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Combined analysis on nature of $X(3960)$, $\chi_{c0}(3930)$, and $X_0(4140)$

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Abstract In this work, a study of the $D_s\bar{D}_s$ interaction and its couplings to the channels $D\bar{D}$ and $J/\psi\phi$ is performed in a quasipotential Bethe–Salpeter equation approach. The $D_s^+D_s^-$ and D^+D^- invariant mass spectra in three-body B decays are investigated in order to understand the origin of $X(3960)$, $\chi_{c0}(3930)$, and $X_0(4140)$ structures reported at LHCb. With the help of effective Lagrangians, the potential kernel can be constructed with meson exchanges, from which the scattering amplitudes can be obtained. By inserting it into the three-body decay processes, the invariant mass spectra can be calculated with an additional Breit–Wigner resonance introduced. The $D_s^+D_s^-$ invariant mass spectrum in the decay process $B^+ \to D_s^+D_s^-K^+$ is well reproduced, and the $X(3960)$ structure can be explained as a molecular state from the $D_s\bar{D}_s$ interaction. The state also exhibits as a peak in the D^+D^- invariant mass spectrum in the decay $B^+ \to D^+D^-K^+$, however, it is too narrow to reproduce the experimental structure around 3930 MeV. The dip around 4140 MeV in the $D_s^+D_s^-$ invariant mass spectrum can be reproduced by the additional resonance with interference effect. However, a small bump instead of a dip is produced from the $D_s\bar{D}_s - J/\psi\phi$ coupled-channel effect, which suggests that the $X_0(4140)$ cannot be interpreted as the $D_s\bar{D}_s - J/\psi\phi$ coupling in the current model.

1 Introduction

Recently, the LHCb collaboration reported two near-threshold structures named $X(3960)$ and $X_0(4140)$ in the $D_s^+D_s^-$ invariant mass spectrum of the decay $B^+ \to D_s^+D_s^-K^+$. The mass, width and quantum numbers of these two structures were measured to be: $M_{X(3960)} = 3956 \pm 5 \pm 10$ MeV, $\Gamma_{X(3960)} = 43 \pm 13 \pm 8$ MeV, $M_{X_0(4140)} = 4133 \pm 6 \pm 6$ MeV, $\Gamma_{X_0(4140)} = 67 \pm 17 \pm 7$ MeV, and both with spin parity and charge parity $J^{PC} = 0^{++}$ [1]. The resonance peak of $X(3960)$ is very close to the $D_s^+D_s^-$ threshold, making it a good candidate of a $D_s^+D_s^-$ hadronic molecule. As suggested by the LHCb collaboration, the $X_0(4140)$ might be caused by either a new resonance or by the $D_s^+D_s^- - J/\psi\phi$ coupled-channel effect, but a determinative conclusion have not been drawn [1].

The $X(3960)$ is certainly not the first state observed in the energy region of 3.9–4.2 GeV. Over the past few decades, several experimental candidates of charmonium-like states have been observed, such as the $X(3915)$, the $\chi_{c0}(3930)$ and the $\chi_{c2}(3930)$. The $X(3915)$ was originally observed at Belle in its $\omega J/\psi$ decay mode, and was produced in both B decay [2] and $\gamma\gamma$ collisions [3] with J^{PC} determined as 0^{++} [4]. In 2020, the LHCb collaboration reported the D^+D^- decay mode of the $\chi_{c0}(3930)$ and $\chi_{c2}(3930)$ using B decays and determined their J^{PC} to be 0^{++} and 2^{++}, respectively [5]. In the current version of the Review of Particle Physics (PDG), the $X(3915)$ decaying to $\omega J/\psi$ and the $\chi_{c0}(3930)$ decaying to D^+D^- are assumed to be both the $\chi_{c0}(3915)$ with 0^{++} [6]. The $X(3960)$ seems to be the same particle as $\chi_{c0}(3930)$ if one considers that the mass and width of the $X(3960)$ state are consistent with those of the $\chi_{c0}(3930)$ meson within 3σ measured by the LHCb Collaboration [1,5]. However, such assumption lead to a partial width ratio $\Gamma(X \to D^+D^-)/\Gamma(X \to D_s^+D_s^-) = 0.29 \pm 0.09 \pm 0.10 \pm 0.08$. It contradicts the expectation that $\Gamma(X \to D^+D^-)$ should be considerably larger than $\Gamma(X \to D_s^+D_s^-)$ if X does not have any intrinsic $s\bar{s}$ content [1]. Hence, the $X(3960)$ and $\chi_{c0}(3930)$ are neither the same resonance, nor the same non-conventional charmonium-like state. It is also puzzled to denote $X(3960)$ as a conventional charmonium-like state because the mass of $\chi_{c0}(3P)$ is around 4131–4292 MeV, which is far away from $X(3960)$.

