Eur. Phys. J. D (2021) 75:68
https://doi.org/10.1140/epjd/s10053-021-00086-8

THE EUROPEAN
PHYSICAL JOURNAL D

Regular Article - Quantum Optics

The effects of an electrolyte solution on screening the Casimir interaction between two symmetric double-layered media

A. Seyedzahedi and A. Moradian[a]

Department of Science, Campus of Bijar, University of Kurdistan, Bijar, Iran

Received 12 December 2020 / Accepted 15 February 2021 / Published online 24 February 2021

Abstract. Using the scattering approach besides the Matsubara formalism, this paper aims at investigating screening and intensifying the thermal Casimir force in an electrolyte solution surrounded by two layers of local media within two semispaces. The electric field in an electrolyte solution is decomposed into its transverse and longitudinal components. We construct the reflection matrix describing the combination of the transverse and the longitudinal modes contribution to the incident wave to make the reflection wave for zero and nonzero Matsubara frequencies, individually. It is shown that the longitudinal modes contribution to the Casimir interaction in the Hamaker coefficient is significant only at zero frequency, and it shows that the presence of layers on the substrates intensifies the transverse modes contribution to the Hamaker coefficient in both the conductor and insulator media. It is illustrated that screening in the Hamaker coefficient shows similar behavior for different layers of the conductor and the insulator. Our calculations reveal that increasing the electrolyte's concentration increases the Hamaker coefficient. Furthermore, the longitudinal modes contribution to the Hamaker coefficient—present due to the ions—is weaker if the electrolyte is surrounded by a conductor rather than being surrounded by a dielectric media. Interestingly, the zero-frequency portion of this coefficient asymptotically reaches its longitudinal contribution at zero frequency for different layers and different concentrations. Our investigation illustrates that in the presence of an electrolyte solution within two dielectric layers surrounded by two other dielectric semispaces, the intensification of the Casimir force per unit area becomes weaker in comparison to the case in which the solution is absent in such a system.

1 Introduction

Since ionic solutions play a main role in biological and electrochemical systems and the electrostatic interaction screening is an essential characteristic of such solutions, many studies have concentrated on these conceptual problems [1–4]. Scattering approach is a beneficial approach to compute the Casimir force or the interaction energy between the two semispaces and among the multilayers [5,6]. Intensifying the Casimir force due to the presence of a layer on the substrate has been investigated [7]. The scattering approach has been applied to calculate the Casimir interaction between two dielectric half spaces separated by an electrolyte solution [8]. The normal component of the field stimulates the charge fluctuations in the TM-polarized wave and the system responses to these fluctuations thorough its longitudinal dielectric function [9]. For an interacting fluid, such as an electrolyte confined between two semispaces of dielectric media, the zero-frequency Matsubara term corresponding to the classical partition function of the Coulomb system may behave very differently from the Lifshitz theory. This term, associated to the fluctuations of the free energy around the mean field in the range of validity of Poisson-Boltzmann approximation, results in the screening of the Hamaker coefficient [10,11]. The Hamaker coefficient relates the van der Waals interaction energy to the particles' separation distance for the interactive forces that are pairwise additive and independent from the intervening media [12]. In these studies, the response of the electrolyte solution has been investigated. The propagation of the longitudinal and transverse modes, and the coupling of these modes in reflection from the surfaces have been investigated. The researchers have shown that the nonzero portion of the Matsubara frequency does not change for typical values of ion concentration. However, the zero frequency of the Hamaker coefficient changes due to the screening contribution of the longitudinal mode. The impact of free ions on the interaction between surfaces and charge fluctuation screening has been investigated [13,14]. It would be interesting to investigate the effect of the electrolyte solution and screening in the scattering approach for a system com-

---

[a] e-mail: a.moradian@uok.ac.ir (corresponding author)

posed of two substrates with a layer on their surface. It will also be compelling to study the impacts of the free ions on the Hamaker coefficient and the Casimir force per unit area for the mentioned system. Studying the dependence of this force on the conductivity and the thickness of the layers and also the concentration of the solution is of importance. In this paper, using the scattering approach, we consider a system composed of two half spaces media with a three-layer array of different materials in between, two layers of which are separated with a thin layer of electrolyte solution.

In this study regarding the non-local effects of the electrolyte solution on charge fluctuations and the electric field in the electrolyte layer, the effect of this solution on screening the Casimir interaction is investigated. Following [7], we consider our model based on Maxwell equations and assume local conductivity for the transverse field and non-local conductivity for the longitudinal field in the electrolyte layer.

We compute the Casimir force per unit area across the solution for different layers and then investigate the effect of the longitudinal channel for zero and nonzero frequencies individually and indicate the effect of screening on this interaction. We also investigate the variations of the Casimir force per unit area through changing the thickness of the layer for different materials and discover the effect of increasing the concentration of the solution in intensifying this interaction. It is shown that the existence of an electrolyte layer between two symmetric double-layered media results in a weaker intensification compared to the absence of such an electrolyte solution. The results of our calculations are presented for the Hamaker coefficient of the system with different layers, various thicknesses, and diverse concentrations.

2 The Casimir interaction force per unit area

In this paper, we investigate the Casimir pressure for the system in Fig. 1 which is composed of an electrolyte solution within two dielectric layers surrounded by two other dielectric semispaces. Considering the scattering approach, the Casimir force per unit area for such a configuration is obtained with the following formula [15,16]

$$\mathcal{F} = \frac{-k_B T}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \big(\det[1-\mathbb{M}_n]\big)^{-1} \partial_L \big(\det[1-\mathbb{M}_n]\big) \tag{1}$$

where k_B is the Boltzmann constant, T refers to the temperature, and the matrix $\mathbb{M}_n = \mathbb{R}e^{-iKL}\mathbb{R}e^{-iKL}$ denotes a full round trip through the electrolyte solution. $\mathbb{R}$, the main component of the scattering theory, is the reflection matrix describing the reflection between the interfaces of the middle medium, and it is constructed with the following recipe

