



# Torsional dark energy in quadratic gauge gravity

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Received: 6 December 2022 / Accepted: 12 March 2023 / Published online: 11 April 2023  
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**Abstract** The covariant canonical gauge theory of gravity (CCGG) is a gauge field formulation of gravity which a priori includes non-metricity and torsion. It extends the Lagrangian of Einstein's theory of general relativity by terms at least quadratic in the Riemann–Cartan tensor. This paper investigates the implications of metric compatible CCGG on cosmological scales. For a totally anti-symmetric torsion tensor we derive the resulting equations of motion in a Friedmann–Lemaître–Robertson–Walker (FLRW) Universe. In the limit of a vanishing quadratic Riemann–Cartan term, the arising modifications of the Friedmann equations are shown to be equivalent to spatial curvature. Furthermore, the modified Friedmann equations are investigated in detail in the early and late times of the Universe's history. It is demonstrated that in addition to the standard  $\Lambda$ CDM behaviour of the scale factor, there exist novel time dependencies, emerging due to the presence of torsion and the quadratic Riemann–Cartan term. Finally, at late times, we present how the accelerated expansion of the Universe can be understood as a geometric effect of spacetime through torsion, rendering the introduction of a cosmological constant redundant. In such a scenario it is possible to compute an expected value for the parameters of the postulated gravitational Hamiltonian/Lagrangian and to provide a lower bound on the vacuum energy of matter.

## 1 Introduction

Mysterious components such as dark energy (DE) caused Einstein's theory of general relativity (GR) to become a

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incomplete and thus unsatisfactory explanation of gravity on cosmological scales. As a result, there has been a plethora of attempts to modify GR, for example by introducing additional degrees of freedom in the form of scalar fields [1–3] or vector fields [4–7].

In this work however, we continue the path taken in [8–11] which is to use the theory of canonical transformations within the De Donder–Weyl formalism [12, 13] and the ideas of gauge theory to arrive at a more general theory of gravity called covariant canonical gauge theory of gravity (CCGG). CCGG is based on merely four postulates [14], namely:

- (i) Hamilton's principle.
- (ii) Non-degeneracy of the total and gravitational Lagrangian.
- (iii) Diffeomorphism invariance.
- (iv) Equivalence principle.

Employing the Palatini approach [15] of treating the metric and the connection independently, these postulates, together with the formalism of canonical transformations, have been shown to result in the so-called CCGG equations [8] which generalise Einstein's field equation. In particular, non-metricity and torsion are not a priori neglected within this approach.

As usual for gauge theories, the Lagrangian, or respectively the Hamiltonian, of the free fields cannot be determined and has to be postulated. Non-degeneracy restricts the choice of the free gravitational Hamiltonian  $\mathcal{H}_{\text{gr}}$  however and forces us [16] to include at least a term quadratic in the Riemann–Cartan tensor. The ansatz for  $\mathcal{H}_{\text{gr}}$  used in CCGG extends the Einstein theory by a trace-free Kretschmann term. This has been shown to be consistent with low-redshift data [17].

The goal of this paper is to study the implications of the CCGG equations on cosmological scales, setting forth earlier investigations [18, 19]. In Sect. 2 a brief review of the underlying theory and the resulting CCGG field equa-

tion is presented. In order to make contact with standard  $\Lambda$ CDM results, a constraint on the torsion tensor and energy–momentum conservation is proposed in Sect. 3. A specific ansatz, satisfying this constraint, is then presented and the resulting equations of motion are derived in Sect. 4 with help of the xAct-package in MATHEMATICA [20]. Thereafter a brief discussion is given in Sect. 5 on the limiting case in which the Kretschmann term vanishes. The key portion of this work is Sect. 6, where the asymptotic equations of motion are investigated in the early and late epochs of the Universe. Lastly, a summary and our conclusions complete this paper in Sect. 7.

Throughout this paper we employ natural units, in which  $\hbar = c = 1$ . Furthermore, the Misner–Thorne–Wheeler convention (+ + +) [21] is used.

## 2 Setup

The gauging process of CCGG has been worked out in detail within the framework of De Donder–Weyl theory in [14] and results in the action integral

$$\begin{aligned}
 S &= \int d^4x \tilde{\mathcal{L}} \\
 &= \int d^4x \left[ \frac{1}{2} \tilde{k}_i^{\mu\nu} \left( \frac{\partial e^i_\mu}{\partial x^\nu} - \frac{\partial e^i_\nu}{\partial x^\mu} + \omega^i_{j\nu} e^j_\mu - \omega^i_{j\mu} e^j_\nu \right) \right. \\
 &\quad \left. + \frac{1}{2} \tilde{q}_i^{\mu\nu} \left( \frac{\partial \omega^i_{j\mu}}{\partial x^\nu} - \frac{\partial \omega^i_{j\nu}}{\partial x^\mu} + \omega^i_{n\nu} \omega^n_{j\mu} - \omega^i_{n\mu} \omega^n_{j\nu} \right) \right. \\
 &\quad \left. - \tilde{\mathcal{H}}_{\text{gr}} + \tilde{\mathcal{L}}_{\text{matter}} \right]. \tag{1}
 \end{aligned}$$

Greek indices denote components with respect to a holonomic basis whereas Latin indices refer to a non-holonomic basis. The tetrads  $e^i_\alpha$  translate between the holonomic and non-holonomic bases. In this regard we have  $g_{\mu\nu} = e^i_\mu e^j_\nu \eta_{ij}$ , where  $\eta_{ij}$  is the Minkowski metric. A tilde always denotes a tensor density. Thus the “momentum tensor density” is given by

$$\tilde{q}_l^{\mu\alpha\beta} \equiv \varepsilon q_l^{\mu\alpha\beta} := \frac{\partial \tilde{\mathcal{L}}}{\partial (\partial_\beta \omega^l_{\mu\alpha})}, \tag{2}$$

and denotes the canonical conjugate of the spin connection  $\omega^l_{\mu\alpha}$ . Further, the conjugate of the tetrads is

$$\tilde{k}_l^{\alpha\beta} \equiv \varepsilon k_l^{\alpha\beta} := \frac{\partial \tilde{\mathcal{L}}}{\partial (\partial_\beta e^l_\alpha)}. \tag{3}$$

