THE EUROPEAN
PHYSICAL JOURNAL C

Unification of inflation and dark matter in the Higgs–Starobinsky model

Daris Samart[1,a]**, Phongpichit Channuie**[3,4,2,b]

[1] High Energy Physics Theory Group, Department of Physics, Faculty of Science, Chulalongkorn University, Phyathai Rd., Bangkok 10330, Thailand
[2] College of Graduate Studies, Walailak University, Thasala, Nakhon Si Thammarat 80160, Thailand
[3] School of Science, Walailak University, Thasala, Nakhon Si Thammarat 80160, Thailand
[4] Thailand Center of Excellence in Physics, Ministry of Education, Bangkok 10400, Thailand

Abstract In this work, we propose unified picture of inflation and dark matter in the Higgs–Starobinsky (HS) model. As pointed out in the literature, Starobinsky R^2 inflation is induced by quantum correction effect from the large Higgs-curvature (graviton) coupling. We start with non-minimal coupling HS action in Jordan frame. We then transform the Jordan frame action into the Einstein one using the conformal transformation. The inflation potential is derived from the gravitational action of non-minimal-Higgs coupling and Starobinsky term in Einstein frame where the R^2 term is dominated in the inflationary phase of the universe. For model of inflation, we compute the inflationary parameters and confront them with Planck 2015 data. We discover that the predictions of the model are in excellent agreement with the Planck analysis. In addition, we find that the HS model is equivalent to a scalar singlet dark matter (SSDM) or Higgs-portal model. The renormalization group equations (RGEs) of HS scenario with standard model at one-loop level is qualitatively analyzed. By using the solutions of parameter spaces from RGE analysis, the coupling constants of the HS model will be verified and can be used to constrain the SSDM using dark matter relic abundance.

1 Introduction

An inflationary scenario is a well-established paradigm describing an early universe and posts an indispensable ingredient of modern cosmology. Regarding degrees of freedom contained in the SM of particle physics or quantum general relativity, Higgs inflation [1] and Starobinsky model [2,3]

a e-mails: dsamart82@gmail.com; daris.s@chula.ac.th

b e-mail: channuie@gmail.com

have received much attraction in the recent years. The preceding scenario technically requires a large non-minimal coupling (ξ) between a Higgs boson H and the Ricci scalar R, i.e. $\xi H^\dagger H R$ which leads to successful inflation and produces the spectrum of primordial fluctuations in good agreement with the observational data. The formulation of the later model is based on R^2 gravity. It is worth noting that these two models are minimalistic and nicely compatible with the latest Planck data [4]. As pointed out in Ref. [5], both operators $\xi H^\dagger H R$ and R^2 are expected to be generated when the SM of particle physics is coupled to general relativity. More importantly, due to a large non-minimal coupling of the Higgs boson and the Ricci scalar, Starobinky inflation can be generated by quantum effects [5,6]. In this situation, the Higgs boson need not to start at a high field value at inflation. In addition, in the HS model, a Higgs potential can be stabilized. Notice that the simplest modification of the Einstein-Hilbert action, R^2-gravity was able to explain the dark matter [7].

Note that the study of inflation and the generation of the gravitational coupling in the context of induced gravity dated back in 1982 [8]. The authors of Ref. [9] demonstrated how a strongly coupled hidden sector could dynamically generate the Planck scale and, at the same time, result in a phase of cosmic inflation. The model of inflation involves the combined dynamics of a nonminimally coupled scalar and the scalaron, which originates from the R^2 term in the effective action can be found in Ref. [9]. However, the true nature of inflation is still unclear. Apart form inflation, the nature of dark matter conveys one of the unsolved problems in physics and also dark energy is still the greatest cosmic mystery. There was a large number of models proposed so far possible to account for DM candidates [10–12] and references therein. It was found that the unification of dark energy and dark matter model in generalized Galileon theories was

proposed in Refs. [13,14] for an alternative picture. One of the most interesting models of DM is a singlet scalar dark matter (SSDM) or Higgs-portal model. The SSDM is the simplest extension of SM by adding massive scalar field into the gauge group of SM and it gives the singlet scalar filed couple to the Higgs field [15–17]. We refer to Refs. [18,19] for recent review and status of the SSDM. By introducing auxiliary field in the action of HS model, one finds that the HS model is equivalent to the SSDM and we will demonstrate its transformation below. The aim of this work is to present a unified description of inflation and dark matter in the context of the HS model. Regrading the present investigation, the feature is multi-fold:

- (i) There is no need of physics beyond the SM of particle physics since the operators here are expected to be generated when general relativity is coupled to the SM of particle physics.
- (ii) We propose a cosmological scenario that unifies comic inflation and dark matter to a single framework. The model is minimalistic.
- (ii) Regarding this two-field scenario, the model dose not suffer from the unitarity problem as that of the Higgs inflation.

This paper is organized as follows: In Sect. 2, we take a short recap of the HS model. We also consider quantum corrections and renormalization group equations. In Sect. 3, we study inflationary implications of the HS model. Here we first construct inflationary model, compute its inflationary observables and then compare with the latest Planck data. In Sect. 4, we present the HS as the SSDM model and make qualitative discussion of the model. Discussions and conclusions are given in the last section.

2 Model set-up

2.1 Inducing Starobinsky, R^2 term by non-minimal Higgs coupling

In this section, the HS action will be considered and constructed. As mentioned earlier, the HS mechanisms is induced by quantum effect with the large coupling of Higgs-curvature [5,6]. we start with the gravitational action of the Higgs-curvature coupling with self-interacting Higgs field, it reads

$$S_J = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R + \frac{1}{2} \xi \sigma^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{\lambda}{4} \sigma^4 \right], \tag{1}$$

where the subscript S_J stands for the action in Jordan frame and $M_P = 1/\sqrt{8\pi G}$ and ξ are Planck mass and non-minimal Higgs coupling constant, respectively. The σ field is scalar field with the conventional Higgs potential and its self-interacting coupling strength λ. Note that inflation and the generation of the gravitational coupling in the context of induced gravity without the gravitational term can be found in Ref. [8]. In the framework of quantum field theory in curved spacetime, it is well known that one needs to introduce the pure higher gravitational terms into the action for making proper renormalization procedure [20,21]. Introducing the higher gravitational correction terms, the gravitational action in this work is written by [22,23],

$$\begin{aligned}
S_J &= \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R - \frac{1}{2} \xi \sigma^2 R \right. \\
&\quad \left. + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{\lambda}{4} \sigma^4 \right] \\
&\quad + \int d^4x \sqrt{-g} \left[-c_1 R^2 - c_2 R_{\mu\nu} R^{\mu\nu} \right. \\
&\quad \left. - c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right] \tag{2} \\
&= \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R - \frac{1}{2} \xi \sigma^2 R \right. \\
&\quad \left. + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{\lambda}{4} \sigma^4 - \alpha R^2 - \beta \mathcal{C}^2 - \gamma \mathcal{G}^2 \right],
\end{aligned}$$

$$\tag{3}$$

where the $\mathcal{C}^2$ and $\mathcal{G}^2$ terms are Wely tensor and Guass-Bonnet term, respectively and they are defined by

$$\begin{aligned}
\mathcal{C}^2 &= \frac{1}{3} R^2 - 2 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}, \\
\mathcal{G}^2 &= R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}. \tag{4}
\end{aligned}$$

