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# DWBA double differential ionization cross sections for positron and electron impact on argon

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**Abstract.** The ionization of the 3p orbital of argon by 100 eV incident electrons and positrons is studied with the DWBA-WM approximation. The variation of our double differential ionization cross sections with the angles of the ejected electrons is found to be in reasonable agreement with the experiment. We found that the inclusion of the post-collision interaction in the theoretical model is necessary to obtain the size of the difference between the experimental double differential cross sections for electron and positron impact ionization.

## 1 Introduction

The ionization of noble gas atoms in coplanar asymmetric geometry is one of the phenomena for which one can study competing interactions by using both electrons and positrons as projectiles. Double and triple differential cross sections (DDCS and TDCS) are particularly suitable for this task. Unfortunately the availability of intense positron beams continues to be a challenge and the number of experiments for these studies is quite small. This is also the reason why many of the positron impact studies dealt with the argon-gas target, where the larger cross section can partially offset the low positron flux.

In a recent paper [1] we presented TDCS results for the ionization of argon by 1 keV electrons and positrons. We found that our distorted-wave Born approximation (DWBA) model agrees reasonably well with the experiment of DuBois et al. [2] after we included the experimental energy resolution.

In this work we present the variation of DDCS with the angles of the ejected electrons for the ionization of argon with 100 eV electrons and positrons. The energy of the ejected electrons is 15 eV. Experimentally this study was performed by Schmitt et al. [3,4].

## 2 Theory

### 2.1 Positron impact

The triple differential cross-section for the ionization of argon by positron impact may be written as

$$\frac{d^3\sigma}{d\hat{k}_f d\hat{k}_e dE_e} = \sum_r \frac{(2\pi)^4}{E_i} |f_r|^2 . \tag{1}$$

Here $E_i$ is the energy of the incident positron, $E_e$ the energy of the ejected electron, while $\hat{k}_e$ and $\hat{k}_f$ stand for the direction of the momenta of the ejected electron and scattered positron, respectively. The summation over $r$ is done over all occupied atomic orbitals. The DWBA results presented in this paper correspond to the ionization of the 3p orbital of argon. The contributions coming from the ionization of the 3s orbital were negligible for the impact energies considered in our calculations.

The amplitude can be written as

$$f_r = \langle \phi_f(\vec{r}_1)\, \phi_e(\vec{r}_2) | V(r_{12}) | \phi_i(\vec{r}_1)\, \phi_r(\vec{r}_2) \rangle, \tag{2}$$

where $\phi_i$ and $\phi_f$ stand for the wavefunction of the incident and scattered positron respectively, $\phi_e$ is the wavefunction of the ejected electron, while $\phi_r$ describes the initial state of the active electron. In the above amplitude $\vec{r}_1$ is the position vector of the positron, while $\vec{r}_2$ stands for the position vector of the active electron.

Our quantum model in the DWBA approximation includes the distortion of the positron and ejected electron waves in various fields seen by these particles. For the incident positron we considered the static field of Ar and the atomic polarization potential. Both were calculated in the Hartree-Fock approximation. The final state channel description depends on the energies of the outgoing leptons. As the energy of the ejected electron is only 15 eV the scattered positron is much faster than the ejected electron. For this reason we considered that the scattered positron sees the same static field and polarization potential as the incident positron. For the ejected electron channel we considered the static potential of the $Ar^+$ ion and a Furness-McCarthy exchange potential [5], where we assumed that the exchange is triplet for all bound electrons.

For the post-collision interaction (PCI) between the scattered positron and the ejected electron we used the

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low energy approximation of Ward and Macek [6] and we shall call the model DWBA-WM. This PCI approximation produces the external factor:

$$|C_{proj\text{-}eject}|^2 = G\left|{}_1F_1\left(\frac{i\gamma}{2\pi}, 1, \frac{i2\pi}{\gamma} r_{ab}^{ave}\right)\right|^2, \qquad (3)$$

where the last parameter of the hypergeometric function ${}_1F_1$ includes:

$$r_{ab}^{ave} = \frac{\pi^2}{16\varepsilon_t}\left(1 + \frac{0.627}{\pi}\sqrt{\varepsilon_t}\ln\varepsilon_t\right)^2. \qquad (4)$$

$\varepsilon_t$ is the total energy of the scattered positron and ejected electron and $G$ is the Gamow factor:

$$G = \frac{\gamma}{\exp(\gamma) - 1}$$

with

$$\gamma = \frac{-2\pi}{|\mathbf{k}_f - \mathbf{k}_e|}. \qquad (5)$$

In this paper we obtained DDCS by integrating the TDCS over the scattered projectile angles. In addition we performed a Gaussian convolution with the experimental angular resolution, which was 6° for the ejected electrons.

### 2.2 Electron impact

For electron impact equation (1) is modified with the inclusion of the exchange amplitude:

$$g_r = \langle\phi_f(\vec{r}_2)\,\phi_e(\vec{r}_1)|V(r_{12})|\phi_i(\vec{r}_2)\,\phi_r(\vec{r}_1)\rangle. \qquad (6)$$

In representing the distortion of the incident electron wave function in addition to the static field of Ar and the atomic polarization potential we added the Furness-McCarthy exchange potential [5].

The scattered electron is faster than the ejected electron and we approximated the potential seen by the scattered electron with the same potential as in the incident channel. For the ejected electron channel we considered the static potential of the $\mathrm{Ar}^+$ ion and a Furness-McCarthy exchange potential.

As far as the PCI between the scattered electron and the ejected electron is concerned the only change from the positron impact case is the change in the sign of $\gamma$.

## 3 Results and discussion

Figure 1 presents our theoretical DDCS for 100 eV projectile impact energy and 15 eV ejected electrons using the DWBA without the inclusion of PCI. We used a full line for the case of the positron impact and short dashed line for the case of electron impact.

