

# Semileptonic decays of $\Lambda_c$ baryons in the relativistic quark model

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**Abstract** Motivated by recent experimental progress in studying weak decays of the  $\Lambda_c$  baryon we investigate its semileptonic decays in the framework of the relativistic quark model based on the quasipotential approach with the QCD-motivated potential. The form factors of the  $\Lambda_c \rightarrow \Lambda l \nu_l$  and  $\Lambda_c \rightarrow n l \nu_l$  decays are calculated in the whole accessible kinematical region without extrapolations and additional model assumptions. Relativistic effects are systematically taken into account including transformations of baryon wave functions from the rest to moving reference frame and contributions of the intermediate negative-energy states. Baryon wave functions found in the previous mass spectrum calculations are used for the numerical evaluation. Comprehensive predictions for decay rates, asymmetries and polarization parameters are given. They agree well with available experimental data.

## 1 Introduction

Recently significant experimental progress has been achieved in studying weak decays of the charm baryons. Thus in 2014, Belle Collaboration [1] measured the branching fraction  $Br(\Lambda_c^+ \rightarrow p K^- \pi^+) = (6.84 \pm 0.24_{-0.27}^{+0.21})\%$  with the larger value and precision improved by a factor of 5 over previous results [2]. This measurement is very important since many of the previously measured  $\Lambda_c$  branching fractions are determined by their ratio with respect to  $Br(\Lambda_c^+ \rightarrow p K^- \pi^+)$  [3]. It leads to the improved value of the semileptonic branching fraction  $Br(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) = (2.9 \pm 0.5)\%$  [5]. Last year the BESIII Collaboration [4] reported the first absolute measurement of the branching fraction of the semileptonic  $\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$  process, which is considerably more precise than previous values [5]. The branching fraction was found to be  $Br(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) = (3.63 \pm 0.38 \pm 0.20)\%$  [4]. Later BESIII provided mea-

surements for 12  $\Lambda_c^+$  decay modes, including the branching fraction for  $\Lambda_c^+ \rightarrow p K^- \pi^+$  decay with the value  $(5.84 \pm 0.27 \pm 0.23)\%$  [6]. The measurements of the branching fractions of the other Cabibbo–Kobayashi–Maskawa (CKM) favored [6] and singly suppressed [7] hadronic decay modes were significantly improved.

Motivated by these experimental achievements we investigate semileptonic  $\Lambda_c$  decays in the relativistic quark model based on the quasipotential approach with the QCD-motivated potential. The mass spectra of heavy and strange baryons calculated in the quark–diquark picture of baryons in our model [8,9] agree well with available experimental data. The proton and neutron masses are also well reproduced. The study of baryon spectroscopy allowed us to determine the baryon wave functions which we use here for the numerical calculations. Recently we considered the semileptonic  $\Lambda_b$  decays [10]. General relativistic expressions for the decay form factors as overlap integrals of the initial and final baryon wave functions were found. They are obtained without application of either heavy quark  $1/m_Q$  or nonrelativistic  $v/c$  expansions. Relativistic effects, including wave function transformations from the rest to moving reference frame and contributions of the intermediate negative-energy states are consistently taken into account. It is important to emphasize that the momentum transfer  $q^2$  dependence of the decay form factors is explicitly determined without additional model assumptions and extrapolations. Here we apply these expressions for the calculation of the form factors of the  $\Lambda_c$  semileptonic decays. On their basis we present predictions for differential and total branching fractions of the  $\Lambda_c \rightarrow \Lambda l \nu_l$  and  $\Lambda_c \rightarrow n l \nu_l$  decays ( $l = e, \mu$ ) and their asymmetries and polarization parameters.

## 2 Relativistic quark model

In the relativistic quark model based on the quark–diquark picture and the quasipotential approach the interaction of two

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quarks in a diquark and the quark–diquark interaction in a baryon are described by the diquark wave function  $\Psi_d$  of the bound quark–quark state and by the baryon wave function  $\Psi_B$  of the bound quark–diquark state, which satisfy the relativistic quasipotential equation of the Schrödinger type [11, 12]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right) \Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_{d,B}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass and the center-of-mass system relative momentum squared on mass shell are

$$\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3},$$

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2},$$

and  $M$  is the bound state mass (diquark or baryon),  $m_{1,2}$  are the masses of quarks ( $q_1$  and  $q_2$ ) which form the diquark or of the diquark ( $d$ ) and quark ( $q$ ) which form the baryon ( $B$ ), and  $\mathbf{p}$  is their relative momentum.

The quasipotentials  $V(\mathbf{p}, \mathbf{q}; M)$  of the quark–quark or quark–diquark interaction are constructed with the help of the QCD-motivated off-mass-shell scattering amplitude, projected onto the positive-energy states. The effective quark interaction is taken to be the sum of the usual one-gluon exchange term and the mixture of long-range vector and scalar linear confining potentials with the mixing coefficient  $\varepsilon$ . It is also assumed that the vector confining potential contains not only the Dirac term but the Pauli term, thus introducing the anomalous chromomagnetic quark moment  $\kappa$ . The explicit expressions for the quasipotentials are given in Refs. [13, 14].

