THE EUROPEAN
PHYSICAL JOURNAL C

Cosmological evolution in bimetric gravity: observational constraints and LSS signatures

Ajay Bassi[1,a], **Shahnawaz A. Adil**[2,b], **Manvendra Pratap Rajvanshi**[1,c], **Anjan A. Sen**[1,d]

[1] Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi 110025, India
[2] Department of Physics, Jamia Millia Islamia, New Delhi 110025, India

Abstract Bimetric gravity is an interesting alternative to standard GR given its potential to provide a concrete theoretical framework for a ghost-free massive gravity theory. Here we investigate a class of Bimetric gravity models for their cosmological implications. We study the background expansion as well as the growth of matter perturbations at linear and second order. We use low-redshift observations from SnIa (Pantheon+ and SH0ES), Baryon Acoustic Oscillations (BAO), the growth ($f\sigma_8$) measurements and the measurement from Megamaser Cosmology Project to constrain the Bimetric model. We find that the Bimetric models are consistent with the present data alongside the ΛCDM model. We reconstructed the " effective dark energy equation of state"(ω_{de}) and "Skewness"(S_3) parameters for the Bimetric model from the observational constraints and show that the current low-redshift data allow significant deviations in ω_{de} and S_3 parameters with respect to the ΛCDM behaviour. We also look at the ISW effect via galaxy-temperature correlations and find that the best fit Bimetric model behaves similarly to ΛCDM in this regard.

1 Introduction

Observed late time acceleration [1–3] of the average expansion of the Universe has been one of the central challenges of modern Cosmology for more than two decades. The two standard approaches to explain such late time acceleration have been: (1) assuming the presence of a new unobserved component, called dark energy in the energy budget of the universe [4] (similar to dark matter that it does not emit any light but with negative pressure instead of zero pressure) (2) modifying the standard theory of Gravitation (Einstein's General Relativity) at large cosmological scales [5,6]. The concordance ΛCDM model [7] has been tremendously successful in explaining observations across time scales of Universe's evolution [8,9] (e.g Big Bang Nucleosynthesis era, matter-radiation decoupling era, structure formation era till the late time accelerated expansion era) despite a number of theoretical problems to construct such model [10]. But recently for the first time, ΛCDM model is facing some serious questions related to observational results. There is a discrepancy in values of Hubble constant (H_0) inferred from nearby cosmological observations [11], and those inferred from CMB observations [8,9]. This tension, termed H_0 tension, has now reached at the level of 5σ and systematical errors as source for this tensions may not be enough to explain this tension [12,13].

This has been the main driving force in the renewed exploration of the theoretical regime with dark energy models beyond ΛCDM or modified theories of gravity. In relation to modified theories of gravity, massive gravity models are one of the most studied modified gravity models. In standard theory of gravity, the gravity is described by massless spin-2 field called gravitons which are still not observed. Fierz and Pauli [14] made the first attempt to make these spin-2 field massive with a linear theory. But it was later shown by Boulware and Deser [15] that the nonlinear extensions of such theories contain a ghost (Boulware–Deser (BW) Ghost). A ghost-free, nonlinear theory for massive spin-2 field in flat space time was first proposed by de Rham, Gabadadze and Trolley in 2011 [16], which is called the dRGT theory. Hassan and Rosen [17] later extended the dRGT theory for Bimetric gravity that describes a gravitating massive spin-2 field by introducing the dynamics in the second metric. Cosmological solutions in Bimetric gravity have been studied which can result in the late time acceleration of the Universe without

[a] e-mail: ajay@ctp-jamia.res.in
[b] e-mail: shazadil14@gmail.com
[c] e-mail: manvendra155@gmail.com
[d] e-mail: aasen@jmi.ac.in (corresponding author)

Springer

any presence of explicit dark energy term. Different families of Bimetric gravity models have been studied [18–23] in light of CMB, BAO and Supernovae data. Gravitational waves in these theories with some constraints on parameter space are studied in [24,25]. Big Bang Nucleosynthesis constraints were obtained in [26].

On the other hand, theories with phantom behaviour(i.e. equation of state w crossing below -1) [27,28] have shown promise in reducing/overcoming the cosmological tensions, more specifically the Hubble tension although the theoretical underpinnings of phantom phenomenology are still under investigations as many of such theories may suffer from instabilities. The viable Bimetric gravity can also show phantom behaviour in some of their parameter space [29,30]. Providing an effective Λ-like effect and the possibility of phantom behaviour can make Bimetric gravity suitable for investigation as a possible solution for the Hubble tension.

In the literature, there are studies related to the effect of Bimetric gravity on the large-scale structure formation in the universe [31–33]. In particular, the effects of modification of gravity at cosmological scales in higher-order clustering are still mostly unknown. One of the important parameters related to higher order clustering is the "skewness" parameter [34]. This is defined as the "normalized third order moment in count-in-cells statistics" which can describe the non-Gaussian feature in the probability distribution of the perturbed matter field. In a purely matter-dominated Einstein–de Sitter Universe, one gets $S_3 \approx 34/7 \approx 4.857$. Any observed deviation from this value can be a signature for a modified gravity model [35].

Our aim in this work is to study the linear and second-order perturbations for a specific class of Bimetric gravity models and illustrate the effects of these modifications to gravity on observables related to perturbed Universe such as $f\sigma_8$, skewness parameter S_3 and Integrated Sachs-Wolfe(ISW) effect and compare the results with concordance ΛCDM models. This can be particularly interesting in the context of results from future surveys like Euclid which can provide an accurate measurement of the skewness parameter and can potentially distinguish ΛCDM model from different modified gravity models including the Bimetric gravity.

This paper is organized as follows: in Sect. 2, we briefly describe the theory of Bimetric gravity; in Sect. 3, we describe the perturbation theory formalism used for the Bimetric gravity and the discuss the effects of these perturbations on observables in respective Sect. 3.2. Data constraints are obtained in Sect. 4. In Sect. 5, we study ISW effect through galaxy-temperature cross-correlations. We conclude with a discussion in the last section.

2 Bimetric gravity and cosmology

Bimetric gravity is the theory of two interacting spin-2 fields, one massive and one massless. An interacting symmetric spin-2 field $f_{\mu\nu}$ is introduced together with the physical metric $g_{\mu\nu}$, which is a linear combination of massless and massive graviton states. The standard matter particles and fields are coupled to the physical metric $g_{\mu\nu}$ only. For ghost-free Bimetric gravity, where both metrics are dynamical, action can be written as [17,19]

$$S = -\int d^4x \sqrt{-g} \frac{R}{2m_g^2} - \int d^4x \sqrt{-f} \frac{\tilde{R}}{2m_f^2}$$
$$+ \int d^4x \sqrt{-g} m^4 V(g_{\mu\nu}, f_{\mu\nu}) + \int d^4x \sqrt{-g} L_m, \quad (1)$$

where R and $\tilde{R}$ are Ricci scalars for $g_{\mu\nu}$ and $f_{\mu\nu}$. m_g and m_f are the Planck's mass for the metrics $g_{\mu\nu}$ and $f_{\mu\nu}$ respectively.. V is the interaction term which has the parametric form as

