

$\mu \rightarrow e\gamma$ in the 2HDM: an exercise in EFT

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Abstract The 2 Higgs Doublet Model of type III has renormalisable Lepton Flavour-Violating couplings, and its one- and two-loop (“Barr–Zee”) contributions to $\mu \rightarrow e\gamma$ are known. In the decoupling limit, where the mass scale M of the second doublet is much greater than the electroweak scale, the model can be parametrised with an Effective Field Theory (EFT) containing dimension-six operators. The $1/M^2$ terms of the exact calculation are reproduced in the EFT, provided that the four-fermion operator basis below the weak scale is enlarged with respect to the SU(2)-invariant Buchmüller–Wyler list. It is found that the dominant two-loop “Barr–Zee” contributions arise mostly in two-loop matching and running, and that dimension-eight operators might be numerically relevant.

1 Introduction

This exercise was born from a puzzle: experiments that search for $\mu \leftrightarrow e$ flavour change constrain a long list of QCD×QED-invariant four-fermion operators, some of which turn out to be of dimension eight when SU(2) invariance is imposed. But it is common, when describing New Physics from above m_W with Effective Field Theory (EFT) [1, 2], to use the SU(2)-invariant basis of dimension 6 operators given by Buchmüller and Wyler [3] and pruned in [4]. To explore when it is justified to neglect the additional four-fermion operators below m_W , we were looking for a model where they might give relevant contributions. The first model we tried was the 2 Higgs Doublet Model (2HDM). It turns out that the additional four-fermion operators (not in the Buchmüller–Wyler list) must be included below m_W to correctly reproduce the $\mathcal{O}(1/M^2)$ terms in the $\mu \rightarrow e\gamma$ amplitude of the 2HDM.

The exercise takes place in a Type III 2HDM in the decoupling limit, where the one-loop and two-loop “Barr–Zee”

contributions to the $\mu \rightarrow e\gamma$ amplitude are known [5]. The aim is to extract the numerically dominant contributions and to identify where they arise in an EFT description.

Section 2 reviews the 2HDM of Type III in the decoupling limit and the calculation in this model of the $\mu \rightarrow e\gamma$ amplitude by Chang et al. [5] (CHK). Type III 2HDMs include charged-lepton flavour-changing couplings; for simplicity, only a $\mu \leftrightarrow e$ flavour-changing interaction is allowed. The decoupling limit is taken by requiring the mass scale M of the second doublet to be $\gtrsim 10 v$, to ensure that EFT can give a reasonable approximation to the $\mu \rightarrow e\gamma$ amplitude in this model. In the final subsections, the $\mathcal{O}(1/M^2)$ and $\mathcal{O}(1/M^4)$ parts of the $\mu \rightarrow e\gamma$ amplitude are compared for the various classes of diagrams.

Section 3 sets up an EFT formalism, based on dimension-six operators and one-loop RGEs, which correctly reproduces all the $\mathcal{O}([\alpha \log]^n/M^2)$ terms of the CHK calculation in the full model. How to obtain the remaining terms of the CHK result is outlined in Appendix B.

Section 4 summarises the less trivial aspects of the calculation and the location in the EFT of the Barr–Zee diagrams. It should be readable independently of the more technical sections, Sects. 2 and 3. There are two-and-a-half issues with the results obtained with dimension-six operators and one-loop RGEs: the four-fermion operator basis below m_W only needs to be invariant under QCD and QED, so it should be enlarged with respect to the list [4] of SU(2)-invariant dimension-six operators. Second, two-loop effects are important, because the one-loop contribution is suppressed by the square of the muon Yukawa coupling. Finally, dimension-eight terms can be enhanced, both by logs and by unknown couplings in the 2HDM (encapsulated in $\tan \beta$).

This exercise overlaps with several studies. Pruna and Signer [6] studied $\mu \rightarrow e\gamma$ in an EFT consisting of the SM extended by a complete set of SU(2)-invariant operators, but they focused on the electroweak running above m_W , rather than the matching at m_W , so they did not extend the oper-

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ator basis below m_W . In the context of B physics, Alonso et al. [7] and Aebischer et al. [8] calculated the coefficients of the enlarged operator basis below m_W , given a selection of SU(2)-invariant operators above m_W . This exercise only agrees approximatively with [7], as discussed in Sect. 4.

2 $\mu \rightarrow e\gamma$ in the Type III 2HDM

2.1 Review of the 2HDM Type III in the decoupling limit

The 2HDM is a minimal extension of the Standard Model, including one extra Higgs doublet with new unknown interactions to the Standard Model fermions and Higgs—for a review, see [9, 10]. In the 2HDM considered here, the extra doublet is taken to be heavy—this is the decoupling limit, and it should be describable with EFT. To allow for LFV, consider a “Type III” model, where there is no discrete symmetry that distinguishes the Higgs, so there is no “symmetry basis” in which to write the Lagrangian (so also no unambiguous definition of $\tan \beta$). For simplicity, the Higgs potential is taken to be CP invariant.

The Lagrangian can be written in the “Higgs basis”, defined such that $\langle H_1 \rangle \neq 0$, and $\langle H_2 \rangle = 0$, so the doublets are written¹

$$H_1 = \left(\frac{G^+}{\sqrt{2}} (v + H_1^0 + iG^0) \right), \quad H_2 = \left(\frac{H^+}{\sqrt{2}} (H_2^0 + iA) \right), \tag{1}$$

where the G s are Goldstones. This shows that the mass eigenstates A and H^\pm are the CP-odd and charged components of the vev-less H_2 , but there can be some mixing between H_1 and H_2 in the CP-even scalars h, H . The convention $\langle H_1 \rangle = v/\sqrt{2}$ implies $v = 246$ GeV,

$$\frac{4G_F}{\sqrt{2}} = \frac{2}{v^2}, \quad \sqrt{2}G_F = \frac{1}{v^2}. \tag{2}$$

In this “Higgs basis”, in the notation of [11], the potential parameters are written in upper case, so the potential is

$$\begin{aligned} \mathcal{V} = & M_{11}^2 H_1^\dagger H_1 + M_{22}^2 H_2^\dagger H_2 - [M_{12}^2 H_1^\dagger H_2 + \text{h.c.}] \\ & \text{(in Higgs basis)} \\ & + \frac{1}{2} \Lambda_1 (H_1^\dagger H_1)^2 + \frac{1}{2} \Lambda_2 (H_2^\dagger H_2)^2 + \Lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) \\ & + \Lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\ & + \left\{ \frac{1}{2} \Lambda_5 (H_1^\dagger H_2)^2 + [\Lambda_6 (H_1^\dagger H_1) \right. \\ & \left. + \Lambda_7 (H_2^\dagger H_2)] H_1^\dagger H_2 + \text{h.c.} \right\}. \end{aligned} \tag{3}$$

The angle $\beta - \alpha$ can be defined from the Higgs potential of a Type III model, unlike β and α . It rotates between the

mass basis of h, H and the Higgs basis:

$$\begin{aligned} h &= H_1^0 s_{\beta-\alpha} + H_2^0 c_{\beta-\alpha}, \\ H &= H_1^0 c_{\beta-\alpha} - H_2^0 s_{\beta-\alpha}, \end{aligned} \tag{4}$$

so that what is here called $\beta - \alpha$ is independent of the angle β , which will later be defined from the Yukawas. If the potential is CP invariant, there is a simple relation for $c_{\beta-\alpha}$ [11]:

$$\cos [(\beta - \alpha)] \sin [(\beta - \alpha)] = \frac{-\Lambda_6 v^2}{m_H^2 - m_h^2}. \tag{5}$$

The masses of the scalars are related to the potential parameters in the Higgs basis as [11]

$$\begin{aligned} m_{H^\pm}^2 &= M^2 + \frac{v^2}{2} \Lambda_3, \\ m_A^2 - m_{H^\pm}^2 &= -\frac{v^2}{2} (\Lambda_5 - \Lambda_4), \\ m_H^2 + m_h^2 - m_A^2 &= +v^2 (\Lambda_1 + \Lambda_5), \\ (m_H^2 - m_h^2)^2 &= [m_A^2 + (\Lambda_5 - \Lambda_1)v^2]^2 + 4\Lambda_6^2 v^4, \end{aligned} \tag{6}$$

and the couplings to W^+W^- are $igm_W C_{\phi WW} g^{\mu\nu}$ with

$$C_{hWW} = s_{\beta-\alpha}, \quad C_{HWW} = c_{\beta-\alpha}, \quad C_{A WW} = 0. \tag{7}$$

In the decoupling limit [12], where $\Lambda_i v^2 \ll M^2$, the exact relations (6) can be expanded in v^2/M^2 to obtain

$$\begin{aligned} m_H^2 - m_A^2 &\simeq +\Lambda_5 v^2 \quad \text{(decoupling limit),} \\ m_h^2 &\simeq +v^2 \Lambda_1 - \frac{v^4}{M^2} \left(\Lambda_6 + \frac{(\Lambda_5 - \Lambda_1)^2}{4} \right) \\ &\quad \text{(decoupling limit);} \end{aligned} \tag{8}$$

and then approximating $s_{\beta-\alpha} \simeq 1$ in Eq. (5) gives

$$\Rightarrow \cos [(\beta - \alpha)] \simeq \frac{-\Lambda_6 v^2}{M^2} \left(1 + \frac{\Lambda_1 v^2}{M^2} \right) + \dots \quad \text{(decoupling limit),} \tag{9}$$

which confirms that, in the decoupling limit, h is mostly H_1 and H is mostly H_2 .