In the nonrelativistic limit the usual Cornell-like potential is reproduced,

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + Ar + B, \tag{2}$$

where the QCD coupling constant with freezing is given by

$$\alpha_s(\mu^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right) \ln \frac{\mu^2 + M_B^2}{\Lambda^2}}, \quad \mu = \frac{2m_1m_2}{m_1 + m_2}, \tag{3}$$

$n_f$  is the number of flavors, and the background mass  $M_B = 2.24\sqrt{A} = 0.95$  GeV,  $\Lambda = 413$  MeV [15, 16].

All parameters of the model were fixed previously from calculations of meson and baryon properties [11–14]. The constituent quark masses  $m_u = m_d = 0.33$  GeV,  $m_s = 0.5$  GeV,  $m_c = 1.55$  GeV and the parameters of the linear potential  $A = 0.18$  GeV<sup>2</sup> and  $B = -0.3$  GeV have the usual values of quark models. The value of the mixing coefficient of

vector and scalar confining potentials  $\varepsilon = -1$  has been determined from the consideration of the heavy quark expansion for the semileptonic heavy meson decays and charmonium radiative decays [11, 12]. The universal Pauli interaction constant  $\kappa = -1$  has been fixed from the analysis of the fine splitting of heavy quarkonia  $^3P_J$ -states [11, 12]. Note that the long-range chromomagnetic contribution to the potential, which is proportional to  $(1 + \kappa)$ , vanishes for the chosen value of  $\kappa = -1$ .

### 3 Semileptonic decay form factors

To calculate the heavy  $\Lambda_c$  baryon decay rate to the  $\Lambda$  hyperon or neutron ( $n$ ) it is necessary to determine the corresponding matrix element of the weak current between baryon states. In the quasipotential approach it is expressed by the relation

$$\langle \Lambda(n)(p_{\Lambda(n)}) | J_\mu^W | \Lambda_c(p_{\Lambda_c}) \rangle = \int \frac{d^3p d^3q}{(2\pi)^6} \bar{\Psi}_{\Lambda(n) \mathbf{p}_{\Lambda(n)}}(\mathbf{p}) \Gamma_\mu(\mathbf{p}, \mathbf{q}) \Psi_{\Lambda_c \mathbf{p}_{\Lambda_c}}(\mathbf{q}), \tag{4}$$

where  $\Gamma_\mu(\mathbf{p}, \mathbf{q})$  is the two-particle vertex function and  $\Psi_{B \mathbf{p}_B}$  are the  $B$  ( $B = \Lambda_c, \Lambda, n$ ) baryon wave functions projected onto the positive-energy states of quarks. The vertex function  $\Gamma$  receives relativistic contributions both from the impulse approximation diagram and from the diagrams with the intermediate negative-energy states which are the consequence of the projection onto the positive-energy states in the quasipotential approach [10]. The boosts of the  $B$  baryon wave functions from the rest to the moving reference frame with momentum  $\mathbf{p}_B$  are also consistently taken into account [10].

The hadronic matrix elements for the semileptonic decay  $\Lambda_c \rightarrow \Lambda(n)l\nu_l$  can be parameterized in terms of six invariant form factors:

$$\begin{aligned} \langle \Lambda(n)(p', s') | V^\mu | \Lambda_c(p, s) \rangle &= \bar{u}_{\Lambda(n)}(p', s') \left[ F_1(q^2) \gamma^\mu + F_2(q^2) \frac{p^\mu}{M_{\Lambda_c}} \right. \\ &\quad \left. + F_3(q^2) \frac{p'^\mu}{M_{\Lambda(n)}} \right] u_{\Lambda_c}(p, s), \\ \langle \Lambda(n)(p', s') | A^\mu | \Lambda_c(p, s) \rangle &= \bar{u}_{\Lambda(n)}(p', s') \left[ G_1(q^2) \gamma^\mu + G_2(q^2) \frac{p^\mu}{M_{\Lambda_c}} \right. \\ &\quad \left. + G_3(q^2) \frac{p'^\mu}{M_{\Lambda(n)}} \right] \gamma_5 u_{\Lambda_c}(p, s), \end{aligned} \tag{5}$$

where  $u_{\Lambda_c}(p, s)$  and  $u_{\Lambda(n)}(p', s')$  are Dirac spinors of the initial and final baryon;  $q = p' - p$ . In the literature another parametrization is often employed [17–19],

$$\langle \Lambda(n)(p', s') | V^\mu | \Lambda_c(p, s) \rangle = \bar{u}_{\Lambda(n)}(p', s') \left[ f_1^V(q^2) \gamma^\mu - f_2^V(q^2) i \sigma^{\mu\nu} \frac{q_\nu}{M_{\Lambda_c}} \right]$$

$$\begin{aligned}
 & + f_3^V(q^2) \frac{q^\mu}{M_{\Lambda_c}} \Big] u_{\Lambda_c}(p, s), \\
 & \langle \Lambda(n)(p', s') | A^\mu | \Lambda_c(p, s) \rangle \\
 & = \bar{u}_{\Lambda(n)}(p', s') \Big[ f_1^A(q^2) \gamma^\mu - f_2^A(q^2) i \sigma^{\mu\nu} \frac{q_\nu}{M_{\Lambda_c}} \\
 & + f_3^A(q^2) \frac{q^\mu}{M_{\Lambda_c}} \Big] \gamma_5 u_{\Lambda_c}(p, s). \tag{6}
 \end{aligned}$$

Relations between the two sets of form factors are given in Ref. [10].