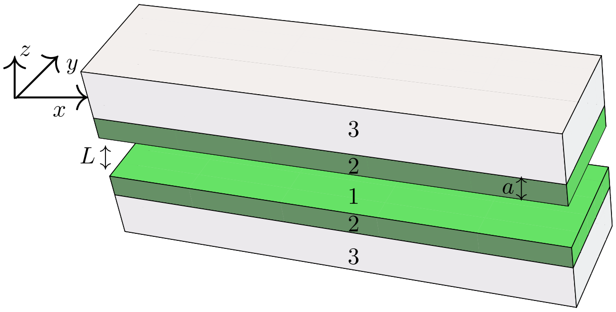

Fig. 1 Configuration for investigating the Casimir force across an electrolyte medium (region 1) with separation distance L between two layers of dielectric materials (region 2) with thickness a within two semispaces of different dielectric media (region 3)

$$\mathbb{R} = \begin{pmatrix} r_{ss} & 0 & 0 \\ 0 & r_{pp} & r_{pl} \\ 0 & r_{lp} & r_{ll} \end{pmatrix} \tag{2}$$

where r_{ll} refers to the longitudinal wave and r_{ss} and r_{pp} are the familiar reflection coefficients analogous to the TE and TM-polarizations or s and p-modes, respectively. The non-diagonal matrix element r_{pl} describes the contribution of the longitudinal modes contribution to the incident wave in the TM-polarization part of the reflected field, while r_{lp} expresses the TM-polarization contribution of the longitudinal incident wave, due to the non-local electric response in the p-mode of the reflected electric field in the inner medium.

e^{-iKL} describes free photon propagation in the inner medium between the two surfaces over the distance L with the following structure

$$e^{-iKL} = \begin{pmatrix} e^{-ik_1 L} & 0 & 0 \\ 0 & e^{-ik_1 L} & 0 \\ 0 & 0 & e^{-ik_l L} \end{pmatrix} \tag{3}$$

where k_1 and k_l are the third components of the transverse and the longitudinal incident waves in the electrolyte solution.

Assuming the effect of finite temperature in Matsubara formalism [17,18], frequency ω is regarded as $\omega = i\zeta_n$ and $\zeta_n = 2\pi n k_B T/\hbar$ where the index n refers to the nth Matsubara frequency.

3 Building determinant expression

Suppose that the inner medium is an electrolyte solution with $\epsilon_1(\omega)$ and $\epsilon_l(\omega)$ as the dielectric response functions of the transverse and longitudinal waves given by [1]

$$\epsilon_1(\omega) = \epsilon_b(\omega) - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

$$\epsilon_l(\omega) = \epsilon_b(\omega) - \frac{\omega_p^2}{\omega(\omega + i\gamma) - v_{th}^2 k^2}. \tag{4}$$

Here, $\epsilon_b(\omega)$ is pure water's dielectric function, $\omega_p = \sqrt{Nq^2/m_q}$ denotes the plasma frequency, and it depends on the free charge density N, ion charge q and ion mass m_q. γ is the damping frequency, and $v_{th} = \sqrt{k_B T/m_q}$ refers to the ions' thermal average velocity in a solution at temperature T.

As mentioned in [8], for the TM-polarized wave, the total electric field in the electrolyte solution includes a transverse part containing both the incident and the reflected TM-polarized terms as well as a longitudinal part due to the presence of ions in the medium. It can be written as

$$\mathbf{E}_{1,p}(\mathbf{r},t) = E_0 e^{-i\omega t} \times \\ \left(e^{i\boldsymbol{\kappa}_p\cdot\mathbf{r}}\ \hat{\mathbf{y}} \times \boldsymbol{\kappa}_p + r_{pp} e^{i\tilde{\boldsymbol{\kappa}}_p\cdot\mathbf{r}}\ \hat{\mathbf{y}} \times \tilde{\boldsymbol{\kappa}}_p + r_{lp} e^{i\boldsymbol{\kappa}_l\cdot\mathbf{r}}\ \boldsymbol{\kappa}_l\right) \tag{5}$$

where the subscript p refers to the p-mode and E_0 is the strength of the incident wave. $\boldsymbol{\kappa}_p = (k, 0, k_1)$ and $\tilde{\boldsymbol{\kappa}}_p = (k, 0, -k_1)$ are the wave vectors of the incident and the reflected parts, respectively, and $\boldsymbol{\kappa}_l = (k, 0, k_l)$ is the wave vector corresponding to the longitudinal reflected wave. r_{pp} and r_{lp} are the reflection coefficients analogous to the TM-reflection and the conversion between p-polarization and the longitudinal wave, respectively. The electric field in the second medium reads

$$\mathbf{E}_{2,p}(\mathbf{r},t) = E_0 e^{-i\omega t}\left(t_{pp} e^{i\boldsymbol{\kappa}_2\cdot\mathbf{r}}\ \hat{\mathbf{y}} \times \boldsymbol{\kappa}_2 + r'_{pp} e^{i\tilde{\boldsymbol{\kappa}}_2\cdot\mathbf{r}}\ \hat{\mathbf{y}} \times \tilde{\boldsymbol{\kappa}}_2\right) \tag{6}$$

where $\boldsymbol{\kappa}_2 = (k, 0, k_2)$ and $\tilde{\boldsymbol{\kappa}}_2 = (k, 0, -k_2)$ are the wave vectors of the incident and reflected p-mode, respectively. t_{pp} refers to the transmission coefficient of the second medium. From now on, we use prime superscript to denote the reflection and the transmission coefficients corresponding to the regions 2 and 3. The TM-polarized electric field in the outer local medium as a semispace only includes the transmission contribution as

$$\mathbf{E}_{3,p}(\mathbf{r},t) = E_0 e^{-i\omega t}\left(t'_{pp} e^{i\boldsymbol{\kappa}_3\cdot\mathbf{r}}\ \hat{\mathbf{y}} \times \boldsymbol{\kappa}_3\right) \tag{7}$$

with $\boldsymbol{\kappa}_3 = (k, 0, k_3)$ which is the wave vector in this medium.