$$V = \sum_{n=0}^{4} \beta_n e_n(\chi). \quad (2)$$

Here χ is a matrix defined in such way that $\chi^2 = g_{\mu\nu} f^{\mu\nu}$. $e_n(\chi)$ is the elementary symmetry polynomials of eigenvalues of the matrix χ which can be written as follows

$$e_0(\chi) = 1, \quad e_1(\chi) = [\chi], \quad e_2(\chi) = \frac{1}{2}\left([\chi]^2 - [\chi^2]\right)$$
$$e_0(\chi) = \frac{1}{6}\left([\chi]^3 - 3[\chi][\chi^2] + 2[\chi^3]\right), \quad e_4(\chi) = \det(\chi)$$
$$(3)$$

where, $[\chi]$ is the trace of the matrix χ and $\det(\chi)$ is the determinant of χ. The Bimetric gravity is characterized by 5 constants β_i. Under the scaling transformation $f_{\mu\nu} \rightarrow \frac{m_g^2}{m_f^2} f_{\mu\nu}$ and $\beta_n \rightarrow \left(\frac{m_f}{m_g}\right)^n \beta_n$, we can make $M_*^2 = 1$ where, $M_* = m_f/m_g$. Hence M_* is not a free parameter. In what follows, we consider $M_*^2 = 1$ and $m_g = m_f$. With such rescaling [36], it is common to work in terms of dimensionless quantities which remain same under these scaling. Here we use the definitions and parameterisation of Dhawan et al. [19] to write the dimensionless parameters as

$$B_i \equiv \frac{m^2 \beta_i}{H_0}. \tag{4}$$

There are various possible models depending on which Bs are non-zero. For simplicity, here we consider models with only B_0 and B_1 nonzero. Cosmological expansion for these models is given by [37]

$$\frac{H^2}{H_0^2} = \frac{\Omega_m(1+z)^3}{2} + \frac{B_0}{6} + \sqrt{\left(\frac{\Omega_m(1+z)^3}{2} + \frac{B_0}{6}\right)^2 + \frac{B_1^2}{3}}, \tag{5}$$

where due to spatial flatness condition, the parameter B_0 can be written in terms of other two parameters Ω_m and B_1 as

$$B_0 = 3\left(1 - \Omega_m - \frac{B_1^2}{3}\right), \tag{6}$$

where Ω_m is the present day matter density parameter. As one can see in Eq. (5), the model naturally gives a cosmological constant term $\frac{B_0}{3}$ in the Universe which can be positive or negative depending on the values of Ω_m and B_1. This originates from the interacting potential itself. For early times $z \to \infty$,

$$\frac{H^2}{H_0^2} \approx \Omega_m(1+z)^3 + \frac{B_0}{3}, \tag{7}$$

which is ΛCDM model. For future infinity ($z \to -1$),

$$\frac{H^2}{H_0^2} \approx constant, \tag{8}$$

and hence a de-Sitter or anti-de-Sitter model.

Writing $H^2/H_0^2 = \Omega_m(1+z)^3 + (1 - \Omega_m)f(z)$, one can compare with Eq. (5) to find the effective dark energy density and subsequently can derive an expression for the effective dark energy equation of state (w_{de}). We plot $w_{de}(z)$ as a function of redshift(z) in Fig. 1 and observe that the Bimetric gravity can show phantom behavior. In the Fig. 2, where we show the dependence of $w_{de}(z = 0)$ on B_1 and Ω_{m0}, it is evident that Bimetric gravity shows phantom behavior at present for a range of parameter values and can provide a possible theoretical basis for parametric phantom models, which people have recently considered [27,38] to alleviate the cosmological tensions.
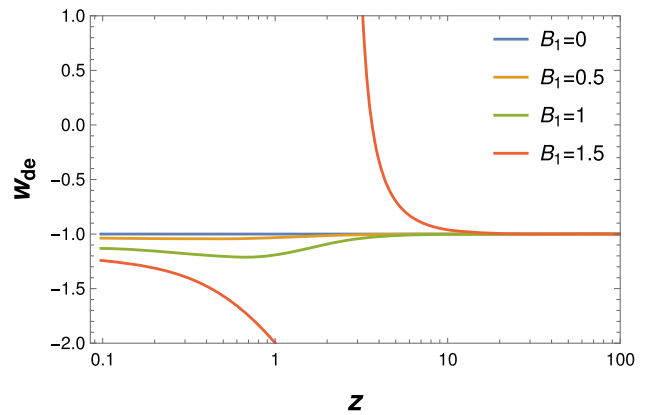

Fig. 1 Equation of state w_{de} for different values of parameter B_1. We set $\Omega_m = 0.3$
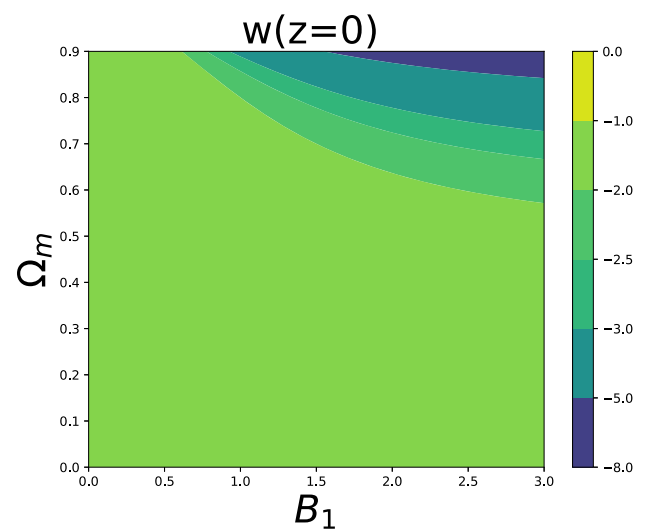

Fig. 2 Present day w as a function of B_1 and Ω_m.

3 Growth of perturbations and large scale structure

Next we study the effects of Bimetric gravity on the growth of matter perturbations. We study linear perturbations as well as second-order perturbations. Second-order perturbations affect the skewness (S_3) of the matter density field. We study the effects of modified theory parameters on skewness. We use the formalism of Multamaki et al. [39] to study the growth of matter perturbations but one can also use the formalism for modified gravity models by Lue et al. [40] and both of them give similar results.

