

# Understanding the internal structures of $X(4140)$ , $X(4274)$ , $X(4500)$ and $X(4700)$

Hua-Xing Chen<sup>1</sup>, Er-Liang Cui<sup>1</sup>, Wei Chen<sup>2,a</sup>, Xiang Liu<sup>3,4,b</sup>, Shi-Lin Zhu<sup>5,6,7,c</sup>

<sup>1</sup> School of Physics and Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China

<sup>2</sup> Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, Saskatchewan S7N 5E2, Canada

<sup>3</sup> School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China

<sup>4</sup> Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China

<sup>5</sup> School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

<sup>6</sup> Collaborative Innovation Center of Quantum Matter, Beijing 100871, China

<sup>7</sup> Center of High Energy Physics, Peking University, Beijing 100871, China

Received: 11 January 2017 / Accepted: 3 March 2017 / Published online: 15 March 2017

© The Author(s) 2017. This article is published with open access at Springerlink.com

**Abstract** We investigate the newly observed  $X(4500)$  and  $X(4700)$  based on the diquark–antidiquark configuration within the framework of QCD sum rules. Both of them may be interpreted as the  $D$ -wave  $cs\bar{c}\bar{s}$  tetraquark states of  $J^P = 0^+$ , but with opposite color structures, which is remarkably similar to the result obtained in Chen and Zhu (Phys Rev D 83:034010, 2011) that  $X(4140)$  and  $X(4274)$  can be both interpreted as the  $S$ -wave  $cs\bar{c}\bar{s}$  tetraquark states of  $J^P = 1^+$ , also with opposite color structures. However, the extracted masses and these suggested assignments to these  $X$  states do depend on these running quark masses where  $m_s(2\text{ GeV}) = 95 \pm 5\text{ MeV}$  and  $m_c(m_c) = 1.23 \pm 0.09\text{ GeV}$ . As a byproduct, the masses of the hidden-bottom partner states of  $X(4500)$  and  $X(4700)$  are extracted to be both around  $10.64\text{ GeV}$ , which can be searched for in the  $\Upsilon\phi$  invariant mass distribution.

## 1 Introduction

It is well known that our world is made from nucleons and electrons while nucleons are made from quarks and gluons. However, we still know little (not enough) on how quarks and gluons compose nucleons, which can be better understood by exploring exotic matter beyond the conventional quark model, such as glueballs, hybrids and multiquark states, etc. [2–5]. With significant experimental progress over the past decade, lots of multiquark candidates have been observed, including dozens of charmonium/bottomonium-

like  $XYZ$  states [2] and the hidden-charm pentaquark states  $P_c(4380)$  and  $P_c(4450)$  [6]. They are new blocks of QCD matter, and provide important hints to deepen our understanding of the non-perturbative quantum chromodynamics (QCD).

Very recently the LHCb Collaboration confirmed  $X(4140)$  and  $X(4274)$  in the  $J/\psi\phi$  invariant mass distribution and determined their spin-parity quantum numbers to be both  $J^P = 1^{++}$  [7–10]. At the same time they investigated the high  $J/\psi\phi$  mass region for the first time, where the results can be described as a nonresonant term plus two new  $J^P = 0^{++}$  resonances, named  $X(4500)$  and  $X(4700)$ . Their masses and widths were measured to be

$$X(4140) : M = 4146.5 \pm 4.5_{-2.8}^{+4.6} \text{ MeV},$$

$$\Gamma = 83 \pm 21_{-14}^{+21} \text{ MeV},$$

$$X(4274) : M = 4273.3 \pm 8.3_{-3.6}^{+17.2} \text{ MeV},$$

$$\Gamma = 56 \pm 11_{-11}^{+8} \text{ MeV},$$

$$X(4500) : M = 4506 \pm 11_{-15}^{+12} \text{ MeV},$$

$$\Gamma = 92 \pm 21_{-20}^{+21} \text{ MeV},$$

$$X(4700) : M = 4704 \pm 10_{-24}^{+14} \text{ MeV},$$

$$\Gamma = 120 \pm 31_{-33}^{+42} \text{ MeV}.$$

The  $X(4140)$  [11] and the  $X(4274)$  [12] were first reported by the CDF Collaboration in 2009 and 2011, respectively. Many theoretical explanations were proposed such as the  $D_s^*\bar{D}_s^*$  and  $D_s\bar{D}_{s0}(2317)$  molecular states [13–22], compact tetraquark states (diquark–antidiquark states) [23, 24], dynamically generated resonances [25, 26], and coupled-channel effects [27, 28], etc.

<sup>a</sup> e-mail: [wec053@mail.usask.ca](mailto:wec053@mail.usask.ca)

<sup>b</sup> e-mail: [xiangliu@lzu.edu.cn](mailto:xiangliu@lzu.edu.cn)

<sup>c</sup> e-mail: [zhul@pku.edu.cn](mailto:zhul@pku.edu.cn)

Among these studies, the results obtained within the framework of QCD sum rules are significant [1, 29–32], which method has been applied to studied many other multiquark candidates [33–35]. In 2010, Chen et al. studied the vector and axial-vector charmonium-like states systematically in Ref. [1], where they used the following two  $J^P = 1^+$  currents to perform QCD sum rule analyses ( $a$  and  $b$  are color indices):

$$J_{3\mu} = s_a^T C \gamma_5 c_b (\bar{s}_a \gamma_\mu C \bar{c}_b^T + \bar{s}_b \gamma_\mu C \bar{c}_a^T) + s_a^T C \gamma_\mu c_b (\bar{s}_a \gamma_5 C \bar{c}_b^T + \bar{s}_b \gamma_5 C \bar{c}_a^T), \tag{1}$$

$$J_{4\mu} = s_a^T C \gamma_5 c_b (\bar{s}_a \gamma_\mu C \bar{c}_b^T - \bar{s}_b \gamma_\mu C \bar{c}_a^T) + s_a^T C \gamma_\mu c_b (\bar{s}_a \gamma_5 C \bar{c}_b^T - \bar{s}_b \gamma_5 C \bar{c}_a^T), \tag{2}$$

which are constructed using diquark and antidiquark fields. There are altogether five diquark fields:  $q_a^T C q_b$ ,  $q_a^T C \gamma_5 q_b$ ,  $q_a^T C \gamma_\mu q_b$ ,  $q_a^T C \gamma_\mu \gamma_5 q_b$  and  $q_a^T C \sigma_{\mu\nu} q_b$  [36, 37]. Among them, the  $S$ -wave diquark fields  $q_a^T C \gamma_5 q_b$  and  $q_a^T C \gamma_\mu q_b$  are favored [3, 38, 39], which can be used to further construct the “good” and “bad” diquarks by demanding their color structure to be antisymmetric  $[\bar{\mathbf{3}}_c]_{qq}$  (by simply adding a totally antisymmetric tensor  $\epsilon^{abc}$ ) [3]. The other three “worse” diquarks all contain  $P$ -wave components [3].