Here  $\tilde{\mathcal{L}}$  denotes the Lagrangian density of the whole system that is obtained by a complete Legendre transformation of the respective total Hamiltonian density. Informa-

tion regarding the matter content is incorporated within  $\tilde{\mathcal{L}}_{\text{matter}}$ . The relevant Jacobi determinant in tetrad formalism is  $\varepsilon := \det(e^i_\alpha) \equiv \sqrt{-g}$ , where  $g := \det(g_{\mu\nu})$ . Varying (1) with respect to the dynamical fields provides us with the set of canonical equations of motion as in [14]. These hold a priori for any arbitrary choice of torsion and non-metricity tensors. In more detail, our choice of the free gravity De Donder–Weyl Hamiltonian density  $\tilde{\mathcal{H}}_{\text{gr}}$  entails, amongst others, information about the appearance of torsion and non-metricity. One can show that the ansatz [19] for the free gravity De Donder–Weyl Hamiltonian density

$$\begin{aligned}
 \tilde{\mathcal{H}}_{\text{gr}} &= -\frac{\tilde{q}_l^{\mu\alpha\beta} \tilde{q}_m^{\lambda\xi\eta}}{4g_1\varepsilon} \eta_{kn} \eta_{ij} e^k_\alpha e^n_\xi e^i_\beta e^j_\lambda \\
 &\quad + g_2 \tilde{q}_l^{\mu\alpha\beta} \eta_{mn} e^l_\alpha e^n_\beta \\
 &\quad - \frac{\tilde{k}_l^{\alpha\beta} \tilde{k}_m^{\xi\lambda}}{2g_3\varepsilon} \eta^{lm} \eta_{kn} \eta_{ij} e^k_\alpha e^n_\xi e^i_\beta e^j_\lambda - g_4 \varepsilon \tag{4}
 \end{aligned}$$

induces the following CCGG field equation [8, 14, 18, 19]:

$$-g_1 Q^{\mu\nu} + \frac{1}{8\pi G} (G^{\mu\nu} + g^{\mu\nu} \Lambda_0) + 2g_3 W^{\mu\nu} = T^{(\mu\nu)}, \tag{5}$$

where  $g_{\mu\nu}$  is the metric, and  $g_i$  are fundamental coupling constants with the dimensions  $[g_1] = 1$ ,  $[g_2] = L^{-2}$ ,  $[g_3] = L^{-2}$  and  $[g_4] = L^{-4}$ . The choice (4) ensures a vanishing non-metricity (chosen here for simplicity) by requesting  $q^{(lm)\alpha\beta} = 0$  but a non-vanishing, a priori arbitrary, torsion.

We defined the symmetric and trace-free Kretschmann term

$$Q^{\mu\nu} := R^{\alpha\beta\gamma\mu} R_{\alpha\beta\gamma}{}^\nu - \frac{1}{4} g^{\mu\nu} R^{\alpha\beta\gamma\lambda} R_{\alpha\beta\gamma\lambda}, \tag{6}$$

with the Riemann–Cartan tensor

$$R^\alpha{}_{\beta\mu\nu} := \partial_\mu \Gamma^\alpha{}_{\beta\nu} - \partial_\nu \Gamma^\alpha{}_{\beta\mu} + \Gamma^\alpha{}_{\lambda\mu} \Gamma^\lambda{}_{\beta\nu} - \Gamma^\alpha{}_{\lambda\nu} \Gamma^\lambda{}_{\beta\mu}, \tag{7}$$

and

$$W^{\mu\nu} := S^{\alpha\beta\mu} S_{\alpha\beta}{}^\nu - \frac{1}{2} S^{\mu\alpha\beta} S^\nu{}_{\alpha\beta} - \frac{1}{4} g^{\mu\nu} S^{\alpha\beta\gamma} S_{\alpha\beta\gamma} \tag{8}$$

with the torsion tensor

$$S^\lambda{}_{\mu\nu} := \frac{1}{2} (\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}). \tag{9}$$

The affine connection  $\Gamma$  can thereby be decomposed as

$$\Gamma^\alpha{}_{\mu\nu} = \bar{\Gamma}^\alpha{}_{\mu\nu} + K^\alpha{}_{\mu\nu}, \tag{10}$$

where  $\bar{\Gamma}$  is the Levi–Civita connection, and  $K^\alpha{}_{\mu\nu}$  the contortion tensor. (Henceforth any object equipped with a bar

shall denote a quantity based on  $\bar{\Gamma}$ .) The contortion tensor is defined as

$$K_{\lambda\mu\nu} := S_{\lambda\mu\nu} - S_{\mu\lambda\nu} + S_{\nu\mu\lambda}. \tag{11}$$

The constant  $\Lambda_0$  arising in Eq. (5) is related to the parameters  $g_1, g_2$ , governing the strength of the quadratic and linear terms in the Hamiltonian (4), and  $g_4$  representing the vacuum energy of matter, as

$$\begin{aligned} g_1 g_2 &= -\frac{1}{16\pi G} \\ 6g_1 g_2^2 + g_4 &= \frac{\Lambda_0}{8\pi G}. \end{aligned} \tag{12}$$

Obviously  $\Lambda_0$  can be identified with the cosmological constant. The parameter  $g_1$  in particular regulates the relative strength of the trace-free Kretschmann term “deforming” Einstein–Cartan gravity.

Care has to be taken with the Einstein tensor  $G^{\mu\nu}$ . Here it is defined to only include the symmetric part of the Ricci tensor

$$G^{\mu\nu} := R^{(\mu\nu)} - \frac{1}{2}g^{\mu\nu}R. \tag{13}$$

Since torsion is present,  $R^{\mu\nu}$  also has an anti-symmetric portion which, together with the anti-symmetric portion of the canonical energy momentum tensor,  $T^{[\mu\nu]}$ , yields another set of equations [8,9] relating torsion and the spin density of matter. However, for our purposes in this paper we can ignore this set of equations and focus solely on the symmetric Eq. (5).

The gauging procedure further reveals that the energy–momentum tensor of spacetime itself corresponds to the l.h.s. of (5) up to a minus sign. Upon defining

$$\Theta^{\mu\nu} := -g_1 Q^{\mu\nu} + \frac{1}{8\pi G} (G^{\mu\nu} + g^{\mu\nu} \Lambda_0) + 2g_3 W^{\mu\nu} \tag{14}$$

the CCGG Eq. (5) are compactly written as

$$\Theta^{\mu\nu} = T^{(\mu\nu)}. \tag{15}$$

### 3 Constraints

It is straightforward to verify that  $\nabla_\nu \Theta^{\mu\nu} = 0$  is not an identity. Thus in general, the energy-momentum conservation  $\nabla_\nu T^{(\mu\nu)} = 0$  is not necessarily satisfied either. In order to determine the evolutionary behaviour of energy densities on cosmological scales, we therefore have to ensure

$$\nabla_\nu \Theta^{\mu\nu} = \nabla_\nu T^{(\mu\nu)}. \tag{16}$$

However, (16) is difficult to solve in general. In order to make the computations analytically tractable we have to make a few simplifying assumptions.