Before describing the detail formalism and the results, we should discuss the issue of instability in the linear perturbations on the sub-horizon scales in the early time. It has been shown that linear perturbations around a background homogeneous and isotropic FLRW solutions are stable in the late times. But in order to define the initial conditions, these perturbations also need to be stable in the early time on sub

horizon scales. However such perturbations are suffered from gradient instability on sub horizon scales in the early time as well as Higuchi Ghost as the Hubble scale exceeds that effective gradient mass m_{eff} [32,33,41–44]. This is similar to the situation in Fierz-Pauli theory and the instability is precisely due to the fact that the massive spin-2 mass in linear theory does not possess a well defined massless limit in curved background. It was shown that the origin of these instabilities can be associated with the Stuckelberg fields arising due to the relation between the physical and fiducial sectors of the Bigravity theory and nonlinear interactions in these Stuckelberg fields may play role for these instabilities [45]. Subsequently it was shown that the nonlinear effects in the scalar graviton mass can solve this instability issue [45]. During the early time in cosmological evolution when $H \gg m_{eff}$, the Stuckelberg fields are nonlinear and due to these nonlinearity, the early growth of perturbations evolve as GR solutions, and as the $H\ m_{efff}$, the bigravity modification to GR starts dominating and for $H \ll m_{eff}$, the linear perturbations are stable on sub-horizon scales [45]. In this regard, we want to stress that the transition from the GR to bigravity behaviour is still an unsettles issue and needs further exploration. There are also certain classes of bigravity models (related to certain choices of the parameters β_i), for which it was shown that the there is no gradient instability in the perturbed Universe in early phases of evolution while treating perturbations at full nonlinear level [46]. Moreover in numerical studies related to inhomogeneous evolution of the Universe, no exponential instabilities has been observed [47]. To settle the issue of linear stability, we may need to adapt more general prescription as described by Ijjas et al. [48]. Although this issue of solving the instability in the linear growth of perturbation at early stage using nonlinear effects, is still an open problem, in this work we rely on the assumption that nonlinear effect during the early phase when $H \gg m_{eff}$ can resolve this of instability and carry forward our subsequent calculations for linear perturbation in the matter sector.

3.1 Formalism and equations

We start with the formalism of Multamaki et al. [39], wherein Raychaudhuri's equation is used to derive a general equation for the growth of perturbations at large scales. We should point out that Bimetric gravity is a modified theory of gravity containing only pressure-less matter in the energy budget of the Universe at late times (one can ignore the contribution from radiation at late times). The formalism developed by Multamaki et al. [39] to study the matter density perturbations for such modified theory of gravity is briefly discussed below.

The Raychaudhuri's equation for a shearless and irrotational velocity field v^μ is given by

$$\dot{\Theta} + \frac{\Theta^2}{3} = R_{\mu\nu}v^\mu v^\nu, \tag{9}$$

where

$$\Theta = \nabla_\mu v^\mu, \qquad \theta = \nabla_i v^i. \tag{10}$$

As shown in [39], this can be related to average Hubble expansion rate $(\bar{H})$ and locally perturbed expansion rate H as

$$\frac{\dot{\theta}}{a} + \frac{\theta}{a}\bar{H} + \frac{\theta^2}{3a^2} = 3(\dot{H} + H^2 - \dot{\bar{H}} - \bar{H}^2). \tag{11}$$

Combining the above equation with the continuity equation for pressure-less matter

$$\frac{\partial \delta}{\partial t} + (1 + \delta)\theta = 0, \tag{12}$$

where δ is the matter overdensity. We get the evolution equation for δ [39] as

$$\frac{d^2\delta}{d\eta^2} + \left(2 + \frac{\dot{\bar{H}}}{\bar{H}^2}\right)\frac{d\delta}{d\eta} - \frac{4}{3}\frac{1}{1+\delta}\left(\frac{d\delta}{d\eta}\right)^2$$
$$= -3\frac{1+\delta}{\bar{H}^2}\left[\left(\dot{H} + H^2\right) - \left(\dot{\bar{H}} + \bar{H}^2\right)\right], \tag{13}$$

where overdot represents derivative w.r.t. time and $\eta \equiv \ln(a)$. Quantities with overbar represent background quantities. Following [39], we can expand the r.h.s of Eq. (13) as

$$3\frac{1+\delta}{\bar{H}^2}\left[\left(\dot{H} + H^2\right) - \left(\dot{\bar{H}} + \bar{H}^2\right)\right] = 3(1+\delta)\sum_{n=1} c_n\delta^n. \tag{14}$$

We further expand δ as [39]

$$\delta = \sum_{i=1}^{\infty} \frac{D_i(\eta)}{i!}\delta_0^i, \tag{15}$$

where δ_0 is the small perturbation. We use Eq. (16) in (15) while expanding δ upto first and second order. Moreover, for background Hubble parameter, we put the expression given in Eq. (6) (we write it in terms of background matter density ρ_m) while for perturbed Hubble parameter, we use Eq. (6) with ρ_m replaced by perturbed matter density $\rho_m(1+\delta)$ (For detail prescription in this regard, please see [39]).

With this we get the linear and second-order perturbation equations as

$$D_1'' + \left(2 + \frac{\dot{\bar{H}}}{\bar{H}^2}\right) D_1' + 3c_1 D_1 = 0, \tag{16}$$

and

$$D_2'' + \left(2 + \frac{\dot{\bar{H}}}{\bar{H}^2}\right) D_2' - \frac{8}{3} D_1'^2 + 3c_1 D_2 + 6(c_1 + c_2) D_1^2 = 0. \tag{17}$$

For Bimetric gravity, which we are considering here, we get

$$c_1 = \frac{-\frac{2\Omega_m}{a^3}\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6}\right)\left(\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6}\right)^2 + \frac{B_1^2}{3}\right) - \frac{2\Omega_m}{a^3}\left(\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6}\right)^2 + \frac{B_1^2}{3}\right)^{3/2} - \frac{\Omega_m^2 B_1^2}{a^6}}{8\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6} + \sqrt{\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6}\right)^2 + \frac{B_1^2}{3}}\right)\left(\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6}\right)^2 + \frac{B_1^2}{3}\right)^{3/2}}, \tag{18}$$

and

$$c_2 = \frac{-\frac{8\Omega_m^2 B_1^2}{a^6}\left(\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6}\right)^2 + \frac{B_1^2}{3}\right) + \frac{9\Omega_m^3 B_1^2}{a^9}\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6}\right)}{96\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6} + \sqrt{\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6}\right)^2 + \frac{B_1^2}{3}}\right)\left(\left(\frac{\Omega_m}{2a^3} + \frac{B_0}{6}\right)^2 + \frac{B_1^2}{3}\right)^{5/2}}, \tag{19}$$

We solve for linear and second-order perturbations for values of B_1 and Ω_m. We set the initial conditions at redshift $z = 1000$, assuming that model reproduces the Einstein–de Sitter Universe at that epoch (specifically we assume D_1 and D_2 and their first derivatives to be same as that for Einstein–de Stitter Universe with $\Omega_m = 1$ at $z = 1000$). Solving these equations, the linear growth rate is shown in Fig. 3 while the evolution of the second-order perturbations are shown in Fig. 4. As shown in Fig. 3, for smaller values of B_1, the linear growth is similar to ΛCDM model. But for higher values, in particular for values $B_1 \geq 1.4$, there is an increase in growth around $z \sim 1$. This can be possibly due to some extra attractive gravitational pull provided by the Bimetric gravity for such values of B_1. The similar behaviour was also found by [46] for a bigravity model with non zero B_1 and B_2.

For the second order perturbation as shown in Fig. 4, we get the similar behaviour where the deviation from ΛCDM increases for larger values of B_1.

