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Stop decay with LSP gravitino in the final state: $\tilde{t}_1 \to \widetilde{G}\, W\, b$

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Abstract In MSSM scenarios where the gravitino is the lightest supersymmetric particle (LSP), and therefore a viable dark matter candidate, the stop $\tilde{t}_1$ could be the next-to-lightest superpartner (NLSP). For a mass spectrum satisfying $m_{\widetilde{G}} + m_t > m_{\tilde{t}_1} > m_{\widetilde{G}} + m_b + m_W$, the stop decay is dominated by the 3-body mode $\tilde{t}_1 \to b\, W\, \widetilde{G}$. We calculate the stop lifetime, including the full contributions from top, sbottom, and chargino as intermediate states. We also evaluate the stop lifetime for the case when the gravitino can be approximated by the goldstino state. Our analytical results are conveniently expressed using an expansion in terms of the intermediate state mass, which helps to identify the massless limit. In the region of low gravitino mass ($m_{\widetilde{G}} \ll m_{\tilde{t}_1}$) the results obtained using the gravitino and goldstino cases turns out to be similar, as expected. However, for higher gravitino masses $m_{\widetilde{G}} \lesssim m_{\tilde{t}_1}$ the results for the lifetime could show a difference of O(100) %.

1 Introduction

The properties of supersymmetric theories, both in the ultraviolet or the infrared domain have had a great impact in distinct domains of particle physics, including model building, phenomenology, cosmology, and formal quantum field theory [1]. In particular, supersymmetric extensions of the Standard Model can include a discrete symmetry, R parity, that guarantees the stability of the lightest supersymmetric particle (LSP) [2,3], which allows the LSP to be a good candidate for dark matter (DM). Candidates for the LSP in the minimal supersymmetric extension of the Standard Model (MSSM) include sneutrinos, the lightest neutralino χ_1^0 and the gravitino $\widetilde{G}$. Most studies has focused on the neutralino LSP [4], while scenarios with the sneutrino LSP seem more constrained [5].

Scenarios with the gravitino LSP as the DM candidate have also been considered [6–9]. In such scenarios, the nature of the next-to-lightest supersymmetric particle (NLSP) determines its phenomenology [10,11].

Possible candidates for NLSP include the lightest neutralino [12,13], the chargino [14], the lightest charged slepton [15], or the sneutrino [16–19]. The NLSP could have a long lifetime, due to the weakness of the gravitational interactions, and this leads to scenarios with a metastable charged sparticle that could have dramatic signatures at colliders [20,21] and it could also affect the Big Bang nucleosynthesis (BBN) [22–26].

Squark species could also be the NLSP, and in such a case natural candidates for NLSP could be the sbottom [27–29] or the lightest stop $\tilde{t}_1$. There are several experimental and cosmological constraints for the scenarios with a gravitino LSP and a stop NLSP that were discussed in [30]. It turns out that the lifetime of the stop $\tilde{t}_1$ could be (very) long, in which case the relevant collider limits are those on (apparently) stable charged particles. For instance the limits available from the Tevatron collider imply that $m_{\tilde{t}_1} > 220$ GeV [31].[1] Thus, knowing in a precise way the stop lifetime is one of the most important issues in this scenario, and this is precisely the goal of our work. In this paper we present a detailed calculation of the stop lifetime, for the kinematical region where the 3-body mode $\tilde{t}_1 \to \widetilde{G}\, W\, b$ dominates.[2] Besides calculating the amplitude using the full wave function for the gravitino, we have also calculated the 3-body decay width (and lifetime) using the gravitino–goldstino equivalence theorem [32]. It should be mentioned that this scenario is not viable within the Constrained Minimal Supersymmetric Standard Model (CMSSM). However, there are regions of parameter space

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[1] The LHC will probably be sensitive to a metastable $\tilde{t}_1$ which is an order of magnitude heavier.

[2] Our calculation of stop lifetime improves the one presented in [30] where an approximation was used for the chargino-mediated contribution that neglected a subdominant term in the expression for the vertex $\chi_i^+\, \widetilde{G}\, W$.

within the Non-Universal Higgs Masses model (NUHM) that pass all collider and cosmological constraints (relic density, nucleosynthesis, CMB mainly) [33].

The organization of our paper is as follows. We begin Sect. 2 by giving some formulas for the stop mass. In Sect. 2.1 we compute the squared amplitudes for the stop decay with gravitino in the final state ($\tilde{t}_1 \to \widetilde{G}\, W\, b$) including the chargino, sbotom and top mediated states. After carefully analyzing the results for the squared amplitude, we have identified a convenient expansion in terms of powers of the intermediate particle mass, which only needs terms of order $O(m_i)$, $O(m_i m_j)$. It is our hope that such expansion could help in order to relate the calculation of the massive and massless cases. In future work we plan to reevaluate this decay using the helicity formalism suited for the spin-$\frac{3}{2}$ case. In Sect. 2.2 we compute the squared amplitudes for the stop decay considering the gravitino–goldstino high energy equivalence theorem that allow us to approximate the gravitino as the derivative of the goldstino. We present in Sect. 3 our numerical results, showing some plots where we reproduce the stop lifetime for the approximate amplitude considered in [30], and compare it with our complete calculation, we also compare these results with the goldstino approximation. Conclusions are included in Sect. 4; finally, all the analytic full results for the squared amplitudes are in Appendices A and B.