For any symmetric tensor  $A^{\mu\nu}$  we have that

$$\nabla_\nu A^{\mu\nu} = \bar{\nabla}_\nu A^{\mu\nu} + K^\mu_{\alpha\nu} A^{\alpha\nu} + K^\nu_{\alpha\nu} A^{\mu\alpha}. \tag{17}$$

Closer inspection of the second term on the r.h.s. reveals  $K^\mu_{\alpha\nu} A^{\alpha\nu} = K^\mu_{(\alpha\nu)} A^{\alpha\nu}$ , due to the symmetry of  $A^{\mu\nu}$ , with

$$\begin{aligned} K^\mu_{(\alpha\nu)} &= \frac{1}{2} (S^\mu_{\alpha\nu} - S^\mu_{\alpha\nu} + S_{\nu\alpha}^\mu \\ &\quad + S^\mu_{\nu\alpha} - S_{\nu\alpha}^\mu + S_{\alpha\nu}^\mu). \end{aligned} \tag{18}$$

Since  $S^\lambda_{\mu\nu} = -S^\lambda_{\nu\mu}$  it follows that

$$K^\mu_{(\alpha\nu)} = 2S_{(\alpha\nu)}^\mu. \tag{19}$$

For the third term in (17) we find

$$K^\nu_{\alpha\nu} = 2S^\nu_{\alpha\nu}. \tag{20}$$

If  $S_{\alpha\nu\mu}$  is additionally anti-symmetric in its first two indices then  $S_{(\alpha\nu)\mu} = 0$  and  $S^\nu_{\alpha\nu} = 0$ . Therefore we conclude that for a totally anti-symmetric torsion tensor the divergence of a symmetric (2, 0)-tensor is equal to the divergence based solely on the Levi-Civita connection  $\nabla_\nu A^{\mu\nu} = \bar{\nabla}_\nu A^{\mu\nu}$ . By assuming a totally anti-symmetric torsion tensor henceforth, relation (16) becomes

$$\bar{\nabla}_\nu \Theta^{\mu\nu} = \bar{\nabla}_\nu T^{(\mu\nu)} \tag{21}$$

due to the symmetry of  $\Theta^{\mu\nu}$ .

In the following we wish to apply this ansatz in a cosmological setting based on the Friedmann–Lemaître–Robertson–Walker (FLRW) metric, and align the analysis with the assumptions of the standard model as far as possible. Hence we require in addition that  $\bar{\nabla}_\nu T^{(\mu\nu)} = 0$  holds, and thus by consistency also  $\bar{\nabla}_\nu \Theta^{\mu\nu} = 0$ . This restriction allows to employ the scaling behaviour of the individual energy densities as known from standard cosmology. The second equation,  $\bar{\nabla}_\nu \Theta^{\mu\nu} = 0$ , must be verified case by case, though. Fortunately this turns out to be rather easy here.

### 4 Modified Friedmann equations

As discussed in the previous section we choose a totally anti-symmetric ansatz for torsion,

$$S_{\alpha\mu\nu} = \epsilon_{\alpha\mu\nu\sigma} s^\sigma, \tag{22}$$

where  $\epsilon_{\alpha\mu\nu\sigma}$  is the Levi-Civita tensor and  $s^\mu = (s_0(t), 0, 0, 0)$  a temporal axial four-vector. This ansatz has already been successfully used in [22]. Since our underlying theoretical framework poses no restrictions on the form of the torsion tensor, the only caveat is that we require (22) to be consistent with the cosmological principle, i.e., a spatially homogeneous and isotropic Universe. In fact, the allowed values for the torsion tensor, given a FLRW-metric, have already been worked out in full generality by [23] and have been employed prominently by [24–26]. Our choice (22) is now seen to be in correspondence with the cosmological principle as it is a subset of the generally allowed values for the torsion tensor in a FLRW-Universe [23]. Moreover, this choice ensures that the auto-parallel and geodesic trajectories of test point particles in curved geometry coincide. The FLRW-metric in spherical coordinates is

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \tag{23}$$

where  $d\Omega^2 = d\theta^2 + \sin^2(\theta)d\phi^2$ . As usual,  $a(t)$  is the scale factor and  $k$  denotes the spatial curvature. Matter is modelled by a set of non-interacting perfect fluids with the energy-momentum tensor

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p g^{\mu\nu}. \tag{24}$$

The total energy density  $\rho$  and pressure  $p$  are the sums of the individual constituents, which in this case are radiation and matter (baryonic and non-baryonic). The vector field  $u^\mu = (1, 0, 0, 0)^\top$  denotes the 4-velocity of the fluid. Since  $K_{\alpha\mu\nu} \equiv S_{\alpha\mu\nu}$  for a totally anti-symmetric torsion tensor, we are now in the position to express the connection (10) and thus the components of the CCGG field equations explicitly.

Equation (15) reduces to only two independent components which give modified versions of the standard Friedmann equations:

$$3g_1 \left[ \frac{k^2}{a^4} + \frac{2k}{a^2} (H^2 - s_0^2) - H^2 (2\dot{H} + 5s_0^2) - \dot{H}^2 + 2Hs_0\dot{s}_0 + \dot{s}_0^2 + s_0^4 \right] + \rho_m + \rho_r + \frac{-3H^2 + \Lambda_0 + 3s_0^2(1 - 8\pi Gg_3)}{8\pi G} - \frac{3k}{8\pi Ga^2} = 0 \tag{25}$$

and

$$g_1 \left[ \frac{k^2}{a^4} + \frac{2k}{a^2} (H^2 - s_0^2) - H^2 (2\dot{H} + 5s_0^2) - \dot{H}^2 + 2Hs_0\dot{s}_0 + \dot{s}_0^2 + s_0^4 \right] + p_r$$

$$+ \frac{3H^2 + 2\dot{H} - \Lambda_0 - s_0^2(1 - 8\pi Gg_3)}{8\pi G} + \frac{k}{8\pi Ga^2} = 0. \tag{26}$$

Here we used  $\rho = \rho_r + \rho_m$  and assumed the usual equation of state  $p_i = w_i \rho_i$  with  $w_m = 0$  for matter and  $w_r = 1/3$  for radiation. The Hubble function is defined as  $H := \dot{a}/a$ .

The trace of (15) is found to be

$$\rho_m + \frac{-3\dot{H} - 6H^2 + 2\Lambda_0 + 3s_0^2(1 - 8\pi Gg_3)}{4\pi G} - \frac{3k}{4\pi Ga^2} = 0. \tag{27}$$

Note that there is no  $g_1$  term appearing in (27), i.e., there is no contribution from the quadratic “radiation like” Riemann term since  $Q^\mu{}_\mu = 0$ .

### 5 Einstein–Cartan Limit

Let us first consider the limit  $g_1 \rightarrow 0$ . This corresponds to a modification of standard GR based solely on the introduction of a non-vanishing torsion tensor. In other words, this case corresponds to Einstein–Cartan theory with a special ansatz for  $S_{\alpha\mu\nu}$ . However, our ansatz for torsion differs from the one used, e.g., in [27]. Therefore we expect to find a different behaviour in this regime.