3.2 Observables

3.2.1 $f\sigma_8$

Linear theory calculations can be used to predict the growth and clustering of structures at appropriate length scales and times. Redshift surveys provide an estimate of a combination of linear growth δ, its derivative given by growth factor f and its rms fluctuation at the length scale of $8h^{-1}$ Mpc given by the parameter σ_8. The combination $f\sigma_8$ [49] is an important observable for perturbed Universe at linear scale. Here the growth factor f and the σ_8 parameters are given as

$$f(a) = \frac{d(log(\delta))}{d(log(a))}, \tag{20}$$

and

$$\sigma_8(a) = \sigma_8(a = 1)\frac{\delta(a)}{\delta(1)}. \tag{21}$$

nt In Fig. 5, we show the $f\sigma_8$ for Bimetric gravity for the different values of the parameter B_1 along with the ΛCDM model. We observe that we always get larger $f\sigma_8$ for larger values of B_1 compared to ΛCDM ($B_1 = 0$). Moreover for larger values of B_1, the $f\sigma_8$ behaviour for Bimetric gravity models are not consistent in the redshift range $z \sim 0.25-0.5$. In this plot, we fix the value of σ_8 at $z = 0$ by Planck-2018 observations [9] assuming a ΛCDM model. Hence for this
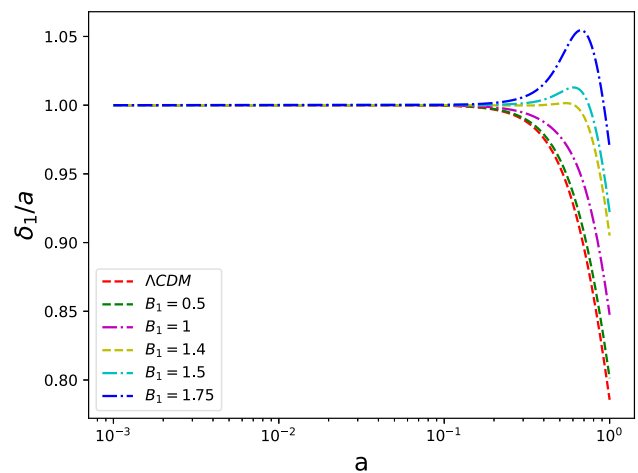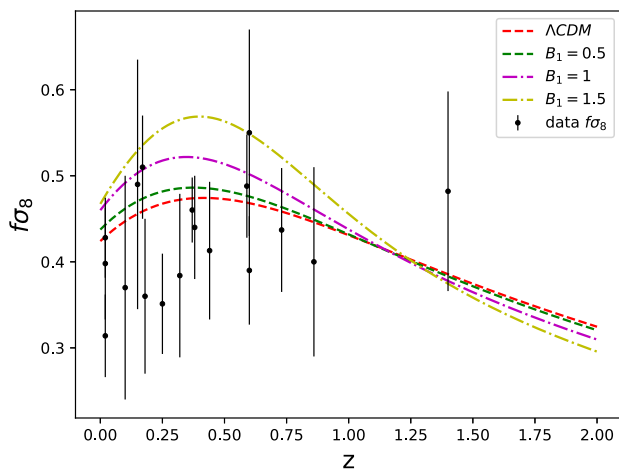
Fig. 3 Linear perturbations for different values of parameter B_1. For higher values of $B_1(> 1.4)$, there is a very distinct feature of a brief epoch with growth faster than ΛCDM as well as Einstein–diSitter
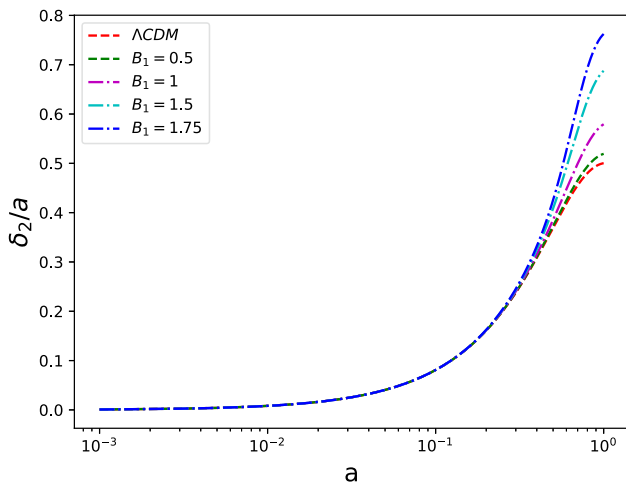
Fig. 4 Second order perturbations for different values of parameter B_1
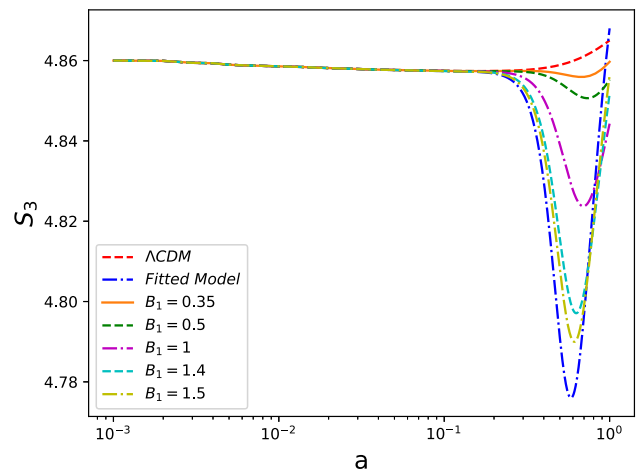
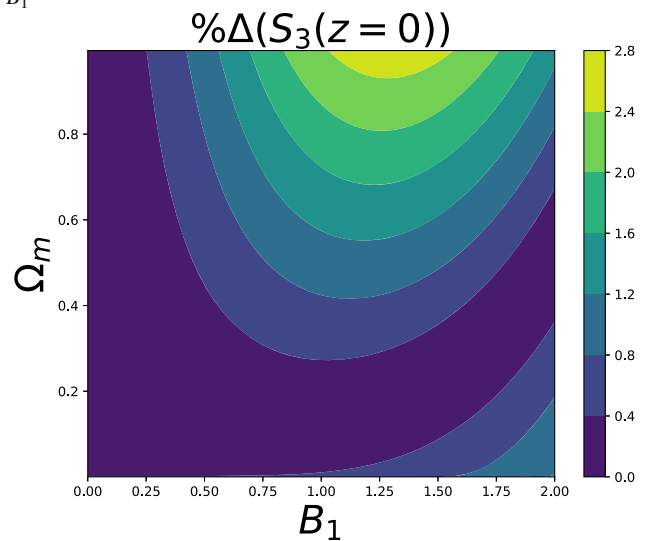**Fig. 6** Skewness S_3 as a function of scale factor for different values of B_1

Fig. 5 Combination $f\sigma_8$ as a function of redshift z for different theories. We also plot the data points from observations [50]

Fig. 7 Deviation of S_3 from ΛCDM model for different values of B_1. While the linear growth rate and second-order perturbations show huge differences, the percentage difference in the combination probed by S_3 is a few percent

figure, one expects that for higher values of B_1, a lower value for σ_8 may be necessary to make the $f\sigma_8$ for Bimetric theory more consistent with the observational data.