The current  $J_{4\mu}$  defined in Eq. (2) has the antisymmetric color structure  $[\bar{\mathbf{3}}_c]_{cs} \otimes [\mathbf{3}_c]_{\bar{c}\bar{s}}$ . Hence, this interpolating current consists of one “good” diquark and one “bad” antidiquark, and is the most favored one among all the  $J^P = 1^+$  currents. Its extracted mass is  $4.07 \pm 0.10$  GeV [1], consistent with the experimental mass of the  $X(4140)$  [2]. The current  $J_{3\mu}$  defined in Eq. (1) consists of one similar diquark and one similar antidiquark, but having the symmetric color structure  $[\mathbf{6}_c]_{cs} \otimes [\bar{\mathbf{6}}_c]_{\bar{c}\bar{s}}$ . Hence, it is less favored but still better than other currents containing “worse” diquarks [3]. Its extracted mass is  $4.22 \pm 0.10$  GeV [1], consistent with the experimental mass of  $X(4274)$  [2].

The  $P$ -wave vector tetraquark states were discussed extensively in Ref. [40]. There also exist investigations of the scalar tetraquark states. In Refs. [29–32] three groups studied the scalar  $D_s^* \bar{D}_s^*$  molecular state through the current composed of two vector meson fields

$$J_{D_s^* \bar{D}_s^*}(x) = \bar{c}_a(x) \gamma_\mu s_a(x) \bar{s}_b(x) \gamma^\mu c_b(x). \tag{3}$$

Two of the three groups obtained similar results using  $J_{D_s^* \bar{D}_s^*}$ ,  $4.14 \pm 0.09$  GeV [29] and  $4.13 \pm 0.10$  GeV [30]. The extracted mass is  $3.91 \pm 0.10$  GeV with the  $D_s \bar{D}_s$  current [30]

$$J_{D_s \bar{D}_s}(x) = \bar{c}_a(x) \gamma_5 s_a(x) \bar{s}_b(x) \gamma_5 c_b(x). \tag{4}$$

The third group extracted a significantly larger mass  $4.43 \pm 0.16$  GeV [31, 32], which is significantly larger than the  $D_s^* \bar{D}_s^*$  threshold, 4.22 GeV.

The  $X(4500)$  and  $X(4700)$  have masses significantly larger than  $X(4140)$  and  $X(4274)$  of  $J^P = 1^+$ . They can

be good candidates of the  $D$ -wave tetraquark states of  $J^P = 0^+$ , whose possible angular momenta are  $\{[cs]_{s=1}[\bar{c}\bar{s}]_{s=1}; L = S = 2, J = 0\}$ . Hence, the “good” diquark of  $S = 0$  cannot be used, but they can still be composed by the “bad” diquark of  $S = 1$ ,  $\epsilon^{abc} q_a^T C \gamma_\mu q_b$ . We also need the  $P$ -wave “bad” diquark field  $\epsilon^{abc} q_a^T C \gamma_{\mu_1} D_{\mu_2} q_b$  and the  $D$ -wave “bad” diquark field  $\epsilon^{abc} q_a^T C \gamma_{\mu_1} D_{\mu_2} D_{\mu_3} q_b$ , as well as their partners having the symmetric color structure  $[\mathbf{6}_c]_{qq}$ .

In this work we will show that  $X(4500)$  and  $X(4700)$  can be both interpreted as  $D$ -wave tetraquark states with the quark content  $cs\bar{c}\bar{s}$  and  $J^P = 0^+$ :  $X(4500)$  consists of one  $D$ -wave “bad” diquark and one  $S$ -wave “bad” antidiquark, having the antisymmetric color structure  $[\bar{\mathbf{3}}_c]_{cs} \otimes [\mathbf{3}_c]_{\bar{c}\bar{s}}$ ; the  $X(4700)$  consists of one similar  $D$ -wave diquark and one similar  $S$ -wave antidiquark, but having the symmetric color structure  $[\mathbf{6}_c]_{cs} \otimes [\bar{\mathbf{6}}_c]_{\bar{c}\bar{s}}$ .

These two interpretations are remarkably similar to those obtained in Ref. [1] that  $X(4140)$  and  $X(4274)$  can be both interpreted as  $S$ -wave tetraquark states with the quark content  $cs\bar{c}\bar{s}$  and  $J^P = 0^+$ :  $X(4140)$  consists of one  $S$ -wave “good” diquark and one  $S$ -wave “bad” antidiquark, having the antisymmetric color structure  $[\bar{\mathbf{3}}_c]_{cs} \otimes [\mathbf{3}_c]_{\bar{c}\bar{s}}$ ; the  $X(4274)$  consists of two similar  $S$ -wave diquarks, but having the symmetric color structure  $[\mathbf{6}_c]_{cs} \otimes [\bar{\mathbf{6}}_c]_{\bar{c}\bar{s}}$ .

To examine these interpretations, we investigate the bottom partner states of  $X(4500)$  and  $X(4700)$ , and we extract their masses to find them to be both around 10.64 GeV. We propose to search for them in the  $\Upsilon\phi$  invariant mass distribution with the running of LHC at 13 TeV and the forthcoming BelleII experiment. If the above interpretations of  $X(4140)$ ,  $X(4274)$ ,  $X(4500)$  and  $X(4700)$  are correct, their dual partner would be quite interesting, such as the  $S$ -wave scalar and the  $D$ -wave axial-vector  $cs\bar{c}\bar{s}$  tetraquark states, consisting of two “bad” diquarks with both symmetric and antisymmetric color structures. All their related studies, both experimentally and theoretically, can deepen our understanding of the non-perturbative QCD. Especially, our present study can be helpful to improve our understanding of the internal structures of exotic hadrons.