The Yukawa couplings in the Higgs basis for the Higgses, and the mass eigenstate basis for the $\{u_R, d_R, e_R, d_L, e_L\}$, are

$$\begin{aligned} -\mathcal{L}_Y = & (\bar{Q}_j \tilde{H}_1 K_{ij}^* Y_i^U U_i + \bar{Q}_i H_1 Y_i^D D_i + \bar{L}_i H_1 Y_i^E E_i) \\ & + \bar{Q}_i \tilde{H}_2 [K^\dagger \rho^U]_{ij} U_j \\ & + \bar{Q}_i H_2 [\rho^D]_{ij} D_j + \bar{L}_i H_2 [\rho^E]_{ij} E_j + \text{h.c.}, \end{aligned} \tag{10}$$

where Q, L are SU(2) doublets, E, U, D are singlets, SU(2) indices are implicit ($\bar{L}H_1 = \bar{\nu}H_1^+ + \bar{e}H_1^0$), $\tilde{H}_i = i\sigma_2 H_i^*$, the generation indices are explicit, and K is the CKM matrix. The Y matrices are flavour diagonal and equal to the SM Yukawas:

$$[Y^P]_{ij} = \sqrt{2} \frac{m_j^P}{v} \delta_{ij} \tag{11}$$

¹ Neglecting the phase ambiguity χ discussed at eqn A.11 of [11].

as a result of being in the fermion mass eigenstate bases.

The ρ^P matrices can be flavour-changing. To obtain $\mu \rightarrow e\gamma$ without other flavour-changing processes, I neglect all off-diagonal elements except $\rho_{\mu e} \simeq \rho_{e\mu} \neq 0$. To obtain a predictive model and make contact with other 2HDM literature, the diagonal elements follow the pattern of one of the types of 2HDM which have a discrete symmetry that ensures flavour conservation:

Model type	ρ^U	ρ^D	ρ^E
Type I	$Y^U \cot \beta$	$-Y^D \tan \beta$	$-Y^E \tan \beta$
Type II	$-Y^U \tan \beta$	$-Y^D \tan \beta$	$-Y^E \tan \beta$
Type X	$Y^U \cot \beta$	$Y^D \cot \beta$	$-Y^E \tan \beta$
Type Y	$-Y^U \tan \beta$	$Y^D \cot \beta$	$-Y^E \tan \beta$

(12)

This requires a definition of $\tan \beta$ which is common to all the fermions. It can for instance be defined from the τ Yukawa: in the mass eigenstate basis for τ_R, τ_L , define H_τ to be the linear combination of Higgses to which couples the τ , and β as the angle in Higgs doublet space between H_1 (the νe) and H_τ :

$$\begin{aligned} H_\perp &= \tilde{H}_1 \sin \beta + \tilde{H}_2 \cos \beta \\ H_\tau &= H_1 \cos \beta - H_2 \sin \beta \end{aligned} \tag{13}$$

Since $\tan \beta$ is defined from the lepton Yukawas, $\rho^E \propto \tan \beta$ in all Types of 2HDM listed above. This is unconventional, but should include the same predictions provided that $\tan \beta$ is allowed to range from $1/50 \rightarrow 50$.

From Eq. (10) the couplings to fermions are

$$\begin{aligned} -\mathcal{L}_Y &= \bar{d} \frac{1}{\sqrt{2}} \left[Y^D (P_R + P_L) s_{\beta-\alpha} + (\rho^D P_R + \rho^{D^\dagger} P_L) c_{\beta-\alpha} \right] dh \\ &+ \bar{d} \frac{1}{\sqrt{2}} \left[Y^D (P_R + P_L) c_{\beta-\alpha} - (\rho^D P_R + \rho^{D^\dagger} P_L) s_{\beta-\alpha} \right] dH \\ &+ \frac{i}{\sqrt{2}} \bar{d} (\rho^D P_R - \rho^{D^\dagger} P_L) dA \\ &\bar{e} \frac{1}{\sqrt{2}} \left[Y^E (P_R + P_L) s_{\beta-\alpha} + (\rho^E P_R + \rho^{E^\dagger} P_L) c_{\beta-\alpha} \right] eh \\ &+ \bar{e} \frac{1}{\sqrt{2}} \left[Y^E (P_R + P_L) c_{\beta-\alpha} - (\rho^E P_R + \rho^{E^\dagger} P_L) s_{\beta-\alpha} \right] \\ &eH + \frac{i}{\sqrt{2}} \bar{e} (\rho^E P_R - \rho^{E^\dagger} P_L) eA \\ &+ \bar{u} \frac{1}{\sqrt{2}} \left[Y^U (P_R + P_L) s_{\beta-\alpha} + (\rho^U P_R + \rho^{U^\dagger} P_L) c_{\beta-\alpha} \right] uh \\ &+ \bar{u} \frac{1}{\sqrt{2}} \left[Y^U (P_R + P_L) c_{\beta-\alpha} - (\rho^U P_R + \rho^{U^\dagger} P_L) s_{\beta-\alpha} \right] uH \\ &- \frac{i}{\sqrt{2}} \bar{u} (\rho^U P_R - \rho^{U^\dagger} P_L) uA \\ &+ \left\{ \bar{u} \left[K \rho^D P_R - \rho^{U^\dagger} K P_L \right] dH^+ + \text{h.c.} \right\} \\ &+ \left\{ \bar{\nu} \left[\rho^E P_R \right] eH^+ + \text{h.c.} \right\}. \end{aligned} \tag{14}$$

This gives Feynman rules of the form $-i F_{ij}^{\phi, X} P_X$, where

$$\begin{aligned} F_{ij}^{h,L} &= \frac{Y_{ij}^P}{\sqrt{2}} s_{\beta-\alpha} + \frac{[\rho^{P^\dagger}]_{ij}}{\sqrt{2}} c_{\beta-\alpha}, \\ F_{ij}^{h,R} &= \frac{Y_{ij}^P}{\sqrt{2}} s_{\beta-\alpha} + \frac{[\rho^P]_{ij}}{\sqrt{2}} c_{\beta-\alpha} \\ F_{ij}^{H,L} &= \frac{Y_{ij}^P}{\sqrt{2}} c_{\beta-\alpha} - \frac{[\rho^{P^\dagger}]_{ij}}{\sqrt{2}} s_{\beta-\alpha}, \\ F_{ij}^{H,R} &= \frac{Y_{ij}^P}{\sqrt{2}} c_{\beta-\alpha} - \frac{[\rho^P]_{ij}}{\sqrt{2}} s_{\beta-\alpha} \\ F_{u_i u_j}^{A,L} &= i \frac{[\rho^{U^\dagger}]_{ij}}{\sqrt{2}}, \\ F_{u_i u_j}^{A,R} &= -i \frac{[\rho^U]_{ij}}{\sqrt{2}} \\ F_{ij}^{A,L} &= -i \frac{[\rho^{P^\dagger}]_{ij}}{\sqrt{2}}, \\ F_{ij}^{A,R} &= i \frac{[\rho^P]_{ij}}{\sqrt{2}}, \quad \text{for } P \in E, D, \end{aligned} \tag{15}$$

where the possible forms for ρ^P are given in Eq. 12.

2.2 $\mu \rightarrow e\gamma$ in the 2HDM

The decay $\mu \rightarrow e\gamma$ can be parametrised by adding the dipole operator to the Standard Model Lagrangian. In the notation of Kuno and Okada [13]

$$\mathcal{L}_{meg} = -\frac{4G_F}{\sqrt{2}} m_\mu (A_R \bar{\mu}_R \sigma^{\alpha\beta} e_L F_{\alpha\beta} + A_L \bar{\mu}_L \sigma^{\alpha\beta} e_R F_{\alpha\beta}), \tag{16}$$

which gives

$$BR(\mu \rightarrow e\gamma) = 384\pi^2 (|A_R|^2 + |A_L|^2) < 5.7 \times 10^{-13} \tag{17}$$

where the upper bound is from the MEG experiment [14]. (The amplification factor of $384\pi^2 = \frac{3}{2}(16\pi)^2$ is because $\mu \rightarrow e\gamma$ is a 2-body decay, and is compared to the usual three-body muon decay.). $|A_R|, |A_L|$ are dimensionless, and if $|A_R| = |A_L|$, then $|A_X| < 8.6 \times 10^{-9}$.

The decay $\mu \rightarrow e\gamma$ has been extensively studied in the 2HDM [3, 15–27], particularly in connection [29–37] with the recent LHC [28, 38] excess in $h \rightarrow \tau^\pm \mu^\mp$. Chang, Hou and Keung [5] (CHK) calculate the contributions to $\mu \rightarrow e\gamma$ of neutral Higgs bosons with flavour-changing couplings. Their calculation can be divided into four classes of diagrams, illustrated in Fig. 1: the one-loop diagrams, then three classes of two-loop diagrams, that is, those with a t -loop, with a b -loop and with a W -loop. I use their results, and later, in matching onto SU(2)-invariant operators, assume that the charged Higgs contribution ensures the SU(2) invariance of

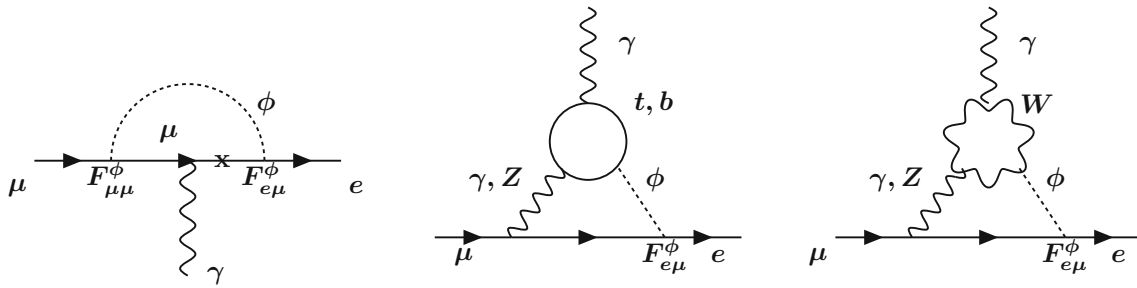


Fig. 1 One- and two-loop diagrams contributing to $\mu \rightarrow e\gamma$ in the 2HDM, in the presence of a flavour-changing Yukawa coupling $F_{\mu e}$ (see Eq. (15))

the results. The results of CHK are referred to as “full-model” results in the following.