Many theoretical efforts have been made into understand the origin and structure of the $X(3960)$. Since its mass is close to the $D_s^+D_s^-$ threshold, the hadronic molecular state

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is a promising picture to explain the $X(3960)$. In Refs. [7,8], the molecular state interpretation was proposed to assign the $X(3960)$ as a $D_s^+D_s^-$ state with $J^{PC}=0^{++}$ in the QCD sum rules approach. In Ref. [9], the $X(3960)$ can be well described by either a bound or a virtual state below the $D_s^+D_s^-$ threshold. Similar conclusion can be found in Ref. [10], where calculation was performed in an effective Lagrangian approach to study the production rate of $X(3960)$ in the B decays utilizing triangle diagrams and assuming the $X(3960)$ as a bound/virtual state from the $D_s^+D_s^-$ interaction. Coupled-channel analysis has also been proposed to reveal the nature of the $X(3960)$ state, which are often combined with the analysis of the $\chi_{c0}(3930)$. Bayar *et al*. performed a coupled-channel calculation of the interaction $D\bar{D}-D_s\bar{D}_s$ in the chiral unitary approach and concluded that the $X(3960)$ in the $D_s^+D_s^-$ mass distribution and the $X(3930)$ in the D^+D^- mass distribution are the same state [11]. Chen et al. performed an analysis on decays $\chi_{c0}(3930)\to D\bar{D}$, $X(3960)\to D_s\bar{D}_s$, and $X(3915)\to J/\psi\omega$ using both a K-matrix approach and a model of Flatte-like parameterization. It was suggested that the $X(3960)$ has probably the mixed nature of a $c\bar{c}$ confining state and $D_s\bar{D}_s$ continuum [12].

There are also several theoretical and experimental works about the $X_0(4140)$ but its origin is still under debate. The LHCb collaboration interpreted it as a new resonance with the 0^{++} assignment or the $D_s\bar{D}_s$-$J/\psi\phi$ coupled-channel effect [1]. Badalian et al. considered $X_0(4140)$ and $X(3960)$ as exotic four-quark states and concluded that the $X_0(4140)$ is formed in the transitions $J/\psi\phi$ into $D_s^{*+}D_s^{*-}$ and back in the framework of the extended recoupling model [13]. Agaev *et al*. studied the process $D_s\bar{D}_s-J/\psi\phi$ by QCD three-point sum rule analyses, and got the conclusion that the structure $X_0(4140)$ may be interpreted as a hadronic molecule $D_s^+D_s^-$, whereas the resonance X(3960) can be identified as a tetraquark $[cs][\bar{c}\bar{s}]$ [14]. Since different interpretations were proposed in the literature, it is important to come up with new ideas that can help us reveal the origin of $X_0(4140)$.

In our previous studies, the $D_s^+D_s^-$ molecular states were studied in a quasipotential Bethe–Salpeter equation (qBSE) approach [15]. Our calculation favors the existence of hidden-heavy bound state $D_s^+D_s^-$ with $J^P=0^+$ at a value of cutoff Λ=1.6 GeV. Inspired by the new experimental observation of $X(3960)$ and $X_0(4140)$, we will explore the $D_s\bar{D}_s$ interaction and its couplings to channels $D\bar{D}$ and $J/\psi\phi$, and compare the theoretical results with the experimental data to discuss the origin of the $X(3960)$, $\chi_{c0}(3930)$ and $X_0(4140)$.

This work is organized as follows. After the introduction, the formalism of three-body decay will be presented in Sect. 2. Lagrangians used to construct the potential kernels, the qBSE approach, and formula of the invariant mass spectrum will be also presented. The numerical results of the corresponding invariant mass spectrums will be given, and the origins of $X(3960)$, $\chi_{c0}(3930)$, and $X_0(4140)$ will be discussed in Sect. 3. A summary of the whole work will be given in Sect. 4.

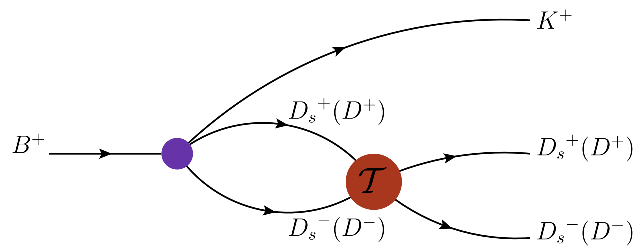

Fig. 1 The diagram for the three-body decay and rescattering process of $B^+\to D^+_{(s)}D^-_{(s)}K^+$

2 Formalism of three-body decay

2.1 Mechanism of three-body decay

In this work, the three-body decay mechanism as illustrated in Fig. 1 in order to investigate the $X(3960)/\chi_{c0}(3930)$ resonance structure observed in the process $B^+\to(X(3960)/\chi_{c0}(3930))K^+\to(D_s^+D_s^-/D^+D^-)K^+$ by LHCb collaboration. The $X(3960)/\chi_{c0}(3930)$ is assumed as an S-wave $D_s^+D_s^-$ molecular state. The B meson decays to $D_s^+D_s^-$ or D^+D^- and K^+ firstly, and the intermediate $D_s^+D_s^-$ molecule takes part in the rescattering process and obtain final products $D_s^+D_s^-/D^+D^-$ subsequently. Besides, to reveal the nature of the structure found around 4140 MeV, the $J/\psi\phi-D_s^+D_s^-$ coupling will be also considered but only in the rescattering process (which is not shown in the diagram) in order to be consistent with experimental analysis [1].

First, we need to treat the direct vertex $B^+\to D^+_{(s)}D^-_{(s)}K^+$ as shown as a blue full circle in Fig. 1. As in Ref. [16], the amplitudes can be constrained by Lorentz invariance to have a form of $\mathcal{A}_{B^+\to D^+_{(s)}D^-_{(s)}K^+}=c_1(c_2)$, where c_1 and c_2 are coupling constants for processes $B^+\to D_s{}^+D_s{}^-K^+$ and $B^+\to D^+D^-K^+$, respectively. The values of c_1 and c_2 can be estimated by the decay width of each process. It can be obtained by multiplying their branching fractions $\mathcal{B}_{B^+\to D^+D^-K^+}=(2.2\pm0.7)\times10^{-4}$ and $\mathcal{B}_{B^+\to D_s^+D_s^-K^+}=(1.15\pm0.07\pm0.06\pm0.38)\times10^{-4}$ [17] with the decay width of B meson obtained from its mean life $\Gamma_B=1/\tau=6.105\times10^{11}\ s^{-1}=4.017\times10^{-10}$ MeV [6]. The values of c_1 and c_2 can be obtained as $c_1=6.027\times10^{-5}$ and $c_2=5.397\times10^{-5}$. Here the rescattering effect is not considered, which effect will be discussed later.