*Interpretation of  $X(4500)$  and  $X(4700)$ .*—As the first step, we use the  $S/P/D$ -waves “axial-vector” diquarks of  $S = 1$  to construct the  $D$ -wave  $cs\bar{c}\bar{s}$  tetraquark currents of  $J^P = 0^+$ . There are two possible ways. One way is to use the combination of one  $P$ -wave diquark and one  $P$ -wave antidiquark:

$$J_{1\pm} = c_a^T C \gamma_{\mu_1} [D_{\mu_3} s_b] (\bar{c}_a \gamma_{\mu_2} C [D_{\mu_4}^\dagger \bar{s}_b^T] \pm \bar{c}_b \gamma_{\mu_2} C [D_{\mu_4}^\dagger \bar{s}_a^T]) \times (g^{\mu_1\mu_3} g^{\mu_2\mu_4} + g^{\mu_1\mu_4} g^{\mu_2\mu_3} - g^{\mu_1\mu_2} g^{\mu_3\mu_4} / 2), \tag{5}$$

where  $J_{1+}$  has the symmetric color structure  $[\mathbf{6}_c]_{cs} \otimes [\bar{\mathbf{6}}_c]_{\bar{c}\bar{s}}$ , and  $J_{1-}$  has the antisymmetric color structure  $[\bar{\mathbf{3}}_c]_{cs} \otimes [\mathbf{3}_c]_{\bar{c}\bar{s}}$ ;

they both have  $l_{cs} = l_{\bar{c}\bar{s}} = 1$  and  $s_{cs} = s_{\bar{c}\bar{s}} = 1$ , and their total momenta are  $L = S = 2$  and  $J = 0$ .

The other way is to use the combination of one  $D$ -wave diquark and one  $S$ -wave antiquark:

$$J_{2\pm} = c_a^T C \gamma_{\mu_1} [D_{\mu_3} D_{\mu_4} s_b] (\bar{c}_a \gamma_{\mu_2} C \bar{s}_b^T \pm \bar{c}_b \gamma_{\mu_2} C \bar{s}_a^T) \times (g^{\mu_1\mu_3} g^{\mu_2\mu_4} + g^{\mu_1\mu_4} g^{\mu_2\mu_3} - g^{\mu_1\mu_2} g^{\mu_3\mu_4} / 2), \tag{6}$$

where  $J_{2+}$  has the symmetric color structure  $[6_c]_{cs} \otimes [\bar{6}_c]_{\bar{c}\bar{s}}$ , and  $J_{2-}$  has the antisymmetric color structure  $[\bar{3}_c]_{cs} \otimes [3_c]_{\bar{c}\bar{s}}$ ; they both have  $l_{cs} = 2, l_{\bar{c}\bar{s}} = 0$  and  $s_{cs} = s_{\bar{c}\bar{s}} = 1$ , and their total momenta are also  $L = S = 2$  and  $J = 0$ .

The  $D$ -wave tetraquark currents can also be constructed by using the  $S/P/D$ -waves mesonic fields,

$$J'_1 = \bar{c}_a \gamma_{\mu_1} [D_{\mu_3} s_a] [D_{\mu_4}^{\dagger} \bar{s}_b] \gamma_{\mu_2} c_b \times (g^{\mu_1\mu_3} g^{\mu_2\mu_4} + g^{\mu_1\mu_4} g^{\mu_2\mu_3} - g^{\mu_1\mu_2} g^{\mu_3\mu_4} / 2), \tag{7}$$

$$J'_2 = \bar{c}_a \gamma_{\mu_1} [D_{\mu_3} D_{\mu_4} s_a] \bar{s}_b \gamma_{\mu_2} c_b \times (g^{\mu_1\mu_3} g^{\mu_2\mu_4} + g^{\mu_1\mu_4} g^{\mu_2\mu_3} - g^{\mu_1\mu_2} g^{\mu_3\mu_4} / 2). \tag{8}$$

These currents have the color structure  $[1_c]_{\bar{c}s} \otimes [1_c]_{\bar{s}c}$ . Moreover, the color-octet quark–antiquark pairs can also be used to construct the tetraquark currents having the hidden-color structure  $[8_c]_{\bar{c}s} \otimes [8_c]_{\bar{s}c}$ . We will not investigate such currents in the present study, but note that we can use the Fierz and color rearrangements to relate the local diquark–antiquark and dimeson currents (see Refs. [5,36,37] for detailed discussions).

In the following we use the currents  $J_{i\pm}$  ( $i = 1, 2$ ) to study the  $D$ -wave  $cs\bar{c}\bar{s}$  tetraquark states of  $J^P = 0^+$ , denoted as  $X$ , using the method of QCD sum rules, which provides a model-independent method to study non-perturbative problems in strong interaction physics [41–45]. We need to deal with the two derivative operators inside  $J_{i\pm}$ , which has been applied to study the  $D$  and  $F$ -wave heavy-light mesons [46–48].  $J_{i\pm}$  couples to  $X$  through

$$\langle 0|J|X\rangle = f_X. \tag{9}$$

Then the two-point correlation function can be written as

$$\Pi(p^2) = i \int d^4x e^{ip \cdot x} \langle 0|T[J(x)J^{\dagger}(0)]|0\rangle, \tag{10}$$

which can be calculated in the QCD operator product expansion (OPE) up to certain order in the expansion, and then matched with a hadronic parametrization to extract information as regards  $X$ .

At the hadron level, Eq. (10) can be written as

$$\Pi(p^2) = \frac{1}{\pi} \int_{s_{<}}^{\infty} \frac{\text{Im}\Pi(s)}{s - p^2 - i\epsilon} ds, \tag{11}$$

where  $s_{<}$  is the physical threshold. We define its imaginary part as the spectral function  $\rho(s)$ , and evaluate it by inserting intermediate hadron states  $\sum_n |n\rangle\langle n|$

$$\rho(s) \equiv \frac{1}{\pi} \text{Im}\Pi(s) = \sum_n \delta(s - M_n^2) \langle 0|\eta|n\rangle\langle n|\eta^{\dagger}|0\rangle = f_X^2 \delta(s - m_X^2) + \text{continuum}, \tag{12}$$

where we only take into account the lowest-lying resonance  $|X\rangle$ , and  $m_X$  and  $f_X$  are its mass and coupling constant, respectively.