Expressions for the form factors  $F_i(q^2)$ ,  $G_i(q^2)$  ( $i = 1, 2, 3$ ) obtained in our model are given in Ref. [10]. They are valid both for the heavy-to-heavy and heavy-to-light baryon decays. The form factors are expressed through the overlap integrals of the baryon wave functions which we take from the mass spectrum calculations. All relativistic effects including transformations of the baryon wave functions from the rest to moving reference frame and contributions of the intermediate negative-energy states are consistently taken into account. It is important to point out that the momentum transfer  $q^2$  behavior is explicitly determined in the whole kinematical range without extrapolations or model assumptions which are used in most of other theoretical considerations. This fact improves reliability of the form factor calculations. Note that in the heavy quark limit these form factors satisfy model independent relations imposed by the heavy quark symmetry [20–23].

Our numerical analysis shows that the weak decay form factors can be approximated with good accuracy in the physical region  $[0 \leq q^2 \leq q_{\max}^2 = (M_{\Lambda_c} - M_{\Lambda(n)})^2]$  by the

following expression:

$$F(q^2) = \frac{F(0)}{\left( 1 - \sigma_1 \frac{q^2}{M_{\Lambda_c}^2} + \sigma_2 \frac{q^4}{M_{\Lambda_c}^4} + \sigma_3 \frac{q^6}{M_{\Lambda_c}^6} + \sigma_4 \frac{q^8}{M_{\Lambda_c}^8} \right)}. \tag{7}$$

The difference between the fitted and numerical values of the form factors is less than 0.5%. The analytical properties of these form factors are discussed in Refs. [24,25].

The values of the form factor parameters  $F(0)$  and  $\sigma_{1,2,3,4}$ , as well as the values at zero recoil  $F(q_{\max}^2)$ , are given in Tables 1, 2. We can estimate the errors of the form factor calculations only within our model, since the uncertainty of the model itself is unknown. They mostly originate from the uncertainties in the baryon wave functions and from the sub-leading contributions in the low recoil region. The latter one is suppressed since the subleading contributions are proportional to the ratio of the small binding energy to the baryon mass. We estimate them to be less than 5%. The  $\Lambda_c$  baryon decay form factors are plotted in Figs. 1 and 2.

We compare our results for the form factors  $f_{1,2,3}^{V,A}$  at the maximum recoil point  $q^2 = 0$  with the predictions of other approaches in Table 3. The covariant confined quark model was used in Refs. [18,19]. The authors of Refs. [26,27] employ QCD light-cone sum rules. The calculations in Ref. [29] are based on full QCD sum rules at light cone. We find reasonable agreement with results of Refs. [18,19,26,27], while the predictions of Ref. [29] are substantially different for most of the form factors.

**Table 1** Form factors of the weak  $\Lambda_c \rightarrow \Lambda$  transition

	$F_1(q^2)$	$F_2(q^2)$	$F_3(q^2)$	$G_1(q^2)$	$G_2(q^2)$	$G_3(q^2)$
$F(0)$	1.14	-0.072	-0.252	0.517	-0.697	0.471
$F(q_{\max}^2)$	1.62	-0.280	-0.420	0.836	-1.16	0.865
$\sigma_1$	1.24	5.15	1.85	1.20	1.23	1.91
$\sigma_2$	-1.58	12.1	1.03	-1.33	-4.07	-1.18
$\sigma_3$	19.3	1.03	4.31	12.1	37.1	21.5
$\sigma_4$	-44.4	-51.2	-13.2	-40.7	-97.6	-54.9

**Table 2** Form factors of the weak  $\Lambda_c \rightarrow n$  transition

	$F_1(q^2)$	$F_2(q^2)$	$F_3(q^2)$	$G_1(q^2)$	$G_2(q^2)$	$G_3(q^2)$
$F(0)$	0.992	-0.079	-0.180	0.503	-0.626	0.354
$F(q_{\max}^2)$	1.72	-0.337	-0.388	0.778	-1.19	0.851
$\sigma_1$	1.39	3.16	2.21	1.14	1.34	2.26
$\sigma_2$	-0.683	0.029	2.17	-0.264	-2.68	0.800
$\sigma_3$	8.43	22.4	-0.039	6.77	23.1	8.84
$\sigma_4$	-14.7	-41.7	-2.09	-14.4	-44.7	-18.3