In addition, the couplings, α, β and γ are linear combinations of the $c_{1,2,3}$ as

$$\alpha = c_1 + \frac{1}{3} (c_2 + c_3), \quad \beta = \frac{1}{2} c_2 + 2 c_3$$

$$\gamma = -\frac{1}{2} c_2 - c_3. \tag{5}$$

We note that the parameters α, β and γ vanish at the classical level. As mentioned, the higher order curvature terms of these parameters are introduced for cancelling the divergence in the energy-momentum tensor when the perturbative expansions of the loop diagrams are taken into account [20,21]. Since we work in a flat FLRW universe, it is worth noting that the Weyl tensor vanishes in the flat FLRW metric and the Guass–Bonnet term is topological invariant in four-dimensional spacetime which is also vanished. Only the relevant higher order curvature term of the action in this work is R^2. Then, the gravitational action of HS model in Jordan frame reads,

$$S_J = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R - \frac{1}{2} \xi \sigma^2 R - \frac{1}{4} \alpha R^2 \right.$$
$$\left. + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \, \partial_\nu \sigma - \frac{\lambda}{4} \sigma^4 \right]. \tag{6}$$

Moreover, the R^2 will contribute to improve the effective potential at one-loop level and we will discuss about its quantum effect and RGEs latter. For more detail about quantum effects of the R^2 term in the curved spacetime, we recommend the readers to Refs. [20–23]. In this work, the HS mechanisms is introduced in Refs. [5,6] where the non-minimal Higgs-curvature coupling is a trigger of inflation with large value of the non-minimal coupling constant. This mechanism generates the Starobinsky inflation. With the HS model, one does not need to set the Higgs field values in the early universe to be large. Then, according to the observed values of the standard model Higgs vacuum, we achieve the corresponding stability of the electroweak vacuum with the experiments [5]. This model dose not suffer from the unitarity problem of the Higgs inflation also. In this work, we will study the inflation by using this picture. The HS model has been studied in Refs. [24–36] and most of the works in the literatures focused in the multi-field inflation framework. With the small value of the Higgs field and in the slow-roll regime during inflation, i.e., suppressing kinetic term of the Higgs field, one can integrate out the non-minimal Higgs coupling term, $\xi \sigma^2 R$ via the equation of motion which gives $\sigma_c^2 = -6 \xi R / \lambda$. This has been shown in Ref. [32] that the action in Eq. (6) becomes,

$$S_J = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R - \frac{M_P^2}{12 M^2} R^2 \right], \tag{7}$$

$$M^2 = \frac{M_P^2}{12 \left(\alpha + 3 \xi^2 / (2 \lambda) \right)}. \tag{8}$$

The above action is usual Starobinsky inflation action. In the Einstein frame, one defines the scalaron mass of the Starobinsky inflaton field as

$$M_\alpha^2 = \frac{M_P^2}{12 \alpha}, \tag{9}$$

and the scalaron (inflation) mass is modified by the following relation [32]

$$M^2 = \frac{M_\alpha^2}{1 + 18 \, (\xi^2 / \lambda) \, M_\alpha^2 / M_P^2}. \tag{10}$$

According to the observational constraints of the amplitudes of the curvature perturbation, one finds $M \approx 1.3 \times 10^{-5} M_P$ [37]. By using the fixing M parameter, we obtain the relation between three parameters ξ, α and λ and we will employ action in Eq. (7) to work out relevant inflation parameters and fix the parameters from the HS model with the observational data in the next section.

2.2 Quantum corrections and renomalization group equations

In this subsection, we will briefly review and discuss the non-minimal Higgs coupling induced the Starobinsky R^2 term. After that we will close this subsection with the renormalization group equations at one-loop of the standard model parameters in the presence of curved spacetime [21,22]. The non-minimal Higgs coupling induced R^2 has been shown in Refs. [5,6]. In this work, we follow the HS mechanism and briefly review that how the R^2 term is induced as shown in [5,23].

2.2.1 Pure gravitational terms and Higgs field

We start with the action in Eq. (2). By using the dimensional regularization scheme via heat kernel technique, the one-loop effective potential (with absorbtion of the wave function renormalization constant) of the Eq. (2) is given by [22,23],

$$V_{\text{eff}}(\sigma) = -\frac{\xi}{2} R \sigma^2 + \frac{\lambda}{4} \sigma^4 - M_P^2 R$$
$$- c_1 R^2 - c_2 R_{\mu\nu} R^{\mu\nu} - c_3 R_{\mu\nu\rho\tau} R^{\mu\nu\rho\tau}$$
$$+ \frac{1}{64\pi^2} \left(3 \lambda \sigma^2 - \left(\xi + \frac{1}{6} \right) R \right)^2$$
$$\times \left[\log \left(\frac{\left| 3 \lambda \sigma^2 - \left(\xi + \frac{1}{6} \right) R \right|}{\mu^2} \right) - \frac{3}{2} \right]$$
$$+ \frac{1}{64\pi^2} \frac{\left(R_{\mu\nu\rho\tau} R^{\mu\nu\rho\tau} - R_{\mu\nu} R^{\mu\nu} \right)}{90}$$
$$\times \left[\log \left(\frac{\left| 3 \lambda \sigma^2 - \left(\xi + \frac{1}{6} \right) R \right|}{\mu^2} \right) \right], \tag{11}$$

where μ is renormalization (subtraction) scale. In this subsection, we note that the couplings appear in above potential are represented for running couplings due to the quantum correction effects as,

$$c_{1,2,3} = c_{1,2,3}(\sigma), \quad \xi = \xi(\sigma), \quad \lambda = \lambda(\sigma). \tag{12}$$

For the classical couplings, they are written by the following forms,

$$c_{1,2,3}^{(0)} = c_{1,2,3}(\sigma = 0), \quad \xi^{(0)} = \xi(\sigma = 0),$$
$$\lambda^{(0)} = \lambda(\sigma = 0). \tag{13}$$