2.2 Flavor wave functions and Lagrangians involved

The potential kernel of rescattering process as shown in Fig. 1 should be constructed with effective Lagrangians and flavor wave functions to find the pole in the complex energy plane within qBSE approach and to calculate the invariant mass spectrum. In Ref. [18], it was explained explicitly how to construct a potential for states with definite isospins under SU(3) symmetry with the corresponding flavor wave functions. For the S-wave ${D_s}^+{D_s}^-$ and D^+D^- states, the wave functions can be constructed as

$$|X^0_{D\bar{D}}\rangle_{I=0} = \frac{1}{\sqrt{2}}\left(|D^+\bar{D}^-\rangle + |D^0\bar{D}^0\rangle\right),$$
$$|X^0_{D_s\bar{D}_s}\rangle = |D_s^- D_s^+\rangle. \tag{1}$$

The meson exchanges were adopted to achieve the potential kernel for the interactions considered. According to the chiral and heavy quark symmetries, the Lagrangians for heavy mesons interacting with light mesons read [19–23]

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\mathbb{V}} = -\sqrt{2}\beta g_V \mathcal{P}_b \mathcal{P}^\dagger_a v\cdot\mathbb{V}_{ba} + \sqrt{2}\beta g_V \widetilde{\mathcal{P}}^\dagger_a \widetilde{\mathcal{P}}_b v\cdot\mathbb{V}_{ab}, \tag{2}$$

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\sigma} = -2g_s\mathcal{P}_b\mathcal{P}^\dagger_b\sigma - 2g_s\widetilde{\mathcal{P}}_b\widetilde{\mathcal{P}}^\dagger_b\sigma, \tag{3}$$

where $\mathcal{P}^T = (D^0, D^+, D_s^+)$. The velocity v should be replaced by $i\overleftrightarrow{\partial}/2\sqrt{m_i m_f}$ with the $m_{i,f}$ being the mass of the initial or final heavy meson. The $\mathbb{V}$ denotes the vector matrix as

$$\mathbb{V} = \begin{pmatrix} \frac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0+\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \tag{4}$$

The parameters involved here were determined in the literature as $\beta = 0.9$, $g_s = 0.76$ and $g_V = 5.9$ [24–27].

In Refs. [28,29], it was suggested that the J/ψ exchange is important in the $D\bar{D}^*$ interaction to reproduce the $Z_c(3900)$. In this work, the couplings of heavy-light charmed mesons to J/ψ are also considered and written with the help of heavy quark effective theory as [23,30],

$$\mathcal{L}_{D_{(s)}\bar{D}_{(s)}J/\psi} = ig_{D_{(s)}D_{(s)}\psi}\,\psi\cdot\bar{D}\overleftrightarrow{\partial}D, \tag{5}$$

where the couplings are related to a single parameter g_2 as $g_{D_{(s)}D_{(s)}\psi}/m_D = 2g_2\sqrt{m_\psi}$, with $g_2 = \sqrt{m_\psi}/(2m_{D_{(s)}}f_\psi)$ and $f_\psi = 405$ MeV.

To investigate whether the structure found around 4140 MeV is caused by the $J/\psi\phi - {D_s}^+{D_s}^-$ coupled-channel effect, which should be taken into consideration in rescattering process. Pseudoscalar $\bar{D}_s$ and D_s mesons and vector $\bar{D}^*_s$ and D^*_s mesons are exchanged during the coupling process $J/\psi\phi - {D_s}^+{D_s}^-$. To describe this process, apart from Eqs. (2) and (5), we also need the Lagrangians as [23,30]

$$\mathcal{L}_{D^*_{(s)}\bar{D}_{(s)}J/\psi}$$
$$= -g_{D^*_{(s)}D_{(s)}\psi}\,\epsilon_{\beta\mu\alpha\tau}\partial^\beta\psi^\mu(\bar{D}\overleftrightarrow{\partial}^\tau D^{*\alpha} + \bar{D}^{*\alpha}\overleftrightarrow{\partial}^\tau D), \tag{6}$$

where $g_{D^*_{(s)}D^*_{(s)}\psi}/m_{D^*} = 2g_2\sqrt{m_\psi}$, and the value of g_2 have been mentioned under Eq. (5).