In the appendix is given a translation dictionary between the notation of CHK and here. The amplitude given by CHK [5]² for the one-loop diagram of Fig. 1, with an internal μ , is

$$\begin{aligned}
 & -2\sqrt{2}G_F m_\mu A_L^{1\text{ loop}} \\
 &= -\frac{em_\mu}{32\pi^2} \left[\sum_{\phi=h,H,A} \frac{F_{\mu\mu}^{\phi,L} F_{e\mu}^{\phi,L}}{m_\phi^2} \left(\ln \frac{m_\mu^2}{m_\phi^2} + \frac{3}{2} \right) \right]^*, \\
 & -2\sqrt{2}G_F m_\mu A_R^{1\text{ loop}} \\
 &= -\frac{em_\mu}{32\pi^2} \left[\sum_{\phi=h,H,A} \frac{F_{\mu\mu}^{\phi,R} F_{e\mu}^{\phi,R}}{m_\phi^2} \left(\ln \frac{m_\mu^2}{m_\phi^2} + \frac{3}{2} \right) \right]^*. \tag{18}
 \end{aligned}$$

Notice that these amplitudes are suppressed by two leptonic Yukawas and an additional muon mass insertion to flip the chirality. In the decoupling limit, with $\phi \in \{H, A\}$ and μ in the loop, Eq. (18) for A_L gives

$$\begin{aligned}
 & -2\sqrt{2}G_F m_\mu A_L^{H,A,1\text{ loop}} \\
 & \simeq -\frac{em_\mu}{32\pi^2 M^2} [\rho^E]_{\mu e} \left[Y_{\mu\mu}^E \frac{\Lambda_6 v^2}{2M^2} - \frac{[\rho^E]_{\mu\mu}}{2M^2} \Lambda_5 v^2 \right]^* \ln \frac{m_\mu^2}{M^2} \tag{19}
 \end{aligned}$$

where $m_H \simeq m_A \simeq M$ was used in the logarithm, and terms suppressed by more than M^{-4} were dropped. The result for A_L is obtained by replacing $[\rho^E]_{ij} \rightarrow [\rho^E]_{ij}^*$ in the above. The one-loop diagram with the light Higgs h and a μ (also in the decoupling limit) gives

² The constant factors given by Omura et al. [28] differ: $-\frac{4}{3}$ for h , H and $-\frac{5}{3}$ for A . However, the constant is irrelevant here because the aim here is only to reproduce the log in EFT. Also, as noted by Omura et al., there are doubtful signs and typos/missing terms in the Barr-Zee formulae in [27], which differ from the formulae here.

$$\begin{aligned}
 & -2\sqrt{2}G_F m_\mu A_L^{h,1\text{ loop}} \\
 & \simeq -\frac{em_\mu}{32\pi^2 m_h^2} [\rho^E]_{\mu e} \left[-Y_{\mu\mu}^E \frac{\Lambda_6 v^2}{2M^2} + 2[\rho^E]_{\mu\mu} \left(\frac{\Lambda_6 v^2}{2M^2} \right)^2 \right]^* \\
 & \quad \times \ln \frac{m_\mu^2}{m_h^2}. \tag{20}
 \end{aligned}$$

As noted long ago by Bjorken and Weinberg [39], there are two-loop diagrams which can be relevant for $\mu \rightarrow e\gamma$. Some examples are illustrated to the right in Fig. 1; a more complete set of diagrams with broken electroweak symmetry can be found in [40]. The resulting two-loop contributions to $\mu \rightarrow e\gamma$ can be numerically larger than the one-loop contributions, because the Higgs attaches only once to the lepton line, via the flavour-changing coupling, and otherwise couples to a W , b or t loop. The result for the top loop (neglecting the diagrams with internal Z exchange, see Fig. 1), is

$$\begin{aligned}
 & -2\sqrt{2}G_F m_\mu A_L^{t\text{ loop}} \\
 &= \frac{e\alpha}{16\pi^3} \frac{1}{m_t} 3Q_t^2 \\
 & \quad \times \left[\sum_{\phi=h,H} F_{e\mu}^{\phi,L} F_{tt}^{\phi,L} f\left(\frac{m_t^2}{m_\phi^2}\right) + F_{e\mu}^{A,L} F_{tt}^{A,L} g\left(\frac{m_t^2}{m_A^2}\right) \right]^* \\
 & \simeq \frac{e\alpha m_t}{32\pi^3 M^2} [\rho^E]_{\mu e} 3Q_t^2 \\
 & \quad \times \left[\left([\rho^U]_{tt} + Y_{tt}^U \frac{\Lambda_6 v^2}{2M^2} \right) \ln^2 \frac{m_t^2}{M^2} - 2Y_{tt}^U \Lambda_6 f\left(\frac{m_t^2}{m_h^2}\right) \right] \tag{21}
 \end{aligned}$$

where the \log^2 terms of the approximate equality are the sum of the heavy H and A contributions, and the third term is due to the light h . The approximate equality is in the decoupling limit, so one uses $m_H \simeq m_A \simeq M$ in the log, neglects terms suppressed by v^2/M^2 , and uses the following, which holds for small z [5]:

$$\begin{aligned}
 f(z) & \simeq g(z) \simeq \frac{z}{2} \ln^2 z, \quad h(z) \simeq z \ln z, \\
 f(z) - g(z) & \simeq z(\ln z + 2). \tag{22}
 \end{aligned}$$

(The function h will appear in the W -loop contribution.) The functions f , g and h are given in CHK, are slowly varying

near $z \sim 1$ and at $z = m_W^2/m_h^2 \simeq 0.4$ they are $f \sim 0.7$, $g \sim 0.9$ and $h \sim 0.5$. The formula for A_R^{loop} is obtained by replacing $F_{ij}^{H,L} \rightarrow F_{ij}^{H,R}$ in the coefficient of f , and $\rho^{F\dagger} \rightarrow \rho^F$ in the coefficient of g . The top-loop amplitude is $\propto m_t$, as expected because a top mass insertion is required both to have an even number of γ -matrices in the loop, and to provide the Higgs leg of the dipole operator.

A b -quark loop should be described by the same formula as the top loop (again neglecting the Z -exchange diagrams), but the A -exchange contribution will subtract from H -exchange, because of the sign difference in the couplings of A to bs and ts (see Eq. (15)):

$$\begin{aligned}
 -2\sqrt{2}G_F m_\mu A_L^{b\ loop} &= \frac{e\alpha}{16\pi^3} \frac{3Q_b^2}{m_b} \\
 &\times \left[\sum_{\phi=H,H} F_{e\mu}^{\phi,L} F_{bb}^{\phi,L} f\left(\frac{m_b^2}{m_\phi^2}\right) + F_{e\mu}^{A,L} F_{bb}^{A,L} g\left(\frac{m_b^2}{m_A^2}\right) \right]^* \\
 &\simeq \frac{e\alpha m_b}{64\pi^3 M^2} 3Q_b^2 \rho_{\mu e}^E \\
 &\times \left[\frac{v^2}{M^2} \left(Y_{bb}^D \Lambda_6 - \rho_{bb}^D \Lambda_5 \right) \ln^2 \frac{m_b^2}{M^2} - \frac{v^2}{m_h^2} \right. \\
 &\left. \times \left(Y_{bb}^D \Lambda_6 - \rho_{bb}^D \frac{\Lambda_6^2 v^2}{M^2} \right) \ln^2 \left(\frac{m_b^2}{m_h^2} \right) \right] \quad (23)
 \end{aligned}$$

where the approximate equality is in the decoupling limit, and uses the approximations of Eq. (22). The last term is the light Higgs contribution. Notice that the $\mathcal{O}(1/M^2)$ contributions of H and A cancelled against each other in the decoupling limit. The formula for A_R is obtained by replacing $F_{ij}^{H,L} \rightarrow F_{ij}^{H,R}$ in the coefficient of f , and $\rho^{F\dagger} \rightarrow \rho^F$ in the coefficient of g .

Finally, there is a two-loop contribution involving a W -loop; the third diagram of Fig. 1 is one of the many that contribute. In the CP-conserving 2HDM considered here, A does not couple to the W or the Goldstones, so it cannot appear in these diagrams. CHK compute separately the diagrams with either photon or Z exchange between the lepton and W (see Fig. 1). The Z -mediated amplitude is proportional to $1 - 4 \sin^2 \theta_W$ and estimated by CHK to be about 10% of the photon-mediated amplitude, so only the γ -amplitude is considered here. The H and h contributions individually are

$$\begin{aligned}
 -2\sqrt{2}G_F m_\mu A_L^{W\ loop} &= \pm \frac{e\alpha}{32\pi^3} \frac{\Lambda_6 v \rho_{\mu e}^E}{M^2 \sqrt{2}} \\
 &\times \left(3f(z_\phi) + 5g(z_\phi) + \frac{3}{4}[g(z_\phi) + h(z_\phi)] \right. \\
 &\left. + \frac{f(z_\phi) - g(z_\phi)}{2z_\phi} \right) \quad (24)
 \end{aligned}$$

where $z_\phi = m_W^2/m_\phi^2$, and $-$ is for $\phi = H$, $+$ is for $\phi = h$. The sum of the contributions, in the decoupling limit, is

$$\begin{aligned}
 -2\sqrt{2}G_F m_\mu A_L^{W\ loop} &\simeq -\frac{e\alpha}{64\pi^3} \frac{\Lambda_6 v \rho_{\mu e}^E}{M^2 \sqrt{2}} \\
 &\times \left[\left(\frac{m_W^2}{M^2} \ln \frac{m_W^2}{M^2} \left(\frac{35}{4} \ln \frac{m_W^2}{M^2} + \frac{3}{2} \right) + \ln \frac{m_W^2}{M^2} + 2 \right) \right. \\
 &\left. - 2 \left(3f(z_h) + 5g(z_h) + \frac{3}{4}[g(z_h) + h(z_h)] \right. \right. \\
 &\left. \left. + \frac{f(z_h) - g(z_h)}{2z_h} \right) \right] \quad (25)
 \end{aligned}$$

where the limiting forms of Eq. (22) were used. The first line of the approximate equality is the contribution of H , and the last line is the contribution of h where $z_h = m_W^2/m_h^2$, and the parentheses term evaluates to ~ 7 . The heavy H exchanged between the lepton line and a Goldstone loop generates an $\mathcal{O}(\log/M^2)$ term, which is carefully discussed by CHK, because the $1/M^2$ suppression arises from the decoupling limit of $c_{\beta-\alpha}$, given in Eq. (9). CHK take the mixing angle $c_{\beta-\alpha}$ as a free parameter, so one refers to this term as a “non-decoupling” contribution.