We refer Refs. [22,23] for detail calculation of the effective potential in Eq. (11) and general concepts and techniques in quantum field in curved spacetime see textbooks [20,21]. Applying one-loop effective potential in Eq. (11) to the Callan–Symanzik equation, one obtains relevant beta functions of the couplings $c_{1,2,3}$ as [22,23],

$$\beta_{c_1} = -\frac{1}{16\pi^2} \frac{1}{2} \left(\xi + \frac{1}{6} \right)^2, \quad \beta_{c_2} = -\frac{1}{16\pi^2} \left(\frac{1}{180} \right),$$

$$\beta_{c_3} = \frac{1}{16\pi^2}\left(\frac{1}{180}\right), \tag{14}$$

where the beta function β_F is defined by,

$$\beta_F = \frac{dF}{dt}, \quad t = \ln\mu. \tag{15}$$

The solutions of the beta functions in Eq. (14) are given by [23],

$$c_1 = c_1^{(0)} - \frac{1}{16\pi^2}\left(\frac{1}{2}\right)\left(\xi + \frac{1}{6}\right)^2 \ln\left(\frac{\sigma}{\mu}\right),$$

$$c_2 = c_2^{(0)} - \frac{1}{16\pi^2}\left(\frac{1}{180}\right)\ln\left(\frac{\sigma}{\mu}\right),$$

$$c_3 = c_3^{(0)} + \frac{1}{16\pi^2}\left(\frac{1}{180}\right)\ln\left(\frac{\sigma}{\mu}\right). \tag{16}$$

Then, we can re-write the gravitational action in Eq. (6) in terms of the running couplings (with quantum corrections at one-loop level) as,

$$S_J = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M_P^2 R - \frac{1}{2}\xi(\sigma)\sigma^2 R - \frac{1}{4}\alpha(\sigma)R^2\right.$$
$$\left. + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\,\partial_\nu\sigma - \frac{\lambda(\sigma)}{4}\sigma^4\right]. \tag{17}$$

Note that the coupling $\alpha(\sigma)$ is linear combinations of the $c_{1,2,3}(\sigma)$ and it reads,

$$\alpha(\sigma) = c_1(\sigma) + \frac{1}{3}\left(c_2(\sigma) + c_3(\sigma)\right) = \alpha^{(0)}$$
$$+ \frac{1}{16\pi^2}\left(\xi + \frac{1}{6}\right)^2 \ln\left(\sqrt{\frac{\mu}{\sigma}}\right). \tag{18}$$

Next we will consider the $\alpha(\sigma)$ parameter in case of $\alpha^{(0)} = 0$ (at tree level of quantum loop expansion). In the case of setting the renormalization scale at Planck mass $\mu \approx M_P$ and at sub-Planckian field $\sigma \ll M_P$, one finds,

$$\alpha(\sigma) = \frac{1}{16\pi^2}\left(\xi + \frac{1}{6}\right)^2 \ln\left(\sqrt{\frac{\sigma}{\mu}}\right). \tag{19}$$

This result shows that the α coupling of the R^2 term is generated by the non-minimal Higgs coupling, ξ at the one-loop level [5,23]. At fixed and small Higgs field regime $\sigma_0^2 \ll M_P^2/\xi$, on one hand, the $\xi\sigma_0^2 R$ in the action (17) is suppressed. On the other hand, the αR^2 dominates the action. It is worth noting here that, for $|\xi| \gg 1$, the quantum correction to α given in Eq. (19) and in the last term in Eq. (18) is still smaller than the effective classical correction to $\alpha \sim \xi^2/\lambda$ that follows from Eq. (8). We finally achieve the usual Starobinsky inflation. As was shown in Eq. (19), in addition, the α coupling is written in terms of the non-minimal coupling parameter. One can tune or fit the ξ rather than α and we will constrain the value of the ξ with inflation from observational data in the latter.

2.2.2 One-loop renormalization group equations for standard model

The Starobinki inflation generated from non-minimal Higgs coupling term has been discussed and demonstrated in the previous subsection. More completely, we will extend our study to the renormalization group equations for the standard model of particle physics in the presence of the curved spacetime. Results in this section discussing below will be very useful for the study of dark matter in the HS model.

Our goal in this subsection is to obtain the running coupling constants of the standard model parameters with the presence of the gravitational couplings, α and ξ. We follow the main results from Refs. [22,38]. By using heat kernel technique and dimensional regularization scheme, the Callan–Symanzik equation of the effective potential at one-loop for the standard model with the Starobinsky R^2 term leads to the renormalization group equations given below [22]. For gravitational part, the beta-functions are given by [22]

$$\beta_\xi = \frac{1}{16\pi^2}\left(\xi + \frac{1}{6}\right)\left(12\lambda + 2Y_2 - \frac{3}{2}(g')^2 - \frac{9}{2}g^2\right), \tag{20}$$

$$\beta_\alpha = \frac{1}{16\pi^2}\left(\frac{1}{3}\right)\left(\xi + \frac{1}{6}\right)^2. \tag{21}$$

We note that the RGE of the α coupling is modified by the presence of standard model with coefficient $\frac{1}{3}$ instead of $\frac{1}{2}$ in the pure gravitational part. The beta-functions of the matters and fields are given by [22]

$$\beta_{y_t} = \frac{1}{16\pi^2}y_t\left(\frac{3}{2}\left(y_t^2 - y_b^2\right)\right.$$
$$\left. + Y_2 - \frac{1}{12}\left(17(g')^2 + 27g^2 + 96g_3^3\right)\right) \tag{22}$$

$$\beta_{y_b} = \frac{1}{16\pi^2}y_b\left(\frac{3}{2}\left(y_b^2 - y_t^2\right)\right.$$
$$\left. + Y_2 - \frac{1}{12}\left(5(g')^2 + 27g^2 + 96g_3^3\right)\right) \tag{23}$$

$$\beta_{y_l} = \frac{1}{16\pi^2}y_l\left(\frac{3}{2}y_l^2 + Y_2 - \frac{1}{12}\left(45(g')^2 + 27g^2\right)\right) \tag{24}$$

$$\beta_\lambda = \frac{1}{16\pi^2}\lambda\left(24\lambda^2 - 3\lambda\left((g')^2 + 3g^2\right)\right.$$
$$\left. + \frac{3}{8}\left((g')^4 + 2(g')^2g^2 + 3g^4\right) + 4\lambda Y_2 - 2Y_4\right) \tag{25}$$

$$\beta_{m^2} = \frac{1}{16\pi^2}m^2\left(12\lambda - \frac{3}{2}(g')^2 - \frac{9}{2}g^2 + 2Y_2\right) \tag{26}$$

$$\beta_{g'} = \frac{1}{16\pi^2}\left(\frac{41}{6}\right)(g')^3, \quad \beta_g = \frac{1}{16\pi^2}\left(-\frac{19}{6}\right)g^3,$$

$$\beta_{g_3} = \frac{1}{16\pi^2}(-7)g_3^4, \tag{27}$$

where

$$Y_2 = 3\left(y_u^2 + y_c^2 + y_t^2\right)$$
$$+ 3\left(y_d^2 + y_s^2 + y_b^2\right) + \left(y_e^2 + y_\mu^2 + y_\tau^2\right), \quad (28)$$

$$Y_4 = 3\left(y_u^4 + y_c^4 + y_t^4\right)$$
$$+ 3\left(y_d^4 + y_s^4 + y_b^4\right) + \left(y_e^4 + y_\mu^4 + y_\tau^4\right). \quad (29)$$

We use $y_{u,d,s,c,t,b}$ for Yukawa coupling of the u, d, s, c, t, b quarks and $y_{l=e,\mu,\tau}$ is Yukawa coupling of electron, muon and tau. The m and λ are mass and self-interaction coupling of the Higgs. The parameters, g', g and g_3 stand for U(1), SU(2) and SU(3) guage couplings. More detail and discussion of the one-loop effective potential and other related quantities, we refer to Refs. [22,38]. We will solve relevant beta-functions given in this subsection for using to study the relic abundances of psuedoscalar Higgs sector as candidate of the dark matter with standard model in the latter section. Notice that first works where the effective potential for arbitrary gauge theory in curved spacetime was found at one-loop level in Refs. [39,40].