We can also evaluate the spectral density  $\rho(s)$  at the quark and gluon level via the QCD operator product expansion. In this work we evaluate it up to dimension ten, including the perturbative term, the quark condensate  $\langle \bar{s}s \rangle$ , the gluon condensate  $\langle g_s^2 GG \rangle$ , the quark–gluon mixed condensates  $\langle g_s \bar{s}\sigma Gs \rangle$  and  $\langle g_s \bar{s}\sigma Gs \rangle^2$ . The full expressions are lengthy and will not be shown here. We have also calculated the condensates  $\langle \bar{s}s \rangle^2$  and  $\langle \bar{s}s \rangle \langle g_s \bar{s}\sigma Gs \rangle$ , which can be important in sum rule studies [1]. However, both of them vanish when the currents  $J_{i\pm}$  ( $i = 1 \dots 2$ ) are used.

After performing the Borel transform at both the hadron and the QCD levels, we can express the two-point correlation function by

$$\Pi^{(all)}(M_B^2) \equiv \mathcal{B}_{M_B^2} \Pi(p^2) = \int_{s_{<}}^{\infty} e^{-s/M_B^2} \rho(s) ds. \tag{13}$$

Then assuming the contribution from continuum states can be approximated well by the OPE spectral density above a threshold value  $s_0$  (duality)

$$\Pi(s_0, M_B^2) = \int_{s_{<}}^{s_0} e^{-s/M_B^2} \rho(s) ds, \tag{14}$$

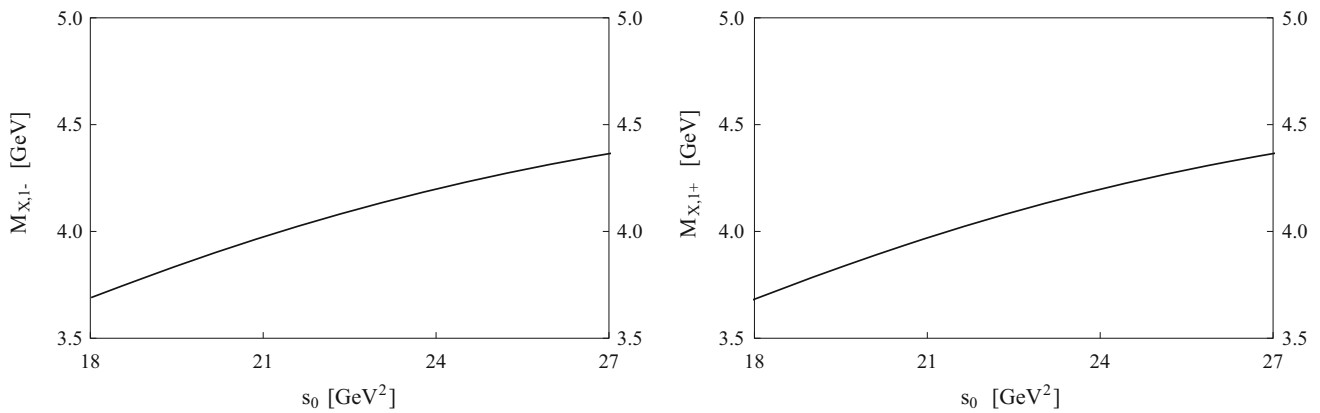
we finally arrive at the sum rule relation:

$$M_X^2(s_0, M_B) = \frac{\int_{s_{<}}^{s_0} e^{-s/M_B^2} \rho(s) s ds}{\int_{s_{<}}^{s_0} e^{-s/M_B^2} \rho(s) ds}. \tag{15}$$

To perform numerical analysis, we use the following QCD parameters of quark masses and various QCD condensates [2, 44, 45, 49–54]:

$$\begin{aligned} \langle \bar{q}q \rangle &= -(0.24 \pm 0.01)^3 \text{ GeV}^3, \\ \langle \bar{s}s \rangle &= (0.8 \pm 0.1) \times \langle \bar{q}q \rangle, \\ \langle g_s^2 GG \rangle &= (0.48 \pm 0.14) \text{ GeV}^4, \\ \langle g_s \bar{s}\sigma Gs \rangle &= -M_0^2 \times \langle \bar{s}s \rangle, \\ M_0^2 &= 0.8 \text{ GeV}^2, \\ m_s(2 \text{ GeV}) &= 95 \pm 5 \text{ MeV}, \\ m_c(m_c) &= 1.23 \pm 0.09 \text{ GeV}, \end{aligned} \tag{16}$$

in which  $\bar{m}_s$  and  $\bar{m}_c$  are the “running masses” of the strange and charm quarks in the  $\overline{\text{MS}}$  scheme. We note that there is



**Fig. 1** Variations of  $M_X$  with respect to the threshold value  $s_0$ , when the Borel mass  $M_B$  is fixed to be  $M_B^2 = 2.0 \text{ GeV}^2$ , obtained using the currents  $J_{1-}$  (left) and  $J_{1+}$  (right)

an additional minus sign in mixed condensates due to the different definition of the coupling constant  $g_s$  compared to that in Ref. [42].

There are two free parameters in Eq. (15): the threshold value  $s_0$  and the Borel mass  $M_B$ . The QCD sum rule prediction of the hadron mass  $M_X$  is only significant and reliable in suitable regions of the parameter space  $(s_0, M_B^2)$ .

First we fix  $M_B^2 = 2.0 \text{ GeV}^2$  and investigate the  $s_0$  dependence. The mass curves obtained using  $J_{1-}$  and  $J_{1+}$  (consisting of one  $P$ -wave diquark and one  $P$ -wave antiquark) are shown in Fig. 1. We find that their results are similar to each other, i.e., the evaluated masses  $M_{X,1\pm}$  monotonically increase with  $s_0$ . We do not want conclude that this is “bad” sum rule results, but it seems difficult to extract the hadron mass  $M_X$  using these two currents. Hence, we shall not discuss  $J_{1-}$  and  $J_{1+}$  any more.

The results obtained using  $J_{2-}$  and  $J_{2+}$  (consisting of one  $D$ -wave diquark and one  $S$ -wave antiquark) are also similar to each other but different from  $J_{1\pm}$ , i.e., the obtained masses  $M_{X,2\pm}$  both have a mass plateau, where the  $s_0$  dependence is weakest [1, 55]. We use the current  $J_{2-}$  as an example and show the mass curves in the left panel of Fig. 2 as a function of the threshold value  $s_0$ . We notice that the  $s_0$  dependence is the weakest around  $s_0 \sim 20 \text{ GeV}^2$ , and the  $M_B$  dependence is the weakest around  $s_0 \sim 24 \text{ GeV}^2$ . Accordingly, we choose the region  $20 \text{ GeV}^2 \leq s_0 \leq 24 \text{ GeV}^2$  as our working region, where the  $s_0$  and  $M_B$  dependence is both acceptable. This is our first criterion to determine  $s_0$ , i.e., the  $s_0$  and  $M_B$  stability.