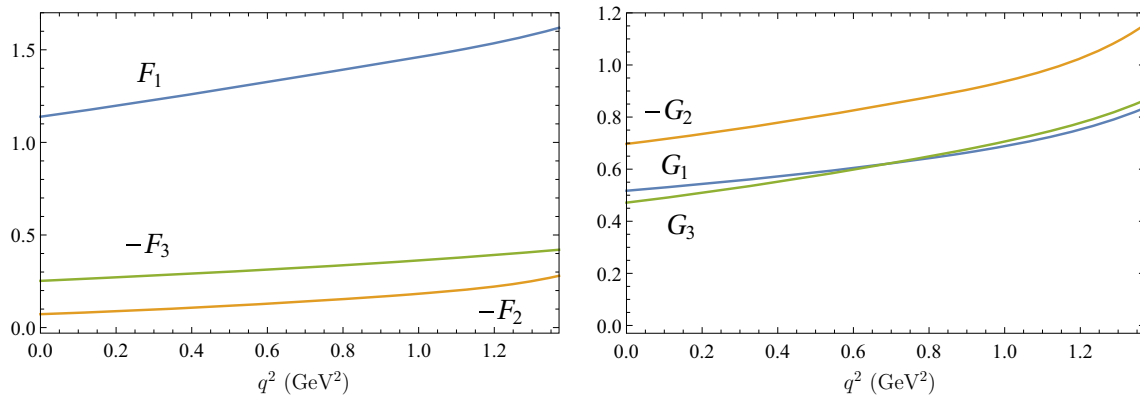


Fig. 1 Form factors of the weak  $\Lambda_c \rightarrow \Lambda$  transition

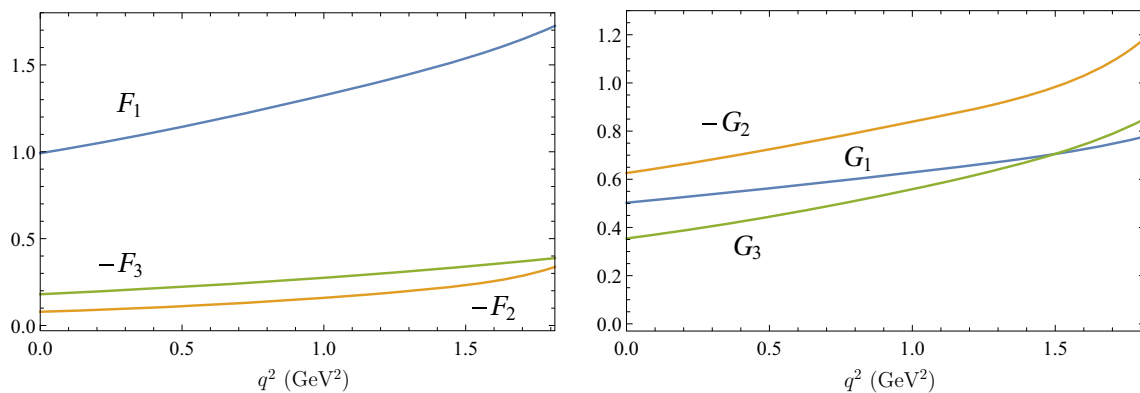


Fig. 2 Form factors of the weak  $\Lambda_c \rightarrow n$  transition

Table 3 Theoretical predictions for the form factors of weak baryon decays at maximum recoil point  $q^2 = 0$

	$f_1^V(0)$	$f_2^V(0)$	$f_3^V(0)$	$f_1^A(0)$	$f_2^A(0)$	$f_3^A(0)$
$\Lambda_c \rightarrow \Lambda$						
This paper	0.700	0.295	0.222	0.448	-0.135	-0.832
[18, 19]	0.511	0.289	-0.014	0.466	-0.025	-0.400
[26]	0.517	0.123		0.517	-0.123	
$\Lambda_c \rightarrow n$						
This paper	0.627	0.259	0.179	0.433	-0.118	-0.744
[18, 19]	0.470	0.246	0.039	0.414	-0.073	-0.328
[27]	$0.59^{+0.15}_{-0.16}$	$0.43^{+0.13}_{-0.12}$		$0.55^{+0.14}_{-0.15}$	$-0.16^{+0.08}_{-0.05}$	
[29]	0.17	1.78	2.95	0.52	0.71	-0.0073

### 4 Semileptonic decay observables

Now we use the calculated  $\Lambda_c$  baryon form factors for the evaluation of the semileptonic decay rates, polarization observables, and decay asymmetries. For this purpose it is convenient to employ the helicity formalism [30]. In it the helicity amplitudes are expressed in terms of the form factors [30] by the following relations:

$$H_{+1/2,0}^{V,A} = \frac{1}{\sqrt{q^2}} \sqrt{2M_{\Lambda_c} M_{\Lambda(n)}(w \mp 1)} \times [(M_{\Lambda_c} \pm M_{\Lambda(n)}) \mathcal{F}_1^{V,A}(w) \pm M_{\Lambda(n)}(w \pm 1) \mathcal{F}_2^{V,A}(w) \pm M_{\Lambda_c}(w \pm 1) \mathcal{F}_3^{V,A}(w)],$$

$$H_{+1/2,1}^{V,A} = -2\sqrt{M_{\Lambda_c} M_{\Lambda(n)}(w \mp 1)} \mathcal{F}_1^{V,A}(w),$$