3 Inflationary implication from the HS model

In this section, we will study of the inflation in the Higgs–Starobinsky model. In the inflation phase of the universe, we have shown in the previous section that the Starobinsky R^2 term dominates the action in Eq. (6). Then, we will use the Starobinsky inflation model in this section. Moreover, the Einstein frame is used as the physical frame in this work. By using an usual conformal transformation, the action in Eq. (7) is written in the Einstein frame in the following form,

$$S_E = \int d^4x \sqrt{-\widetilde{g}} \left[-\frac{1}{2} M_P^2 \widetilde{R} + \frac{1}{2} \widetilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right], \quad (30)$$

where all quantities with "~" are represented quantities in the Einstein frame. The conformal factor, Ω^2 plays important role on transformation of the gravitational action from Jordan frame to Einstein frame. The relation between metric tensors of Jordan and Einstein frames reads,

$$g_{\mu\nu} = \Omega^2 \widetilde{g}_{\mu\nu}. \quad (31)$$

The corresponding conformal factor of Eq. (6) is given by

$$\Omega^2 = \frac{2}{M_P^2} \frac{\partial}{\partial R} \left(\frac{1}{2} M_P^2 R + \frac{M_P^2}{12 M^2} R^2 \right) = 1 + \frac{R}{3 M^2}, \quad (32)$$

where the definition of the effective mass M is given in Eq. (10). The Ricci scalar in Jordan frame is written in terms of quantities in Einstein frame as

$$R = \Omega^2 \left(\widetilde{R} + 3 \widetilde{g}^{\mu\nu} \partial_\mu \partial_\nu \ln \Omega^2 - \frac{3}{2} \widetilde{g}^{\mu\nu} \partial_\mu \ln \Omega^2 \partial_\nu \ln \Omega^2 \right). \quad (33)$$

More importantly, the scalaron field, φ of the Starobinsky inflation is introduced via

$$\varphi = M_P \sqrt{\frac{3}{2}} \ln \Omega^2. \quad (34)$$

Using the definition of the scalaron field, one can write the effective potential of the scalaron in Einstein frame as

$$V(\varphi) = \frac{3}{4} M_P^2 M^2 \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\varphi}{M_P}} \right)^2. \quad (35)$$

This is usual Starobinsky scalaron potential in Einstein frame and we will employ this potential in the analysis of inflation below. According to Higgs–Starobinsky mechanism as shown in Eq. (18), the condition $\varphi < M_P$ is kept throughout this analysis.

3.1 Slow-roll approximation

We are ready to study inflation in this subsection. A flat homogeneous and isotropic FLRW metric is used as the background of the universe and it is written by

$$ds^2 = dt^2 - a(t)^2 \left(d^2x + d^2y + d^2z \right), \quad (36)$$

where t in this subsection is cosmic time and $a(t)$ is the scale factor of the universe. The corresponding Friedman equation, its cosmic time derivative and the Klein–Gordon equation of the scalaron from above metric and the action in Eq. (30) are

$$H^2 = \frac{1}{3 M_P^2} \left(\frac{1}{2} \dot{\varphi}^2 + V(\phi) \right), \quad (37)$$

$$\dot{H} = -\frac{1}{3 M_P^2} \left(\frac{1}{2} \dot{\varphi}^2 \right), \quad (38)$$

$$\ddot{\varphi} = -3 H \dot{\varphi} - \partial_\varphi V(\varphi), \quad (39)$$

where $H \equiv \dot{a}/a$ and $\dot{a} \equiv da/dt$. In the slow-roll regime, the kinetic term of the scalaron (inflaton) is varying very slow with respect to the cosmic time and it is suppressed. The Friedman equation can be re-written as

$$H^2 \approx \frac{1}{3 M_P^2} V(\phi) = \frac{1}{4} M^2 \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\varphi}{M_P}} \right)^2. \quad (40)$$

Next we recall the definitions of the slow roll parameters and they read

$$\epsilon = \frac{M_P^2}{2} \left(\frac{1}{V(\varphi)} \frac{dV(\varphi)}{d\varphi} \right)^2,$$

$$\eta = M_P^2 \left(\frac{1}{V(\varphi)} \frac{d^2V(\varphi)}{d\varphi^2} \right),$$

$$\zeta = M_P^2 \left(\frac{1}{V(\varphi)} \frac{dV(\varphi)}{d\varphi} \frac{d^3 V(\varphi)}{d\varphi^3} \right),$$

$$N = \frac{1}{M_P^2} \int_{\varphi_{\text{end}}}^{\varphi_N} \frac{V(\varphi)}{dV(\varphi)/d\varphi} d\varphi. \tag{41}$$

Applying the scalaron potential in Eq. (35) to the slow roll parameters (41), we obtain

$$\epsilon = \frac{4}{3 \left[e^{\sqrt{\frac{2}{3}} \frac{\varphi}{M_P}} - 1 \right]^2}, \quad \eta = -\frac{4 \left[e^{\sqrt{\frac{2}{3}} \frac{\varphi}{M_P}} - 2 \right]}{3 \left[e^{\sqrt{\frac{2}{3}} \frac{\varphi}{M_P}} - 1 \right]^2},$$

$$\zeta = \frac{16 \left[e^{\sqrt{\frac{2}{3}} \frac{\varphi}{M_P}} - 4 \right]}{9 \left[e^{\sqrt{\frac{2}{3}} \frac{\varphi}{M_P}} - 1 \right]^3}. \tag{42}$$

At inflation end i.e., $\epsilon_{\text{end}} = 1$, we find $\varphi_{\text{end}} = 0.764 \, M_P$. The number of e-folding number from φ_{end} to φ_N is given by

$$N = \frac{3}{4} \left[e^{\sqrt{\frac{2}{3}} \frac{\varphi_N}{M_P}} - e^{\sqrt{\frac{2}{3}} \frac{\varphi_{\text{end}}}{M_P}} - \frac{\sqrt{6}}{4 \, M_P} (\varphi_N - \varphi_{\text{end}}) \right]$$

$$\approx \frac{3}{4} e^{\sqrt{\frac{2}{3}} \frac{\varphi_N}{M_P}}, \tag{43}$$

where we have used the field values $\varphi_N \gg \varphi_{\text{end}}$. We approximately find $\varphi_N \sim \sqrt{3/2} M_P \ln(4N/3)$. With $N = 60$, we obtain $\varphi_N \approx 5.37 \, M_P$.