Assuming the second medium to be a non-penetrable dielectric medium to ion and the fact that ions are not accumulated on the boundaries [19], the ions velocity perpendicular to the electrolyte–dielectric interface is zero. Therefore, there is no electric current density perpendicular to the interfaces and the condition $J_z = 0$ is assumed at the interface $z = 0$ on the ionic current density. The continuity boundary conditions for the tangential components of the electric and the magnetic fields are:

$$\begin{aligned} E_{1,\sigma} &= E_{2,\sigma} \\ H_{1,\sigma} &= H_{2,\sigma} \end{aligned} \tag{8}$$

Regarding the non-local electric response of the electrolyte solution, besides the transverse modes, there appeared to be longitudinal ones, as well [1]. The longitudinal incident wave reads

$$\mathbf{E}_{1,l}(\mathbf{r},t) = E_0 e^{-i\omega t}\left(e^{i\tilde{\boldsymbol{\kappa}}_l\cdot\mathbf{r}}\ \tilde{\boldsymbol{\kappa}}_l + r_{ll} e^{i\boldsymbol{\kappa}_l\cdot\mathbf{r}}\ \boldsymbol{\kappa}_l + r_{pl} e^{i\tilde{\boldsymbol{\kappa}}_p\cdot\mathbf{r}}\ \hat{\mathbf{y}} \times \tilde{\boldsymbol{\kappa}}_p\right) \tag{9}$$

with $\tilde{\boldsymbol{\kappa}}_l = (k, 0, -k_l)$. In the second medium, there only exist two transverse contributions: a transmitted contribution as well as a reflected one

$$\mathbf{E}_{2,l}(\mathbf{r},t) = E_0 e^{-i\omega t}\left(t_{pl} e^{i\tilde{\boldsymbol{\kappa}}_2\cdot\mathbf{r}}\ \hat{\mathbf{y}} \times \boldsymbol{\kappa}_2 + r'_{ll} e^{i\tilde{\boldsymbol{\kappa}}_2\cdot\mathbf{r}}\ \hat{\mathbf{y}} \times \tilde{\boldsymbol{\kappa}}_2\right) \tag{10}$$

with t_{pl} as the transmission coefficient associated to the longitudinal wave in this medium. The transmitted wave in the third medium reads

$$\mathbf{E}_{3,l}(\mathbf{r},t) = E_0 e^{-i\omega t}\left(t'_{pl} e^{i\tilde{\boldsymbol{\kappa}}_3\cdot\mathbf{r}}\ \hat{\mathbf{y}} \times \boldsymbol{\kappa}_3\right). \tag{11}$$

With respect to the boundary conditions of Eq. (8), in addition to the condition $J_z = 0$, this time at the interface $z = a$, the analogous reflection coefficients are obtained in this case. Finally, considering the TE-polarized electric wave, we have

$$\begin{aligned} \mathbf{E}_{1,s}(\mathbf{r},t) &= E_0 e^{-i\omega t}\left(e^{i\boldsymbol{\kappa}_p\cdot\mathbf{r}} + r_{ss} e^{i\tilde{\boldsymbol{\kappa}}_p\cdot\mathbf{r}}\right)\ \hat{\mathbf{y}} \\ \mathbf{E}_{2,s}(\mathbf{r},t) &= E_0 e^{-i\omega t}\left(t_{ss} e^{i\boldsymbol{\kappa}_2\cdot\mathbf{r}} + r'_{ss} e^{i\tilde{\boldsymbol{\kappa}}_2\cdot\mathbf{r}}\right)\ \hat{\mathbf{y}} \\ \mathbf{E}_{3,s}(\mathbf{r},t) &= E_0 e^{-i\omega t}\left(t'_{ss} e^{i\boldsymbol{\kappa}_3\cdot\mathbf{r}}\right)\ \hat{\mathbf{y}}. \end{aligned} \tag{12}$$

Since the electric fields inside the electrolyte do not have any interactions with this polarization, the s-mode reflection coefficient in the electrolyte solution r_{ss} is the same as the standard Fresnel coefficient for the TE-polarization. Here, t_{ss} denotes the transmission coefficient in the second medium for this polarization.

Considering the frequency dependence of the dielectric functions as well as the probable different behaviors of the Casimir force, the Hamaker coefficient and the screening effect of the non-local media at zero and nonzero frequencies, we investigate these two contributions individually. For the nonzero Matsubara frequencies $\omega \gg \omega_p$, according to Eq. (4), we presume $\epsilon_1 \simeq \epsilon_b$ and the reflection coefficients take the following simple form

$$r_{lp} = 0\,, \qquad r_{ss} = \frac{e^{2iak_2}(k_1+k_2)(k_2-k_3)+(k_1-k_2)(k_2+k_3)}{e^{2iak_2}(k_1-k_2)(k_2-k_3)+(k_1+k_2)(k_2+k_3)}$$

$$r_{ll} = 1\,, \qquad r_{pp} = \frac{e^{2iak_2}(\epsilon_2 k_1+\epsilon_1 k_2)(-\epsilon_3 k_2+\epsilon_2 k_3)-(\epsilon_2 k_1-\epsilon_1 k_2)(\epsilon_3 k_2+\epsilon_2 k_3)}{e^{2iak_2}(\epsilon_2 k_1-\epsilon_1 k_2)(-\epsilon_3 k_2+\epsilon_2 k_3)-(\epsilon_2 k_1+\epsilon_1 k_2)(\epsilon_3 k_2+\epsilon_2 k_3)} \tag{13}$$

where ϵ_2 and ϵ_3 are the dielectric functions of the dielectric media of regions 2 and 3 (Fig. 1) and the third components of the wave vectors are

$$k_{mn} = i\sqrt{k^2+\epsilon_m(i\zeta_n)\frac{\zeta_n^2}{c^2}}, \quad m = 1, 2, 3. \tag{14}$$