2.2.1 The relative importance of the $\mathcal{O}(1/M^2)$ and $\mathcal{O}(1/M^4)$ terms

Since EFT is an expansion in the operator dimension, it is interesting to see under what conditions the $\mathcal{O}(1/M^2)$ terms give a good approximation to the full answer. These terms should arise in a relatively simple EFT using dimension-six operators; a more extended EFT would be required to reproduce the $\mathcal{O}(1/M^4)$ terms. So this section compares the magnitude of the $\mathcal{O}(1/M^2)$ and $\mathcal{O}(1/M^4)$ terms in the four classes of diagrams (one-, b -, t - and W -loop classes), with little attention to signs and 2s.

For the one-loop diagrams, Eqs. (19,20) give the ratio of the $\mathcal{O}(1/M^4)$ parts to the $\mathcal{O}(1/M^2)$ part as

$$\begin{aligned}
 &\left(\frac{v^2}{M^2} \right) \\
 &\times \frac{2[\rho^E]_{\mu\mu} \Lambda_6^2 + [Y_{\mu\mu}^E \Lambda_6 - [\rho^E]_{\mu\mu} \Lambda_5]}{2Y_{\mu\mu}^E \Lambda_6} \left(1 + \ln \frac{m_h^2}{M^2} / \ln \frac{m_\mu^2}{m_h^2} \right) \quad (26)
 \end{aligned}$$

where I used $v^2/m_h^2 \simeq 4$. Recall that, in this paper, $\tan \beta$ is defined referring to the leptons, see Eq. (13), so $[\rho^E]_{\mu\mu} / [Y_{\mu\mu}^E]_{\mu\mu} \equiv \tan \beta$. Therefore, the $\mathcal{O}(1/M^2)$ parts are larger than the $\mathcal{O}(1/M^4)$ contributions, provided that³

³ There is an $\mathcal{O}(1/M^4)$ term from the heavy Higgs exchange diagrams that benefits from an additional log enhancement—it runs from $M \rightarrow m_\mu$ rather than from $m_h \rightarrow m_\mu$ like the $\mathcal{O}(1/M^2)$ light Higgs contribution. However, this is insignificant, because $M/m_h \ll m_h/m_\mu$.

$$\Lambda_6 \tan \beta \frac{v^2}{M^2} < 1, \quad \tan \beta \frac{\Lambda_5}{4\Lambda_6} \frac{v^2}{M^2} < 1$$

$$\Rightarrow \mathcal{O}\left(\frac{1}{M^2}\right) > \mathcal{O}\left(\frac{1}{M^4}\right) \tag{27}$$

If $1/50 < \tan \beta < 50$, $v^2/M^2 < 0.01$, and $\Lambda_5 \sim \Lambda_6 < 1$, then the $\mathcal{O}(1/M^2)$ terms give the correct order of magnitude, but the $\mathcal{O}(1/M^4)$ terms can be required to get two significant figures for large $\tan \beta$.

For the top loop, the ratio of $\mathcal{O}(1/M^4)$ over the $\mathcal{O}(1/M^2)$ terms is

$$\left(\frac{v^2}{M^2}\right) \frac{Y_{tt}^U \Lambda_6}{[\rho^U]_{tt} - Y_{tt}^U \Lambda_6 / \ln^2 \frac{m_t^2}{M^2}} \tag{28}$$

where $f\left(\frac{m_t^2}{m_h^2}\right) \simeq 1$ was used. This shows that the $\mathcal{O}(1/M^2)$ terms are always larger (neglecting the possible cancellation of $[\rho^U]_{tt} \ln^2 \frac{m_t^2}{M^2}$ against Λ_6), however, in 2HDMs where the top couples mostly to the SM Higgs such that $[\rho^U]_{tt} \ln^2 \frac{m_t^2}{M^2} \ll \Lambda_6$, the dimension-eight contribution is only suppressed by $\frac{m_t^2}{M^2} \ln^2 \frac{m_t^2}{M^2}$ which is $\simeq .2$ for $\frac{m_t^2}{M^2} = 0.01$. So the dimension-eight contribution can be numerically relevant in some areas of parameter space.

For the b -loop, the ratio of $\mathcal{O}(1/M^4)$ over $\mathcal{O}(1/M^2)$ terms is

$$\left(\frac{v^2}{M^2}\right) \times \frac{\left[4\Lambda_6 \rho_{bb}^{D\dagger} + \left(Y_{bb}^D - \rho_{bb}^D \frac{\Lambda_5}{\Lambda_6}\right) \left(1 + \ln^2 \frac{m_h^2}{M^2} / \ln^2 \left(\frac{m_b^2}{m_h^2}\right)\right)\right]}{4Y_{bb}^D} \tag{29}$$

So the $\mathcal{O}(1/M^2)$ parts are larger than the $\mathcal{O}(1/M^4)$ contributions, provided that

$$\frac{\Lambda_5}{4\Lambda_6} \frac{\rho_{bb}^D}{Y_{bb}^D} \frac{v^2}{M^2} < 1, \quad \Lambda_6 \frac{\rho_{bb}^D}{Y_{bb}^D} \frac{v^2}{M^2} < 1 \Rightarrow \mathcal{O}\left(\frac{1}{M^2}\right) > \mathcal{O}\left(\frac{1}{M^4}\right) \tag{30}$$

where $\frac{\rho_{bb}^D}{Y_{bb}^D} \sim 35$ in 2HDMs where $\rho_{bb}^D \simeq 1$. As in the case of the one-loop contribution, the logarithm multiplying the $\mathcal{O}(1/M^4)$ heavy Higgs exchange diagrams runs from $M \rightarrow m_b$ rather than from $m_h \rightarrow m_b$, which could give a factor 2.

For the W -loop, the ratio of $\mathcal{O}(1/M^4)$ over $\mathcal{O}(1/M^2)$ terms is

$$\frac{\frac{m_W^2}{M^2} \ln \frac{m_W^2}{M^2} \left(\frac{35}{4} \ln \frac{m_W^2}{M^2} - \frac{3}{2}\right)}{\ln \frac{m_W^2}{M^2} - 12} \sim \frac{m_W^2}{M^2} \ln^2 \frac{m_W^2}{M^2} \tag{31}$$

where the estimate neglects cancellations and shows that the $\mathcal{O}(1/M^4)$ contribution is only mildly suppressed, because $z \ln^2 z$ does not decrease rapidly.

So in summary, provided that

$$\Lambda_6 \sim \Lambda_5, \quad \frac{v^2}{M^2} \cot \beta < 1, \quad \frac{v^2}{M^2} \tan \beta < 1 \tag{32}$$

in all four types of 2HDM, the $\mathcal{O}(1/M^2)$ terms give the correct order of magnitude, but the $\mathcal{O}(1/M^4)$ terms can be required to get two significant figures in some areas of parameter space.

2.2.2 The relative size of the four classes of diagram

The previous section suggested that it would be reasonable to use an EFT with dimension-six operators. So the next question is to determine the loop order to which the matching and running should be performed in the EFT, in order to reproduce the numerically dominant $\mathcal{O}(1/M^2)$ terms. So below, the magnitudes of the $\mathcal{O}(1/M^2)$ terms of the four classes of diagrams are compared.

The $\mathcal{O}(1/M^2)$ part of the one-loop contribution arises from h exchange:

$$2\sqrt{2}G_F m_\mu A_L^{h,1 loop} \sim \frac{em_\mu}{\pi M^2} [\rho^E]_{\mu e} Y_{\mu\mu}^E \Lambda_6 \tag{33}$$

where $v^2/m_h^2 \simeq 4$, and $\frac{1}{16\pi} \ln \frac{m_\mu^2}{m_h^2} = 0.998 \simeq 1$.

Normalising the $\mathcal{O}(1/M^2)$ part of the two-loop diagrams to Eq. (33) gives

$$t \text{ loop} \sim \frac{\alpha m_t^2}{32\pi^2 m_\mu^2} \frac{3Q_t^2}{\Lambda_6} \left[[\rho^U]_{tt} \ln^2 \frac{m_t^2}{M^2} - \Lambda_6 \right], \tag{34}$$

$$b \text{ loop} \sim \frac{\alpha m_b^2}{16\pi^2 m_\mu^2} 3Q_b^2 \ln^2 \left(\frac{m_b^2}{m_h^2}\right), \tag{35}$$

$$W \text{ loop} \sim \frac{\alpha}{64\pi^2} \frac{m_t^2}{m_\mu^2} \left[\ln \frac{m_W^2}{M^2} - 12 \right] \tag{36}$$

where $f(m_t^2/m_h^2) \simeq 1$ was used. Since $\frac{\alpha}{64\pi^2} \frac{m_t^2}{m_\mu^2} \simeq 32$, this reproduces the well-known dominance of the top and W -loops. Interestingly, only the top loop can be enhanced (or suppressed) by the angle β —the relative magnitude of the other terms is mostly controlled by Standard Model parameters.