The Friedmann Eqs. (25) and (26) reduce to

$$H^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda_0}{3} - \frac{k}{a^2} + s_0^2(1 - 8\pi Gg_3) \tag{28}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p_r) + \frac{\Lambda_0}{3}. \tag{29}$$

Furthermore, (25) and (26) in the limit of  $g_1 \rightarrow 0$  reveal that the remaining contribution of the torsion may be absorbed into the energy-momentum tensor of a perfect fluid with energy density

$$\rho_s = \frac{3s_0^2(1 - 8\pi Gg_3)}{8\pi G} \tag{30}$$

and pressure

$$p_s = -\frac{s_0^2(1 - 8\pi Gg_3)}{8\pi G}. \tag{31}$$

Thus torsion admits the equation of state with  $w_s = -1/3$ , same as for the spatial curvature  $k$ . Since we were able to

define an appropriate energy density and pressure for the torsion terms, conservation of the torsion contribution is automatically ensured via the standard energy–momentum conservation  $\bar{\nabla}_\nu T^{(\mu\nu)} = 0$ . Hence we know that the energy density associated to  $s_0$  scales as  $\rho_s \propto a^{-2}$  and thus  $s_0 \propto a^{-1}$ .

The l.h.s. of the CCGG equations in this case just reduces to the Einstein tensor (based solely on the Levi–Civita connection) together with the cosmological constant term and hence satisfies  $\bar{\nabla}_\nu (G^{\mu\nu} + g^{\mu\nu} \Lambda_0) = 0$  due to the Bianchi identities and metricity. This therefore ensures that

$$\bar{\nabla}_\nu \Theta^{\mu\nu} = 0 \tag{32}$$

is consistently satisfied.

As  $s_0$  enters into the connection (10) linearly, it has to admit real values. By setting  $s_0 \sqrt{1 - 8\pi G g_3} = b/a$  for some  $b$  with  $[b] = L^{-1}$ , equation (28) becomes

$$H^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda_0}{3} - \frac{k - b^2}{a^2}. \tag{33}$$

Obviously, the contribution of torsion counteracts the spatial curvature [18, 19]. Therefore it is possible to misinterpret in the standard  $\Lambda$ CDM model the geometry type as open, albeit  $k \geq 0$ , as long as  $k - b^2 < 0$ , or flat with  $k = b^2 > 0$ . A closed Universe still appears only with a positive  $k$ , however with the slightly stronger constraint  $k > b^2$ .

With density parameters defined by

$$\begin{aligned} \Omega_r &= \frac{8\pi G}{3H^2} \rho_r \\ \Omega_m &= \frac{8\pi G}{3H^2} \rho_m \\ \Omega_\Lambda &= \frac{\Lambda_0}{3H^2} \\ \Omega_k &= -\frac{k}{a^2 H^2} \\ \Omega_s &= \frac{s_0^2 (1 - 8\pi G g_3)}{H^2}, \end{aligned} \tag{34}$$

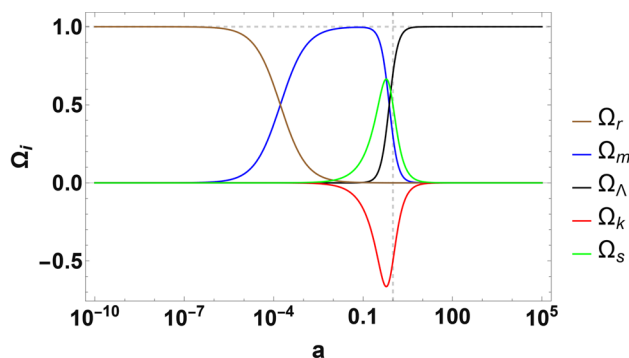
Equation (28) is equivalent to

$$1 = \Omega_r + \Omega_m + \Omega_\Lambda + \Omega_k + \Omega_s. \tag{35}$$

Thus, torsion dominates if  $s_0^2 (1 - 8\pi G g_3) \rightarrow H^2$  and vanishes trivially for  $s_0 \rightarrow 0$  or  $g_3 \rightarrow 1/(8\pi G)$ .

Let us choose  $a(t = t_0) = 1$ , where  $t_0$  denotes today. The Hubble constant is denoted by  $H_0 := H(1)$ . Then

$$\frac{H^2}{H_0^2} = \Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} + \Omega_{\Lambda,0} + \Omega_{k,0} a^{-2} + \Omega_{s,0} a^{-2}, \tag{36}$$



**Fig. 1** Qualitative plot of the density parameters as a function of the scale factor with logarithmic scale in  $a$ . The chosen values for the density parameters today are  $\Omega_{r,0} = 5 \times 10^{-5}$ ,  $\Omega_{m,0} = 0.3$ ,  $\Omega_{\Lambda,0} = 0.7$  and  $\Omega_{k,0} = -0.5$ . The inferred value for the torsion density parameter today is therefore  $\Omega_{s,0} = -(\Omega_{k,0} + \Omega_{r,0}) = 0.49995$

where  $\Omega_{i,0}$  is the  $i$ -th density parameter evaluated at  $t = t_0$ . Using

$$\Omega_i = \Omega_{i,0} \frac{H_0^2}{H^2} a^{-n_i} \tag{37}$$

with  $n_r = 4$ ,  $n_m = 3$ ,  $n_\Lambda = 0$  and  $n_k = n_s = 2$ , we may then compute the behaviour of the density parameters as a function of the scale factor for given values of  $\Omega_{i,0}$ . In Fig. 1 these behaviours are shown for  $\Omega_{r,0} = 5 \times 10^{-5}$ ,  $\Omega_{m,0} = 0.3$ ,  $\Omega_{\Lambda,0} = 0.7$  and (for illustrative purposes)  $\Omega_{k,0} = -0.5$ . As expected, the contribution of the torsion counteracts the contribution of the spatial curvature and radiation today, i.e.  $\Omega_{k,0} + \Omega_{r,0} = -\Omega_{s,0}$  in this case.

### 6 Evolution of the universe

Let us now consider the general case  $g_1 \neq 0$  and focus on investigating the asymptotic behaviour of the Friedmann Eqs. (25) and (27) (which is equivalent to considering (25) and (26)).