3.2 Contact with observational constraints

Next we constrain our scalaron potential with the COBE normalization condition for fixing parameters in the HS model. We use reads,

$$\frac{V(\varphi)}{\epsilon} \simeq (0.0276 \, M_P)^4. \tag{44}$$

Using the potential in (35), we get,

$$\xi \simeq 2.3 \times 10^{-3} \sqrt{\lambda} \sqrt{1.8475 \times 10^{10} N^2 - 128648.\alpha}/, . \tag{45}$$

In order for ξ to satisfy the usual Higgs inflation, i.e. $\xi \sim 10000$, we discover

$$\lambda \sim -\frac{1.5 \times 10^8}{1.\alpha - 5.16993 \times 10^8}, \tag{46}$$

where we have used $N = 60$. Using typical values of λ, a parameter α can be fixed by the CMB constraint:

$$\alpha \sim -2.5 \times 10^9 \quad \text{for} \quad \lambda = 0.05, \tag{47}$$

$$\alpha \sim -1.45 \times 10^{10} \quad \text{for} \quad \lambda = 0.01. \tag{48}$$

Notice that a successful prediction of the density perturbation requires α to be large similar to those found in, e.g., Refs. [5,43]. The spectral index of curvature perturbation n_s and

the tensor-to-scalar ratio r are given to the leading order of the slow-roll approximation as:

$$n_s := 1 - 6\epsilon + 2\eta \simeq 1 - \frac{2}{N}, \tag{49}$$

$$r := 16\epsilon \simeq \frac{12}{N^2}. \tag{50}$$

Notice that the predictions of our present model are in agreement with those of Ref. [41] where $n_s \simeq 0.966$ and $r = 0.0033$ for $N = 60$. Remarkably, our inflationary predictions given in Eqs. (49,50) nicely coincide with those of the universal alpha-attractors scenarios [44–53] in case $\alpha \to 1$. It is worth adding here that the quantitatively correct predictions for the scalar power spectrum P_ζ and the tensor-to-scalar ratio r in the pure $R + R^2$ model were first obtained in Ref. [54].

4 Dark matter from the HS model

In this section, we solve the renormalization group equations for demonstrating residual effect of the inflation to dark matter in the HS model. In addition, we will demonstrate how the HS is equivalent to the SSDM. The results of the SSDM from thermal relic abundance constraints are discussed in terms of the couplings of the HS framework.

4.1 The HS model as the SSDM

We start with the the gravitation action of the HS model in Jordan frame and follow a framework proposed in Ref. [30]. Let us introduce an auxiliary field χ so that the action in Eq. (6) is rewritten as.

$$S_J = \int d^4 x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R - \frac{1}{2} \xi \sigma^2 R - \frac{1}{2} \alpha \chi^2 R \right.$$

$$\left. -\frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \, \partial_\nu \sigma - \frac{1}{2} m_\sigma \sigma^2 - \frac{\lambda}{4} \sigma^4 - \frac{\alpha}{4} \chi^4 + \mathcal{L}_{SM} \right]. \tag{51}$$

Note that the variation with respect to χ gives $\chi^2 = R$. We restore the original action (17) after substituting it to Eq. (51). The field χ corresponds to a spin-zero mode of the graviton that is dynamical due to the presence of the Ricci scalar squared term so called a scalaron [32]. It is worth mentioning here that the word "scalaron" was first introduced by Starobinsky in Refs. [2,3]. Here if we generalize the model and include one-loop contributions of massive quantum fields then scalarons will decay into pairs of massive particles (see [2,3] for more details). Instead of Eq. (51), we here introduce the coupling between the fields χ and σ, and consider the following action:

$$S_J = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R - \frac{1}{2} \widetilde{\xi} \sigma^2 R - \frac{1}{2} \widetilde{\alpha} \chi^2 R \right.$$
$$+ \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \, \partial_\nu \sigma - \frac{1}{2} m_\sigma \sigma^2 - \frac{\widetilde{\lambda}}{4} \sigma^4$$
$$\left. - \frac{\widetilde{c}}{4} \chi^4 - \frac{\widetilde{\kappa}}{2} \chi^2 \sigma^2 - \mathcal{L}_{SM} \right]. \tag{52}$$

More importantly, the couplings ξ, α and λ in Eq. (6) are linear combinations of the new coupling constants, $\widetilde{\xi}$, $\widetilde{\alpha}$, $\widetilde{c}$, $\widetilde{\kappa}$ and $\widetilde{\lambda}$ as,

$$\xi = \widetilde{\xi} - \frac{\widetilde{\alpha}\,\widetilde{\kappa}}{\widetilde{c}}, \quad \alpha = \frac{\widetilde{\alpha}^2}{\widetilde{c}}, \quad \lambda = \widetilde{\lambda} - \frac{\widetilde{\kappa}^2}{\widetilde{c}}. \tag{53}$$

Using Eq. (53), the action (52) reproduces the original action of the HS model in Eq. (6) after integrating out the scalaron χ [30]. We note that there is no kinetic term for the field χ in Eq. (52). According to the principle of effective field theory, the reason is that the χ is not a dynamical field and is freezed above the Electroweak scale, Λ_{EW}. The kinetic term $\frac{1}{2} Z_\chi \partial_\mu \chi \partial^\mu \chi$ emerges due to renormalization effect and Z_χ is the wave-function renormalization constant. At high energy scales $\mu \to \Lambda_{EW}$, we require that $Z_\chi = 0$. After symmetry breaking or below the Electroweak scale, the χ plays the role as the dynamical field in this energy regime and the χ field starts to propagate and the kinetic term of the χ is generated, $Z_\chi \to 1$. Taking the Electroweak symmetry breaking of the Higgs filed, i.e., $\sigma \to \sigma + v$ and emergence of the dynamical field, χ into account, one rewrites the action in Eq. (52) as,

$$S_J \in \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R - \frac{1}{2} \widetilde{\xi} \sigma^2 R - \frac{1}{2} \widetilde{\alpha} \chi^2 R \right.$$
$$+ \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \, \partial_\nu \sigma - \frac{1}{2} m_\sigma \sigma^2 + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \, \partial_\nu \chi$$
$$\left. - \frac{1}{2} m_\chi \chi^2 - \frac{\widetilde{\lambda}}{4} \sigma^4 - \frac{\widetilde{c}}{4} \chi^4 - \frac{\widetilde{\kappa}}{2} \chi^2 \sigma^2 - \frac{\widetilde{\kappa}}{2} \chi^2 v \sigma + \mathcal{L}_{SM} \right]. \tag{54}$$