$$H_{+1/2,t}^{V,A} = \frac{1}{\sqrt{q^2}} \sqrt{2M_{\Lambda_c} M_{\Lambda(n)}(w \pm 1)} \times [(M_{\Lambda_c} \mp M_{\Lambda(n)}) \mathcal{F}_1^{V,A}(w) \pm (M_{\Lambda_c} - M_{\Lambda(n)}w) \mathcal{F}_2^{V,A}(w) \pm (M_{\Lambda_c} w - M_{\Lambda(n)}) \mathcal{F}_3^{V,A}(w)], \tag{8}$$

where

$$w = \frac{M_{\Lambda_c}^2 + M_{\Lambda(n)}^2 - q^2}{2M_{\Lambda_c} M_{\Lambda(n)}},$$

the upper (lower) sign corresponds to  $V(A)$  and  $\mathcal{F}_i^V \equiv F_i$ ,  $\mathcal{F}_i^A \equiv G_i$  ( $i = 1, 2, 3$ ).  $H_{\lambda', \lambda_W}^{V,A}$  are the helicity amplitudes for weak transitions induced by vector ( $V$ ) and axial vector ( $A$ ) currents, where  $\lambda'$  and  $\lambda_W$  are the helicities of the final baryon and the virtual  $W$ -boson, respectively. The amplitudes for negative values of the helicities are related to the ones with the positive values by

$$H_{-\lambda', -\lambda_W}^{V,A} = \pm H_{\lambda', \lambda_W}^{V,A}.$$

The total helicity amplitude for the  $V - A$  current can be written as

$$H_{\lambda', \lambda_W} = H_{\lambda', \lambda_W}^V - H_{\lambda', \lambda_W}^A.$$

It is convenient to introduce the following set of helicity structure functions [17–19]:

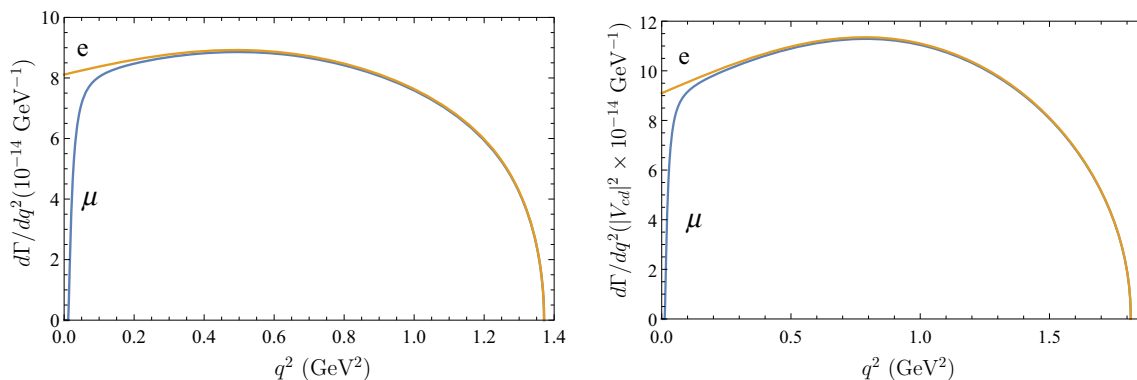
$$\begin{aligned} \mathcal{H}_U &= |H_{+1/2,+1}|^2 + |H_{-1/2,-1}|^2, \\ \mathcal{H}_L &= |H_{+1/2,0}|^2 + |H_{-1/2,0}|^2, \\ \mathcal{H}_S &= |H_{+1/2,t}|^2 + |H_{-1/2,t}|^2, \\ \mathcal{H}_P &= |H_{+1/2,+1}|^2 - |H_{-1/2,-1}|^2, \\ \mathcal{H}_{Lp} &= |H_{+1/2,0}|^2 - |H_{-1/2,0}|^2, \\ \mathcal{H}_{Sp} &= |H_{+1/2,t}|^2 - |H_{-1/2,t}|^2. \end{aligned} \tag{9}$$

Then the differential decay rate for the semileptonic  $\Lambda_c$  baryon decay to  $\Lambda(n)$  is given by [18, 19]

$$\frac{d\Gamma(\Lambda_c \rightarrow \Lambda(n)l\nu_l)}{dq^2} = \frac{G_F^2}{(2\pi)^3} |V_{cq}|^2 \frac{\lambda^{1/2}(q^2 - m_l^2)^2}{48M_{\Lambda_c}^3 q^2} \mathcal{H}_{\text{tot}}, \tag{10}$$

where  $G_F$  is the Fermi constant,  $V_{cq}$  ( $q = s, d$ ) is the CKM matrix element,  $\lambda \equiv \lambda(M_{\Lambda_c}^2, M_{\Lambda(n)}^2, q^2) = M_{\Lambda_c}^4 + M_{\Lambda(n)}^4 + q^4 - 2(M_{\Lambda_c}^2 M_{\Lambda(n)}^2 + M_{\Lambda(n)}^2 q^2 + M_{\Lambda_c}^2 q^2)$ ,  $m_l$  is the lepton mass and

$$\mathcal{H}_{\text{tot}} = (\mathcal{H}_U + \mathcal{H}_L) \left(1 + \frac{m_l^2}{2q^2}\right) + \frac{3m_l^2}{2q^2} \mathcal{H}_S. \tag{11}$$



**Fig. 3** Differential decay rates of the  $\Lambda_c^+ \rightarrow \Lambda^+ l \nu_l$  (left) and  $\Lambda_c^+ \rightarrow n^+ l \nu_l$  (right) semileptonic decays

The other useful observables for the semileptonic  $\Lambda_c$  decays are the following.