#### 6.1 Radiation dominated epoch

In the radiation dominated epoch (RDE) we choose the ansatz  $a = \beta t^\alpha$  with some constants  $\alpha$  and  $\beta$ , where  $[\beta] = L^{-\alpha}$ . Then  $H = \alpha t^{-1}$  and  $\dot{H} = -\alpha t^{-2}$ . Assuming that the energy content of the Universe is dominated by radiation and relativistic particles, we may neglect the cosmological constant and matter energy density. For simplicity we further assume the spatial curvature to vanish, and leave the case with  $k \neq 0$  to a forthcoming numerical study (J. Kirsch et al.). This allows us to find an analytic solution of Eq. (27):

$$s_0 = \pm \sqrt{\frac{\alpha(2\alpha - 1)}{1 - 8\pi G g_3}} \frac{1}{t}. \tag{38}$$

We directly infer the special cases  $\alpha = 0$  and  $\alpha = 1/2$ , which both yield a vanishing torsion contribution. As it turns out, the remaining consistent solutions for  $s_0$  will be such that  $\alpha(2\alpha - 1) \geq 0$ . Hence we need to impose  $g_3 < 1/(8\pi G)$  in order to obtain physical solutions. Aforementioned approximations together with (38) and  $\rho_r = \rho_{r,0}a^{-4}$  transform the first Friedmann Eq. (25) into

$$\frac{\rho_{r,0}}{\beta^4 t^{4\alpha}} + \frac{C_1(\alpha)}{t^2} - g_1 \frac{C_2(\alpha)}{t^4} = 0. \tag{39}$$

The coefficient

$$C_1(\alpha) := \frac{3\alpha(\alpha - 1)}{8\pi G} \tag{40}$$

contains the explicit contributions from the Einstein tensor and the quadratic torsion tensor  $W^{\mu\nu}$ , whereas

$$C_2(\alpha) := \frac{3\alpha(2\alpha - 1)}{(1 - 8\pi G g_3)^2} \left[ (\alpha + 1)(3\alpha - 1) - 8\pi G g_3(5\alpha^2 - 1) - 64\pi^2 G^2 g_3^2 \alpha \right], \tag{41}$$

comprises the contributions from the quadratic Riemann–Cartan term. Naturally,  $C_2$  implicitly carries the former contributions via (38).

Equation (39) has to hold as a polynomial equation for small  $t$  with  $t \neq 0$  and thus the respective coefficients of all monomials have to be zero. Due to the appearance of  $\alpha$  in the exponent we have to distinguish between different cases. The cases are  $\alpha = 1$ ,  $\alpha = 1/2$  and all other real values, i.e.,  $\alpha \notin \{1, 1/2\}$ . However, for  $\alpha \notin \{1, 1/2\}$  we know from (39) that  $C_1(\alpha) = 0$  and  $C_2(\alpha) = 0$  have to hold, which is only true for  $\alpha = 0$ . This implies  $\rho_{r,0} = 0$ , which contradicts our assumption and hence we are left with

$$\alpha \in \left\{ 1, \frac{1}{2} \right\}. \tag{42}$$

For  $\alpha = 1/2$  we find  $s_0 = 0$  and  $C_2(1/2) = 0$ . Furthermore we need to have

$$\frac{\rho_{r,0}}{\beta^4} + C_1\left(\frac{1}{2}\right) \stackrel{!}{=} 0. \tag{43}$$

But  $C_1(1/2) = -3/32\pi G$  and thus  $\beta^4 = 32\pi G \rho_{r,0}/3$ . Since only non-negative, real solutions for the scale factor are admissible we find

$$a = \sqrt[4]{\frac{32\pi G}{3} \rho_{r,0}} \sqrt{t}. \tag{44}$$

Note that this result corresponds exactly to the scale factor in the RDE of standard cosmology. This is to be expected,

since for  $\alpha = 1/2$  the contribution of the quadratic Riemann–Cartan term and the torsion vanishes and hence we find ourselves in the regime of Einstein’s GR.

For the last case  $\alpha = 1$  we have

$$s_0 = \pm \frac{1}{\sqrt{1 - 8\pi G g_3}} \frac{1}{t} \tag{45}$$

and  $C_1(1) = 0$ . Thus we are left with

$$\frac{\rho_{r,0}}{\beta^4} - g_1 C_2(1) \stackrel{!}{=} 0. \tag{46}$$

With

$$C_2(1) = \frac{12 - 96\pi G g_3(1 + 2\pi G g_3)}{(1 - 8\pi G g_3)^2} \tag{47}$$

we get  $\beta^4 = \rho_{r,0}/(g_1 C_2(1))$  and therefore

$$a = \sqrt[4]{\frac{\rho_{r,0}}{g_1 C_2(1)}} t, \tag{48}$$

which requires  $g_1 C_2(1) > 0$  and  $g_3 \neq (-1 \pm \sqrt{2})/(4\pi G)$ . This solution yields an interesting time dependence of the scale factor resulting in particular from the presence of the quadratic Riemann–Cartan term and the torsion. The linear time dependence is in correspondence with that of a Milne Universe. However, it is achieved in a very different manner. Namely, in contrast to a Milne Universe we did assume  $k = 0$  and  $\rho_{r,0} \neq 0$ . In addition we have a torsion contribution that decreases as the Universe expands.

To understand how the different ingredients of the CCGG equations contribute to the total energy density, we define the density parameter

$$\Omega_{\text{geo}} = \frac{8\pi G g_1}{H^2} \left[ -H^2(2\dot{H} + 5s_0^2) - \dot{H}^2 + 2s_0\dot{s}_0 H + \dot{s}_0^2 + s_0^4 \right]. \tag{49}$$

The parameters  $\Omega_r$  and  $\Omega_s$  that are relevant in this scenario are defined exactly in the same manner as in the Einstein–Cartan limit (34). They incorporate the energy density of radiation and the energy density of torsional contributions from  $G^{\mu\nu}$  and  $W^{\mu\nu}$ . The novel  $\Omega_{\text{geo}}$  on the other hand describes the energy density of the Kretschmann term  $Q^{\mu\nu}$ . The first Friedmann Eq. (25), in this RDE epoch, thus reduces to

$$1 = \Omega_r + \Omega_s + \Omega_{\text{geo}}. \tag{50}$$

We further have

$$\Omega_r = \Omega_{r,0} a^{-4} \frac{H_0^2}{H^2}$$

$$\Omega_s = \Omega_{s,0} a^{-2} \frac{H_0^2}{H^2} \tag{51}$$

with

$$\frac{H_0^2}{H^2} = a^2 H_0 \sqrt{\frac{8\pi G g_1 C_2(1)}{3\Omega_{r,0}}} \tag{52}$$

for the case  $a \propto t$ .

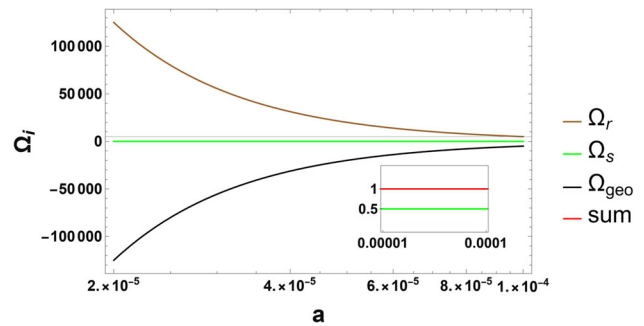
Now notice that our assumptions for the RDE are only valid in the very early Universe. Nevertheless we still extrapolate the solution until the present time by referencing today’s values of the density parameters (51) and the Hubble constant  $H_0$ . This is obviously not consistent. We should rather normalise  $a = 1$  at a time in the very early Universe, and reference the density parameters to that same time. Unfortunately we neither know the exact value of the Hubble parameter at times in the very early Universe, nor do we know the radiation density. Thus, to limit the amount of unknown parameters, we stick to the procedure above. The result will not provide us with true numerical values, but give us merely a qualitative idea of the contributions of the different ingredients in the RDE.