The two-loop top and b contributions contain \log^2 terms, which should arise at second order in the one-loop RGEs of dimension-six operators. However, there are significant terms of the top and W contributions without a log, which presumably arise in two-loop matching. The log term of the W amplitude should be generated by two-loop RGEs. So we see that a two-loop analysis would be required to reproduce the dominant $\mathcal{O}(1/M^2)$ terms.

3 The EFT version

The aim of this section is to obtain, in EFT, the “leading” $\mathcal{O}([\alpha \log]^n/M^2)$ parts of the $\mu \rightarrow e\gamma$ amplitude. Appendix B discusses where some other parts of the full-model calculation would arise. EFT is transparently reviewed in [1,2], and the EFT construction here attempts to follow the recipe given there. For simplicity, the EFT has only three scales: the heavy Higgs scale M , the weak scale m_W (taken $\simeq m_Z, m_h, m_t$) and a low scale m_μ (taken $\simeq m_b$). Between M and m_W , the theory and operators are $SU(3) \times SU(2) \times U(1)$ invariant; below m_W , they are $SU(3) \times U(1)$ invariant.

The EFT constructed here contains dimension-six operators, which should reproduce the $\mathcal{O}(1/M^2)$ terms of the $\mu \rightarrow e\gamma$ amplitude. In Eqs. (39)–(42) the dimension-six, $SU(2)$ -invariant operators required above m_W are listed. Equation (47) gives some additional $SU(3) \times U(1)$ -invariant, dimension-six operators which are required below m_W (but which would have been of dimension eight in the $SU(2)$ -invariant formulation appropriate above m_W). As expected, the operator basis below m_W should include all four-fermion operators that are of dimension six in a $QED \times QCD$ -invariant theory. If instead the basis is restricted to $SU(2)$ -invariant “Buchmuller–Wyler” operators, one cannot obtain all the $\mathcal{O}([\alpha \log]^n/M^2)$ terms of the $\mu \rightarrow e\gamma$ amplitude (only the top loop is included).

The EFT studied here is at “lowest order” in the loop expansion: tree-level matching of operator coefficients, and one-loop RGEs. This should reproduce the $\mathcal{O}(\alpha^n \log^n)$ terms⁴ in the amplitude. I do not calculate the two-loop matching, or the two-loop RGEs, which would allow one to reproduce the dimension-six contributions of the top and W .

3.1 Setting up the EFT calculation

The aim of this top-down EFT calculation is to reproduce the amplitude for $\mu \rightarrow e\gamma$ in the 2HDM. So the first step is to match out the heavy Higgses H and A at the “New Physics scale” M . The H^\pm are neglected because the full-model calculation (to which the EFT is compared) includes only neutral Higgses. Presumably the H^\pm contribution ensures $SU(2)$ invariance. The 2HDM from above M is matched onto the (unbroken) Standard Model, with its full particle content, and a selection of $SU(3) \times SU(2) \times U(1)$ -invariant dimension-six operators. A list of possible operators was given by Buchmuller and Wyler [3], and slightly reduced in [4]. Here, only those operators which are required to reproduce the $\mathcal{O}(1/M^2)$ part of the $\mu \rightarrow e\gamma$ amplitude are selected. In

⁴ Since the dipole operator has only two fermion legs but an external photon, the anomalous dimension mixing four-fermion operators into the dipole is $\propto 1/e$. So in counting powers of $\alpha \log$, sometimes one should multiply by e .

matching out, the operator coefficients of the effective theory are assigned so as to reproduce the tree-level Green’s functions of the full theory, at zero external momentum.

The second step is to run the operator coefficients down to the scale m_W . This running should be performed with electroweak RGEs, which are given in [6,41]. However, since CHK separate photon and Z diagrams, and only the photon contributions were retained in the previous section, the running from $M \rightarrow m_W$ is performed with the RGEs of QED. The operator coefficients evolve [42] with scale μ as

$$\mu \frac{\partial}{\partial \mu} \vec{C} = \frac{\alpha_{em}}{4\pi} \vec{C} \Gamma \tag{37}$$

where the operator coefficients have been organised into a row vector \vec{C} , and $\frac{\alpha_{em}}{4\pi} \Gamma$ is the anomalous dimension matrix. The algorithm to calculate Γ is given, for instance, in [43]. For a square Γ , this equation can be perturbatively solved to give the components of the vector $\vec{C}(\mu)$ at a lower scale μ :

$$\begin{aligned} C_A(M) & \times \left(\delta_{AB} - \frac{\alpha(\mu)}{4\pi} [\Gamma]_{AB} \ln \frac{M}{\mu} + \frac{\alpha^2(\mu)}{32\pi^2} [\Gamma\Gamma]_{AB} \ln^2 \frac{M}{\mu} + \dots \right) \\ & = C_B(\mu) \end{aligned} \tag{38}$$

At m_W , the W, Z, h and t should be matched out. The theory below m_W should be QED and QCD for all the SM fermions except the top, augmented by a complete set of $QED \times QCD$ -invariant dimension-six operators. For simplicity, I consider only QED for the b, μ and e , plus those dimension-six operators required in matching onto the tree-level Green’s functions of the $SU(2)$ -invariant EFT from above m_W .

Finally, the operator coefficients are run down to m_μ , where the dipole coefficient can be used to calculate the $\mu \rightarrow e\gamma$ amplitude. In principle, the RGEs of QED and QCD should be used. However, QCD is neglected because it is not included in the full-model calculation of CHK.

3.2 Matching at M and one-loop running to m_W

The operator for $\mu \rightarrow e\gamma$ is given in Eq. (16). It is convenient to rescale the coefficient as

$$\begin{aligned} 2\sqrt{2} G_F m_\mu A_R^{ij} (\bar{e}_i \sigma^{\alpha\beta} P_R e_j) F_{\alpha\beta} \\ = \frac{Y_{\mu\mu}^E}{M^2} C_{D,R}^{ij} (\bar{L}_i H_1 \sigma^{\alpha\beta} P_R E_j) F_{\alpha\beta} \equiv \frac{Y_{\mu\mu}^E}{M^2} C_{D,R}^{ij} O_{D,R}^{ij} \end{aligned} \tag{39}$$

so that the dimensionful operator coefficient is suppressed by the heavy Higgs scale M . Above m_W , there is a hypercharge dipole and an $SU(2)$ dipole; here only the linear combination corresponding to the photon dipole is used, because only the QED part of the $SU(2)$ running is included. To obtain the $\mathcal{O}([\alpha \log]^n/M^2)$ parts of the $\mu \rightarrow e\gamma$ amplitude, only three

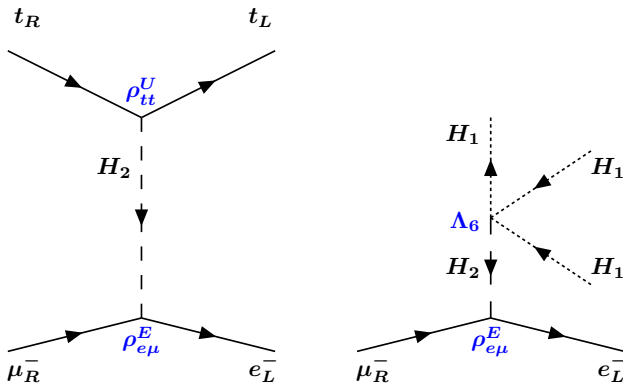


Fig. 2 The left diagram generates the dimension-six $(\bar{Q}_3 U_t)(\bar{L}_e E_\mu)$ operator, by matching out the heavy doublet Higgs H_2 (dashed line). The right diagram generates the dimension-six $H_1^\dagger H_1(\bar{L}_e H_1 E_\mu)$ operator

additional operators from the pruned Buchmuller–Wyler list are required between M and m_W :

$$O_{LEQU}^{e\mu tt} = (\bar{L}_e^A E_\mu)\epsilon_{AB}(\bar{Q}_t^B U_t),$$

$$O_{LEQU}^{\mu ett} = (\bar{L}_\mu^A E_e)\epsilon_{AB}(\bar{Q}_t^B U_t), \tag{40}$$

$$O_{T,LEQU}^{e\mu tt} = (\bar{L}_e^A \sigma^{\mu\nu} E_\mu)\epsilon_{AB}(\bar{Q}_t^B \sigma_{\mu\nu} U_t)$$

$$O_{T,LEQU}^{\mu ett} = (\bar{L}_\mu^A \sigma^{\mu\nu} E_e)\epsilon_{AB}(\bar{Q}_t^B \sigma_{\mu\nu} U_t), \tag{41}$$

$$O_{eH}^{e\mu} = H_1^\dagger H_1 \bar{L}_e H_1 E_\mu,$$

$$O_{eH}^{\mu e} = H_1^\dagger H_1 \bar{L}_\mu H_1 E_e, \tag{42}$$

where σ is the anti-symmetric tensor $\frac{i}{2}[\gamma, \gamma]$, and ϵ provides an SU(2) contraction. These operators appear in the Lagrangian as

$$\delta\mathcal{L} = - \sum_{\text{operators}} \frac{C_Z^{ijmn}}{M^2} O_Z^{ijmn} + h.c. \tag{43}$$

so the coefficients C are dimensionless, and the four-fermion operators are normalised such that the Feynman rule is $-iC/M^2$.

The last pair of operators, Eq. (42) will give the flavour-changing light Higgs interaction required for the one-loop, b -loop and W -loop amplitudes. As can be seen from the right diagram of Fig. 2, matching at the scale M of the tree-level Green’s functions of the full theory onto those of the SM + dimension-six operators gives coefficients

$$\frac{C_{eH}^{e\mu}}{M^2} = -\frac{[\rho^E]_{e\mu}\Lambda_6}{M^2}, \quad \frac{C_{eH}^{\mu e}}{M^2} = -\frac{[\rho^E]_{\mu e}\Lambda_6}{M^2}. \tag{44}$$

The running of these coefficients between M and m_W is neglected, because it is not required to reproduce the CHK results.