After fixing  $s_0$ , we use two extra criteria to constrain the Borel mass  $M_B$ : (a) to ensure the convergence of the OPE series, we require that the mixed condensate  $\langle g_s \bar{s} \sigma G s \rangle$  be less than 30% to determine its lower limit  $M_B^{\text{min}}$  (the contribution from the highest condensate  $\langle g_s \bar{s} \sigma G s \rangle^2$  is negligible, so we do not use it in this criterion):

$$\text{Convergence (CVG)} \equiv \left| \frac{\Pi_{\langle g_s \bar{s} \sigma G s \rangle}(\infty, M_B)}{\Pi(\infty, M_B)} \right| \leq 30\%; \tag{17}$$

(b) to ensure that the one-pole parametrization in Eq. (12) is valid, we require that the pole contribution (PC) be larger than 20% to determine the upper limit on  $M_B^2$ :

$$\text{PC} \equiv \frac{\Pi(s_0, M_B)}{\Pi(\infty, M_B)} \geq 20\%. \tag{18}$$

The small pole contribution is due to the large powers of  $s$  in the spectral function (see other sum rule analyses for the six-quark state  $d^*(2380)$  [56] and the  $F$ -wave heavy mesons [47]).

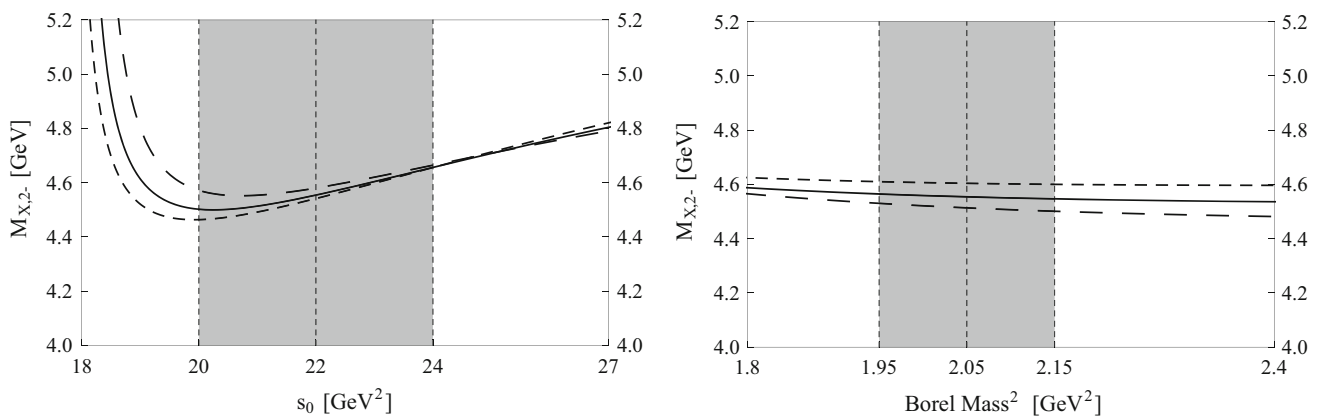
Using these two criteria we obtain the working region of the Borel mass to be  $1.95 \text{ GeV}^2 < M_B^2 < 2.15 \text{ GeV}^2$  for the current  $J_{2-}$  with  $s_0 = 22 \text{ GeV}^2$  (there exist Borel windows only when  $s_0 \geq 22 \text{ GeV}^2$ ). The variation of  $M_X$  with respect to the Borel mass  $M_B$  is shown in the right panel of Fig. 2, where the mass curves are very stable not only inside this Borel window but also in a larger nearby area.

Taking this together we obtain the working regions for the current  $J_{2-}$  to be  $20 \text{ GeV}^2 \leq s_0 \leq 24 \text{ GeV}^2$  and  $1.95 \text{ GeV}^2 < M_B^2 < 2.15 \text{ GeV}^2$ , where  $M_X$  can be extracted to be

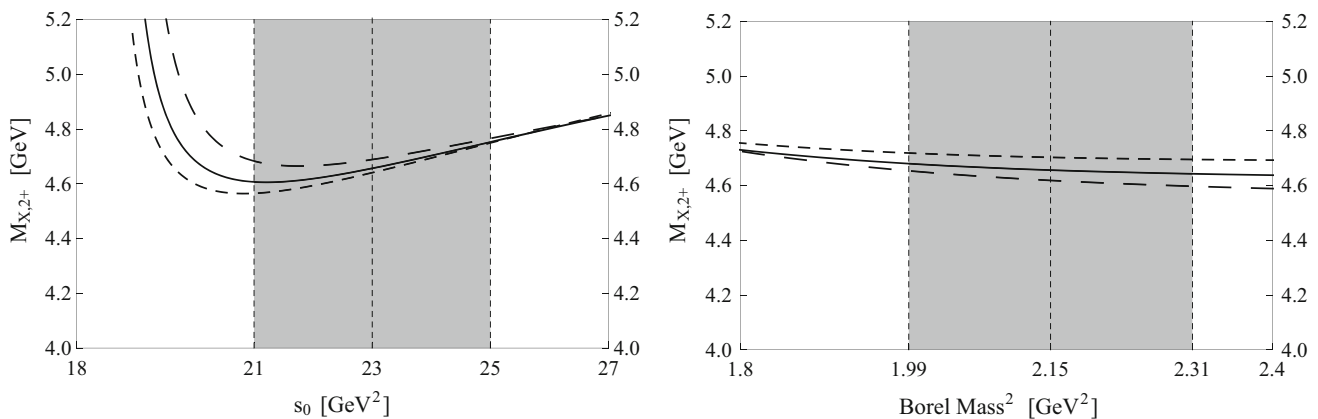
$$M_{X,2-} = 4.55_{-0.13}^{+0.19} \text{ GeV}, \tag{19}$$

Here the central value corresponds to  $M_B^2 = 2.05 \text{ GeV}^2$  and  $s_0 = 22 \text{ GeV}^2$ , and the uncertainty comes from the Borel mass  $M_B$ , the threshold value  $s_0$ , the strange and charm quark masses, and the various condensates. This value is consistent with the experimental mass of the  $X(4500)$  [7–10], supporting it to be a  $D$ -wave  $cs\bar{c}\bar{s}$  tetraquark state of  $J^P = 0^+$ . It consists of one  $D$ -wave “bad” diquark and one  $S$ -wave “bad” antiquark, having the antisymmetric color structure  $[\bar{\mathbf{3}}_c]_{cs} \otimes [\mathbf{3}_c]_{\bar{c}\bar{s}}$ .