The third component of the longitudinal wave in the electrolyte solution is

$$k_{ln} = i\sqrt{k^2+\frac{1}{v_{th}^2}\left(\zeta_n(\zeta_n+\gamma)+\frac{\omega_p^2}{\epsilon_b(i\zeta_n)}\right)}. \tag{15}$$

The contribution to the force due to nonzero Matsubara frequencies does not depend on the longitudinal dielectric function, as it holds in [8]. For these frequencies, the reflection matrix has consequently taken a diagonal form and the determinant in Eq. (1) appears as

$$\det[1-\mathbb{M}_n] = (1-e^{2ik_{ln}L})(1-r_{pp}^2 e^{2ik_{1n}L})(1-r_{ss}^2 e^{2ik_{1n}L}), \quad n \neq 0. \tag{16}$$

$$\det[1-\mathbb{M}_n] = 1-e^{2ik_{ln}L}r_{ll}^2-2e^{i(k+k_{ln})L}r_{lp}r_{pl} -e^{2ikL}r_{pp}^2+e^{2i(k+k_{ln})L}(r_{lp}r_{pl}-r_{ll}r_{pp})^2 \tag{17}$$

the product of the non-diagonal terms appears and it is negligible. In fact, at n=0 the ions are so slow compared to field frequencies to couple transverse and longitudinal waves. Therefore, the reflection matrix is approximately diagonal and the effect of the electrolyte appears in the longitudinal reflection corresponding to the longitudinal incident. We have

$$\det[1-\mathbb{M}_n] = (1-r_{ll}^2 e^{2ik_{ln}L})(1-r_{pp}^2 e^{2ikL}), \quad n = 0 \tag{18}$$

where

$$r_{ll} = \frac{e^{2iak}\left(\dfrac{\epsilon_2(0)-\epsilon_3(0)}{\epsilon_2(0)+\epsilon_3(0)}\right)+\left(\dfrac{\epsilon_b(0)k_l-\epsilon_2(0)k}{\epsilon_b(0)k_l+\epsilon_2(0)k}\right)}{e^{2iak}\left(\dfrac{\epsilon_2(0)-\epsilon_3(0)}{\epsilon_2(0)+\epsilon_3(0)}\right)\left(\dfrac{\epsilon_b(0)k_l-\epsilon_2(0)k}{\epsilon_b(0)k_l+\epsilon_2(0)k}\right)+1},$$

$$r_{pp} = -1 \tag{19}$$

in which $\epsilon_2(0)$ and $\epsilon_3(0)$ are the zero-frequency dielectric functions of the layers and the substrates of Fig. 1, respectively, and $\epsilon_b(0)$ refers to the zero-frequency dielectric function of pure water.

4 Results for numerical calculations of the Casimir force per unit area

In this section, we apply formalism of Eq. (1) to obtain the Casimir interaction force per unit area for the system in Fig. 1. Due to the Debye–Huckel theory at dilute ionic solutions, the Debye screening length is independent of the ion size and increasing the ion concentration decreases this screening length [2]. Considering that the Debye screening length is $\lambda_D = \sqrt{\epsilon_b(0)k_BT/Nq^2}$ in such solutions, we suppose a concentration of $0.9\,mM$ for salt aqueous solution and $T = 300\ K$. Regarding the common use of silicon as a substrate, we assume the semispaces to be silicon with Drude–Lorentz model dielectric function [20] and then perform our calculations to obtain different contributions of the Casimir pressure in the salt aqueous solution surrounded by layers of different materials with the dielectric model presented in [21]. To investigate the effect of screening on the intensified Casimir force of the configuration in Fig. 1, we compute the contribution of nonzero Matsubara frequencies to the Casimir force per unit area in the vacuum between the two silicon semispaces with polyester layer on the surface. As it is illustrated in Fig. 2 even for a dilute electrolyte solution, the Casimir force is weaker than the investigated force in the absence of such a solution. This weakening is due to the thermal ionic motion.

The longitudinal modes contribution to this pressure is negligible even in nanometer distances. We have also compared the transverse modes contribution for different materials. Being presented in Fig. 3, one can see that the Casimir pressure increases through rising the conductivity of the layers, as we expected and depicted in [7].

It is worth mentioning that the transverse part of the contribution of the zero Matsubara frequency to the Casimir pressure in the solution is independent of the layers' material and it only depends on the electrolyte solution properties and the separation distance. In the following table, the results of our calculations are presented for the zero-frequency longitudinal Casimir pressure. This part of the Casimir pressure, rising from the presence of the ions, has an interesting behavior. As

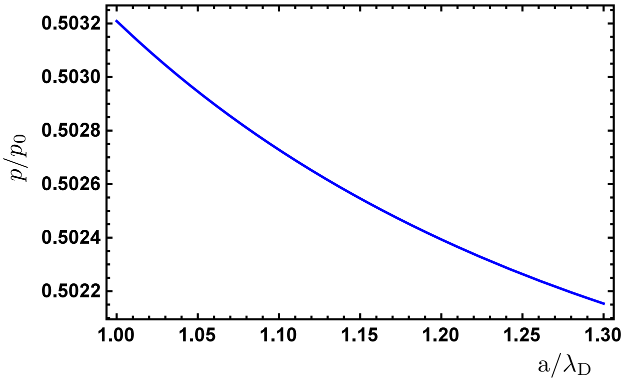

Fig. 2 The ratio of the Casimir pressure between two silicon semispaces with polyester layer on the surface in the presence of the electrolyte solution with $N = 0.9nM$ concentration (p) and in the vacuum (p_0), for separation distance $L = 0.1\,\lambda_D$ at temperature $T = 300K$. This ratio illustrates that the presence of the electrolyte solution in such a system causes the intensification of the Casimir pressure to be weaker