Evaluating now (52) at today yields for a consistency relation between different, still undetermined parameters:

$$g_1 = \frac{3\Omega_{r,0}}{8\pi G H_0^2 C_2(1)}. \tag{53}$$

In particular, given  $\Omega_{r,0}$  and  $H_0$ , the value of  $g_1$  is fixed if we provide a specific value for  $g_3$ . For  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $\Omega_{r,0} = 5 \times 10^{-5}$ ,  $\Omega_{s,0} = 0.5$  and  $g_3 = 0.34 M_p^2$ , where  $M_p := 1/\sqrt{8\pi G} = 2.44 \times 18 \text{ GeV}$  is the reduced Planck mass, the density parameters are shown in Fig. 2. Beside the negligible impact of  $\Omega_s$  in the early times, we see in particular the important contribution from the Kretschmann term  $\Omega_{\text{geo}}$  that counters the radiation energy density. The contributions of  $\Omega_{\text{geo}}$  increase with decreasing time/scale factor. This is expected since the Kretschmann term is believed to yield high contributions in high density environments.

From (25) and (26) it is evident that since the Kretschmann term is trace-free, then, if considered as a perfect fluid, it admits an equation of state parameter  $w = 1/3$ , same as that of radiation. Based on the qualitative discussion above we argue that at sufficiently early times,  $\Omega_{\text{geo}}$  must admit negative values and thus behave like negative radiation, i.e., radiation with negative energy density. Friedmann cosmologies with such negative energy densities have been investigated for instance in [28,29]. At a certain point in time, depending on the exact value of  $\Omega_s$ ,  $\Omega_{\text{geo}}$  might have to switch its sign and become positive in order to satisfy the Friedmann equation. In general we thus conclude that the



**Fig. 2** Behaviour of the density parameters as a function of the scale factor with logarithmic scale in  $a$ . The values of the density parameters today were chosen to be  $\Omega_{r,0} = 5 \times 10^{-5}$  and  $\Omega_{s,0} = 0.5$ . Furthermore,  $g_3 = 0.34 M_p^2$  was taken, meaning  $g_1 = 2.3 \times 10^{115}$  via (53)

trace-free Kretschmann term is ghost-like, and its dynamics can provide both, positive or negative energy density.

Let us lastly investigate whether the consistency with energy-momentum conservation, i.e., Eq. (32) is fulfilled. It is straightforward to see that with the given assumptions,  $\bar{\nabla}_\nu \Theta^{\mu\nu} = 0$  reduces, for  $\mu = 0$ , to

$$3\alpha(\alpha - 1)(2\alpha - 1) \times \left\{ (1 - 8\pi G g_3)^2 t^2 - 16\pi G g_1 \left[ (\alpha + 1)(3\alpha - 1) + 8\pi G g_3 (1 - 5\alpha^2 - 8\pi G g_3 \alpha) \right] \right\} = 0. \tag{54}$$

The other three equations for  $\mu \in \{1, 2, 3\}$  are trivial as in the Einstein–Cartan limit. Indeed we see that (42) satisfies this consistency check. Equation (54) reveals two more possible values for  $\alpha$ , namely, the roots of the expression inside the square bracket. However, these roots are time dependent and hence contradict our assumption of  $\alpha$  being a constant.

### 6.2 Dark energy dominated epoch

Here we wish to address the late time accelerated expansion of the Universe and its relation to torsion and the quadratic Riemann–Cartan term. Note that torsion alone is not able to account for an accelerated expansion in the Einstein–Cartan limit. This is most easily seen in the second Friedmann equation (29) where no torsion term appears and thus cannot alter  $\ddot{a}$  in a torsion dominated epoch. On the other hand, as we will see, the full CCGG theory allows for such a scenario. Thus the quadratic Riemann–Cartan term is vital for the following steps.

#### 6.2.1 General setup

An accelerated expansion is achieved if

$$\ddot{a} > 0 \quad \text{and} \quad \dot{a} > 0. \tag{55}$$

Now let us assume that torsion is dominant in such an epoch, possibly interacting with the cosmological constant. Then (27) becomes

$$-3\dot{H} - 6H^2 + 2\Lambda_0 + 3s_0^2(1 - 8\pi Gg_3) = 0. \tag{56}$$

Hence  $\ddot{a} > 0$  means that we need

$$s_0^2(1 - 8\pi Gg_3) > H^2 - \frac{2}{3}\Lambda_0. \tag{57}$$

From (56), for  $g_3 \neq 1/(8\pi G)$ , we also find that

$$s_0^2 = \frac{1}{(1 - 8\pi Gg_3)} \left( \dot{H} + 2H^2 - \frac{2}{3}\Lambda_0 \right), \tag{58}$$

which, plugged into the first Friedmann equation (25) results in a differential equation for  $H$ :

$$\begin{aligned} & \frac{-3\dot{H} + (\Lambda_0 - 3H^2)(1 + 8\pi Gg_3)}{8\pi G(1 - 8\pi Gg_3)} \\ & - g_3 \frac{(2\Lambda_0 - 6H^2 - 3\dot{H})}{1 - 8\pi Gg_3} - 3g_1 \left\{ H^4 - (H^2 + \dot{H})^2 \right. \\ & + \frac{1}{1 - 8\pi Gg_3} \left[ H(4H\dot{H} + \ddot{H}) - 5H^2 \right. \\ & \times \left. \left. \left( 2H^2 + \dot{H} - \frac{2}{3}\Lambda_0 \right) \right] + \frac{(2H^2 + \dot{H} - 2\Lambda_0/3)^2}{(1 - 8\pi Gg_3)^2} \right. \\ & \left. \left. + \frac{(4H\dot{H} + \ddot{H})^2}{4(1 - 8\pi Gg_3)(2H^2 + \dot{H} - 2\Lambda_0/3)} \right\} = 0. \end{aligned} \tag{59}$$

Obviously now we are not allowed to have

$$\dot{H} + 2H^2 - \frac{2}{3}\Lambda_0 = 0. \tag{60}$$

This is however equivalent to  $s_0 = 0$  by (58) and hence would be a contradiction to our assumption.

In theory, the solution of (59) governs the time evolution of the scale factor and via (58) also the time evolution of the torsion  $s_0$ . But it is not easy to solve (59) in general. Hence let us consider a specific example.