The first four operators of the list above will generate the top loop contribution. Matching at M via the diagram given on the left in Fig. 2 gives coefficients for the scalar $LEQU$

operators,

$$\frac{C_{LEQU}^{e\mu tt}}{M^2} = -\frac{[\rho^E]_{e\mu}[\rho^{U\dagger}]_{tt}}{M^2}, \quad \frac{C_{LEQU}^{\mu ett}}{M^2} = -\frac{[\rho^E]_{\mu e}^*[\rho^{U\dagger}]_{tt}}{M^2}, \tag{45}$$

where the negative sign is from the scalar propagator. Then, between the scales M and m_t , the RGEs of QED mix the scalar operator $O_{LEQU}^{e\mu tt}$ to the tensor $O_{T,LEQU}^{e\mu tt}$, and the tensor to the dipole, such that the coefficient of $O_{D,L}^{e\mu}$ is

$$-m_\mu \frac{C_{D,L}^{e\mu}}{M^2} = -\frac{C_{LEQU}^{e\mu tt}}{M^2} \frac{e\alpha}{128\pi^3} [(2Q_t)(8N_c Q_t m_t)] \log^2 \frac{M}{m_t}$$

$$= \frac{e\alpha}{32\pi^3 M^2} 3Q_t^2 m_t [\rho^E]_{e\mu} [\rho^{U\dagger}]_{tt} \log^2 \frac{m_t}{M^2} \tag{46}$$

where in the brackets after the first equality is the product of the scalar \rightarrow tensor and tensor \rightarrow dipole elements of the anomalous dimension matrix Γ . This agrees with the $\mathcal{O}(1/M^2)$ part of Eq. (21) that is generated by heavy Higgs exchange.

3.3 Matching at m_W and one-loop running to m_μ

At the weak scale $\simeq m_W$, the h , W and t should be matched out of the theory, onto a basis of dimension-six operators. These operators should respect the gauge symmetries below m_W , which, in the absence of the h , W and Z , are QCD and QED. So there is no reason to impose SU(2) on the operator basis below m_W .

1. If the matching is performed at tree or one loop, then matching out the top leaves only the contribution to the dipole operator given in Eq. (46). The top-loop contribution with a light Higgs, which is $\mathcal{O}(1/M^2)$, could be included by matching at two-loop.
2. In matching out the Higgs h , the one-loop, and b -loop $\mathcal{O}(1/M^2)$ -contributions can be obtained by matching onto the scalar and tensor operators:

$$O_S^{e\mu bb} = (\bar{e} P_R \mu)(\bar{b} P_R b), \quad O_S^{\mu e bb} = (\bar{\mu} P_R e)(\bar{b} P_R b),$$

$$O_T^{e\mu bb} = (\bar{e} \sigma P_R \mu)(\bar{b} \sigma P_R b),$$

$$O_T^{\mu e bb} = (\bar{\mu} \sigma P_R e)(\bar{b} \sigma P_R b),$$

$$O_S^{e\mu\mu\mu} = (\bar{e} P_R \mu)(\bar{\mu} P_R \mu), \quad O_S^{\mu e\mu\mu} = (\bar{\mu} P_R e)(\bar{\mu} P_R \mu),$$

$$O_T^{e\mu\mu\mu} = (\bar{e} \sigma P_R \mu)(\bar{\mu} \sigma P_R \mu),$$

$$O_T^{\mu e\mu\mu} = (\bar{\mu} \sigma P_R e)(\bar{\mu} \sigma P_R \mu). \tag{47}$$

If SU(2) were imposed, these operators would be of dimension eight (for instance, the first operator could be

written as $(^8)O_{LEQD}^{e\mu bb} = (\bar{L}_e H E_\mu)(\bar{Q}_3 H D_b)$. However, they are of dimension six in the QCD×QED-invariant EFT below m_W , and they are required in the 2HDM to correctly reproduce the $\mathcal{O}(\alpha^n \log^n / M^2)$ terms that dimension-six, one-loop EFT should obtain. The operators of Eq. (47) are not included in the EFT analysis of $\mu \rightarrow e\gamma$ performed by Pruna and Signer [6], who make the restriction to dimension-six SU(2)-invariant operators.

From the diagrams illustrated in Fig. 3, one obtains

$$\frac{C_S^{e\mu\mu\mu}}{M^2} = \frac{[Y^E]_{\mu\mu}[\rho^E]_{e\mu}\Lambda_6 v^2/2}{m_h^2 M^2}$$

$$\frac{C_S^{\mu e\mu\mu}}{M^2} = \frac{[Y^E]_{\mu\mu}[\rho^E]_{\mu e}\Lambda_6 v^2/2}{m_h^2 M^2}, \tag{48}$$

$$\frac{C_S^{e\mu bb}}{M^2} = \frac{[Y^D]_{bb}[\rho^E]_{e\mu}\Lambda_6 v^2/2}{m_h^2 M^2}$$

$$\frac{C_S^{\mu e bb}}{M^2} = \frac{[Y^D]_{bb}[\rho^E]_{\mu e}\Lambda_6 v^2/2}{m_h^2 M^2}, \tag{49}$$

so the coefficients are $\mathcal{O}(1/M^2)$ because $v^2/m_h^2 \simeq 4$. Then, between the scales m_W and m_μ , the RGEs of QED mix the scalar operator $O_S^{e\mu bb}$ to the tensor $O_T^{e\mu bb}$, which mixes to the dipole, exactly as in the previously discussed case of tops. So with the anomalous dimensions as in Eq. (46), the dipole coefficient

$$-m_\mu \frac{C_{D,L}^{e\mu}}{M^2} = -\frac{C_S^{e\mu bb}}{M^2} \frac{e\alpha m_b}{32\pi^3} 3Q_b^2 \log^2 \frac{m_\mu^2}{m_h^2}$$

$$= -\frac{e\alpha m_b}{64\pi^3 M^2} 3Q_b^2 \frac{[Y^D]_{bb}[\rho^E]_{e\mu}\Lambda_6 v^2}{m_h^2} \log^2 \frac{m_\mu^2}{m_h^2} \tag{50}$$

is generated, in agreement with the $\mathcal{O}(1/M^2)$ part of the b -loop contribution to $\mu \rightarrow e\gamma$, given in Eq. (23).

In matching out the Higgs h , the flavour-changing operator $O_{eH}^{e\mu}$ can also generate the dimension-six operator $O_{LEQD}^{e\mu bb} = (\bar{L}_e H E_\mu)(\bar{D}_b Q_b)$ with coefficient $\propto \rho_{\mu e}^E \Lambda_6 v^2 / (m_h^2 M^2)$. However, this operator is not useful for generating $\mu \rightarrow e\gamma$ via one-loop RGEs, because there is no tensor operator for it to mix to, on the way to the dipole. The reason that there is no tensor is that σ and $\sigma\gamma_5$ are related: $\sigma_{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\gamma_5$, which implies that $(\bar{e}\sigma^{\alpha\beta}\gamma_5\mu)(\bar{b}\sigma_{\alpha\beta}\gamma_5 b) = (\bar{e}\sigma_{\mu\nu}\mu)(\bar{b}\sigma^{\mu\nu}b)$ or $(\bar{e}\sigma^{\alpha\beta}P_L\mu)(\bar{b}\sigma_{\alpha\beta}P_Rb) = 0$.

The scalar operator with three muons, $O_S^{e\mu\mu\mu}$, mixes directly to the dipole via a penguin diagram, so the coefficient of $O_{D,L}^{e\mu}$ is

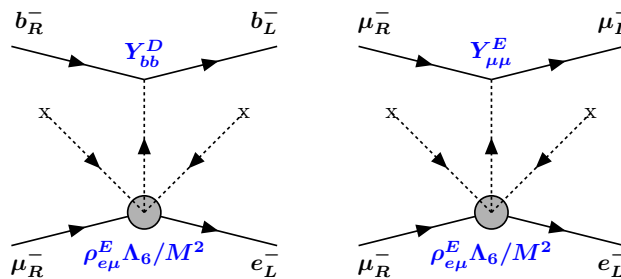


Fig. 3 The right diagram generates the QCD×QED-invariant, dimension-six operator $(\bar{e}P_R\mu)(\bar{\mu}P_R\mu)$, by matching out the light Higgs h (dotted line), which has a SU(2)-invariant dimension-six LFV vertex represented by the grey circle. The free Higgs legs attach to the vev. The left diagram generates a similar operator $(\bar{b}P_Rb)(\bar{e}P_R\mu)$ involving b quarks

$$-m_\mu \frac{C_{D,L}^{e\mu}}{M^2} = C_S^{\mu e\mu\mu} \frac{\alpha}{16\pi^2} \left[\frac{-1}{e} \right] \log \frac{m_h}{m_\mu}$$

$$= \frac{em_\mu}{64\pi^2 M^2} \frac{[Y^E]_{\mu\mu}[\rho^E]_{\mu e}\Lambda_6 v^2}{m_h^2} \log \frac{m_\mu^2}{m_h^2} \tag{51}$$

where in the brackets are the scalar→dipole element of the anomalous dimension matrix Γ . This agrees with the $\mathcal{O}(1/M^2)$ part of the one-loop, light Higgs diagrams; see Eq. (20).

3. In matching out the W at tree or one-loop, none of the W -loop contribution to $\mu \rightarrow e\gamma$ is included, because the light Higgs part arises in two-loop matching. In the full-model result of Eq. (25), there is also an $\mathcal{O}(1/M^2)$ heavy Higgs part, but it is only enhanced by one log and presumably arises in the two-loop RGEs.

4 Discussion

The CP-conserving 2HDM studied here is a minimal extension of the Standard Model—it has only one LFV coupling $[\rho^E]_{\mu e} \simeq [\rho^E]_{e\mu}$ (see Eq. 14), the magnitude of the other new flavoured couplings is controlled by the single parameter $\tan\beta$, defined in Eq. (13), and the only new mass scale is the heavy doublet mass M , taken $\gtrsim 10v$. The one- and two-loop contributions to $\mu \rightarrow e\gamma$ of the neutral Higgses were calculated by Chang et al. [5], and their result, in the decoupling limit, is given in Eqs. (20), (19), (21), (23) and (25).