2.3 Potential kernel

The potential interaction can be constructed by the meson exchange as [15,31,32],

$$\mathcal{V}_{\mathbb{P},\sigma} = I_i\Gamma_1\Gamma_2 P_{\mathbb{P},\sigma}f(q^2),\quad \mathcal{V}_{\mathbb{V}} = I_i\Gamma_{1\mu}\Gamma_{2\nu}P^{\mu\nu}_{\mathbb{V}}f(q^2), \tag{7}$$

Here the vertices Γ can be obtained by the standard Feynman rule from the Lagrangians given in the above. In the current work, the SU(3) symmetry is considered. Hence, with the help of the wave function in Eq. (1) and the flavor part in Lagrangians and the matrix in Eq. (4), a flavor factor can attracted for a state with a certain exchanged meson based on the Feynman diagram of the meson exchanges [15,18,29,32]. Except the flavor factor $I_\rho = 3/2$, $I_\omega = 1/2$ for the $D\bar{D}$ state, all other flavor factor has values of 1 as $I_\sigma = I_{J/\psi} = I_\phi = I_{D_s} = I_{\bar{D}_s} = I_{D^*_s} = I_{\bar{D}^*_s} = 1$. The value of the flavor factor I_{K^*} equals to 1 theoretically, but we take it equals to 0.3, which will be discussed later. The propagators are defined as usual as

$$P_{\mathbb{P},\sigma} = \frac{i}{q^2 - m^2_{\mathbb{P},\sigma}},\quad P^{\mu\nu}_{\mathbb{V}} = i\frac{-g^{\mu\nu} + q^\mu q^\nu/m^2_{\mathbb{V}}}{q^2 - m^2_{\mathbb{V}}}, \tag{8}$$

with q being the momentum of the exchanged meson. A form factor $f(q^2) = \Lambda_e^2/(\Lambda_e^2 - q^2)$ is introduced to compensate the off-shell effect of exchanged meson with a cutoff Λ_e.The current form factor can avoid overestimation of the contribution of J/ψ exchange.

2.4 Quasipotential Beth–Salpeter approach

In the above, we construct the potential of the interactions considered in the current work. The rescattering amplitude can be obtained with the qBSE [18,29,33–35]. After the partial-wave decomposition, the qBSE can be reduced to a 1-dimensional equation with a spin-parity J^P as [29],

$$i\hat{\mathcal{T}}^{J^P}_{m,n}(\mathrm{p},\mathrm{p}') = i\hat{\mathcal{V}}^{J^P}_{m,n}(\mathrm{p},\mathrm{p}') + \sum_k\int\frac{\mathrm{p}''^2 d\mathrm{p}''}{(2\pi)^3}$$
$$\cdot\, i\hat{\mathcal{V}}^{J^P}_{m,k}(\mathrm{p},\mathrm{p}'')G_0(\mathrm{p}'')i\hat{\mathcal{T}}^{J^P}_{k,n}(\mathrm{p}'',\mathrm{p}'), \tag{9}$$

where $\hat{\mathcal{T}}^{J^P}$ and $\hat{\mathcal{V}}^{J^P}$ are partial-wave rescattering amplitude and potential. The n, m, k denotes the independent helicities $\lambda_{(2,3)}$ of two constituents for the initial, final and intermediate mesons, respectively. If two constituents have helicities $\lambda_1 = \lambda_2 = 0$, such as the case of the $D_s\bar{D}_s$ interaction in the current work, a factor $f = 1/\sqrt{2}$ should be introduced for the partial wave amplitude $\hat{\mathcal{T}}^{J^P}$ and potential $\hat{\mathcal{V}}^{J^P}$, otherwise, $f = 1$. The explicit explanations about the independent amplitudes and the factor can be find in Ref. [29]. $G_0(\mathrm{p}'')$ is reduced propagator with the spectator approximation in the center-of-mass frame with $P = (M, \mathbf{0})$ as

$$G_0 = \frac{\delta^+(p_h''^{\,2} - m_h^2)}{p_l''^{\,2} - m_l^2} = \frac{\delta^+(p_h''^0 - E_h(\boldsymbol{p}''))}{2E_h(\boldsymbol{p}'')[(W - E_h(\boldsymbol{p}''))^2 - E_l^2(\boldsymbol{p}'')]}. \tag{10}$$

As required by the spectator approximation, the heavier particle (remarked as h) is put on shell, which has $p_h''^0 = E_h(\mathrm{p}'') = \sqrt{m_h^{\,2} + \mathrm{p}''^2}$. The $p_l''^0$ for the lighter particle (remarked as l) is then $W - E_h(\mathrm{p}'')$ with W being the total energy of the system of 2 and 3 particles. Here and hereafter we define the value of the momentum $\mathrm{p} = |\boldsymbol{p}|$.

The potential kernel $\hat{\mathcal{V}}^{J^P}_{m,n}/(f_m f_n) = \mathcal{V}^{J^P}_{\lambda_2\lambda_3,\lambda_2'\lambda_3'}$ can be obtained from the potential in Eq. (7) as

$$i\mathcal{V}^{J^P}_{\lambda_2\lambda_3,\lambda_2'\lambda_3'}(\mathrm{p},\mathrm{p}') = 2\pi \int d\cos\theta\, [d^J_{\lambda_{32}\lambda_{32}'}(\theta) i\mathcal{V}_{\lambda_2\lambda_3,\lambda_2'\lambda_3'}(\boldsymbol{p},\boldsymbol{p}') + \eta d^J_{-\lambda_{32}\lambda_{32}'}(\theta) i\mathcal{V}_{\lambda_2\lambda_3,-\lambda_2'-\lambda_3'}(\boldsymbol{p},\boldsymbol{p}')], \tag{11}$$

where $\eta = PP_2P_3(-1)^{J-J_2-J_3}$ with P and J being parity and spin for system and constituent 2 or 3. $\lambda_{32} = \lambda_3 - \lambda_2$. The initial and final relative momenta are chosen as $\boldsymbol{p}' = (0, 0, \mathrm{p}')$ and $\boldsymbol{p} = (\mathrm{p}\sin\theta, 0, \mathrm{p}\cos\theta)$. The $d^J_{\lambda\lambda'}(\theta)$ is the Wigner d-matrix. An exponential regularization was also introduced as a form factor into the reduced propagator as $G_0(\mathrm{p}'') \to G_0(\mathrm{p}'')e^{-2(p_l''^2 - m_l^2)^2/\Lambda_r^4}$ with the m_l and Λ_r being the mass of light constituent and a cutoff, respectively [29]. The cutoff Λ_r and the cutoff Λ_e for exchanged meson provide similar effect on the results, hence, we take them as a unified parameter Λ.