The partner of  $X(4500)$  having the symmetric color structure,  $[\mathbf{6}_c]_{cs} \otimes [\bar{\mathbf{6}}_c]_{\bar{c}\bar{s}}$ , can be investigated using the current  $J_{2+}$ . We use this current to perform sum rule analyses, and show the obtained mass  $M_{X,2+}$  in Fig. 3 as a function of



**Fig. 2** The variation of  $M_X$  with respect to the threshold value  $s_0$  (left) and the Borel mass  $M_B$  (right), calculated using the current  $J_{2-}$  of  $J^P = 0^+$ . In the left figure, the long-dashed, solid and short-dashed curves are obtained by fixing  $M_B^2 = 1.85, 2.05$  and  $2.25 \text{ GeV}^2$ , respectively. In the right figure, the long-dashed, solid and short-dashed curves are obtained for  $s_0 = 21, 22$  and  $23 \text{ GeV}^2$ , respectively



**Fig. 3** The variation of  $M_X$  with respect to the threshold value  $s_0$  (left) and the Borel mass  $M_B$  (right), calculated using the current  $J_{2+}$  of  $J^P = 0^+$ . In the left figure, the long-dashed, solid and short-dashed curves are obtained by fixing  $M_B^2 = 1.95, 2.15$  and  $2.35 \text{ GeV}^2$ , respectively. In the right figure, the long-dashed, solid and short-dashed curves are obtained for  $s_0 = 22, 23$  and  $24 \text{ GeV}^2$ , respectively

$s_0$  and  $M_B$ . We find that the  $s_0$  dependence is the weakest around  $s_0 \sim 21 \text{ GeV}^2$ , and the  $M_B$  dependence is the weakest around  $s_0 \sim 25 \text{ GeV}^2$ . Accordingly, we fix our working regions to be  $21 \text{ GeV}^2 \leq s_0 \leq 25 \text{ GeV}^2$  and  $1.99 \text{ GeV}^2 \leq M_B^2 \leq 2.31 \text{ GeV}^2$ , where the  $s_0$  and  $M_B$  dependence is in both cases acceptable. The mass can be extracted to be

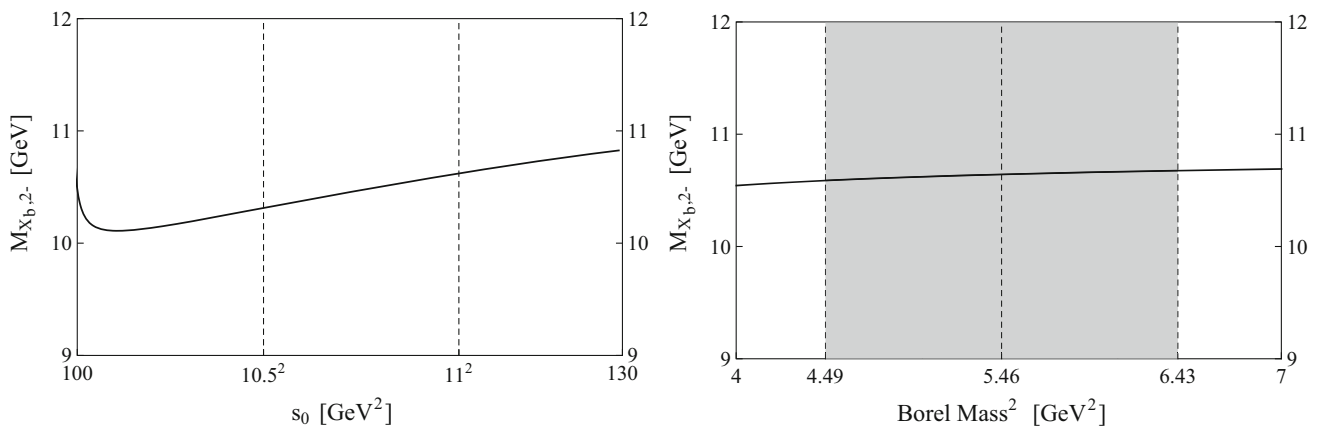
$$M_{X,2+} = 4.66^{+0.20}_{-0.14} \text{ GeV}, \tag{20}$$

where the central value corresponds to  $M_B^2 = 2.15 \text{ GeV}^2$  and  $s_0 = 23 \text{ GeV}^2$ . We find that there exist Borel windows only when  $s_0 \geq 22 \text{ GeV}^2$ , which threshold value is the same as that for  $J_{2-}$ . However, if we choose  $s_0 = 22 \text{ GeV}^2$ , the Borel window would be quite narrow ( $1.99 \text{ GeV}^2 \leq M_B^2 \leq 2.03 \text{ GeV}^2$ ), but the mass extracted would not change much ( $M_{X,2+} = 4.66^{+0.36}_{-0.19} \text{ GeV}$ ). The value listed in Eq. (20) is consistent with the experimental mass of the  $X(4700)$  [7–10], suggesting that it can also be interpreted as a  $D$ -wave

$cs\bar{c}\bar{s}$  tetraquark state of  $J^P = 0^+$ . It consists of one  $D$ -wave diquark and one  $S$ -wave antiquark, having the symmetric color structure  $[\mathbf{6}_c]_{cs} \otimes [\bar{\mathbf{6}}_c]_{\bar{c}\bar{s}}$ .