(a) Forward–backward asymmetry of the charged lepton

$$\begin{aligned} A_{\text{FB}}(q^2) &= \frac{\frac{d\Gamma}{dq^2}(\text{forward}) - \frac{d\Gamma}{dq^2}(\text{backward})}{\frac{d\Gamma}{dq^2}} \\ &= \frac{3}{4} \frac{\mathcal{H}_P - 2\frac{m_l^2}{q^2}(H_{+1/2,0}H_{+1/2,t}^\dagger + H_{-1/2,0}H_{-1/2,t}^\dagger)}{\mathcal{H}_{\text{tot}}}. \end{aligned} \tag{12}$$

(b) The convexity parameter

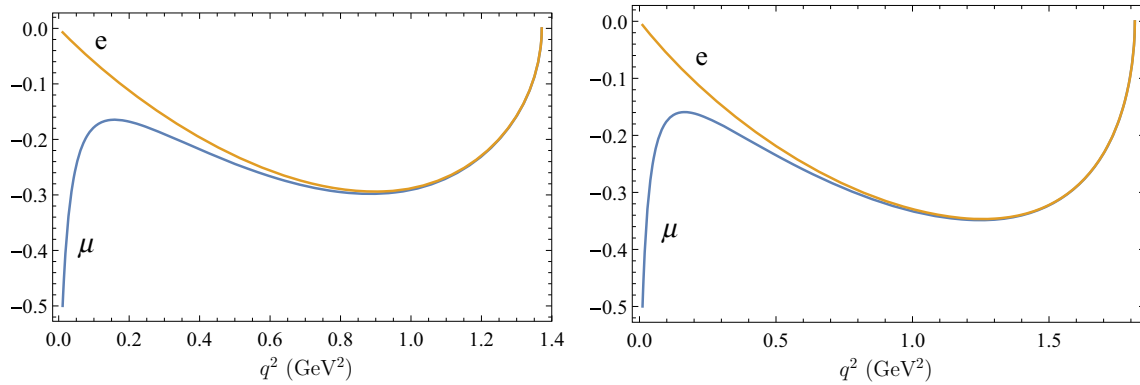
$$C_F(q^2) = \frac{3}{4} \left(1 - \frac{m_l^2}{q^2}\right) \frac{\mathcal{H}_U - 2\mathcal{H}_L}{\mathcal{H}_{\text{tot}}}. \tag{13}$$

(c) The longitudinal polarization of the final  $\Lambda(n)$  baryon

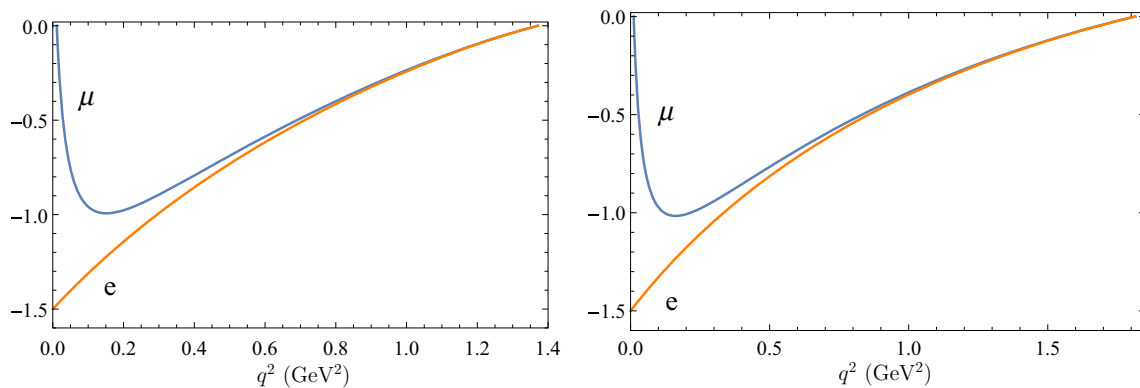
$$P_L(q^2) = \frac{(\mathcal{H}_P + \mathcal{H}_{Lp}) \left(1 + \frac{m_l^2}{2q^2}\right) + 3\frac{m_l^2}{2q^2} \mathcal{H}_{Sp}}{\mathcal{H}_{\text{tot}}}. \tag{14}$$

The forward–backward asymmetry of the charged lepton  $A_{\text{FB}}$  and the convexity parameter  $C_F$  are the linear and quadratic in  $\cos \theta$  terms of the twofold angular distribution  $d\Gamma/dq^2 d\cos \theta$  for the decay  $\Lambda_c \rightarrow \Lambda(n)W^+(\rightarrow l^+\nu_l)$ , where  $\theta$  is the angle between lepton and  $W$  [17]. Detailed experimental study of such distribution can lead to the measurement of their values. On the other hand, the longitudinal polarization of the final baryon can be extracted from the experimental study of the fourfold angular distributions  $\Lambda_c \rightarrow \Lambda(\rightarrow p\pi^-)W^+(\rightarrow l^+\nu_l)$  [17–19]. Such a complicated analysis can in principle be done by the BESIII Collaboration.