### 6.2.2 Exponential expansion

One special case for accelerated expansion is the exponential ansatz  $a \propto \exp(Ct)$  for some  $C > 0$  with  $[C] = L^{-1}$ . We then have that  $H = C$  and  $\dot{H} = 0$ . This is most commonly used in  $\Lambda$ CDM cosmology in conjunction with the cosmological constant as the sole source of dark energy.

In this approximation, Eq. (27) becomes

$$-6H^2 + 2\Lambda_0 + 3s_0^2(1 - 8\pi Gg_3) = 0 \tag{61}$$

at late times. This is solved for  $s_0$  as

$$s_0 = \pm \sqrt{\frac{2}{1 - 8\pi Gg_3} \left( H^2 - \frac{1}{3}\Lambda_0 \right)}. \tag{62}$$

By a similar analysis as before, using that  $H$  and  $s_0$  are constant, Eq. (25) is reduced to

$$\begin{aligned} 3g_1(-5H^2s_0^2 + s_0^4) + \frac{-3H^2 + \Lambda_0}{8\pi G} \\ + \frac{3s_0^2(1 - 8\pi Gg_3)}{8\pi G} = 0. \end{aligned} \tag{63}$$

With (62) we then get

$$\begin{aligned} \left( H^2 - \frac{1}{3}\Lambda_0 \right) \left[ -g_1 \left( 6H^2 + \frac{4}{3}\Lambda_0 \right) + \frac{1}{8\pi G} \right. \\ \left. - 2g_3 \left( 1 - 4\pi Gg_3 - 40\pi GH^2g_1 \right) \right] = 0. \end{aligned} \tag{64}$$

This equation is solved by either  $H^2 = \Lambda_0/3$  or

$$H^2 = \frac{16\pi G[-2g_1\Lambda_0 + 3g_3(4\pi Gg_3 - 1)] + 3}{48\pi Gg_1(3 - 40\pi Gg_3)} \tag{65}$$

with  $g_3 \neq 3/(40\pi G)$ . If  $H^2 = \Lambda_0/3$ , then it follows from (62) that  $s_0 = 0$ , which contradicts our assumption. The only remaining case is (65), giving

$$s_0 = \pm \frac{1}{2} \sqrt{\frac{8\pi G(10\Lambda_0g_1 + 3g_3) - 3}{6\pi Gg_1(40\pi Gg_3 - 3)}}. \tag{66}$$

This case also ensures that the conservation equation (32) is satisfied. Applying all previously stated approximations in this limit and using (62), Eq. (32) becomes

$$\begin{aligned} H \left( \Lambda_0 - 3H^2 \right) \left[ 3 - 16\pi g_1 G \left( 2\Lambda_0 + 9H^2 \right) \right. \\ \left. - 48\pi Gg_3(1 - 4\pi Gg_3 - 40\pi GH^2g_1) \right] = 0. \end{aligned} \tag{67}$$

The l.h.s. of this equation indeed vanishes for  $H^2$  as in (65).

Since the Universe already entered a phase of accelerated expansion, the value for the Hubble parameter can be set equal to the Hubble constant  $H_0 \approx 70 \text{ Km s}^{-1}\text{Mpc}^{-1}$  [30]. For a given  $g_3$ , Eq. (65) then reveals a simple inverse proportionality between the parameter  $g_1$  and the cosmological constant  $\Lambda_0$

$$g_1 = \frac{-3(1 - 8\pi Gg_3)^2}{16\pi G[-2\Lambda_0 + 3H_0^2(40\pi Gg_3 - 3)]}. \tag{68}$$



Likewise are the behaviours of  $g_2$  and  $g_4$  as a function of  $\Lambda_0$  obtained from Eq. (12):

$$g_2 = \frac{-2\Lambda_0 + 3H_0^2(40\pi Gg_3 - 3)}{3(1 - 8\pi Gg_3)^2} \tag{69}$$

$$g_4 = \frac{-1}{8\pi G} \left[ -\Lambda_0 + \frac{3H_0^2(3 - 40\pi Gg_3) + 2\Lambda_0}{(1 - 8\pi Gg_3)^2} \right]. \tag{70}$$

Let us consider the special circumstance where the expansion of the Universe is solely driven by torsion. In other words, we set  $\Lambda_0 = 0$  as is suggested by the ‘‘Zero-Energy-Universe’’ conjecture discussed in [31]. In that case the combination of the coupling constants  $g_1$  and  $g_2$  cancels the vacuum energy of matter,  $g_4$ . The trace Eq. (62) then yields

$$s_0 = \pm \sqrt{\frac{2}{1 - 8\pi Gg_3}} H_0, \tag{71}$$

whereas (65) reduces to

$$H_0^2 = \frac{48\pi Gg_3(4\pi Gg_3 - 1) + 3}{48\pi Gg_1(3 - 40\pi Gg_3)}, \tag{72}$$

which further means

$$s_0 = \pm \frac{1}{2} \sqrt{\frac{1 - 8\pi Gg_3}{2\pi Gg_1(3 - 40\pi Gg_3)}}. \tag{73}$$

Equations (72) and (73) again emphasize the relevance of the quadratic Riemann–Cartan term via the appearance of  $g_1$ .

The parameter  $g_3$  only ever appears multiplied by multiples of  $M_p^{-2}$  and is subsequently added to constants of order unity. The torsional contribution coming from the tensor (8) is thus only relevant if  $g_3 \gtrsim M_p^2$ . For  $|g_3| \ll M_p^2$  the parameters  $g_1, g_2$  and  $g_4$  admit a very simple form. From (68) we get

$$g_1 = \frac{1}{48\pi GH_0^2} = \frac{M_p^2}{6H_0^2}. \tag{74}$$

We hence find the expected value of  $g_1$  to be

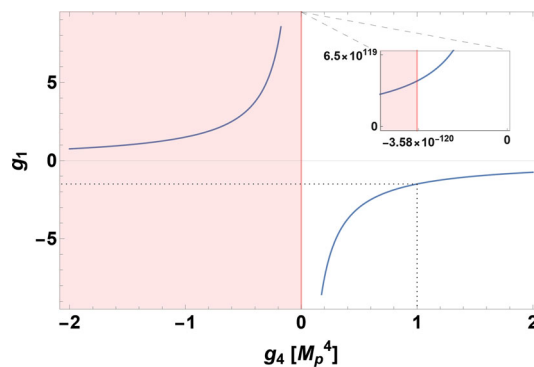
$$g_1 \approx 4.45 \times 10^{119}. \tag{75}$$

For the parameters  $g_2$  and  $g_4$  we find

$$\begin{aligned} g_2 &= \frac{-1}{16\pi Gg_1} = -3H_0^2 \\ g_4 &= \frac{-3}{128\pi^2 G^2 g_1} = -9M_p^2 H_0^2 \end{aligned} \tag{76}$$

and therefore with (75) we obtain

$$g_2 \approx -6.69 \times 10^{-84} GeV^2$$



**Fig. 3** Dependence of the parameter  $g_1$  on the vacuum energy  $g_4$  for the late times exponential expansion with  $\Lambda_0 = 0$ . The red region indicates the forbidden values for  $g_4$ . The dotted line marks the naive cutoff at Planck scales, for which  $g_1 = -3/2$

$$g_4 \approx -1.19 \times 10^{-46} GeV^4. \tag{77}$$

As the parameter  $g_2$  is related to the Riemann curvature tensor of the maximally symmetric spacetime via  $\hat{R}_{\eta\alpha\xi\beta} = -g_2(g_{\eta\xi}g_{\alpha\beta} - g_{\eta\beta}g_{\alpha\xi})$  [19], a positive (negative)  $g_2$  therefore implies an AdS (dS) geometry of the ground state of spacetime. Hence for  $|g_3| \ll M_p^2$  the ground state of spacetime admits a dS geometry.