Section 3 tries to reproduce the amplitude for $\mu \rightarrow e\gamma$, in a simple EFT with three scales: M, m_W, m_μ . At the scale M , the heavy Higgses were “matched out” onto dimension-six, SU(2)-invariant operators. Between M and m_W , the operator coefficients should run according to electroweak RGEs, but

I used those of QED (because I compare only to the photon diagrams of [5]). At m_W , the W, Z, h and t are matched out, so below m_W is an EFT containing all Standard Model fermions but the top, interacting via QCD, QED, and various four-fermion operators. Finally the operator coefficients run to m_μ according to the RGEs of QED.

There were three issues in reproducing the amplitude for $\mu \rightarrow e\gamma$ in EFT:

1. In order to obtain the $\mathcal{O}(1/M^2)$ terms of the full-model calculation, the operator basis below the weak scale needed to include QED \times QCD-invariant four-fermion operators which are dimension six, but would have been dimension eight in the SU(2)-invariant basis appropriate above m_W .⁵

This was the case for the one-loop contribution of the light Higgs to $\mu \rightarrow e\gamma$: the light Higgs has a flavour-changing dimension six interaction $H_1^\dagger H_1 \bar{L}_e H_1 E_\mu$. At m_h , the light Higgs can be matched onto the operator $v^2(\bar{e}P_R\mu)(\bar{\mu}P_R\mu) = (\bar{L}_e H E_\mu)(\bar{L}_\mu H E_\mu)$ with coefficient $1/(M^2 m_h^2)$, as illustrated on the right in Fig. 3. Then, in QED running down to m_μ , this operator mixes to the dipole by a penguin diagram obtained by closing a muon loop, inserting a mass and attaching a photon. So the contribution to $\mu \rightarrow e\gamma$ is $\propto v^2/(M^2 m_h^2) \log(m_\mu^2/v^2) \sim \log(m_\mu^2/v^2)/M^2$.

However, although the coefficients of $(\bar{e}P_R\mu)(\bar{\mu}P_R\mu)$ and $(\bar{e}P_R\mu)(\bar{b}P_Rb)$ are formally $\mathcal{O}(1/M^2)$, they are also small, because proportional to light fermion Yukawa couplings. As discussed in Sect. 2.2.2, they do not give a numerically relevant contribution to $\mu \rightarrow e\gamma$ in the decoupling limit of the 2HDM. So although these extra operators are required below m_W to obtain all the $\mathcal{O}(1/M^2)$ terms, they are not necessary for getting a reasonable approximation to the answer.⁶

2. There are two-loop contributions involving a top or W -loop, initially discussed by Bjorken and Weinberg [3], and illustrated in Fig. 1. As discussed in Sect. 2.2.2, these always dominate the one-loop contribution to $\mu \rightarrow e\gamma$ in the decoupling limit. This disorder in the loop expansion occurs because the one-loop contribution is suppressed

by the square of the muon Yukawa coupling, whereas the two-loop diagrams are proportional to the square of the gauge coupling or of the top Yukawa. In the EFT, two-loop matching and running would be required to reproduce the W -loop, and part of the top loop.

This “disorder” is partly a feature of the dipole operator, and partly of the 2HDM. The dipole operator has a Higgs leg, and at one loop, that Higgs can attach to the fermion line, or to the boson line, if the boson of the loop has a dimension-three coupling to the Higgs (such as the $\mu^* H_\mu^* L E$ interaction in supersymmetry). However, at two loops, there are many more possibilities for attaching the external Higgs leg with an $\mathcal{O}(1)$ coupling. In some models, it might be possible to have $\mathcal{O}(1)$ coupling of the external Higgs leg to the fermion or boson of the one-loop diagram. However, in the case of the 2HDM, there are no new fermions so the fermion of the one-loop diagram is at best a τ . The boson is a Higgs without dimension-three interactions, so the one-loop diagram is suppressed by small Yukawa couplings. This issue could be addressed by running and matching at two-loop,⁷ as is done for $b \rightarrow s\gamma$ [43].

3. Section 2.2.1 checked that the $\mathcal{O}(1/M^2)$ terms in the amplitude are larger than the $\mathcal{O}(1/M^4)$ terms, so a parametrisation in terms of dimension-six operators should work. However, dimension-eight operators can be enhanced by logs and factors of $\tan\beta$, so that it may be necessary to include them for numerical accuracy.

It is convenient to neglect dimension-eight operators, and they are expected to be suppressed by $\sim z = v^2/M^2$. However, if dimension-eight operators are log-enhanced in running, and dimension-six operators only contribute in matching, then dimension eight are only suppressed by $z \log^n(z)$. This was the case in the 2HDM for the two-loop contributions of the W s and b s. Since the high scale is not so high in the 2HDM, (e.g. $v/M \simeq 1/10$), $z = v^2/M^2 \simeq 0.01$, but $z \log^2(z) \simeq .2$.

There is also the inevitable ignorance, in EFT, as regards the magnitude of operator coefficients in the full theory. This uncertainty is parametrised by $\tan\beta$ in the 2HDM. To justify neglecting the dimension-eight operators, the restrictions $\cot\beta, \tan\beta < v^2/M^2$ had to be imposed. However, since flavour physics is about the hierarchy of

⁵ It is reasonable to use a basis of QED \times QCD-invariant four-fermion operators below m_W , because the EFT recipe [1, 2] says that one should use a complete basis of operators consistent with the symmetries of the theory—which, below m_W , is QED \times QCD (but not SU(2)).

⁶ This distinction is maybe related to a study in B physics [7], which gave the restrictions on the coefficients of QED \times QCD operators below m_W , obtained by assuming that New Physics can be described above the weak scale by SU(2)-invariant, dimension-six operators. The authors conclude that operators of the structure $(\bar{Q}HD)(\bar{Q}HD)$ can be neglected—perhaps because the operator $H_1^\dagger H_1 \bar{Q} H_1 D$, was not included above m_W (this operator is included in [8]), or perhaps because the Yukawa suppression makes these operators irrelevant.

⁷ The canonical EFT recipe says that one should match at loop order j , and compute RGEs to loop order $j + 1$, so as to reproduce all terms up to order $\alpha^n \log^{n-j}$. However, the dipole operator can only be generated with loops, so “leading order” matching, for the dipole, is at one loop rather than at tree. It is therefore debatable [44] whether j -loop RGEs should be combined with matching at order j or $j - 1$ for the dipole. In order to reproduce the numerically significant parts of the $\mu \rightarrow e\gamma$ amplitude in the 2HDM, it seems one should run and match at the same order, that is, at two loops.

couplings, it may not be sensible to assume that all New Physics couplings are $\mathcal{O}(1)$ at the New Physics scale.

This exercise located the ‘‘Barr–Zee’’ diagrams (see the right two diagrams of Fig. 1) in an EFT description.

1. The top loop with heavy Higgses contributes at second order in the one-loop RGEs for dimension-six operators, as sketched in Fig. 4, so it is included in the EFT used here. (There is also a dimension-eight contribution.) The top loop with the light Higgs would appear in two-loop matching at m_W .
2. The W -loop with heavy Higgs contributes in two-loop RGEs of dimension-six operators, and at second order in the one-loop RGEs for dimension-eight operators. The W -loop with the light Higgs would contribute in two-loop matching at m_W . As a result, none of the W contribution is included in the EFT used here.
3. The b -loop contribution is small in the decoupling limit. The light Higgs contribution, which appears at second order in the one-loop RGEs for dimension-six operators below m_W , is suppressed by the square of the SM Yukawa coupling of the b . The heavy Higgs contribution, which could be $\tan \beta$ enhanced, however, only contributes at dimension eight, via second order terms in the one-loop RGEs. This is because the tensor operator $(\bar{\psi}_1 \sigma P_L \psi_2)(\bar{\psi}_3 \sigma P_R \psi_4)$ vanishes, so the $SU(2)$ -invariant dimension-six scalar operator that can be constructed with leptons and down quarks, $(\bar{Q}D)(\bar{E}L)$, cannot mix via RGEs to a tensor and then the dipole.

5 Summary

This paper used Effective Field Theory (EFT) to calculate the amplitude for $\mu \rightarrow e\gamma$ in the decoupling limit of a 2 Higgs Doublet Model (2HDM) of Type III. A ‘‘leading order’’ EFT was used, with one-loop running, tree matching, dimension-six operators, and three scales (the heavy doublet mass M , the electroweak scale m_W , and m_μ). Comparing this EFT to a calculation performed in the 2HDM showed two things: first, that this EFT reproduces (as it should) the $\mathcal{O}([\alpha \log]^n/M^2)$ terms of the full-model calculation, *provided that* the operator basis below m_W is constructed with dimension-six $SU(3) \times U(1)$ invariant operators (some of which could be dimension eight in an $SU(2)$ -invariant construction). Secondly, to obtain the numerically dominant contributions to $\mu \rightarrow e\gamma$ in the 2HDM, two-loop RGEs and two-loop matching are required, but the enlarged operator basis below the weak scale is not.

The 2HDM also illustrates that higher dimensional operators may be numerically relevant, because they can be enhanced by unknown large couplings of the high-scale model (e.g. $\tan \beta$), or by logarithms.