3.2.2 Skewness (S3)

Second-order perturbations provide further connection with statistics of observed perturbations. Gaussian initial conditions evolve into non-gaussian distribution with time and the extent of non-gaussianity depends on dynamics of the individual fluid components of the universe or the theory of gravity which evolves the whole system. Mode coupling leads to an imbalance in the distribution of voids and overdense regions [51]. Second-order perturbations play a role in this and can be related to the skewness of the density field as [39,51]

$$S_3 = \frac{\langle \delta^3 \rangle}{\langle \delta^2 \rangle^2}, \qquad (22)$$

which can be written as

$$S_3 = 3\frac{D_2}{D_1^2}. \qquad (23)$$

S_3 can be sensitive to underlying dark energy characteristics [35] or modification of GR. Here we show that the growth of perturbations is sensitive to parameter B_1 as we illustrate in Figs. 3 and 4. In Fig. 6, we show the evolution of S_3 for different values of B_1. In Fig. 7 we show the present day percentage difference of S_3 from ΛCDM model as a function of B_1 and Ω_{m0}. This can be used to distinguish the Bimetric

Table 1 The range of the uniform priors for the parameters used for the MCMC analysis.

Parameter	Prior
Ω_m	[0.0, 0.9]
h	[0.6, 0.8]
B_1	[0, 5]
r_d	[130, 160]
σ_8	[0.01, 0.9]
M	[−19.40, −19.00]

models from the ΛCDM using the S_3 measurements in near future.

4 Observational constraints on bimetric model

Given the behaviour of Bimetric gravity in terms of different observables, we now study the observational constraints on Bimetric gravity using low redshift cosmological observations. In this regard, we do the Markov Chain Monte Carlo (MCMC) analysis using different latest cosmological observational data to put constraints on the model parameters for the Bimetric gravity. The analysis is performed using the EMCEE hammer [52], a PYTHON implementation of the MCMC sampler.

We use the following data:

- Pantheon+ and SH0ES data [53] ;
- The Gold-2017 set for the $f\sigma_8$ data [50,54];
- The Baryon acoustic oscillations (BAO) measurements by the completed Sloan Digital Sky Survey (SDSS) lineage of experiments on large scales [55];
- The angular diameter distances measured using water megamasers under the Megamaser Cosmology Project [56].

We use the uniform priors given in Table 1 for the model parameters for the Bimetric gravity. The posterior probability distributions and their corresponding confidence contours for different parameters are shown in Fig. 8. As one can see from this figure, a substantial deviation from ΛCDM model in terms of the parameter B_1 is allowed by the data although the ΛCDM behaviour ($B_1 = 0$) is still allowed. The reconstructed effective dark energy equation of state and the reconstructed S_3 parameters as a function of redshifts are shown in Figs. 9 and 10. The constrained Bimetric model gives a phantom-like effective dark energy equation of state. The data also allows substantial deviation in the parameter S_3 from ΛCDM model behaviour although the ΛCDM behaviours for S_3 is also allowed in the constrained behaviour of S_3.

5 Integrated Sachs Wolfe (ISW) effect

Given the observational constraints on the Bimetric gravity considered in this analysis, we study how far the ISW signal in this model deviates from the ΛCDM model within that constraints. CMB photons traveling through evolving spacetime, traverse potentials created by matter inhomogeneity and undergo changes in their wavelengths. This contributes to anisotropies of CMB spectrum and is dubbed ISW effect [57,58]. The effect can be detected by cross-correlation of Large Scale Structure (LSS) tracers with CMB anisotropies [40,58–60] which can be used as a probe for theories giving dark energy effects. Here we follow the formalism of Lue et al. [40] who gave a general prescription for studying the ISW effect in modified gravity theories. One of the criteria for applying this prescription for modified gravity models is the existence of static spacetime outside a spherically symmetric mass distribution. The important criteria for Birkhoff's theorem is that the outside of a spherically symmetric mass distribution should be an empty space given by $R_{\mu\nu} = 0$ which can be extended to spacetime with constant curvature. In bimetric theory, this criteria does not hold true except the special choice of parameters for which the spacetime reduces to Schwarzschild A(dS). The question is whether one can get a static spacetime outside a spherically symmetric mass distribution for the choice of parameters used in this work. Kocic et al. [61] have obtained a non static spherically symmetric solution in Bimetric theory for a certain choice of the parameters β_i which apparently violates the Birkhoff's theorem. Our specific choice, where only β_0 and β_1 are non zero and rest of the β_i are zero, is different from the constraints on β_i for which Kocic et al. [61] have obtained the non static solution. With our choice of parameters, we have not found any non static spherically symmetric solution vacuum solution in Bimetric gravity theory in the literature (In [46], it was argued, although without details, that the Birkhoff's theorem does not hold in bigravity theory for any combinations of β_i parameters). On the other hand, Volkov has obtained a whole class of Black Hole solutions in Bimetric gravity which are static spacetimes outside a spherically symmetric mass distribution and they can be both asymptotically flat [62] or asymptotically non flat [63]. Just recently Rahman et al. [64] found a new static wormhole solution outside a spherically symmetric mass distribution in bimetric gravity with same choice of parameters used in this work. Keeping all these in mind, we assume a static spherically symmetric metric outside a spherical overdensity in order to apply the prescription given by Lue et al. [40].

We calculate the evolution of time derivatives of potentials for our Bimetric model and compare it with the standard ΛCDM model.

Following the prescription by Lue et al. [40], we characterize background expansion in Bimetric gravity theory by a
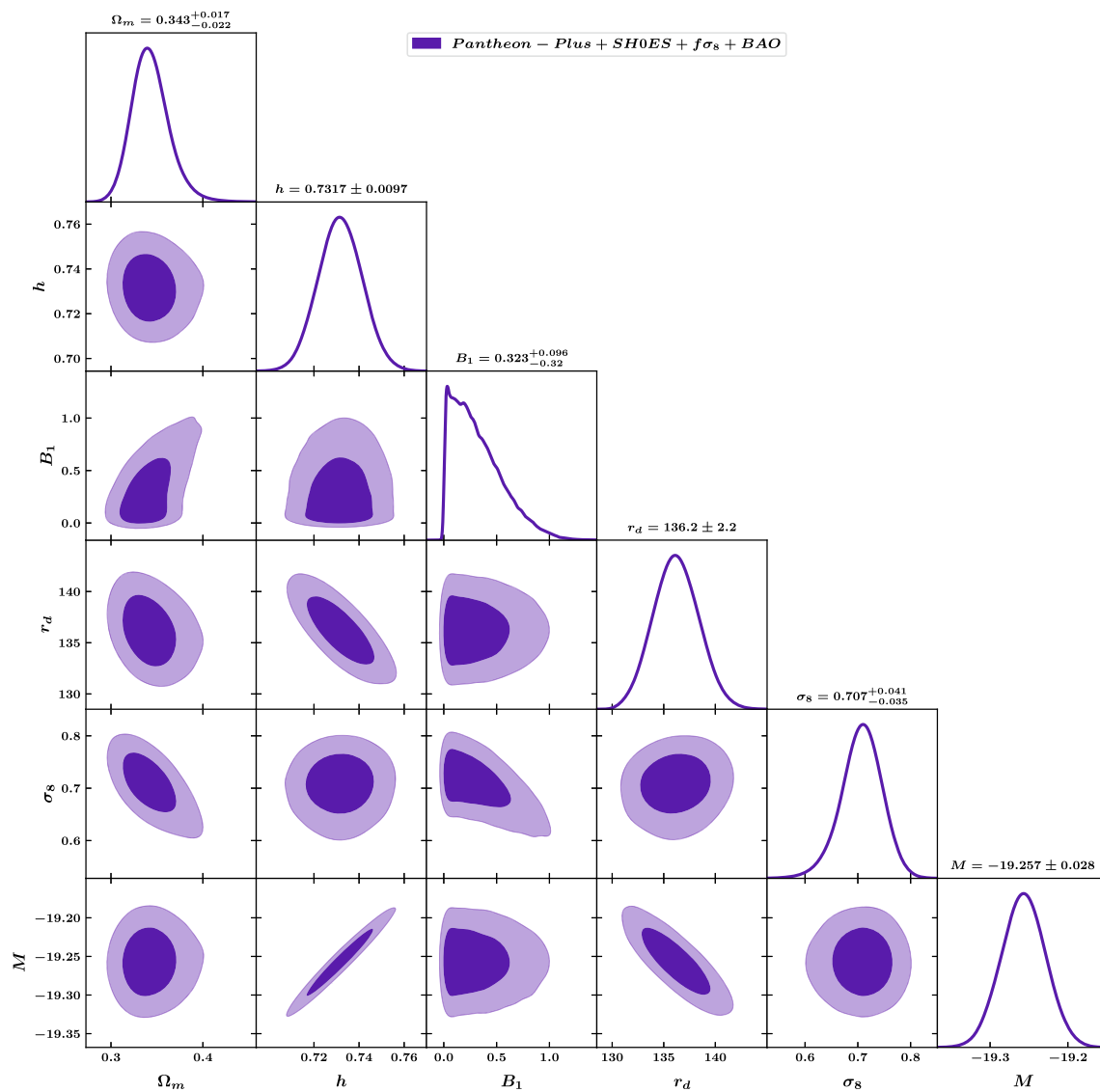
Fig. 8 Marginalized posterior distribution of the set of parameters Ω_m, h, B_1, r_d, σ_8 and M and their corresponding 2D confidence contours (68% and 95%), obtained from the MCMC analysis for the Bimetric gravity utilizing all the data sets mentioned in Sect. 4