The rescattering amplitude $\mathcal{T}$ can be obtained by discretizing the momenta p', p, and p'' in the integral equation (9) by the Gauss quadrature with a weight $w(\mathrm{p}_i)$. After such treatment, the integral equation can be transformed to a matrix equation [29],

$$T_{ik} = V_{ik} + \sum_{j=0}^{N} V_{ij} G_j T_{jk}. \tag{12}$$

The propagator G is a diagonal matrix as

$$G_{j>0} = \frac{w(\mathrm{p}_j'')\mathrm{p}_j''^2}{(2\pi)^3} G_0(\mathrm{p}_j''),$$
$$G_{j=0} = -\frac{i\mathrm{p}_o''}{32\pi^2 W} + \sum_j \left[\frac{w(\mathrm{p}_j)}{(2\pi)^3} \frac{\mathrm{p}_o''^2}{2W(\mathrm{p}_j''^2 - \mathrm{p}_o''^2)}\right], \tag{13}$$

with on-shell momentum $\mathrm{p}_o'' = \lambda^{\frac{1}{2}}(W, m_2, m_3)$ with the $\lambda(x, y, z) = [x^2 - (y+z)^2][x^2 - (y-z)^2]$.

2.5 Invariant mass spectrum

With the preparation above, the invariant mass spectrum of processes considered can be given with direct three-body decay amplitude $\mathcal{A}^{J^P}_{m,\lambda_B} = \mathcal{A}_{B^+\to D_{(s)}{}^+ D_{(s)}{}^- K^+}$ (here we remark it as $\hat{\mathcal{A}}^{J^P}_{m,\lambda_B} = f_m \mathcal{A}^{J^P}_{m,\lambda_B}$ with pseudoscalar B meson having a helicity $\lambda_B = 0$) and the rescattering amplitude $\mathcal{T}^{J^P}$ as Ref. [33],

$$\frac{d\Gamma}{dM_{23}} = \frac{1}{16M} \frac{1}{(2\pi)^5} \frac{\check{\mathrm{p}}_1 \mathrm{p}_3^{cm}}{M} \sum_{m,\lambda_B;J^P} \frac{1}{N_J^2} |\hat{\mathcal{M}}^{J^P}_{m;\lambda_B}(M_{23})|^2, \tag{14}$$

with

$$\hat{\mathcal{M}}^{J^P}_{m,\lambda_B}(M_{23}) = \hat{\mathcal{A}}^{J^P}_{m,\lambda_B}(M_{23}) + \sum_l \int \frac{d\mathrm{p}_3'^{cm} \mathrm{p}_3'^{cm2}}{(2\pi)^3} i\hat{\mathcal{T}}^{J^P}_{m;n}(\mathrm{p}_3'^{cm}, M_{23}) G_0(\mathrm{p}_3'^{cm}) \hat{\mathcal{A}}^{J^P}_{n;\lambda_B}(\mathrm{p}_3'^{cm}, M_{23}), \tag{15}$$

where M is the mass of the B meson. $N_J^2 = (2J+1)/4\pi = 1/4\pi$ with $J = 0$ here. p_3^{cm} and $\mathrm{p}_3'^{cm}$ are the center of mass momentum in the 23 and $2'3'$ system, which can be expressed as $\mathrm{p}_{3^{(\prime)}}^{cm} = \sqrt{\lambda\left(M^2_{2^{(\prime)}3^{(\prime)}}, m^2_{3^{(\prime)}}, m^2_{2^{(\prime)}}\right)}/2M_{2^{(\prime)}3^{(\prime)}}$. And $\check{\mathrm{p}}_1$ satisfies $M_{23}^2 = (M - \check{\mathrm{E}}_1)^2 - \check{\mathrm{p}}_1^2$.

Equation (15) can be abbreviated as matrix form $M = A + TGA$ by the same discretizing in Eq. (12), where T satisfies $T = V + VGT$. The rescattering amplitude T can be solved as $T = (1 - VG)^{-1}V$. Since we focus on the pole of the rescattering amplitude, we need to find the position where $|1 - VG| = 0$ with $z = E_R + i\Gamma/2$ corresponding to the total energy and width at the real axis.

3 Numerical results and discussions

With the above preparation, the invariant mass spectra of B decays considered in the current work will be calculated and compared with the experimental data. In this section, the $D_s\bar{D}_s$ interaction and its couplings to channels $D\bar{D}$ and $J/\psi\phi$ will be considered.