## 2 Conclusion and discussions

To summarize, we have used the method of QCD sum rule to investigate  $X(4500)$  and  $X(4700)$  of  $J^P = 0^+$  based on the diquark–antidiquark configuration within the framework of QCD sum rules. We find that  $X(4500)$  and  $X(4700)$  can both be interpreted as  $D$ -wave tetraquark states with the quark content  $cs\bar{c}\bar{s}$  and  $J^P = 0^+$ :  $X(4500)$  consists of one  $D$ -wave “bad” diquark and one  $S$ -wave “bad” antiquark, with the antisymmetric color structure  $[\mathbf{3}_c]_{cs} \otimes [\mathbf{3}_c]_{\bar{c}\bar{s}}$ ;  $X(4700)$  consists of similar diquarks, but with the symmetric color structure  $[\mathbf{6}_c]_{cs} \otimes [\bar{\mathbf{6}}_c]_{\bar{c}\bar{s}}$ . These two interpretations are remark-



**Fig. 4** The variation of  $M_X$  with respect to the threshold value  $s_0$  (left) and the Borel mass  $M_B$  (right), calculated using the current  $J_{2-}(bs\bar{b}\bar{s})$  of  $J^P = 0^+$ . In the left figure, the Borel mass  $M_B$  is

fixed to be  $M_B^2 = 5.0 \text{ GeV}^2$ . In the right figure, the threshold value  $s_0$  is fixed to be  $s_0 = 11^2 \text{ GeV}^2$ , and the Borel window is  $4.49 \text{ GeV}^2 \leq M_B^2 \leq 6.43 \text{ GeV}^2$

ably similar to those obtained in Ref. [1] that  $X(4140)$  and  $X(4274)$  can be both interpreted as  $S$ -wave  $cs\bar{c}\bar{s}$  tetraquark states of  $J^P = 0^+$ , but with opposite color structures.

The possible decay channels of  $X(4500)$  and  $X(4700)$  can be investigated by performing the Fierz and color rearrangements on the currents  $J_{2\pm}$  and changing them to mesonic-mesonic structures [5,36,37]:

$$\begin{aligned}
 J_{2\pm} \rightarrow & \left( [\bar{s}_a \gamma_{\mu_1} c_b][\bar{c}_a \gamma_{\mu_2} D_{\mu_3} D_{\mu_4} s_b \pm \{a \leftrightarrow b\}] \right. \\
 & \oplus [\bar{s}_a \gamma_{\mu_1} \gamma_5 c_b][\bar{c}_a \gamma_{\mu_2} \gamma_5 D_{\mu_3} D_{\mu_4} s_b \pm \{a \leftrightarrow b\}] \\
 & \left. \oplus [\bar{s}_a \sigma_{\mu_1 \rho} c_b][\bar{c}_a \sigma_{\mu_2 \rho} D_{\mu_3} D_{\mu_4} s_b \pm \{a \leftrightarrow b\}] \right) \\
 & \times (g^{\mu_1 \mu_3} g^{\mu_2 \mu_4} + g^{\mu_1 \mu_4} g^{\mu_2 \mu_3} - g^{\mu_1 \mu_2} g^{\mu_3 \mu_4} / 2).
 \end{aligned}
 \tag{21}$$

Besides these structures, their similar/relevant  $[\bar{s}s][\bar{c}c]$  structures are also possible. Accordingly, we obtain the possible decay channels of  $X(4500)$  and  $X(4700)$  to be  $S$ -wave  $D_s^{*+} D_s^{*-}$ ,  $D_s^{*+} D_{s1}^{*-}$  (2860),  $D_s^{*+} D_{s3}^{*-}$  (2860),  $J/\psi\phi$ ,  $J/\psi\phi_3$  (1850),  $P$ -wave  $D_s^{*+} D_{s0}^{*-}$ ,  $D_s^{*+} D_{s1}^{*-}$ ,  $D_s^{*+} D_{s2}^{*-}$ , and  $D$ -wave  $D_s^{*+} D_s^{*-}$  and  $J/\psi\phi$ , etc. The  $X(4500)$  and  $X(4700)$  were observed by LHCb in the  $J/\psi\phi$  channel, which probably contain both  $S$ -wave and  $D$ -wave components. However, the overlap of the  $S$ -wave  $J/\psi\phi$  channel (as well as the  $S$ -wave  $D_s^{*+} D_s^{*-}$  channel) and the  $J_{2\pm}$  (containing the  $D$ -wave antiquark) is quite small, which makes the widths of  $X(4500)$  and  $X(4700)$  not very large.

To examine these interpretations, we have also studied the bottom partners of  $X(4500)$  and  $X(4700)$  by simply replacing charm quarks to be bottom quarks. We evaluate their masses using the bottom quark mass  $m_b(m_b) = 4.20 \pm 0.07 \text{ GeV}$  in the  $\overline{\text{MS}}$  scheme [1,2]. We show the mass obtained using  $J_{2-}(bs\bar{b}\bar{s})$  in Fig. 4 as a function of

the threshold values  $s_0$  and the Borel mass  $M_B$ . There is a mass plateau around  $s_0 \sim 10^2 \text{ GeV}^2$ , but it depends on the bottom quark mass, which has large uncertainty [2]. We choose  $s_0 = 11^2 \text{ GeV}^2$  [1] (there exist Borel windows when  $s_0 \geq 10.5^2 \text{ GeV}^2$ ), and the mass obtained is around 10.64 GeV. The mass obtained using  $J_{2+}(bs\bar{b}\bar{s})$  is also around 10.64 GeV. We propose to search for them in the  $\Upsilon\phi$  invariant mass distribution with the running of LHC at 13 TeV and the forthcoming BelleII experiment.

Besides the above dependence on the bottom quark mass, the extracted masses of  $X(4500)$  and  $X(4700)$  in the present work also depend on the running strange and charm quark masses, which further depend on the energy scale. Therefore, our results still have some extra theoretical uncertainties not included in Eqs. (20) and (20), and more theoretical and experimental studies are necessary to understand their internal structures. Especially, the determination/confirmation of their spin-parity quantum numbers in experiments can be essential. We also note that  $X(4140)$ ,  $X(4274)$ ,  $X(4500)$  and  $X(4700)$  can have many partner states. If their interpretations in this letter are correct, their dual partner states would be quite interesting, such as the  $S$ -wave scalar and the  $D$ -wave axial-vector tetraquark states with the quark content  $cs\bar{c}\bar{s}$ . Especially in the diquark-antidiquark configuration, the  $S$ -wave scalar  $cs\bar{c}\bar{s}$  tetraquark state consisting of two “bad” diquarks is the dual partner state of both  $X(4140)$  (by replacing one “good” diquark by one “bad” diquark) and  $X(4500)$  (as its ground state), which may also exist.

To end this paper, we note that we can also use the  $S/P/D$ -waves diquarks and antidiquarks to construct many other states, and we plan to use QCD sum rules to systematically study them. Although QCD sum rule studies cannot predict their existence, our studies can still be helpful to experimental searching of new exotic hadrons.

**Acknowledgements** This project is supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) and the National Natural Science Foundation of China under Grants Nos. 11205011, 11375024, 11222547, 11175073, 11575008, and 11261130311; the Ministry of Education of China (the Fundamental Research Funds for the Central Universities), 973 program. Xiang Liu is also supported by the National Youth Top-notch Talent Support Program (“Thousands-of-Talents Scheme”).