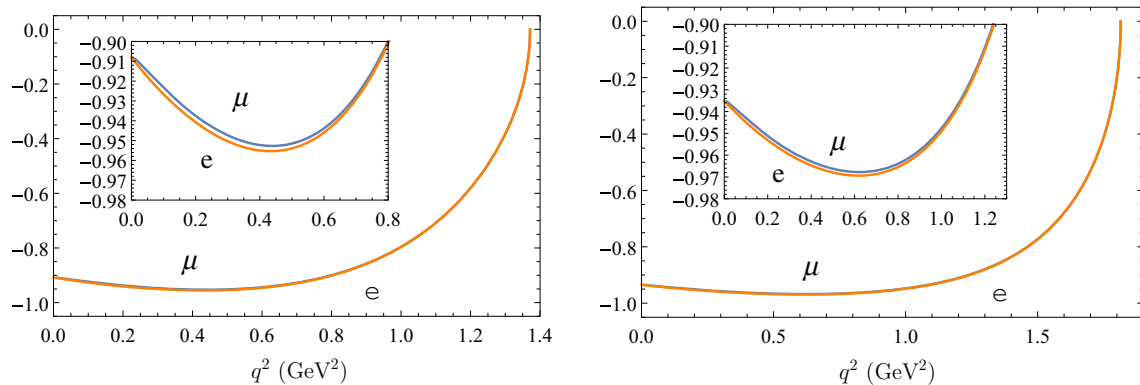
We plot these observables in Figs. 3, 4, 5, 6 both for  $\Lambda_c \rightarrow \Lambda l \nu_l$  and  $\Lambda_c \rightarrow n l \nu_l$  ( $l = e, \mu$ ) semileptonic decays. From these figures we see that the plots for decays involving



**Fig. 4** The forward–backward asymmetry  $A_{FB}(q^2)$  in the  $\Lambda_c^+ \rightarrow \Lambda l^+ \nu_l$  (left) and  $\Lambda_c^+ \rightarrow n l^+ \nu_l$  (right) semileptonic decays



**Fig. 5** The convexity parameter  $C_F(q^2)$  in the  $\Lambda_c^+ \rightarrow \Lambda l^+ \nu_l$  (left) and  $\Lambda_c^+ \rightarrow n l^+ \nu_l$  (right) semileptonic decays



**Fig. 6** The longitudinal polarization  $P_L(q^2)$  of the final baryon in the  $\Lambda_c^+ \rightarrow \Lambda l^+ \nu_l$  (left) and  $\Lambda_c^+ \rightarrow n l^+ \nu_l$  (right) semileptonic decays

electron and muon almost coincide near the zero recoil point  $q^2 = q_{\max}^2 \equiv (M_{\Lambda_c} - M_{\Lambda(n)})^2$ , while the differential decay rates, the forward–backward asymmetry  $A_{FB}$  and convexity parameter  $C_F$  have significantly different behavior near maximum recoil  $q^2 = q_{\min}^2 \approx 0$  ( $q_{\min}^2 = 0$  for electron, which we consider to be massless, and  $q_{\min}^2 = m_\mu^2$  for muon). Thus the forward–backward asymmetry  $A_{FB}(q^2)$  for  $q^2 \rightarrow q_{\min}^2$  is going to 0 for the decay  $\Lambda_c \rightarrow \Lambda(n)e\nu_e$  and to  $-0.5$  for  $\Lambda_c \rightarrow \Lambda(n)\mu\nu_\mu$ , while the convexity parameter  $C_F(q^2)$  for  $q^2 \rightarrow q_{\min}^2$  is going to  $-1.5$  and  $0$ , respectively. On the other

hand the plots for the longitudinal polarization  $P_L$  of the final baryon are almost indistinguishable in the whole kinematical range.

The total decay rates and branching fractions are obtained by integrating the differential decay rates (10) over accessible kinematical region of the momentum transfer  $q^2$ . We present the obtained results in Table 4 together with the average values of the forward–backward asymmetry of the charged lepton ( $\langle A_{FB} \rangle$ ), the convexity parameter ( $\langle C_F \rangle$ ) and the longitudinal polarization of the final baryon ( $\langle P_L \rangle$ ). The quantities  $\langle A_{FB} \rangle$ ,



**Table 4** Baryon decay rates, branching fractions, and asymmetry parameters

Decay	$\Gamma$ (ns <sup>-1</sup> )	$\Gamma/ V_{cq} ^2$ (ps <sup>-1</sup> )	$Br$ (%)	$Br/ V_{cq} ^2$	$\langle A_{FB} \rangle$	$\langle C_F \rangle$	$\langle P_L \rangle$
$\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$	162	0.167	3.25	0.033	-0.209	-0.65	-0.86
$\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu$	157	0.162	3.14	0.032	-0.242	-0.55	-0.86
$\Lambda_c^+ \rightarrow n e^+ \nu_e$	13.4	0.265	0.268	0.053	-0.251	-0.60	-0.91
$\Lambda_c^+ \rightarrow n \mu^+ \nu_\mu$	13.1	0.260	0.262	0.052	-0.276	-0.52	-0.90