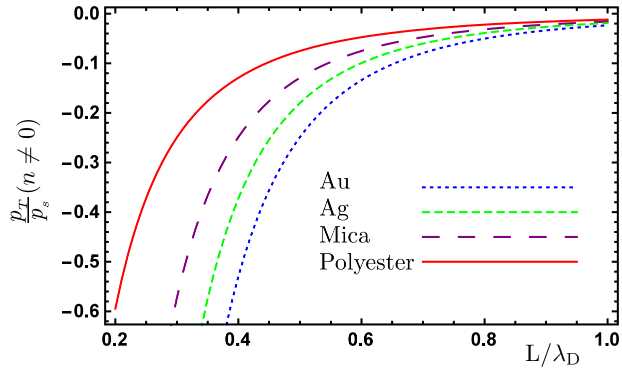

Fig. 3 Plot of the contribution of nonzero Matsubara frequencies to the scaled transverse Casimir pressure in the aqueous solution for different layers on the silicon substrates. As it is illustrated in this figure, increasing the electric conductivity of the dielectric layer causes an upward trend in the Casimir pressure. Here, the pressure scale is $p_s = (2\pi)^2 k_B T/\lambda_D^3$ at $T = 300K$ and $N = 0.9\,mM$

it is displayed in Table 1, increasing the conductivity of the layers results in a slow downward behavior of this pressure. This behavior can be interpreted according to the formation of image charges caused by the conductivity characteristic of the layer. Despite the fact that *Ag* is a conductor and mica is an insulator, the longitudinal contribution of the zero Matsubara frequency to the Casimir pressure for *Ag* is greater than mica since the zero-frequency dielectric constant of *Ag* is less than mica. The manifestation of such a considerable zero-frequency longitudinal pressure declares that in addition to the effect of the existence of the substrates in a two-slab configuration and also the presence of the layers on the substrates in intensifying the Casimir force [7], the presence of an electrolyte solution causes a longitudinal contribution at zero frequency that intensifies the Casimir pressure with the same order of magnitude as the transverse part does, see Fig. 4.

The Casimir pressure is the same for different concentrations with the scale pressure p_s corresponding to that concentration, namely the forces are the same with different scale factors. Our calculations show that increasing the concentration of salt in the solution leads to a significant growth in the Casimir pressure in the electrolyte solution. Introducing an increasing factor, $p(N_2)/p(N_1)$, i.e. the ratio of the Casimir pressure for two different concentrations, we have exhibited that increasing the concentration from $N_1 = 0.9\,mM$ to $N_2 = 90\,mM$ in Fig. 5 leads to such an intensification with an increment behavior.

Our calculations show that the transverse part of the contribution of the zero Matsubara frequency to the Casimir pressure is independent of the layers thickness. This contribution for the nonzero frequencies decreases through increasing the thickness of the layer just similar to the results of the insulators in [7], while the longitudinal modes contribution to the zero Matsubara frequency to the Casimir pressure shows an upward trend as it is depicted for the polyester layer in Fig. 6. It is worth mentioning that since the transverse nonzero-frequency contribution of the Casimir pressure is the dominant part, this pressure as a whole decreases due to the increase in the thickness of the polyester layer. The noticeable decreasing behavior of the longitudinal modes to the Casimir force per unit area at $n = 0$ for $L > \lambda_D$ can be associated to the ions' screening effect in the electrolyte solution.

5 Results for numerical calculations of the Hamaker coefficient

The interactions between particles in a medium depend on the macroscopic properties such as the dielectric constant. The Hamaker coefficient has been used as a useful instrument for surface interfacial energies [22,23] to investigate the screening effect on the Casimir effect in the presence of substrates for the insulator and conductor layers. For the interactions between two different bodies in a liquid, this coefficient can be calculated as

$$H(L) = -12\pi L^2 \mathcal{E}(L) \tag{20}$$

where $\mathcal{E}(L)$ is the Casimir energy per unit area for the separation distance L. Our calculations for different layers show that the longitudinal modes contribution to the Hamaker coefficient for nonzero frequencies is negligible; however, it results in a remarkable contribution at zero frequency as it is depicted in Fig. 7. This portion of the Hamaker coefficient, rising from the longitudinal scattering channel is important for the separation distances shorter than the Debye screening length and for longer separation distances this contribution has been screened. One can interpret this downward

Table 1 The longitudinal contribution of the zero Matsubara frequency to the scaled Casimir pressure in an electrolyte solution surrounded by two layers of polyester, mica, silver and gold on the silicon substrates. We consider $a = 0.1\lambda_D$ for the thickness of the dielectric layer in between and $0.9\,mM$ for the concentration of the solution. In this table, the separation distance L is presented versus λ_D as the length scale and correspondingly define p_s as the scale of pressure for the Casimir pressure

L/λ_D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$p_{l,Polyester}$	− 0.984	− 0.121	− 0.0352	− 0.0145	− 0.00713	− 0.00394	− 0.00235	− 0.00147	− 0.000968	− 0.000654
$p_{l,Mica}$	− 0.932	− 0.115	− 0.0337	− 0.0139	− 0.00687	− 0.00381	− 0.00227	− 0.00144	− 0.000943	− 0.000638
$p_{l,Ag}$	− 0.946	− 0.117	− 0.0341	− 0.0140	− 0.00692	− 0.00384	− 0.00229	− 0.00145	− 0.000948	− 0.000641
$p_{l,Au}$	− 0.869	− 0.109	− 0.0323	− 0.0134	− 0.00667	− 0.00371	− 0.00223	− 0.00141	− 0.000926	− 0.000628