It is worth to mention that the χ field couples to Higgs via three-point and four-point interaction terms with the same coupling constant, $\widetilde{\kappa}$. This coupling will is the connection between HS and SM of particle physics. It has been also demonstrated in Ref. [30] that there is dual description to choose the choices of the invariant under shift and re-scaling transformation of the auxiliary field χ^2. In addition, the χ field is a scalar mode of the graviton which is called scalaron [55,56]. We come to the crucial result of this work and it is important to note that the action in Eq. (54) has the same structure of that given by the singlet scalar dark matter (SSDM) model [15–17]. The SSDM model has been demonstrated to be the unification of inflation and dark matter [57,58] or Higgs-portal paradigms [60] as well as the composite NJL model of inflation and dark matter [41]. Moreover, the coupling between scalaron χ and Higgs σ naturally

emerges with the choice of re-define auxiliaiary field and new parameters in (53), respectively. In order to see the behavior of the couplings in different energy scales, the renormalization group analysis is required. This subsection is to study the HS model as candidate of the dark matter. The new scalaron χ plays the role as SSDM via thermal relic abandance as we will discuss in the next subsection. We impose the energy scale in this study is the Elecetroweak scale at the given values of the top quark mass $m_t = 173$ GeV and vacuum expectation value $v = 246$ GeV. The beta functions of the standard model coupling parameters are given in Eqs. (22), (23), (24), (27) except for Eqs. (25), (26) need to modified due to our new tilde parameters $\widetilde{\lambda}$ and $\widetilde{\alpha}$. The beta functions of the parameters in Eq. (53) are given by [57,58]

$$\beta_{\widetilde{\alpha}} = \frac{1}{16\pi^2} \left[(3 + c_\sigma) \widetilde{\kappa} \left(\widetilde{\xi} + \frac{1}{6} \right) + 6 c_\chi \left(\widetilde{\alpha} + \frac{1}{6} \right) \right], \tag{55}$$

$$\beta_{\widetilde{\xi}} = \frac{1}{16\pi^2} \left[\left((6 + c_\sigma) \widetilde{\lambda} + 6 y_t^2 \right. \right.$$
$$\left. - \frac{3}{2} \left(3 g^2 + (g')^2 \right) \right) \left(\widetilde{\xi} + \frac{1}{6} \right) + c_\chi \left(\widetilde{\alpha} + \frac{1}{6} \right) \right], \tag{56}$$

$$\beta_{\widetilde{c}} = \frac{1}{16\pi^2} \left[\frac{1}{2} (c_\sigma^2 + 3) \widetilde{\kappa} + 18 c_\chi \widetilde{c}^2 \right], \tag{57}$$

$$\beta_{\widetilde{\kappa}} = \frac{1}{16\pi^2} \left[4 c_\sigma c_\chi + 6 (c_\sigma^2 + 1) \widetilde{c} \widetilde{\kappa} \frac{3}{2} (3 g^2 + (g')^2) \widetilde{\kappa} \right.$$
$$\left. + 6 y_t^2 \widetilde{\kappa} + 6 c_\sigma \widetilde{c} \widetilde{\kappa} \right], \tag{58}$$

$$\beta_{\widetilde{\lambda}} = \frac{1}{16\pi^2} \left[18 (c_\sigma^2 + 6) \widetilde{\lambda}^2 - 6 y_t^2 + \frac{3}{8} \left(2 g^2 + (g + (g'))^2 \right) \right.$$
$$\left. + \left(12 y_t^2 - 9 g^2 - 3 (g')^2 \right) \widetilde{\lambda} + \frac{1}{2} c_\chi^2 \widetilde{\kappa}^2 \right], \tag{59}$$

where c_σ and c_χ are the suppression factor for the Higgs (σ) and the scalaron (χ) and we will set them to 1 in the discussion below (see Refs. [41,57,58] for definitions and detail descriptions). Here we consider the real part of the χ field only. By solving the beta functions for $\beta_{\widetilde{\xi}}$ and $\beta_{\widetilde{\alpha}}$ in Eqs. (56) and (55) respectively, it has been demonstrated in Ref. [41] (in the framework of the composite NJL model but it has the same field configurations) that the renormalization group running analysis of the $\widetilde{\xi}$ and $\widetilde{\alpha}$ confirms the dominance of the χ field inflation over the Higgs field with the relation,

$$\widetilde{\xi} \approx 0.019 \, \widetilde{\alpha}. \tag{60}$$

Before closing this subsection, we turn to discuss about the Electroweak vacuum stability and its perturbative properties of the HS model in the form of action (53). According to Ref. [58], we find the constraints,

$$\widetilde{c} > 0, \quad \widetilde{\lambda} > 0, \quad \widetilde{\kappa} > 0, \quad \text{or} \quad \widetilde{\kappa}^2 < \widetilde{c} \widetilde{\lambda}. \tag{61}$$

Together with the perturbative conditions up to the Planck scale, one obtains [58],

$$\widetilde{c} < \frac{2\pi}{3}, \quad \widetilde{\lambda} < \frac{2\pi}{3}, \quad \widetilde{\kappa} < 4\pi, \tag{62}$$

which are helped to constrain the couplings in SSDM of the HS model.

In general, one may expect that scalaron and Higgs boson are unstable and decay quickly. This implies that the scalaron may not work as dark matter candidate for our proposal in this model. However, in any case, there are other possibilities to overcome this problem. For instance, Ref. [7], here the author has introduced new mode of the scalaron from higher order gravity (i.e., R^2 term) and the new scalar mode oscillates around the minimum of its potential. Then, the oscillations might correspond to non-zero-momentum condensate and be able to associate with standard non-relativistic matter. This process is a so-called "vacuum misalignment" mechanism which mimics a famous mechanism of the axion production as a cold DM candidate [62–64]. For the details about the mechanism of the scalaron stability and its main decay channels and how the scalaron can work as DM we refer to Ref. [7]. In addition, similar approaches for the scalaron of the higher order gravity as DM candidate have been investigated in the literature [65–67].

4.2 Relic abundance

By using the action in Eq. (54), it has been shown that the observed mass of dark matter is reproduced by the thermal relic abundance of the χ field [41,57,58]. We closely follow an analysis and employ the relevant values and results of the parameters for relic abundance in Ref. [41]. The current observed mass of the dark matter is given by [59],

$$\Omega_{DM} h^2 = 0.1199 \pm 0.0027, \tag{63}$$

where $h \approx 0.7$. The crucial parameter for estimating the mass of dark matter is $\widetilde{\kappa}$ as shown in [41,57,58]. According to the analysis of [41], the mass formula of dark matter mass is given by

$$M_{DM}^2 = m_\chi^2 + \widetilde{\kappa} \, v^2/2, \tag{64}$$

In case of a light DM mass, we can constrain a lower bound of a mass M_{DM}. Here we use the best-fit value of a physical mass M_{DM} located around $\tilde{\kappa} = 6.5 \times 10^{-4}$, $M_{DM} = 62.51$ GeV [18] and find $m_\chi \approx 62.35$ GeV. In case of a heavy DM mass, we consider for instance $\tilde{\kappa} = 0.5$, $M_{DM} = 410$ GeV [41] and find $m_\chi \approx 391$ GeV. Notice that in the region above $m_\sigma/2$, the authors of Ref. [42] show that the relic density constrains the coupling as a function of the DM mass which can be approximately describe by the dependence $\log_{10} \tilde{\kappa} > -3.36 + 1.04 \log_{10}(M_{DM}/\text{GeV})$. We stress here that thorough analysis of the present work can be done by