3.1 $D_s^+D_s^-$ invariant mass spectrum with $D_s\bar{D}_s$ interaction

First, we consider the simplest model, that is, only the $D_s\bar{D}_s$ interaction is included in the rescattering to provide a basic picture of molecular state from the interaction. In such model, the intermediate and final products are all $D_s\bar{D}_s$. And in the current work, we only consider the S-wave state with spin parity $J^P = 0^+$. The corresponding cutoff Λ is adjusted to 1.8 GeV, which is a little bigger than the one in our previous work [15], which makes the result fit better with the new experimental data. Besides the obvious peak near the $D_s\bar{D}_s$ threshold, a structure can be seen around 4140 MeV. Obviously, such structure can not be produced from the $D_s\bar{D}_s$ interaction as shown in Eq. (14). Here, as suggested by LHCb collaboration [1], we introduce a Breit–Wigner resonance, which is used to fit the dip around 4140 MeV as

$$M_{BW} = e^{i\theta} \cdot \frac{a\Gamma_0 M_0}{M^2 - {M_0}^2 + i\Gamma_0 M_0}, \tag{16}$$

where M_0 and Γ_0 are the mass and width of the structure obtained from the experimental results. The strength factor a and phase angle θ are free parameters. The theoretical results for events can be obtained by multiplying the theoretical decay distribution on efficiency, which equals to $N_{sig}/(N_{B^+}\mathcal{B}_{B^+\to D_s{}^+D_s{}^-K^+})$ with N_{sig}, N_{B^+} and $\mathcal{B}$ being the numbers of signal events, the total number of B candidates, and the branching fraction, respectively. Unfortunately, though the N_{sig} and $\mathcal{B}$ were provided by LHCb collaboration in Refs. [1,17], the N_{B^+} was not reported, so we have to introduce a parameter multiplying on the theoretical decay distribution to fit the experimental data. After such treatment, the comparison between the theoretical and experimental results can be carried out. We would like to remind that such treatment make the absolute values of the coupling constants c_1 and c_2 meaningless.

The pole and $D_s^+D_s^-$ invariant mass spectrum for decay $B^+ \to D_s^+D_s^-K^+$ are presented in Fig. 2 and compared with the experiment. In the upper panel of Fig. 2, a pole is found at 3932.8 GeV in the first Riemann sheet on the real axis near $D_s\bar{D}_s$ threshold, which suggests a very weakly bound state from the $D_s\bar{D}_s$ interaction and close to the mass of $X(3960)$ measured by the LHCb Collaboration. To confirm their relation, the invariant mass spectrum in energy region of 3800–4800 MeV is calculated and compared with the experiment. In the lower panel of Fig. 2, a sharp peak arises near the $D_s\bar{D}_s$ threshold, which is from the $D_s\bar{D}_s$ rescattering. With increase of the energies, a dip can be found around 4140 MeV, which is due to the Breit–Wigner resonance introduced in Eq. (16). We evaluate the χ^2 between the experimental data

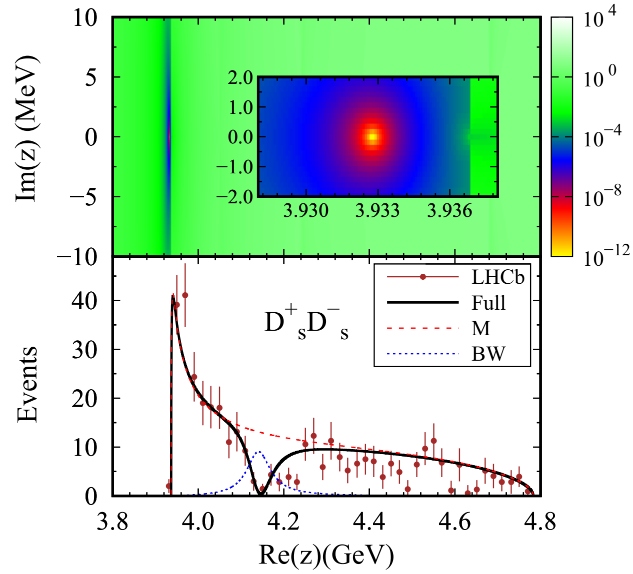

Fig. 2 The pole (upper panel) and the $D_s^+D_s^-$ invariant mass spectrum for decay $B^+ \to D_s^+D_s^-K^+$ (lower panel) with the $D_s\bar{D}_s$ interaction. The black (full), red (dashed), and blue (dotted) curves are for the total, rescattering as $\mathcal{M}$ in Eq. (15), and Breit–Wigner resonance in Eq. (16). The red points with error bars are the data from the LHCb experiment cited from Ref. [1]

and our theoretical results in two energy regions of 3800–4170 MeV and of 3800–4800 MeV. The former one is predominantly relevant to the $X(3960)$ and $X_0(4140)$ structures, and the latter one is relevant to all energy region. The χ^2 in energy region of 3800–4170 MeV is equal to 13.45 for 13 data points, while the χ^2 in energy region of 3800–4800 MeV is equal to 53.06 for 43 data points, which indicates that our theoretical data are in a good agreement with experimental data, and favors the assumption of the $X(3960)$ as a $D_s\bar{D}_s$ molecular state.

3.2 Discussion about relation between $X(3960)$ and $\chi_{c0}(3930)$

In this subsection, we perform a study of the B meson decay with rescattering of $D_s\bar{D}_s$-$D\bar{D}$ in order to discuss the relation between $X(3960)$ and $\chi_{c0}(3930)$. The intermediate and final products can be $D_s\bar{D}_s$ and $D\bar{D}$ with spin parity $J^P = 0^+$. The value of flavor factor I_{K^*} is adjusted to 0.3 instead of 1. It is interesting to found that if we take I_{K^*}= 1 as its theoretical value, the $D\bar{D}$ state will remain but the $D_s\bar{D}_s$ state will disappear, only if decreasing the value of I_{K^*} will these two states appear, which is analogous to Ref. [11] and may cause by SU(3) symmetry breaking for the K^* meson. A Breit–Wigner resonance is still added into the amplitude of $D_s\bar{D}_s$–$D\bar{D}$ rescattering to match the dip found around 4140 MeV in Refs. [1,17]. The corresponding cutoff Λ will be also chosen as 1.8 GeV for both $D_s\bar{D}_s$ and $D\bar{D}$ channels.