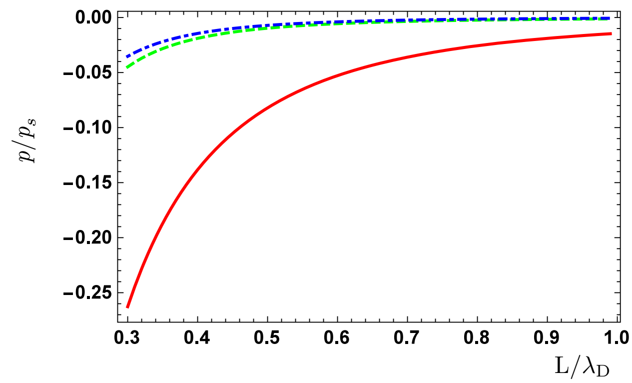

Fig. 4 A comparison between different contributions of the scaled Casimir pressure of Fig. 1 configuration for layers of polyester with $a = 0.1\lambda_D$ thickness on the silicon substrates. Here, the pressure scale is $p_s = (2\pi)^2 k_B T/\lambda_D^3$ at $T = 300K$ and $N = 0.9\,mM$. The red diagram (solid line) refers to the transverse contribution of the nonzero Matsubara frequencies to the Casimir pressure, the green diagram (dashed line) presents the transverse contribution of the zero Matsubara frequency to the pressure, and the blue one (dash-dotted line) displays the longitudinal modes contribution to this pressure. The longitudinal contribution of nonzero Matsubara frequencies to the Casimir pressure is negligible. This is while the longitudinal contribution of the zero Matsubara frequency to the pressure that rises from the longitudinal channel and has an intensifying role in the Casimir pressure is in the same order of magnitude as the transverse contribution of zero Matsubara frequency to the Casimir pressure and the transverse portion of this pressure for the nonzero Matsubara frequencies is the dominant part of the Casimir pressure

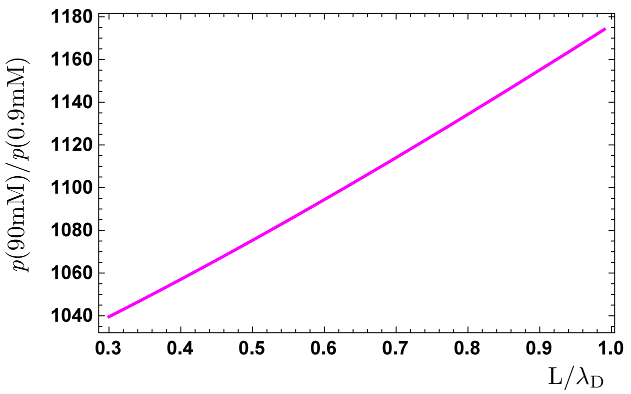

Fig. 5 The result of our calculations for the ratio of the Casimir pressure in an electrolyte solution between two layers of polyester on the silicon substrates, due to an increase in salt concentration from $N_1 = 0.9\,mM$ to $N_2 = 90\,mM$

behavior through increasing the conductivity of the layers according to the longitudinal channel origin.

It is worth mentioning that the transverse contribution of the zero-frequency Hamaker coefficient equals $3\,\zeta(3)\,k_B T/4$ and it is independent from the material of the layer, the substrate and even the concentration of the solution. The significant contribution of this coefficient is the nonzero-frequency transverse part. Considering its different contributions, the Hamaker coefficient presents an interesting behavior as it is illustrated in Fig. 8. For insulators, this diagram has a peak before the Debye screening wave length and this peak appears in shorter separation distances if the conductivity increases. As it is expected for the conductors, the diagram shows a monotonic trend and screening happens after λ_D. The diagram in Fig. 8 shows a downward trend for $L > \lambda_D$ for both insulators and conductors. A comparison between Fig. 8a and b reveals that the Hamaker coefficient shows a significant intensification with more concentration of the solution in the inner medium in Fig. 1.

It is worth noting that increasing the thickness of the layer increases the transverse part of the nonzero frequencies of the Hamaker coefficient for conductors, while it decreases for the insulators. The longitudinal modes contribution to the zero frequency of the Hamaker coefficient shows an opposite behavior. Since the nonzero frequencies of the transverse part of the Hamaker coefficient is the dominant portion, the Hamaker coefficient generally shows an upward trend after increasing the thickness of the conductor layer and a downward trend through increasing the thickness of the insulator layer, as illustrated in Fig. 9.

Note that the Hamaker coefficient reaches its asymptotic value $3\,\zeta(3)\,k_B T/4$ at thermal wave length $\hbar c/k_B T = 7.6 \times 10^{-4}\,cm$ corresponding to $747\lambda_D$ for $N = 0.9\,mM$ concentration. This reduction occurs exponentially.

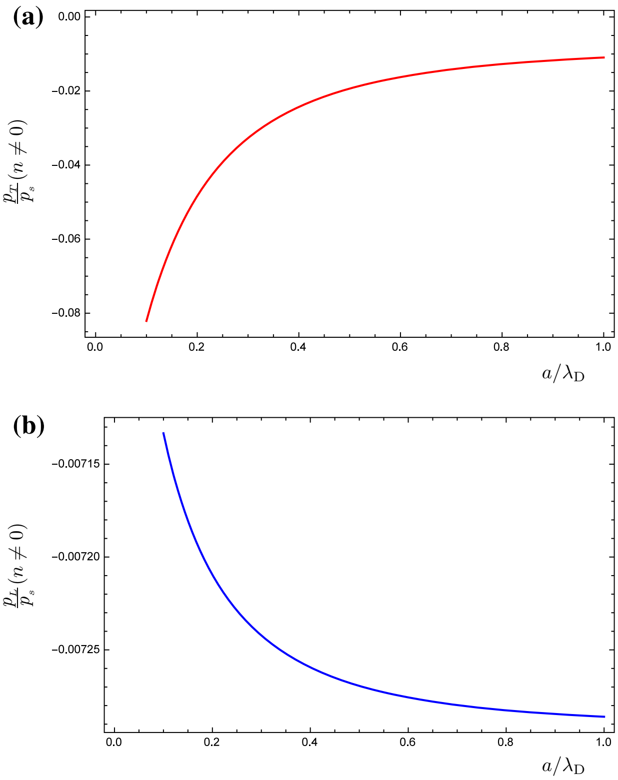

Fig. 6 Plot of the thickness-dependent contributions of the scaled Casimir pressure for the fixed separation distance $L = 0.5\,\lambda_D$. As it is already illustrated, the nonzero-frequency transverse part shows a downward behavior after increasing the thickness of the polyester layer in (**a**), while the longitudinal zero-frequency mode shows an upward trend in (**b**)