following Ref. [41]. To see the values of the couplings of the original action of the HS model, we employ the values of the relevant input parameters in the action (52) which have been estimated by [58] i.e., $\widetilde{c} = 0.025$ or $\widetilde{c} = 0.2$. Moreover, the solutions of the beta-functions in terms of $\widetilde{\xi}$ and $\widetilde{\alpha}$ have been done by Ref. [41] and given by

$$\widetilde{\xi} = \frac{\widetilde{\kappa}_M \, \widetilde{\alpha}_0}{18 \, \widetilde{c}_0} \left[1 - \left(1 - \frac{9 \, \widetilde{c}_0}{8 \, \pi^2} t \right)^{1/3} \right], \tag{65}$$

$$\widetilde{\alpha} = \widetilde{\alpha}_0 \left(1 - \frac{9 \, \widetilde{c}_0}{8 \, \pi^2} t \right)^{-1/3} \tag{66}$$

where we have used $t \equiv \ln \mu/m_t$ and $\widetilde{\kappa}_M$, $\widetilde{\alpha}_0$ and $\widetilde{c}_0$ are an average of $\widetilde{\kappa}$ and initial values of $\widetilde{\alpha}$ and $\widetilde{c}$, respectively. We will use above relations to rewrite a parameter $\widetilde{\xi}$ in terms of $\widetilde{\alpha}$ and we will estimate the values of the ξ, α and λ parameters by using the definitions in Eq. (53).

At given renormalization scale of the top quark mass, $\mu = m_t \rightarrow t = 0$ in Eqs. (65) and (66), one finds $\widetilde{\xi} = 0$ and $\widetilde{\alpha} = \widetilde{\alpha}_0$ where the value of $\widetilde{\alpha}_0$ is of the order of $\approx 10^5$, for example Ref. [58] obtained $\widetilde{\alpha}_0 = 2.5 \times 10^5$ with $\widetilde{c}_0 = \widetilde{c}(m_t) = 0.2$ and $\widetilde{\kappa}(m_t) = \widetilde{\kappa}_0 = 0.1$. Although the value of $\widetilde{\alpha}$ is rather large, this term is suppressed by the Planck mass. Then the non-minimal coupling $\widetilde{\alpha} \chi^2 R$ does not affect to dark matter-Higgs coupling, $\widetilde{\kappa} \chi^2 \sigma^2$. The relevant parameter for study the relic abundance in the original action of the HS is Higgs self-interacting coupling, λ. To achieve the require dark matter relic abundance, we present the possibilities of the λ coupling with the given values of the $\widetilde{c}$, $\widetilde{\kappa}$ and $\widetilde{\lambda}$ as

$$\lambda = \widetilde{\lambda} - \frac{\widetilde{\kappa}^2}{\widetilde{c}} = \begin{cases} \widetilde{\lambda} - 2 \times 10^{-6}; & \text{for } \widetilde{\kappa} = 6.25 \times 10^{-4}, \quad \text{Ref. [18]}, \\ \widetilde{\lambda} - 490; & \text{for } \widetilde{\kappa} = 9.9, \quad \text{Ref. [18]}, \\ \widetilde{\lambda} - 0.15; & \text{for } \widetilde{\kappa} = 0.1, \quad \text{Ref. [42]}, \\ \widetilde{\lambda} - 1.05; & \text{for } \widetilde{\kappa} = 0.5, \quad \text{Ref. [42]}, \end{cases} \tag{67}$$

where we fixed the couplings $\widetilde{c} = 0.2$ as done in Ref. [58]. This value is constrained by solving the renormalized group equations. We have employed the results of the dark matter mass from SSDM with GAMBIT collaboration [18] and the composite NJL model [41]. According to above results, we can have both light DM mass and the heavy one in the HS model. For the lighter one, we find $M_{DM} = m_\sigma/2$ GeV with $\widetilde{\kappa} = 6.25 \times 10^{-4}$. This ensures to reproduce the reasonable and closer value of the Higg self-interacting coupling, $\lambda = m_\sigma^2/2 v^2 \approx 0.1$ for $m_\sigma = 125$ GeV and $v = 246$ GeV. In the heavy DM mass, we can obtain the same mass value $M_{DM} = 410$ GeV with $\widetilde{\kappa} = 0.5$ as that of Ref. [41]. In this later case, we find $\lambda = \widetilde{\lambda} - 1.05$.

5 Conclusion

In this work, we presented a unified description of inflation and dark matter in the context of the HS model. The

salient feature of this work is to demonstrate that the HS scenario can simultaneously describe inflation and dark matter without introducing new physics beyond standard model. We considered the original action describing the HS model. We started with non-minimal coupling HS action in Jordan frame and transformed it to the Einstein frame using the conformal transformation. We also derived the inflation potential from the gravitational action of non-minimal-Higgs coupling and Starobinsky term in Einstein frame where the R^2 term is dominated in the inflationary phase of the universe. For model of inflation, we computed the inflationary parameters and confronted them with Planck 2015 data. We discovered that the predictions of the model are in excellent agreement with the Planck analysis.

In addition, we considered the HS model as a candidate for dark matter. We analyzed the renormalization group equations (RGEs) of HS scenario with the standard model at one-loop level. More importantly, we discovered that the HS model is equivalent to the SSDM. We made qualitative discussions to identify the coupling constants from dark matter relic abundance constraints. Employing results of the SSDM with GAMBIT collaboration and composite NJL model, the DM mass in the HS model flavors the light mass around $M_{DM} = m_\sigma/2$ in order to reproduce the reasonable values of the Higgs self-interaction coupling.

However, there are some limitations in the present work - for example, one should complete the RGEs for all scales and solve them numerically. Moreover, regarding this single framework, another crucial issue for successful models of inflation is the (pre)reheating mechanism. We plan to investigate this mechanism, within our framework, by following a composite inflationary scenario [61]. As mentioned in the previous section, the stable properties of the scalaron DM are worth investigating. As a result, the cosmological consequences of the present work can be further considered. These also include, e.g., preheating and reheating processes. In addition, cosmic history of chameleon mechanism of the DM is also interesting to be investigated, see for example [65]. We hope to address these issues with future investigations.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This manuscript has no associated data.]