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Appendix A: Translating between notations

To present the results of CHK, a translation dictionary between their notation and the notation here is useful. CHK give the Lagrangian as

$$\mathcal{L} = -\bar{t} \frac{m_t}{v} \left[\Delta_{tt}^{\phi*} P_R + \Delta_{tt}^{\phi} P_L \right] t \phi - \bar{e} \frac{\sqrt{m_e m_\mu}}{v} \times \left[\Delta_{e\mu}^{\phi R} P_R + \Delta_{e\mu}^{\phi L} P_L \right] \mu + g m_W \cos \theta_\phi W^+ W^- \phi, \tag{52}$$

where we recall $\frac{m_t}{v} = \frac{Y_{tt}^U}{\sqrt{2}}$. Defining $\Delta_{tt}^{\phi*} = \Delta_{tt}^{\phi R}$, $\Delta_{tt}^{\phi} = \Delta_{tt}^{\phi L}$ gives

$$\Delta_{ij}^{\phi X} = \frac{v}{\sqrt{m_i m_j}} F_{ij}^{\phi X} \cos \theta_\phi = \begin{cases} c_{\beta-\alpha} & \phi = H \\ s_{\beta-\alpha} & \phi = h \\ 0 & \phi = A \end{cases} \tag{53}$$

CHK give their two-loop amplitudes $A_{L,R}$ in a different normalisation from Eq. (17); the relation is

$$-A_{L,R}^{KO} = \frac{e\alpha}{64\pi^3} \sqrt{\frac{m_e}{m_\mu}} \left(A_{L,R}^{CHK} \right)^* \tag{54}$$

where the negative sign is because Kuno–Okada subtract their dipole operator from the SM Lagrangian, and the hermitian conjugate is because Kuno–Okada write an operator that mediates $\mu^+ \rightarrow e^+ \gamma$, and CHK compute amplitudes for $\mu^- \rightarrow e^- \gamma$.

Appendix B: Finding the remaining terms in an EFT calculation

This appendix discusses where to find, in an EFT, the missing parts Eqs. (20), (19), (21), (23) and (25).

B.1. The full-model one-loop contribution

All the terms in Eqs. (20) and (19) are log-enhanced, so they should arise in the one-loop RGEs, however, most of the terms are $\mathcal{O}(1/M^4)$. This is because they arise in matching onto the operator $O_S^{e\mu\mu\mu}$, which would be of dimension eight

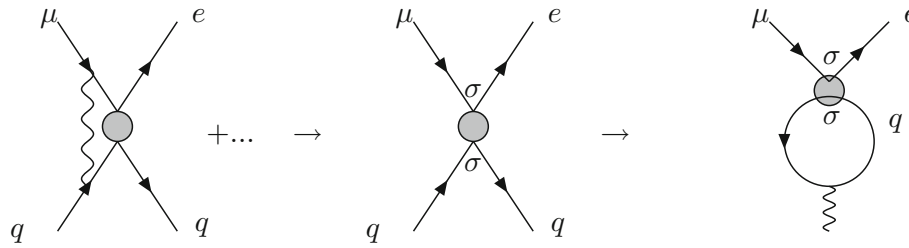


Fig. 4 Obtaining the Barr–Zee diagrams at second order in the one-loop RGEs: the scalar operator (*grey circle* of the left diagram) is mixed to the tensor (*grey circle* of the next diagram) via photon exchange

between the leptons and quarks (only one of the four diagrams is drawn). Then the quark loop of the tensor operator is closed to mix it to the *dipole*

if SU(2) was imposed: $v^2 O_S^{e\mu\mu\mu} = (\bar{L}_e H E_\mu)(\bar{L}_\mu H E_\mu)$. For instance, in matching at the scale M , the two diagrams illustrated on the left in Fig. 5 will contribute to the coefficient of the dimension-eight operator:

$$\frac{^{(8)}C^{e\mu\mu\mu}}{M^4} \sim \frac{[\rho^E]_{e\mu} \Lambda_5 [\rho^E]_{\mu\mu}}{M^4}, \frac{[\rho^E]_{\mu e} \Lambda_6 [Y^E]_{\mu\mu}}{M^4} \quad (55)$$

The operator $O_S^{e\mu\mu\mu}$ can then mix via a penguin to the dipole operator.

Then there is also a light Higgs contribution to the same operator at $\mathcal{O}(1/M^4)$, which can nonetheless by relatively enhanced by $\tan \beta$ with respect to the $\mathcal{O}(1/M^2)$ term. The corresponding diagram contracts two of the diagrams illustrated on the right in Fig. 2, so it has four external H_1 lines.

B.2. The top loop

The two-loop contribution of H and A is \log^2 enhanced, so it arises in the one-loop RGEs. In addition to the previously discussed dimension-six term, that can be suppressed by $\cot \beta$, the third diagram of Fig. 5 matches onto either of two dimension-eight operators $H_1^\dagger H_1 (\bar{L}_e^A E_\mu) \epsilon_{AB} (\bar{Q}_t^B U_t)$ and $(\bar{L}_e H_1 E_\mu) (\bar{Q}_t H_1 U_t)$. The second arises when the H_1 contracted with H_2 is an external leg. In both cases, the coef-

ficient is of order

$$\frac{^{(8)}C^{e\mu t t}}{M^4} \sim \frac{[\rho^E]_{e\mu} \Lambda_6 [Y^U]_{t t}}{M^4} \quad (56)$$

and such a term appears in the full-model calculation of Eq. (21).

The top-loop contribution of the light h , which can contribute a significant part of the $\mu \rightarrow e\gamma$ amplitude, is a two-loop matching contribution at the weak scale.

B.3. The b -loop

The b -loop differs from the top loop, in that the heavy Higgs only contribute at $\mathcal{O}(1/M^4)$, because there is no dimension-six scalar operator for bs that can mix to a tensor. Matching out the heavy Higgs onto the dimension-eight operator $(\bar{L}_\mu H_1 E_e) (\bar{Q}_3 H_1 D_b)$ would give

$$\frac{^{(8)}C^{e\mu b b}}{M^4} \sim \frac{[\rho^E]_{e\mu} \Lambda_5 [\rho^D]_{b b} [\rho^E]_{\mu e} \Lambda_6 [Y^D]_{b b}}{M^4} \quad (57)$$

then between the scales M and m_b , the RGEs of QED mix the dimension-eight scalar operator $(\bar{L}_e H_1 E_\mu) (\bar{Q}_b H_1 D_b)$ to the dimension-eight tensor $(\bar{L}_e H_1 \sigma^{\mu\nu} E_\mu) (\bar{Q}_b H_1 \sigma_{\mu\nu} D_b)$ and the tensor to the dimension-eight dipole, which reproduces that heavy Higgs part of Eq. (23).

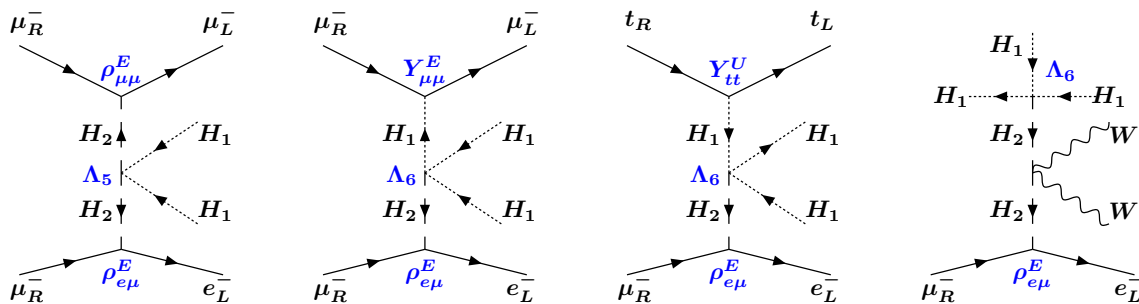


Fig. 5 The left pair of diagrams generate the dimension eight $(\bar{L}_\mu H_1 E_\mu)(\bar{L}_e H_1 E_\mu)$ operator, by matching out the heavy doublet Higgs H_2 (*dashed line*). The third diagram generates a dimension-eight

relative of the $(\bar{Q}_3 U_t)(\bar{L}_e E_\mu)$ operator; see Eq. (56). The last diagram gives a $e\mu WWHHH$ interaction by matching out a heavy doublet Higgs H_2

As in the one-loop contribution, there is an $\mathcal{O}(1/M^4)$ term in the light Higgs exchange amplitude, which can be $\tan\beta$ enhanced, and arises due to two appearances of the dimension-six O_{eH} , with indices $e\mu$ and bb .

B.4. The W loop

Despite the fact that the W -loop can give the dominant contribution to $\mu \rightarrow e\gamma$ in the 2HDM, none of it was obtained in a one-loop EFT calculation using dimension-six operators.

The heavy Higgs part has terms of $\mathcal{O}(\log/M^2)$, $\mathcal{O}(\log^2/M^4)$, and $\mathcal{O}(\log/M^4)$. Consider here the first two: CHK refer to the dimension-six part as a “non-decoupling” contribution, because they take the mixing angle $c_{\beta-\alpha}$ as a free parameter, rather than using the decoupling limit dependence given in Eq. (9). They say this contribution arises from Goldstone loops, so in EFT it could be generated by matching out the heavy doublet onto $O_{eH}^{e\mu}$, followed by the mixing of $O_{eH}^{e\mu}$ to the dipole operator in two-loop RGEs.

Consider now the $\mathcal{O}(\log^2/M^4)$ terms, which should arise at second order in the 1-loop RGEs of dimension-eight operators. These could be generated by tree matching onto $(D_\mu H_1)^\dagger (D^\mu H_1) \bar{L} H_1 E$ as in the last diagram of Fig. 5 (notice this operator is symmetric on interchange of the Lorentz indices of the W), then mixing in the one-loop RGEs of QED to a tensor operator such as $[(D_\mu H_1)^\dagger (D_\nu H_1) - (D_\nu H_1)^\dagger (D_\mu H_1)] (\bar{L} H_1 \sigma^{\mu\nu} E)$, which could then mix to a dimension-eight dipole $(H_1^\dagger H_1) (\bar{L} H_1 \tau^a \sigma^{\mu\nu} E) W_{a,\mu\nu}$.

Finally, there is the light Higgs contribution, which is $\mathcal{O}(1/M^2)$, and arises in two-loop matching at the weak scale.

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