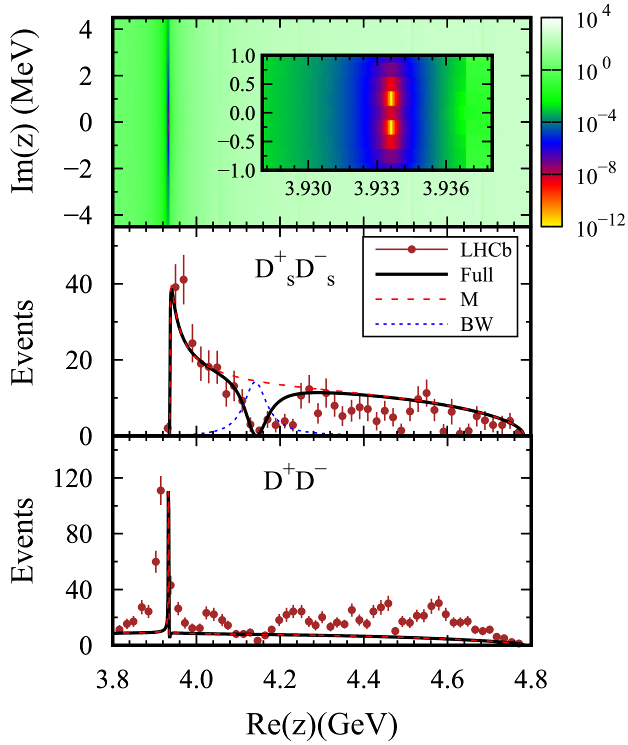

Fig. 3 The pole (upper panel) and the $D_s^+D_s^-$ (middle panel) and D^+D^- (lower panel) invariant mass spectra with the $D\bar{D}$-$D_s\bar{D}_s$ coupled-channel interaction. The black (full), red (dashed), and blue (dotted) curves are for the total, rescattering as $\mathcal{M}$ in Eq. (15), and Breit–Wigner resonance in Eq. (16). The red points with error bars are the data of $X(3960)$ (middle panel) and $X(3930)$ (lower panel) from the LHCb experiment [24]

The $D_s^+D_s^-$ and D^+D^- invariant mass spectra are presented in Fig. 3 and compared with the experiment.

In the upper panel of Fig. 3, the poles leave the real axis and become two conjugate poles in the complex energy plane after including the coupled-channel effect. These two conjugate poles are located at 3933.6±0.25i MeV, which means a molecular state with a mass of 3933.6 MeV and a width of 0.5 MeV close to the corresponding $D_s\bar{D}_s$ threshold. The $D_s^+D_s^-$ invariant mass spectrum in the middle panel of Fig. 3 looks similar to the single-channel results in energy region of 3800–4170 MeV in Fig. 2. The χ^2 between the experimental data and our theoretical data in energy region of 3800–4170 MeV is equal to 14.88 for 13 data points, which is close to the above single-channel results. However, the χ^2 in energy region of 3800–4800 MeV is equal to 76.13 for 43 data points, which is a little larger than the one obtained in Sect. 3.1. Since in the higher energy region, the real mechanism should be more complex, the current result is still accepted if we focus on the structure near the $D_s\bar{D}_s$ threshold.

In the lower panel of Fig. 3, the D^+D^- invariant mass spectrum are presented and compared with the data of

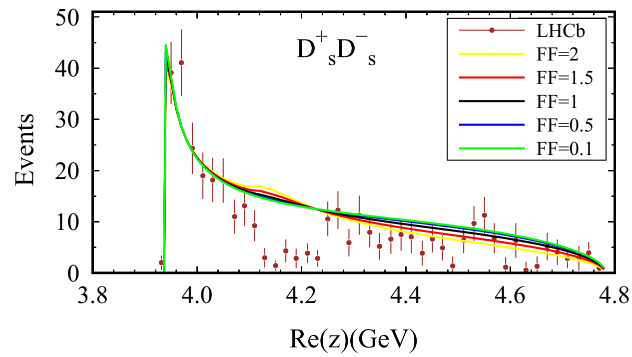

Fig. 4 The $D_s^+D_s^-$ invariant mass spectrum with the $D_s\bar{D}_s$-$J/\psi\phi$ coupled-channel interaction. The yellow, red, black, blue and green curves are for the mass distribution with flavor factor (FF) equal to 2, 1.5, 1, 0.5 and 0.1, respectively. The red points with error bars are the data of $X(3960)$ from the LHCb experiment [1]

$X(3930)$ from LHCb experiment [5]. As shown in Fig. 3, the peak near 3930 MeV is extremely narrow, seems too sharp compared with the experimental structure. If a wider peak in the D^+D^- invariant mass spectrum is reached, the peak in the $D_s^+D_s^-$ invariant mass spectrum will disappear. Our calculation suggests that though peaks can be produced simultaneously near the $D_s\bar{D}_s$ threshold in both $D_s^+D_s^-$ and D^+D^- invariant mass spectrum, the explicit shape of the experimental structure can not be well-fitted simultaneously. In the experimental article [5], such structure is suggested to be formed by two states $\chi_{c0}(3930)$ and $\chi_{c2}(3930)$, and the $\chi_{c0}(3930)$ has a smaller width. Hence, it is still possible to assign the $\chi_{c0}(3930)$ as a $D_s\bar{D}_s$ molecular state but with a very small width. If so, the $\chi_{c2}(3930)$ may play a more important role to form the structure around 3930 MeV in the D^+D^- invariant mass spectrum.