**Table 5** Theoretical predictions for the  $\Lambda_c$  baryon semileptonic decay parameters and available experimental data

Parameter	Theory						Experiment	
	This paper	[18,19]	[28]	[26]	[29]	[31]	PDG [5]	BESIII [4]
$\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$								
$\Gamma$ (ns <sup>-1</sup> )	162	139	236					
$Br$ (%)	3.25	2.78	4.72	3.0 ± 0.3			2.9 ± 0.5	3.63 ± 0.43
$\langle A_{FB} \rangle$	-0.209	-0.21						
$\langle C_F \rangle$	-0.65	-0.62						
$\langle P_L \rangle$	-0.86	-0.87						
$\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu$								
$\Gamma$ (ns <sup>-1</sup> )	157	135	236					
$Br$ (%)	3.14	2.69	4.72	3.0 ± 0.3			2.7 ± 0.6	
$\langle A_{FB} \rangle$	-0.242	-0.24						
$\langle C_F \rangle$	-0.55	-0.54						
$\langle P_L \rangle$	-0.86	-0.87						
$\Lambda_c^+ \rightarrow n e^+ \nu_e$								
$\Gamma$ (ns <sup>-1</sup> )	13.4		13.5					
$\frac{\Gamma}{ V_{cd} ^2}$ (ps <sup>-1</sup> )	0.265	0.20			8.21 ± 2.80			
$Br$ (%)	0.268	0.207	0.27		8.69 ± 2.89	0.293 ± 0.034		
$\langle A_{FB} \rangle$	-0.251	-0.236						
$\Lambda_c^+ \rightarrow n \mu^+ \nu_\mu$								
$\frac{\Gamma}{ V_{cd} ^2}$ (ps <sup>-1</sup> )	0.260	0.19			8.3 ± 2.85			
$Br$ (%)	0.262	0.202			8.78 ± 2.89			
$\langle A_{FB} \rangle$	-0.276	-0.260						

$\langle C_F \rangle$  and  $\langle P_L \rangle$  are calculated by separately integrating the numerators and denominators in (12)–(14) over  $q^2$ . We estimate the errors of our calculations of the decay rates and branching fractions divided by the square of the corresponding CKM matrix element  $|V_{cq}|^2$ , to be about 10%. For the calculation of the absolute values of the branching fractions we use the average experimental values of the CKM matrix elements  $|V_{cs}| = 0.986 \pm 0.016$  and  $|V_{cd}| = 0.225 \pm 0.008$  [5].

In Table 5 we compare our results for various  $\Lambda_c$  semileptonic decay observables with the recent predictions of other theoretical approaches [18,19,26,28,29,31] (references to previous results can be found e.g. in Refs. [18,19]) and available experimental data [4,5]. The most detailed predictions are given in the framework of the covariant confined quark model [18,19]. The authors present the values not only of decay rates and branching fractions but also of

different asymmetries and polarization parameters. We find good agreement with their results. The semirelativistic quark model is used in Ref. [28]. Different versions of QCD light-cone sum rules are employed in Refs. [26,29]. The predictions of Ref. [31] are based on the application of the flavor SU(3) symmetry to  $\Lambda_c$  decays, where experimental data for  $\Lambda_c \rightarrow \Lambda e \nu_e$  is taken as an input. We compare theoretical predictions with experimental values given in PDG [5] and with recent BESIII data [4], which are available for the branching fractions of  $\Lambda_c \rightarrow \Lambda l \nu_l$  decays. All theoretical predictions reasonably agree with data. For  $\Lambda_c \rightarrow n l \nu_l$  no data are available at present. Most of the theoretical predictions [18,19,26,28,31], except the light-cone QCD sum rule approach [29] (which predicts significantly different values of decay form factors, see Table 3), give close values for the branching fractions  $BR(\Lambda_c \rightarrow n l \nu_l) = 0.2$ – $0.3\%$ .

## 5 Conclusions

In this paper we studied the  $\Lambda_c$  semileptonic decays in the framework of the relativistic quark model based on the quasipotential approach and QCD. The decay form factors were calculated in the whole accessible kinematical range without additional model assumptions and extrapolations. The relativistic effects were consistently taken into account including wave function transformations from the rest to moving reference frame and contributions of the intermediate negative-energy states. They were expressed through the overlap integrals of the baryon wave functions, which are known from the mass spectrum calculations. Such self consistent approach significantly improves the reliability of the obtained results. Further improvements can be achieved by considering deviations from the quark–diquark picture of the baryons.

Using the helicity formalism and calculated form factors we got detailed predictions for the differential and total  $\Lambda_c \rightarrow \Lambda l \nu_l$  and  $\Lambda_c \rightarrow \Lambda n \nu_l$  decay rates as well as asymmetry and polarization parameters. The obtained results agree well with most of the previous ones [18, 19, 26, 28, 31] and available experimental data. Our model predicts currently unmeasured branching fraction of the semileptonic  $\Lambda_c \rightarrow n l \nu_l$  decay to be  $(0.27 \pm 0.03)\%$ .

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