C_{60}^-$ [21–23]), and $m_- < m_+$ (i.e., $Ar^+ - F^-$ [3, 4]). So, in our present investigation, we have graphically observed the variation of the electrostatic positive potential with α_1 under the consideration of $m_- > m_+$ (i.e., $\alpha_1 > 1$) and $\mu_e > \mu_{ec}$ in Fig. 9, and it is obvious from this figure that (a) the amplitude of the positive potential decreases with an increase in the value of the negative ion mass but increases with an increase in the value of the positive ion mass for a fixed value of their charge state; (b) the height of the IASHWs with positive potential increases (decreases) with negative (positive) ion charge state for a constant mass of positive and negative ion species. So, the mass and charge state of the PI play an opposite role for the formation of positive shock structure. Figure 10 describes the nature of the electrostatic negative potential with α_1 under the consideration of $m_- < m_+$ (i.e., $\alpha_1 < 1$) and $\mu_e > \mu_{ec}$. It is clear

from this figure that (a) due to the $m_- < m_+$ (i.e., $\alpha_1 < 1$), we have observed negative potential profile even though we have considered $\mu_e > \mu_{ec}$ (i.e., $A > 0$); (b) the existence of the heavy positive ion change the dynamics of the plasma system; and (c) in this case, the magnitude of the amplitude of negative potential increases (decreases) with negative (positive) ion mass when other plasma parameters are constant. So, the dynamics of the PI plasma rigorously changes with these conditions $m_- > m_+$ (i.e., $\alpha_1 > 1$) and $m_- < m_+$ (i.e., $\alpha_1 < 1$).

5 Conclusion

We have studied IASHWs in a three-component magnetized PI plasma by considering kinematic viscosities of both inertial warm positive and negative ion species, and inertialess non-extensive electrons. The reductive perturbation method is used to derive the Burgers' equation. The results that have been found from our investigation can be summarized as follows:

- The parametric regimes for the formation of positive (i.e., $\psi > 0$) and negative (i.e., $\psi < 0$) potential shock structures can be found corresponding to $A > 0$ and $A < 0$.
- The magnitude of the amplitude of positive and negative shock structures increases with the oblique angle (δ) which arises due to the external magnetic field.
- The magnitude of the amplitude of positive and negative shock profiles is not affected by the variation of the ion kinematic viscosity but the steepness of the shock profile decreases with ion kinematic viscosity.

It may be noted here that the gravitational effect is very important but beyond the scope of our present work. In future and for better understanding, someone

can investigate the nonlinear propagation in a three-component PI plasma by considering the gravitational effect. The results of our present investigation will be useful in understanding the nonlinear phenomena both in astrophysical environments such as upper regions of Titan's atmosphere [1–8], cometary comae [9], (H^+ , O_2^-) and (H^+ , H^-) plasmas in the D- and F-regions of Earth's ionosphere [2–7], and also in the laboratory experiments, namely (Ar^+ , F^-) plasma [10], (K^+ , SF_6^-) plasma [11, 12], neutral beam sources [13], plasma processing reactors [14], (Ar^+ , SF_6^-) plasma [15–18], combustion products [19], plasma etching [19], (Xe^+ , F^-) plasma [20], (Ar^+ , O_2^-) plasma, fullerene (C_{60}^+ , C_{60}^-) plasma [21–23], etc.

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Author contributions

All authors contributed equally to complete this work.

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