function $g(x)$ as

$$g(x) = \left(\frac{H}{H_0}\right)^2, \tag{24}$$

with x defined as

$$x \equiv \frac{8\pi G \rho_m}{3H_0^2}. \tag{25}$$

For example, in ΛCDM, the function takes the form

$$g(x) = x + \Omega_\Lambda. \tag{26}$$

For the Bimetric gravity, $g(x)$ is given by

$$g(x) = \frac{1}{2}x + \frac{B_0}{6} + \sqrt{\left(\frac{1}{2}x + \frac{B_0}{6}\right)^2 + \frac{B_1^2}{3}}. \tag{27}$$

We start with the following convention for the perturbed metric [40]

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)(dr^2 + r^2 d\Omega^2). \tag{28}$$

The ISW effect is proportional to a combination of temporal derivatives of potentials as

$$A(\dot{\Psi} - \dot{\Phi})$$
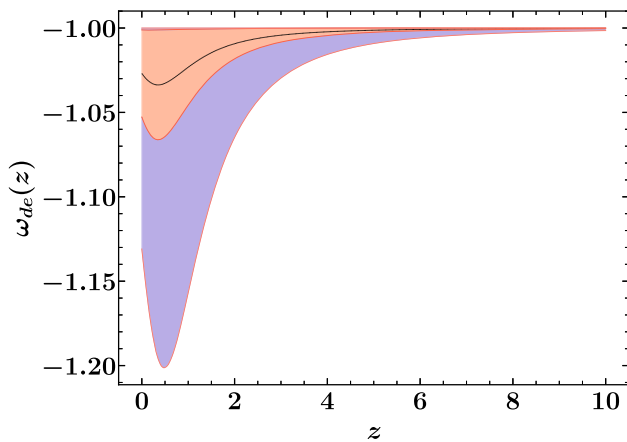
Fig. 9 Reconstructed equation of state ω_{de} as a function of redshift z. Black line is the best fit value with shaded regions as 1σ and 2σ for the inner and the outer shaded region respectively
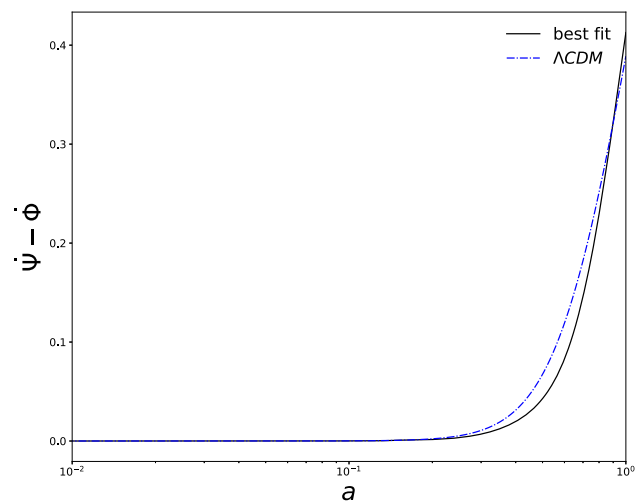

Fig. 11 Comparison of the evolution of temporal derivatives of the potentials. Differences in linear growth rate are translated here as well

Fig. 10 Reconstructed skewness S_3 as a function of redshift z. Black line is the best fit value with shaded regions as 1σ, 2σ and 3σ for the innermost to the outermost shaded region. Blue line is for the ΛCDM model

$$= \left[(1 - f^{MG})(g' + (3/2)g''x) + \frac{3}{2}(5xg'' + 3x^2g''') \right] D_+, \tag{29}$$

wherein D_+ is linear first order perturbation and f is growth rate defined as $\frac{d(lnD_+)}{d(lna)}$. In Fig. 11, we compare this term for our constrained Bimetric gravity (for best fitted parameter values) with ΛCDM model as constrained by Planck-2018. We see that the observationally constrained Bimetric model does not differ much from the ΛCDM model.

Finally, the cross-correlated ISW signal (w_{gT}) between LSS and CMB can be calculated as

$$w_{gT} = 3T_0\Omega_{m0}b(2\pi)^2 \frac{H_0}{c^3} \int dz \sqrt{(g)} D_+^2$$
$$\times \left[(1 - f)(g' + (3/2)g''x) + \frac{3}{2}(5xg'' + 3x^2g''') \right]$$

Fig. 12 Galaxy temperature correlation for the best fit Bimetric model along with ΛCDM. The two are very similar

$$\times w_g(z) \int \frac{dk}{k} P(k) J_0(k\theta\chi). \tag{30}$$

Here T_0 is the present CMB temperature, b is the bias factor (assumed constant here), $P(k)$ is present-day matter power spectrum, χ is comoving distance as a function of z, J_0 is zeroth Bessel function and $w_g(z)$ is the survey-dependent galaxy selection function. For T_0, we take the value $2.725\mu K$, b is taken from Lue et al. [40] that is 5.47. We use the $w_g(z)$ of Takada and Jain [65] with mean redshift of 0.49.

This correlation function for best fitted Bimetric gravity as well as for ΛCDM model are plotted in Fig. 12 and it is evident that the cross-correlated ISW signal in Bimetric gravity is similar to ΛCDM model.