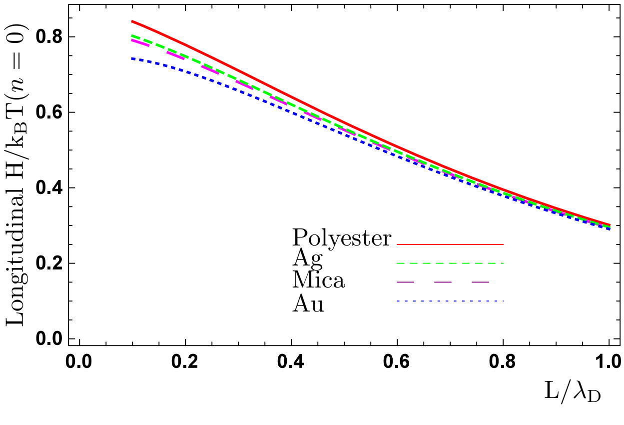

Fig. 7 Plot of the longitudinal modes contribution of the Hamaker coefficient for different layers. We consider silicon semispaces and assume $a = 0.1\,\lambda_D$ and $N = 0.9\,mM$. Increasing the zero-frequency dielectric constant of the layer decreases this part of the Hamaker coefficient

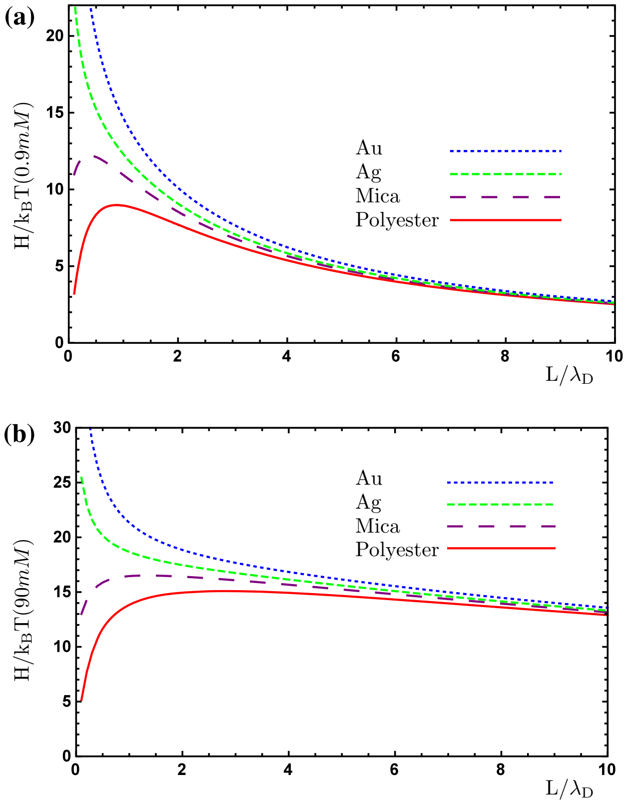

Fig. 8 Plot of the Hamaker coefficient for different layers. We set $N = 0.9\,mM$ in (**a**) and $N = 90\,mM$ in (**b**). It is clear that screening happens for $L > \lambda_D$. The diagrams both asymptote to $3\,\zeta(3)/4$; that is, the longitudinal modes contribution of the Hamaker coefficient at zero frequency has scaled with k_BT

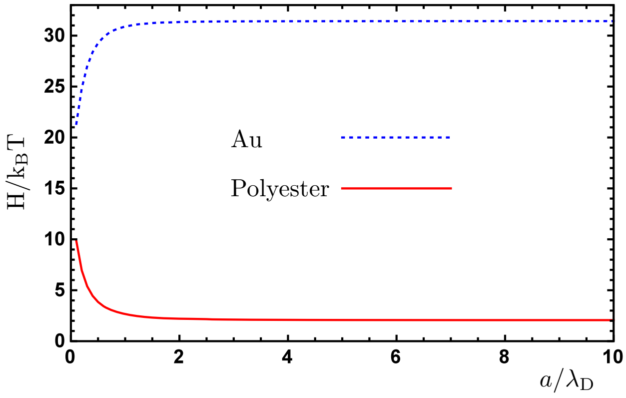

Fig. 9 Diagram presenting the variations of the Hamaker coefficient with the layers' thickness. We assume $N = 0.9\,mM$ and the fixed separation distance $L = 0.5\,\lambda_D$. It is obvious that polyester and gold show opposite behaviors, as instances of insulators and conductors, respectively. It should be noted that for thicknesses greater than the Debye wavelength, screening occurs and increasing the thickness of the layer does not result in any further intensifications