**Open Access** This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP<sup>3</sup>.

## References

- W. Chen, S.L. Zhu, Phys. Rev. D **83**, 034010 (2011)
- K.A. Olive et al., [Particle Data Group Collaboration], Review of Particle Physics. Chin. Phys. C **38**, 090001 (2014)
- R.L. Jaffe, Phys. Rept. **409**, 1 (2005)
- X. Liu, Chin. Sci. Bull. **59**, 3815 (2014)
- H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Phys. Rept. **639**, 1 (2016)
- R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **115**, 072001 (2015)
- R. Aaij et al. [LHCb Collaboration], [arXiv:1606.07895](https://arxiv.org/abs/1606.07895) [hep-ex]
- R. Aaij et al. [LHCb Collaboration], [arXiv:1606.07898](https://arxiv.org/abs/1606.07898) [hep-ex];
- T. Skwarnicki, on behalf of the LHCb Collaboration at Meson2016. <http://meson.if.uj.edu.pl/indico/event/3/session/1/contribution/16/material/slides/0>
- T. Britton, on behalf of the LHCb Collaboration at APS April Meeting 2016. <https://absuploads.aps.org/presentation.cfm?pid=11733>
- T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. **102**, 242002 (2009)
- T. Aaltonen et al. [CDF Collaboration], [arXiv:1101.6058](https://arxiv.org/abs/1101.6058) [hep-ex]
- X. Liu, Z.G. Luo, Y.R. Liu, S.L. Zhu, Eur. Phys. J. C **61**, 411 (2009)
- X. Liu, S.L. Zhu, Phys. Rev. D **80**, 017502 (2009). Erratum: [Phys. Rev. D **85**, 019902 (2012)]
- N. Mahajan, Phys. Lett. B **679**, 228 (2009)
- T. Branz, T. Gutsche, V.E. Lyubovitskij, Phys. Rev. D **80**, 054019 (2009)
- G.J. Ding, Eur. Phys. J. C **64**, 297 (2009)
- X. Liu, Z.G. Luo, S.L. Zhu, Phys. Lett. B **699**, 341 (2011). Erratum: [Phys. Lett. B **707**, 577 (2012)]
- Z.G. Wang, Int. J. Mod. Phys. A **26**, 4929 (2011)
- S.I. Finazzo, M. Nielsen, X. Liu, Phys. Lett. B **701**, 101 (2011)
- J. He, X. Liu, Eur. Phys. J. C **72**, 1986 (2012)
- C. Hidalgo-Duque, J. Nieves, M.P. Valderrama, Phys. Rev. D **87**, 076006 (2013)
- F. Stancu, J. Phys. G **37**, 075017 (2010)
- S. Patel, M. Shah, P.C. Vinodkumar, Eur. Phys. J. A **50**, 131 (2014)
- R. Molina, E. Oset, Phys. Rev. D **80**, 114013 (2009)
- T. Branz, R. Molina, E. Oset, Phys. Rev. D **83**, 114015 (2011)
- I.V. Danilkin, Y.A. Simonov, Phys. Rev. D **81**, 074027 (2010)
- E. van Beveren, G. Rupp, [arXiv:0906.2278](https://arxiv.org/abs/0906.2278) [hep-ph]
- R.M. Albuquerque, M.E. Bracco, M. Nielsen, Phys. Lett. B **678**, 186 (2009)
- J.R. Zhang, M.Q. Huang, J. Phys. G **37**, 025005 (2010)
- Z.G. Wang, Eur. Phys. J. C **63**, 115 (2009)
- Z.G. Wang, Z.C. Liu, X.H. Zhang, Eur. Phys. J. C **64**, 373 (2009)
- H.X. Chen, A. Hosaka, S.L. Zhu, Phys. Lett. B **650**, 369 (2007)
- H.X. Chen, W. Chen, X. Liu, T.G. Steele, S.L. Zhu, Phys. Rev. Lett. **115**, 172001 (2015)
- H.X. Chen, D. Zhou, W. Chen, X. Liu, S.L. Zhu, Eur. Phys. J. C **76**, 602 (2016)
- H.X. Chen, A. Hosaka, S.L. Zhu, Phys. Rev. D **74**, 054001 (2006)
- H.X. Chen, A. Hosaka, S.L. Zhu, Phys. Rev. D **76**, 094025 (2007)
- L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, Phys. Rev. D **71**, 014028 (2005)
- R.T. Kleiv, T.G. Steele, A. Zhang, I. Blokland, Phys. Rev. D **87**, 125018 (2013)
- R.F. Lebed, A.D. Polosa, Phys. Rev. D **93**, 094024 (2016)
- M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**, 385 (1979)
- L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rept. **127**, 1 (1985)
- M. Nielsen, F.S. Navarra, S.H. Lee, Phys. Rept. **497**, 41 (2010)
- P. Colangelo, A. Khodjamirian, *At the Frontier of Particle Physics/Handbook of QCD*, vol. 3 (World Scientific, Singapore, 2001), p. 1495
- S. Narison, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **17**, 1 (2002)
- D. Zhou, E.L. Cui, H.X. Chen, L.S. Geng, X. Liu, S.L. Zhu, Phys. Rev. D **90**, 114035 (2014)
- D. Zhou, H.X. Chen, L.S. Geng, X. Liu, S.L. Zhu, Phys. Rev. D **92**, 114015 (2015)
- W. Chen, Z.X. Cai, S.L. Zhu, Nucl. Phys. B **887**, 201 (2014)
- M. Eidemuller, M. Jamin, Phys. Lett. B **498**, 203 (2001)
- V. Gimenez, V. Lubicz, F. Mescia, V. Porretti, J. Reyes, Eur. Phys. J. C **41**, 535 (2005)
- M. Jamin, Phys. Lett. B **538**, 71 (2002)
- B.L. Ioffe, K.N. Zybalyuk, Eur. Phys. J. C **27**, 229 (2003)
- A.A. Ovchinnikov, A.A. Pivovarov, Sov. J. Nucl. Phys. **48**, 721 (1988)
- A.A. Ovchinnikov, A.A. Pivovarov, Yad. Fiz. **48**, 1135 (1988)
- S. Narison, Nucl. Phys. B **509**, 312 (1998)
- H.X. Chen, E.L. Cui, W. Chen, T.G. Steele, S.L. Zhu, Phys. Rev. C **91**, 025204 (2015)