This graph shows that the DWBA approximation gives quite similar DDCS curves for positron and electron impact. Both curves show an unexpected minimum around 60°.

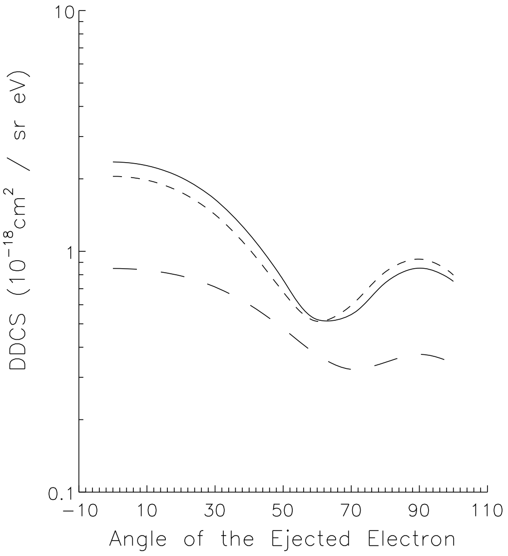

**Fig. 1.** Double differential cross sections for Ar ionization by 100 eV projectiles and for 15 eV ejected electrons. The continuous curve corresponds to our DWBA model for positron impact and the short dashed curve to our DWBA model for electron impact. The large dashed curve shows the effect of eliminating electron exchange in the electron impact calculation.

We tried to determine the origin of this minimum by modifying the representation of leptons in various channels. For instance we did not including polarization, electron exchange, and we used an ion potential for the scattered electron and plane waves for the incoming projectiles. The only effect that seemed to affect that minimum was the electron exchange with Furness-McCarthy. We found that the only the distortion due to the inclusion of the exchange potential in the electron channels is somewhat related to that minimum. The longer dashed curve in Figure 1 corresponds to an electron impact ionization model in which we eliminated the Furness-McCarthy exchange potential from all electron channels. In this case the shape of the DDCS curve shows a less pronounced minimum around 70°. In the positron impact case the model contains only the exchange potential for the ejected electron and its elimination has little impact on the DWBA results.

Figure 2 presents our theoretical DDCS obtained with DWBA and the low energy approximation of Ward and Macek [6] for the post-collision leptons interaction (model DWBA-WM). Our results are compared with the experimental data of Schmitt et al. [3,4], which were normalized to our electron impact DDCS for the angle of the ejected electrons equal to 20°.

Our calculations confirm that the positron impact DDCS are significantly larger than the electron impact DDCS. Also we confirm the experimental finding that the DDCS decrease with the angle of the ejected electron for the positron impact case and increase in the electron impact case. The agreement between our theoretical DDCS and the experiment is quite good for all angles. Figures 1 and 2 show that only with the inclusion of the PCI between the scattered projectile and the

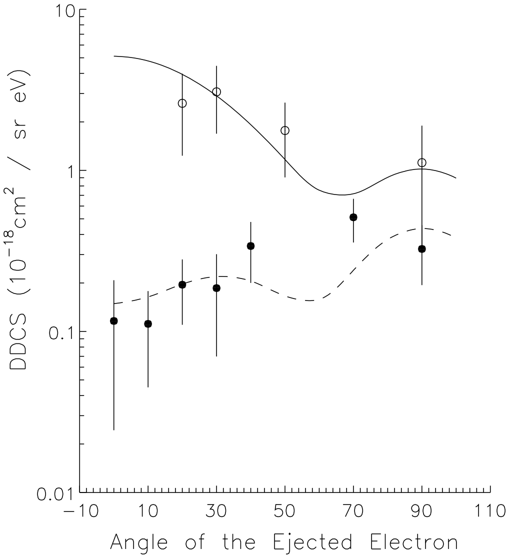

**Fig. 2.** Double differential cross sections for Ar ionization by 100 eV projectiles and for 15 eV ejected electrons. The continuous curve corresponds to our DWBA-WM model for positron impact and the dashed curve to our DWBA-WM model for electron impact. The experimental results are the relative measurements of Schmitt et al. [3,4] scaled to fit our electron impact DWBA-WM data at an angle of the ejected electron equal to 20°.

ejected electron one can obtain the correct size difference between the DDCS for positron and the electron impact.

We should mention that the experimental cross sections reported by Schmitt et al. [3,4] were 5.2 times lower than the experimental results of Figure 2. In the paper by Schmitt et al. [3,4] the experimental data was normalized to the theoretical DDCS obtained for the electron impact ionization of atomic hydrogen. The same normalization factor was used for the positron impact experimental data.

The normalization to the theory for the atomic hydrogen was based on: (i) the absence of theoretical DDCS for argon and (ii) the observation that for hydrogen and argon in their ground states the asymptotic shapes of their wave functions are very similar.

Our current work shows that the normalization used by Schmitt et al. [3,4] provides a reasonable variation of DDCS with the angle of the ejected electrons but it provides an incorrect size for the DDCS.

## 4 Conclusions

We conclude that our DWBA-WM model produces a variation of the DDCS with the angles of the ejected electrons which agrees quite well with the variation of the experimental data of Schmitt et al. [3,4]. Our work shows that the experimental data of Schmitt et al. [3,4] should be increased by a factor of 5.2 to agree with our theoretical results.

Our DWBA-WM model included the distortion of the positron and electron waves in the static field of the target. We also included where suitable the argon polarization and the Furness-McCarthy exchange potential. In this work we found that the addition of the PCI between the scattered positron and the ejected electron through the Ward-Macek PCI approximation is very important. Only by including the PCI one obtains the size difference between the experimental DDCS for electron and positron impact ionization.

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