With the assumptions and restrictions applied here, an exponential expansion in a torsion dominated epoch is thus seen to be possible even if  $\Lambda_0 = 0$ . Requesting in addition  $|g_3| \ll M_p^2$  and consistency with the observed Hubble constant  $H_0$  yields for the parameters  $g_i$  the specific values given in Eqs. (75) and (77). (Notice that the vacuum energy inferred is in the  $meV^4$  range, which is the order of magnitude discussed in [32].)

If  $|g_3| \gtrsim M_p^2$  the contribution from (8) is relevant and leaves us with an additional free parameter. Here it is possible to express all parameters as functions of the vacuum energy  $g_4$ :

$$\begin{aligned} g_1 &= \frac{-3M_p^4}{2g_4} \\ g_2 &= \frac{g_4}{3M_p^2} \end{aligned} \tag{78}$$

and

$$g_3 = M_p^2 \left[ \frac{15M_p^2 H_0^2 \pm M_p H_0 \sqrt{3(75M_p^2 H_0^2 + 8g_4)}}{2g_4} + 1 \right]. \tag{79}$$

In particular we thus need  $g_4 \geq -75M_p^2 H_0^2/8 \sim -10^{-46} GeV^4$  and  $g_4 \neq 0$ . The dependence of the parameter  $g_1$  on the vacuum energy  $g_4$  is illustrated in Fig. 3. Depending on the sign of  $g_4$ , the case  $|g_3| \gtrsim M_p^2$  also allows for an AdS geometry of the ground state of spacetime.

## 7 Conclusion

For a specific ansatz of the free gravitational De Donder–Weyl Hamiltonian we investigated the corresponding CCGG equations which extend Einstein’s field equations by a trace-free Kretschmann term and torsion contributions. After choosing a totally anti-symmetric, temporal ansatz for the torsion tensor, it was possible to align our theory with the Cosmological Principle and the energy density scaling behaviours of standard  $\Lambda$ CDM cosmology.

In a FLRW-Universe with perfect fluids accounting for the stress-energy density, the modified Friedmann equations were derived and given explicitly. Consistency has been ensured by verifying the conservation of the energy–momentum tensor of gravity on the l.h.s. of the CCGG equations. In the Einstein–Cartan limit, with the Kretschmann term set to zero, the modifications due to torsion turned out to be equivalent to spatial curvature contributions. We thus argue that in such a case it would be possible to misinterpret the observationally inferred value of the spatial curvature  $k$  in presence of torsion. The geometry type of the Universe is not solely bound to the value of  $k$  anymore but depends also on the torsional parameter  $s_0$ .

Finally, with the Kretschmann term invoked, the modified Friedmann equations were investigated in early and late times, by making appropriate assumptions for the scale factor dependence. In the RDE, we identified, in addition to the standard cosmology solution, a further, novel linear time dependence of the scale factor by virtue of the interplay between radiation, the Kretschmann term and torsion. Such a linear dependence has already appeared in Jordan–Brans–Dicke cosmology [33] and has further been shown to be in agreement with data in the so-called Power Law Universes [34]. Linear Coasting cosmologies appear very alluring in this regard as they do not suffer from the horizon problem nor the flatness problem while complying with data from SNIa, gravitational lensing and high-redshift galaxies [35,36].

Moreover, we learned that in the late epoch torsion can account for a phase of accelerated expansion even without the presence of a cosmological constant. For a specific case, namely exponential expansion, as expected in the dark energy era, the values of the parameters  $g_1$ ,  $g_2$  and  $g_4$  were derived from the observed value of the current late-time Hubble constant in the case that  $|g_3| \ll M_p^2$ . For the vacuum energy of matter,  $g_4$ , we find a value in the  $\text{meV}^4$  range. We therefore claim that our model is able to provide a geometrical explanation for the late time accelerated expansion of the Universe through torsion without resorting to the introduction of a cosmological constant.

For  $|g_3| \gtrsim M_p^2$  we were additionally able to provide a lower bound on the vacuum energy of matter. We have yet to investigate the implications of this finding.

Work in this area is still in progress. A numerical evaluation of the modified Friedmann equations as shown in this paper will be presented in the near future. That numerical analysis is not restricted to the simplified monomial ansatz of the scale factor in the RDE which we used in this paper. Indeed polynomial behaviour is seen for certain solutions which allows for more freedom in the exploration of the expansion history of the Universe. Early dark energy (EDE) solutions appear as possible candidates in this regard [37–39]. Ultimately, in order to check the consistency of our ansatz with the full set of cosmological observations, we envisage to carry out an MCMC analysis for the late data, but also work out the perturbation theory in detail to include the CMB data.

Ideas to be followed beyond that certainly include modifying the free gravitational Hamiltonian and the models of the torsion tensor in compliance with the cosmological principle [24,25]. Last but not least: discovering an independent way of measuring the parameter  $g_1$  is one of the key pending tasks for the future.

**Acknowledgements** The authors are indebted to the “Walter Greiner-Gesellschaft zur Förderung der physikalischen Grundlagenforschung e.V.” (WGG) in Frankfurt for their support. AvdV, DV and JK especially thank the Fueck Stiftung for support. The authors also wish to thank David Benisty, Eduardo Guendelman, Horst Stöcker, Yilin Cheng, Wei Li and Peter Hess for valuable discussions.

**Author contributions** All authors listed on the title page contributed equally to this work.

**Data Availability Statement** This is a theoretical study and all the related data can be found in the references.

### Declarations

**Conflict of interest** This work was supported by the “Walter Greiner-Gesellschaft zur Förderung der physikalischen Grundlagenforschung e.V.” (WGG). A. van de Venn, D. Vasak and J. Kirsch are further supported by the Fueck Stiftung.

**Ethics approval** Not applicable.

**Consent to participate** Not applicable.

**Consent for publication** Not applicable.

**Code availability** Not applicable.

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Funded by SCOAP<sup>3</sup>. SCOAP<sup>3</sup> supports the goals of the International Year of Basic Sciences for Sustainable Development.

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