3.3 Discussion about origin of $X(4140)$

As suggested by the LHCb collaboration, the additional structure found around 4140 MeV in the $D_s^+D_s^-$ invariant mass spectrum might be caused either by a new resonance with the 0^{++} assignment or by a $D_s\bar{D}_s$-$J/\psi\phi$ coupled-channel effect [1]. In the above, we adopt the former assumption by introducing a Breit–Wigner resonance. For better understanding of the origin of this structure, the $D_s\bar{D}_s$-$J/\psi\phi$ coupled-channel effects with spin parity $J^P = 0^+$ is introduced to replace the Breit–Wigner resonance. The corresponding cutoff Λ will be chosen as 1.8 GeV as above calculation. We also study effect of flavor factors of exchanged meson I_{D_s}, $I_{\bar{D}_s}$, $I_{D_s^*}$ and $I_{\bar{D}_s^*}$ by changing their values. The result is shown in Fig. 4.

The black curve is the mass distribution obtained by theoretical values of flavor factor of the exchanged mesons, that is, I_{D_s}=$I_{\bar{D}_s}$=$I_{D_s^*}$=$I_{\bar{D}_s^*}$=1. The peak near 3960 MeV is analogous to Figs. 2 and 3, which also favors the assumption of the $X(3960)$ as a $D_s\bar{D}_s$ molecular state, whereas a small bump is obtained instead of a dip at around 4140 MeV. This bump-like structure in the $D_s^+D_s^-$ invariant mass spectrum is caused by the $D_s\bar{D}_s$-$J/\psi\phi$ coupling. However, we cannot reproduce a dip from this hump-like structure by adjusting the parameters. In the current model, the coupled-channel effect with coupling to the $J/\psi\phi$ channel is constrained by the heavy quark symmetry as shown in the Lagrangians. In the experimental treatment by the LHCb collaboration [1], the free parameters were introduced to adjust the contributions of the coupled-channel effect. Hence, our results do not conflict with experimental analysis, but are obtained with more theoretical constraint.

Besides, adjusting other parameters, such as the value of flavor factor, doesn't work either. To prove this point, we take values of flavor factor (FF) of the exchanged mesons I_{D_s}, $I_{\bar{D}_s}$, $I_{D_s^*}$ and $I_{\bar{D}_s^*}$ in this process all equal to 0.1, 0.5, 1, 1.5 and 2, respectively. From Fig. 4 we can see although the value of the peak for the bump-like structure around 4140 MeV gradually decrease with the decreasing of the value of FF, a dip structure still cannot be reproduced in our model. Based on these calculations, one can say that, given the fact that the coupled channel $D_s\bar{D}_s$-$\phi J/\psi$ cannot reproduce a dip around 4140 MeV in our model, the $X_0(4140)$ could not be easily interpreted as a $D_s\bar{D}_s$-$\phi J/\psi$ coupling.

4 Summary

Inspired by the newly observed $X(3960)$, we study the B meson decay with $D_s\bar{D}_s$ rescattering and couplings to channels $D\bar{D}$ and $J/\psi\phi$ in a qBSE approach with spin parity 0^+. With the help of effective Lagrangian, the potential kernel can be constructed by meson exchanges and the invariant mass spectra are calculated and compared with the LHCb experiment.

With the $D_s\bar{D}_s$ rescattering, a pole is found at 3932.8 MeV on the real axis near $D_s\bar{D}_s$ threshold, which shows that a very weakly bound state can be produced from the $D_s\bar{D}_s$ interaction. A peak structure near the $D_s\bar{D}_s$ threshold is found in the $D_s^+D_s^-$ invariant mass spectrum and have a good agreement with the experimental results, favoring the assumption of the $X(3960)$ as a $D_s\bar{D}_s$ molecular state. After adding $D\bar{D}$ as intermediate and final channel in the process, the theoretical $D_s^+D_s^-$ invariant mass spectrum is still in agreement with the relevant experimental data.

After adding $D\bar{D}$ channel, an extremely sharp peak can be obtained near 3930 MeV in D^+D^- invariant mass spectrum. The $\chi_{c0}(3930)$ can be assigned as the same $D_s\bar{D}_s$ state in the $D_s^+D_s^-$ invariant mass spectrum, but with a very small width. The experimental structure around 3930 MeV in the D^+D^- invariant mass spectrum can not be reproduced in the current model only with this molecular state with 0^{++} from the $D_s\bar{D}_s$ interaction. The experimental analysis of such structure suggests that it may be formed by states with spins 0 and 2, $\chi_{c0}(3930)$ and $\chi_{c2}(3930)$. The current results indict that the $\chi_{c2}(3930)$ may play a more important role to form the structure near 3930 MeV in the D^+D^- invariant mass spectrum [5].

With an additional Breit–Wigner resonance, the dip can be well reproduced around 4140 MeV in the $D_s^+D_s^-$ invariant mass spectrum. The possible role of the $D_s\bar{D}_s$-$J/\psi\phi$ coupled-channel effect on the appearance of the dip $X(4140)$ is also discussed in the current model. However, a small bump instead of a dip is found around 4140 MeV. Moreover, the dip can not be reproduced by adjusting the parameters, which does not support the assignment of $X_0(4140)$ as the $D_s\bar{D}_s$-$J/\psi\phi$ coupled-channel effect. It suggests that the $X(4140)$ has an independent origin different from the peak $X(3960)$ which can be well reproduced from the $D_s\bar{D}_s$ interaction in the current work.

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