6 Conclusions

We study the cosmological evolution in a subclass of Bimetric gravity model, where only the parameters B_0 and B_1 are nonzero. We show that the effective dark energy behaviour in such a modified gravity theory can be phantom-like for a large range of parameter values. Moreover the model admits a cosmological constant that can be positive or negative depending on the values of the parameter B_1 and Ω_m. This can naturally mimic a cosmological evolution where the Universe contains a phantom like dark energy plus a negative Cosmological Constant apart from the standard matter. As shown recently such set up can be useful to solve the Hubble Tension [27,28,38].

We also study the linear and second order growth of matter fluctuations in the Bimetric gravity. We find that the growth of both linear and second order perturbations are strongly dependent on the values of parameter B_1 that signifies the deviation from the corresponding ΛCDM limit. This results in significant deviations of observables like "$f\sigma_8$" and "Skewness" parameter S_3 from the ΛCDM behaviour for higher values of the parameter B_1.

With these observations, we subsequently constrain the Bimetric model with low-redshift observational data from SnIa Observation (Pantheon+ and SH0ES), BAO observations as well as Growth measurements. It shows that the data allow significant deviation from ΛCDM behavior although ΛCDM limit of Bimetric theory ($B_1 = 0$) is also consistent with the data.

Finally we calculate the ISW signal by cross-correlating the CMB and LSS signals for our best fit Bimetric gravity model and show that it is mostly similar to the ΛCDM model as constrained by Planck-2018.

We want to stress that the issue of solving the instability in the linear perturbations on the sub-horizon scales in the early Universe with nonlinear effects is still an unresolved issue. Hence one should keep that in mind when considering observables in bimetric gravity that are related to the linear perturbations. This is also true for the validity of Birkhoff's theorem in bimetric gravity which is still not properly resolved at present although there are hints that the Birkhoff's theorem does not hold in bimetric gravity for any choice of parameters β_i. We need to keep in mind all these issues while studying perturbed Universe in bimetric gravity under the assumptions that the instability issue can be taken care of using nonlinear or other effects as well as assumption of validity of Birkhoff's theorem, as done in the present work. Concrete validation of breaking down of any one of these assumptions in future, will need new approach for the present study.

To conclude, we show that the low-redshift observations allow Bimetric gravity that behaves differently than ΛCDM. This motivates us to study the behaviour of CMB fluctuations in such models and see whether they are consistent with the Planck-2018 measurements. We plan to study this in the near future.

References

1. A.G. Riess et al., Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astron. J. **116**, 1009–1038 (1998)
2. S. Perlmutter et al., Measurements of Ω and Λ from 42 High-Redshift Supernovae. ApJ **517**, 565–586 (1999)
3. B.P. Schmidt, N.B. Suntzeff, M.M. Phillips, R.A. Schommer, A. Clocchiatti, R.P. Kirshner, P. Garnavich, P. Challis, B. Leibundgut, J. Spyromilio, A.G. Riess, A.V. Filippenko, M. Hamuy, R.C. Smith, C. Hogan, C. Stubbs, A. Diercks, D. Reiss, R. Gilliland, J. Tonry, J. Maza, A. Dressler, J. Walsh, R. Ciardullo, The high-Z supernova search: measuring cosmic deceleration and global curvature of the universe using type IA supernovae. ApJ **507**, 46–63 (1998)
4. L. Amendola, S. Tsujikawa, Dark Energy: Theory and Observations (2010)
5. T. Clifton, P.G. Ferreira, A. Padilla, C. Skordis, Modified gravity and cosmology. Phys. Rep. **513**, 1–189 (2012)
6. S. Nojiri, S.D. Odintsov, Unified cosmic history in modified gravity: from F(R) theory to Lorentz non-invariant models. Phys. Rep. **505**, 59–144 (2011)
7. L. Perivolaropoulos, F. Skara, Challenges for ΛCDM: an update. New Astron. Rev. **95**, 101659 (2022)
8. Planck Collaboration, P.A.R. Ade et al., Planck 2015 results. XIII. Cosmological parameters. A&A **594**, A13 (2016)
9. Planck Collaboration, N. Aghanim et al., Planck 2018 results. VI. Cosmological parameters. A&A **641**, A6 (2020)

10. S. Weinberg, The Cosmological Constant Problems (Talk given at Dark Matter 2000, February, 2000), arXiv e-prints. arXiv:astro-ph/0005265 (2000)

11. A.G. Riess, W. Yuan, L.M. Macri, D. Scolnic, D. Brout, S. Casertano, D.O. Jones, Y. Murakami, G.S. Anand, L. Breuval, T.G. Brink, A.V. Filippenko, S. Hoffmann, S.W. Jha, W.D. Kenworthy, J. Mackenty, B.E. Stahl, W. Zheng, A comprehensive measurement of the local value of the Hubble constant with 1 km/s/mpc uncertainty from the Hubble space telescope and the SH0es team. Astrophys. J. Lett. **934**, L7 (2022)

12. B. Follin, L. Knox, Insensitivity of the distance ladder Hubble constant determination to Cepheid calibration modelling choices. MNRAS **477**, 4534–4542 (2018)

13. S. Dhawan, S.W. Jha, B. Leibundgut, Measuring the Hubble constant with Type Ia supernovae as near-infrared standard candles. A&A **609**, A72 (2018)

14. M. Fierz, W. Pauli, On relativistic wave equations for particles of arbitrary spin in an electromagnetic field. Proc. R. Soc. Lond. Ser. A **173**, 211–232 (1939)

15. D.G. Boulware, S. Deser, Can gravitation have a finite range? Phys. Rev. D **6**, 3368–3382 (1972)

16. C. de Rham, G. Gabadadze, A.J. Tolley, Resummation of massive gravity. Phys. Rev. Lett. **106**, 231101 (2011)

17. S.F. Hassan, R.A. Rosen, Bimetric gravity from ghost-free massive gravity. J. High Energy Phys. **2012**, 126 (2012)

18. M. von Strauss, A. Schmidt-May, J. Enander, E. Mörtsell, S. Hassan, Cosmological solutions in bimetric gravity and their observational tests. J. Cosmol. Astropart. Phys. **2012**, 042 (2012)

19. E. Mortsell, S. Dhawan, Does the Hubble constant tension call for new physics? J. Cosmol. Astropart. Phys. **2018**, 025 (2018)

20. S. Dhawan, A. Goobar, E. Mörtsell, R. Amanullah, U. Feindt, Narrowing down the possible explanations of cosmic acceleration with geometric probes. J. Cosmol. Astropart. Phys. **2017**, 040 (2017)

21. M. Lüben, A. Schmidt-May, J. Weller, Physical parameter space of bimetric theory and SN1a constraints. J. Cosmol. Astropart. Phys. **2020**, 024 (2020)

22. M. Högas, E. Mörtsell, Constraints on bimetric gravity. Part II. Observational constraints. J. Cosmol. Astropart. Phys. **2021**, 002 (2021)

23. M. Högas, E. Mörtsell, Constraints on bimetric gravity. Part I. Analytical constraints. J. Cosmol. Astropart. Phys. **2021**, 001 (2021)

24. K. Max, M. Platscher, J. Smirnov, Gravitational wave oscillations in bigravity. Phys. Rev. Lett. **119**, 111101 (2017)