References

1. F.L. Bezrukov, M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008)
2. A.A. Starobinsky, Phys. Lett. B **91**, 99 (1980) [Phys. Lett. **91B**, 99 (1980)]
3. A.A. Starobinsky, Adv. Ser. Astrophys. Cosmol. **3**, 130 (1987)
4. P.A.R. Ade et al., [Planck Collaboration]. Astron. Astrophys. **594**, A20 (2016)
5. X. Calmet, I. Kuntz, Eur. Phys. J. C **76**(5), 289 (2016)
6. A. Salvio, A. Mazumdar, Phys. Lett. B **750**, 194 (2015)
7. J.A.R. Cembranos, Phys. Rev. Lett. **102**, 141301. https://doi.org/10.1103/PhysRevD.24.3338 (2009)
8. F. Cooper, G. Venturi, Phys. Rev. D **24**, 3338 (1981). https://doi.org/10.1103/PhysRevD.24.3338
9. J. Kubo, M. Lindner, K. Schmitz, M. Yamada, arXiv:1811.05950 [hep-ph]
10. J.L. Feng, Ann. Rev. Astron. Astrophys. **48**, 495 (2010)
11. J.L. Feng, A. Rajaraman, F. Takayama, Phys. Rev. D **68**, 063504 (2003)
12. M. Viel, J. Lesgourgues, M.G. Haehnelt, S. Matarrese, A. Riotto, Phys. Rev. D **71**, 063534 (2005)
13. G. Koutsoumbas, K. Ntrekis, E. Papantonopoulos, E.N. Saridakis, JCAP **1802**, 003 (2018)
14. O. Luongo, M. Muccino, Phys. Rev. D **98**(10), 103520 (2018)
15. V. Silveira, A. Zee, Phys. Lett. **161B**, 136 (1985)
16. J. McDonald, Phys. Rev. D **50**, 3637 (1994)
17. C.P. Burgess, M. Pospelov, T. ter Veldhuis, Nucl. Phys. B **619**, 709 (2001)
18. P. Athron et al. [GAMBIT Collaboration], Eur. Phys. J. C **77**(8), 568 (2017)
19. P. Athron et al. [GAMBIT Collaboration]. arXiv:1808.10465 [hep-ph]
20. N.D. Birrell, P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1984)
21. L.E. Parker, D. Toms, *Quantum Field Theory in Curved Space-time: Quantized Field and Gravity* (Cambridge University Press, Cambridge, 2009)
22. T. Markkanen, S. Nurmi, A. Rajantie, S. Stopyra, JHEP **1806**, 040 (2018)
23. D.M. Ghilencea, Phys. Rev. D **98**(10), 103524 (2018)
24. F.L. Bezrukov, D.S. Gorbunov, Phys. Lett. B **713**, 365 (2012)
25. S. Kaneda, S.V. Ketov, Eur. Phys. J. C **76**(1), 26 (2016)
26. M.S. Ruf, C.F. Steinwachs, Phys. Rev. D **97**(4), 044050 (2018)
27. A. Salvio, Phys. Lett. B **780**, 111 (2018)
28. J.L.F. Barbon, J.A. Casas, J. Elias-Miro, J.R. Espinosa, JHEP **1509**, 027 (2015)
29. Y.C. Wang, T. Wang, Phys. Rev. D **96**(12), 123506 (2017)
30. Y. Ema, Phys. Lett. B **770**, 403 (2017)
31. S. Pi, Yl Zhang, Q.G. Huang, M. Sasaki, JCAP **1805**(05), 042 (2018)
32. M. He, A.A. Starobinsky, J. Yokoyama, JCAP **1805**(05), 064 (2018)
33. D. Gorbunov, A. Tokareva, Phys. Lett. B **788**, 37 (2019)
34. A. Gundhi, C.F. Steinwachs. arXiv:1810.10546 [hep-th]
35. A. Karam, T. Pappas, K. Tamvakis. arXiv:1810.12884 [gr-qc]
36. V.M. Enckell, K. Enqvist, S. Rasanen, L.P. Wahlman. arXiv:1812.08754 [astro-ph.CO]
37. T. Faulkner, M. Tegmark, E.F. Bunn, Y. Mao, Phys. Rev. D **76**, 063505 (2007)

38. T. Markkanen, A. Rajantie, S. Stopyra. arXiv:1809.06923 [astro-ph.CO]
39. E. Elizalde, S.D. Odintsov, Phys. Lett. B **303**, 240 (1993)
40. E. Elizalde, S.D. Odintsov, Phys. Lett. B **321**, 199 (1994)
41. P. Channuie, C. Xiong, Phys. Rev. D **95**(4), 043521 (2017)
42. J.M. Cline, K. Kainulainen, P. Scott, C. Weniger, Phys. Rev. D **88**, 055025 (2013) Erratum: [Phys. Rev. D **92**, no. 3, 039906 (2015)]
43. TdP Netto, A.M. Pelinson, I.L. Shapiro, A.A. Starobinsky, Eur. Phys. J. C **76**(10), 544 (2016)
44. R. Kallosh, A. Linde, JCAP **1307**, 002 (2013)
45. R. Kallosh, A. Linde, JCAP **1310**, 033 (2013)
46. R. Kallosh, A. Linde, D. Roest, Phys. Rev. Lett. **112**, 011303 (2014)
47. R. Kallosh, A. Linde, D. Roest, JHEP **1408**, 052 (2014)
48. M. Galante, R. Kallosh, A. Linde, D. Roest, Phys. Rev. Lett. **114**(14), 141302 (2015)
49. S. Cecotti, R. Kallosh, JHEP **1405**, 114 (2014)
50. Z. Yi, Y. Gong, Phys. Rev. D **94**(10), 103527 (2016)
51. J.J.M. Carrasco, R. Kallosh, A. Linde, Phys. Rev. D **92**(6), 063519 (2015)
52. J.J.M. Carrasco, R. Kallosh, A. Linde, D. Roest, Phys. Rev. D **92**(4), 041301 (2015)
53. K. Karwan, P. Channuie. arXiv:1811.03006 [gr-qc]
54. A.A. Starobinsky, Sov. Astron. Lett. **9**, 302 (1983)
55. B. Whitt, Phys. Lett. **145B**, 176 (1984)
56. K i Maeda, Phys. Rev. D **39**, 3159 (1989)
57. T.E. Clark, B. Liu, S.T. Love, T. ter Veldhuis, Phys. Rev. D **80**, 075019 (2009)
58. R.N. Lerner, J. McDonald, Phys. Rev. D **80**, 123507 (2009)
59. P.A.R. Ade et al., [Planck Collaboration]. Astron. Astrophys. **571**, A16 (2014)
60. H. Han, S. Zheng, JHEP **1512**, 044 (2015)
61. P. Channuie, P. Koad, Phys. Rev. D **94**(4), 043528 (2016)
62. L.F. Abbott, P. Sikivie, Phys. Lett. B **120**, 133 (1983)
63. J. Preskill, M.B. Wise, F. Wilczek, Phys. Lett. B **120**, 127 (1983)
64. M. Dine, W. Fischler, Phys. Lett. B **120**, 137 (1983)
65. T. Katsuragawa, S. Matsuzaki, Phys. Rev. D **97**, no. 6, 064037 (2018) Erratum: [Phys. Rev. D **97**, no. 12, 129902 (2018)]
66. S. Nojiri, S.D. Odintsov, TSPU Bull. N **8**(110), 7 (2011)
67. S. Choudhury, M. Sen, S. Sadhukhan, Eur. Phys. J. C **76**(9), 494 (2016)