6 Conclusion

In this work, the scattering approach has been used to study the effects of an electrolyte solution on screening the Casimir interaction force across a dilute electrolyte solution surrounded by two layers of dielectric materials within two semispaces. The electric field in the solution has been decomposed into a common transverse part and a longitudinal component which is existing due to the non-local electric response of the electrolyte solution. In this regard, the continuity boundary condition has been imposed on the tangential components of the electric and the magnetic fields on the surfaces and a reflection matrix has been constructed which describes the conversion between the longitudinal and the transverse distributions of the electric field in the electrolyte solution. We have investigated zero and nonzero frequencies individually, since the Casimir pressure, the Hamaker coefficient, and the screening effect may display different behaviors in these frequencies. We have assumed $0.9\,mM$ as a typical value for the salt concentration in the aqueous solution at temperature $T = 300K$. Layers of Au, Ag, mica, and polyester have been supposed surrounding the aqueous solution, with silicon substrates. Our calculations depict that the longitudinal component of the nonzero frequencies' contribution to the Casimir pressure is insignificant and its corresponding transverse component is less intensified in the presence of the ions. The longitudinal modes contribution to the Casimir effect at zero frequency displays an interesting behavior. This portion, caused by the presence of the ions, is a function of the zero-frequency dielectric constant of the layers' materials and this effects the intensification of the pressure. This longitudinal modes contribution to the Casimir pressure at zero frequency shows a downward trend when increasing the zero-frequency dielectric constant of the layers' material. It seems that the formation of the image charges in the conductor layers results in such a behavior. In a comparison between the zero-frequency longitudinal portion of the Casimir pressure for the configuration in Fig. 1 with Ag and mica layers, an undesirable behavior has been found. Since the zero-frequency dielectric constant of mica is greater than Ag, despite the fact that mica is an insulator, this contribution for Ag as a conductor is stronger than mica. This is while the transverse part of the zero frequency of the force is independent from the layers' material. A perspective of different contributions of the force indicates that the longitudinal and transverse modes contribution to this force at $n = 0$ are of the same order of magnitude, and the longitudinal modes contribution to $n \neq 0$ can be ignored and its corresponding transverse part takes the largest proportion. Generally, with the growth in the conductivity of the layers, the Casimir pressure increases as already expected. We would like to mention that increasing the concentration of the salt in the solution leads to a remarkable increase in the Casimir pressure. The transverse contribution of the zero Matsubara frequency to the Casimir pressure is independent from the layers' thickness and increasing the thickness of the insulator layer causes a downward trend in this contribution for the nonzero frequencies, while it increases the longitudinal modes contribution to the zero Matsubara frequency to the Casimir pressure. Regarding the screening effect of ions in the electrolyte solution at $n = 0$ for $L \gg \lambda_D$, a noticeable fall occurs in the longitudinal modes contribution to the Casimir pressure. The Hamaker coefficient has also been investigated for the configuration in Fig. 1 with layers of different materials and silicon semispaces. A significant contribution has been found for the longitudinal modes contribution to the Hamaker coefficient at $n = 0$. This portion, caused by the presence of ions in the electrolyte, decreases through increasing the conductivity of the layers. Its corresponding transverse part is $3\,\zeta(3)\,k_BT/4$ and it only depends on the temperature. Just like the Casimir pressure, the nonzero-frequency transverse contribution of the Hamaker coefficient has a significant contribution. The Hamaker coefficient illustrates an interesting behavior; it has got a maximum value. This extremum takes place in shorter separation distances if the layers become more conductive. For conductor layers, we will have a monotonic function and for $L \gg \lambda_D$ there appears a remarkable fall due to the screening effect. It seems that this coefficient shows an upward trend after increasing the concentration. Assuming the same materials for the regions 2 and 3 in Fig. 1, our results match the results obtained in [8]. It seems that the transverse part of the nonzero frequencies of the Hamaker coefficient for conductors shows an upward behavior with the growth of the layers thickness, while it displays an opposite behavior for layers of insulator materials. However, the longitudinal portion of this coefficient as a result of the longitudinal channel at $n = 0$ shows a completely opposite trend. Generally, increasing the thickness of the conductor layer results in increasing the Hamaker coefficient, while it ends in decreasing the coefficient for the insulator layers.

At thermal wavelength $\hbar c/k_BT = 7.6 \times 10^{-4}\,cm$ that is equal to $747\lambda_D$ for $N = 0.9\,mM$ concentration, the zero-frequency contribution of the Hamaker coefficient exponentially takes its asymptotic value $3\zeta(3)\,k_BT/4$.

Acknowledgements We would like to thank Paulo A. Maia Neto for his useful comment.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical study and no experimental data has been listed.]

References

1. B. Davies, B.W. Ninham, J. Chem. Phys. **56**, 5797 (1972)
2. P.W. Debye, E. Huckel, Phys. Z. **24**, 185 (1923)
3. Y. Levin, Rep. Prog. Phys. **65**, 1577 (2002)

4. T. Markovich, D. Andelman, R. Podgornik, in *Handbook of Lipid Membranes*, edited by C. Safinya and J. Radler (Taylor and Francis Group, in press)
5. A. Lambrecht, P.A. Neto, S. Reynaud, New J. Phys. **8**, 243 (2006)
6. S.J. Rahi, T. Emig, N. Graham, R.L. Jaffe, M. Kardar, Phys. Rev. D **80**, 085021 (2009)
7. A. Seyedzahedi, A. Moradian, M.R. Setare, Phys. Lett. A **380**, 1475 (2016)
8. P.A.M. Neto, F.S. Rosa, L.B. Pires, A.B. Marim, A. Canaguier-Durand, R. Guerout, A. Lambrecht, S. Reynaud, Eur. Phys. J. D **73**, 178 (2019)
9. R. Esquivel, V.B. Svetovoy, Phys. Rev, A **69**, 062102 (2004)
10. A. Naji, M. Kandu, J. Forsman, R. Podgornik, J. Chem. Phys. **139**, 150901 (2013)
11. R.R. Netz, Eur. Phys. J. E **5**, 557 (2001)
12. E.C. Donaldson, W. Alam, *Wettability* (TIPS Technical Publishing Inc, United States of America, 2008)
13. J.N. Israelachvili, *Intermolecular and Surface Forces* (Academic Press, Waltham, 2011)
14. H.J. Butt, M. Kappl, *Surface and Interfacial Forces* (Wiley-VCH, Weinheim, 2010)
15. T. Emig, N. Graham, R.L. Jaekel, M. Kardar, Phys. Rev. Lett. **99**, 170403 (2007)
16. A. Lambrecht, P.A.M. Neto, S. Reynaud, New J. Phys. **8**, 243 (2006)
17. J.I. Kapusta, *Finite Temperature Field Theory* (Cambridge University Press, 1989)
18. A. Das, *Quantum Field Theory, A 20th Century Profile*, edited by A. N. Mitra (Indian National Science Academy 2000)
19. F. Garcia-Moliner, F. Flores, J. Phys. France **38**, 851 (1977)
20. L. Bergstrom, Adv. in Colloid. and Interf. Sci. **70**, 125 (1997)
21. V.A. Parsegian, *van der Waals Forces: A Handbook for Biologists, Chemists, Engineers and Physicists* (Cambridge University Press, Cambridge, 2005)
22. H.C. Hamaker, Physica 4 **10**, 1058 (1973)
23. W.B. Russel, D.A. Saville, W.R. Schowalter, *Colloidal Dispersions* (Cambridge University Press, Cambridge, 1989)