25. P. Brax, A.-C. Davis, J. Noller, Gravitational waves in doubly coupled bigravity. Phys. Rev. D **96**, 023518 (2017)

26. M. Högås, E. Mörtsell, Constraints on bimetric gravity from Big Bang nucleosynthesis, arXiv e-prints. arXiv:2106.09030 (2021)

27. E. Di Valentino, A. Mukherjee, A.A. Sen, Dark energy with phantom crossing and the H0 tension. Entropy **23**, 404 (2021)

28. A. Chudaykin, D. Gorbunov, N. Nedelko, Exploring ΛCDM extensions with SPT-3G and Planck data: 4σ evidence for neutrino masses, full resolution of the Hubble crisis by dark energy with phantom crossing, and all that, arXiv e-prints. arXiv:2203.03666 (2022)

29. E. Mörtsell, Cosmological histories in bimetric gravity: a graphical approach. J. Cosmol. Astropart. Phys. **2017**, 051 (2017)

30. M. Högas, E. Mörtsell, Constraints on bimetric gravity. Part I. Analytical constraints. J. Cosmol. Astropart. Phys. **2021**, 001 (2021)

31. N. Khosravi, H.R. Sepangi, S. Shahidi, Massive cosmological scalar perturbations. Phys. Rev. D **86** (2012)

32. Y. Akrami, S. Hassan, F. Koennig, A. Schmidt-May, A.R. Solomon, Bimetric gravity is cosmologically viable. Phys. Lett. B **748**, 37–44 (2015)

33. A.D. Felice, A.E. Gumrukcuglu, S. Mukohyama, N. Tanahashi, T. Tanaka, Viable cosmology in bimetric theory. J. Cosmol. Astropart. Phys. **2014**, 037 (2014)

34. L. Amendola, C. Quercellini, Skewness as a test of the equivalence principle. Phys. Rev. Lett. **92** (2004)

35. R. Emy Fazolo, L. Amendola, H. Velten, Skewness as a test of dark energy perturbations, arXiv e-prints. arXiv:2202.08355 (2022)

36. M. Luben, A. Schmidt-May, J. Weller, Physical parameter space of bimetric theory and SN1a constraints. J. Cosmol. Astropart. Phys. **2020**, 024 (2020)

37. S. Dhawan, D. Brout, D. Scolnic, A. Goobar, A.G. Riess, V. Miranda, Cosmological model insensitivity of local H_0 from the Cepheid distance ladder. ApJ **894**, 54 (2020)

38. A.A. Sen, S.A. Adil, S. Sen, Do cosmological observations allow a negative λ? Mon. Not. R. Astron. Soc. **518**, 1098–1105 (2022)

39. T. Multamaki, E. Gaztanaga, M. Manera, Large scale structure in nonstandard cosmologies. Mon. Not. R. Astron. Soc. **344**, 761 (2003)

40. A. Lue, R. Scoccimarro, G. Starkman, Differentiating between modified gravity and dark energy. Phys. Rev. D **69**, 044005 (2004)

41. F. Koennig, L. Amendola, Instability in a minimal bimetric gravity model. Phys. Rev. D**90** (2014)

42. F. Koennig, Y. Akrami, L. Amendola, M. Motta, A.R. Solomon, Stable and unstable cosmological models in bimetric massive gravity. Phys. Rev. D **90** (2014)

43. F. Koennig, Higuchi ghosts and gradient instabilities in bimetric gravity. Phys. Rev. D **91** (2015)

44. D. Comelli, M. Crisostomi, L. Pilo, FRW cosmological perturbations in massive bigravity. Phys. Rev. D **90** (2014)

45. K. Aoki, K. Ichi Maeda, R. Namba, Stability of the early universe in bigravity theory. Phys. Rev. D **92** (2015)

46. M. Högas, F. Torsello, E. Mörtsell, On the stability of bimetric structure formation. J. Cosmol. Astropart. Phys. **2020**, 046 (2020)

47. M. Kocic, F. Torsello, M. Hogas, E. Mörtsell, Spherical dust collapse in bimetric relativity: bimetric polytropes, arXiv e-prints. arXiv:1904.08617 (2019)

48. A. Ijjas, F. Pretorius, P.J. Steinhardt, Stability and the gauge problem in non-perturbative cosmology. J. Cosmol. Astropart. Phys. **2019**, 015 (2019)

49. Y.-S. Song, W.J. Percival, Reconstructing the history of structure formation using redshift distortions. J. Cosmol. Astropart. Phys. **2009**, 004 (2009)

50. S. Nesseris, G. Pantazis, L. Perivolaropoulos, Tension and constraints on modified gravity parametrizations of G_{eff}(z) from growth rate and Planck data. Phys. Rev. D **96**, 023542 (2017)

51. F. Bernardeau, S. Colombi, E. Gaztanaga, R. Scoccimarro, Large-scale structure of the universe and cosmological perturbation theory. Phys. Rep. **367**, 1–248 (2002)

52. D. Foreman-Mackey, D.W. Hogg, D. Lang, J. Goodman, emcee: the MCMC hammer. Publ. Astron. Soc. Pac. **125**, 306–312 (2013)

53. D. Brout et al., The pantheon+ analysis: cosmological constraints. Astrophys. J. **938**(2), 110 (2022)

54. E. Macaulay, I.K. Wehus, H.K. Eriksen, Lower growth rate from recent redshift space distortion measurements than expected from Planck. Phys. Rev. Lett. **111**(16), 161301 (2013)

55. S. Alam et al., Completed SDSS-IV extended baryon oscillation spectroscopic survey: cosmological implications from two decades of spectroscopic surveys at the Apache Point Observatory. Phys. Rev. D **103**(8), 083533 (2021)

56. M.J. Reid, J.A. Braatz, J.J. Condon, L.J. Greenhill, C. Henkel, K.Y. Lo, The Megamaser Cosmology Project: I. VLBI observations of UGC 3789. Astrophys. J. **695**, 287–291 (2009)

57. R.K. Sachs, A.M. Wolfe, Perturbations of a cosmological model and angular variations of the microwave background. ApJ **147**, 73 (1967)

58. A.J. Nishizawa, The integrated Sachs-Wolfe effect and the Rees–Sciama effect. Prog. Theor. Exp. Phys. **2014**, 06B110 (2014)

59. R.G. Crittenden, N. Turok, Looking for a cosmological constant with the Rees–Sciama effect. Phys. Rev. Lett. **76**, 575–578 (1996)

60. A. Krolewski, S. Ferraro, The Integrated Sachs Wolfe effect: unWISE and Planck constraints on dynamical dark energy. J. Cosmol. Astropart. Phys. **2022**, 033 (2022)

61. M. Kocic, M. Högas, F. Torsello, E. Mörtsell, On Birkhoff's theorem in ghost-free bimetric theory. arXiv:1708.07833 (2017)

62. R. Gervalle , M.S. Volkov, Asymptotically flat hairy black holes in massive bigravity. Phys. Rev. D **102** (2020)

63. M.S. Volkov, Hairy black holes in the ghost-free bigravity theory. Phys. Rev. D **85**, (2012)

64. M. Rahman, A.A. Sen, S.S. Bohra, Traversable wormholes in bimetric gravity (2023)

65. M. Takada, B. Jain, The impact of non-Gaussian errors on weak lensing surveys. MNRAS **395**, 2065–2086 (2009)