2 The stop lifetime within the MSSM

We start by giving some relevant formulas for the input parameters that appear in the Feynman rules of the gravitino within the MSSM. The (2×2) stop mass matrix can be written as

$$\widetilde{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^{2\,\dagger}_{LR} & M^2_{RR} \end{pmatrix}, \tag{1}$$

where the entries take the form

$$\begin{aligned} M^2_{LL} &= M^2_L + m^2_t + \frac{1}{6}\cos 2\beta\,(4m^2_W - m^2_Z), \\ M^2_{RR} &= M^2_R + m^2_t + \frac{2}{3}\cos 2\beta \sin^2\theta_W\, m^2_Z, \\ M^2_{LR} &= -m_t(A_t + \mu\,\cot\beta) \equiv -m_t X_t\,. \end{aligned} \tag{2}$$

The corresponding mass eigenvalues are given by

$$m^2_{\tilde{t}_1} = m^2_t + \frac{1}{2}(M^2_L + M^2_R) + \frac{1}{4}m^2_Z \cos 2\beta - \frac{\Delta}{2} \tag{3}$$

and

$$m^2_{\tilde{t}_2} = m^2_t + \frac{1}{2}(M^2_L + M^2_R) + \frac{1}{4}m^2_Z \cos 2\beta + \frac{\Delta}{2}, \tag{4}$$

where $\Delta^2 = \left(M^2_L - M^2_R + \frac{1}{6}\cos 2\beta(8m^2_W - 5m^2_Z)\right)^2 + 4\,m^2_t |A_t + \mu\cot\beta|^2$. The mixing angle $\theta_{\tilde{t}}$ appears in the mixing matrix that relate the weak basis $(\tilde{t}_L, \tilde{t}_R)$ and the mass eigenstates $(\tilde{t}_1, \tilde{t}_2)$, and it is given by $\tan\theta_{\tilde{t}} = \frac{(m^2_{\tilde{t}_1} - M^2_{LL})}{|M^2_{LR}|}$. From these expressions it is clear that in order to obtain a very light stop one needs to have a very large value for the trilinear soft supersymmetry-breaking parameter [29,34]. It turns out that such a scenario helps to obtain a Higgs mass value in agreement with the mass measured at LHC (125–126 GeV) in a consistent way within the MSSM.

Following Ref. [35], we derived the expressions for all the relevant interactions vertices that appear in the amplitudes for the decay width ($\tilde{t}_1 \to \widetilde{G}\, W\, b$), whose Feynman graphs are shown in Figs. 1–3. We shall need the following vertices:

$$V_1(\tilde{t}_1\, t\, \widetilde{G}) = -\frac{1}{\sqrt{2}M}(\gamma^\nu\gamma^\mu p_\nu)(\cos\theta_{\tilde{t}} P_R + \sin\theta_{\tilde{t}} P_L), \tag{5}$$

$$V_2(t\, b\, W) = \frac{i g_2}{\sqrt{2}}\gamma_\rho P_L, \tag{6}$$

$$V_3(\tilde{t}_1\, W\, \tilde{b}_i) = -\frac{i g_2 \kappa_i}{\sqrt{2}}(p + q_1)_\mu\,, \tag{7}$$

$$V_4(\tilde{b}_i\, b\, \widetilde{G}) = -\frac{1}{\sqrt{2}M}(\gamma^\nu\gamma^\mu q_{2\nu})(a_i P_R + b_i P_L), \tag{8}$$

$$V_5(\tilde{t}_1\, b\, \chi^+_i) = -i(S_i + P_i\gamma_5), \tag{9}$$

$$\begin{aligned} V_6(\chi^+_i\, W\, \widetilde{G}) = -\frac{1}{\sqrt{2}M}\Big(&-\frac{1}{4}\not{p}\gamma^\rho\gamma^\mu(V_{1i}P_R - U_{i1}P_L) \\ &- m_W\gamma^\nu\gamma^\mu(V_{i2}\sin\beta\, P_R + U_{i2}\cos\beta\, P_L)\Big), \end{aligned} \tag{10}$$

where $\tilde{t}_1$ denotes the lightest stop, while t is the top quark and $\widetilde{G}$ denotes the gravitino. With b we denote the bottom quark, while W is the gauge boson, χ^+_i denotes the chargino and $\tilde{b}_i$ is the sbottom. With P_R and P_L corresponding to the left and right projectors, $a_i\, b_i$, S_i, P_i are defined in Appendices A and B, as well the mixing matrices V_{1i}, U_{1i}, which diagonalize the chargino factor.

For the case when the gravitino approximates the goldstino state, the interaction vertices that will appear in the amplitudes for the decay width ($\tilde{t}_1 \to G\, W\, b$) are the following:

$$\widetilde{V}_1(\tilde{t}_1\, t\, G) = \left(\frac{m^2_t - m^2_{\tilde{t}_1}}{2\sqrt{3}Mm_{\widetilde{G}}}\right)(\cos\theta_{\tilde{t}} P_R + \sin\theta_{\tilde{t}} P_L), \tag{11}$$

$$\widetilde{V}_4(\tilde{b}_i\, b\, G) = \left(\frac{m^2_b - m^2_{\tilde{b}_i}}{2\sqrt{3}Mm_{\widetilde{G}}}\right)(a_i P_R + b_i P_L), \tag{12}$$

$$\widetilde{V}_6(\chi^+_i\, W\, G) = -\frac{m_{\chi^+_i}}{\sqrt{6}Mm_{\widetilde{G}}}[\not{p}\gamma^\rho(V_{1i}P_R - U_{i1}P_L)], \tag{13}$$

whereas the vertices $V_2(t\, b\, W)$, $V_3(\tilde{t}_1\, W\, \tilde{b}_i)$, and $V_5(\tilde{t}_1\, b\, \chi^+_i)$ remain the same as in the gravitino case.

2.1 The Amplitude for $\tilde{t}_1 \to \widetilde{G}\, W\, b$

The decay lifetime of the stop was calculated in Ref. [30], where the chargino contribution was approximated by including only the dominant term. Here we shall calculate the full amplitude and determine the importance of the neglected term for the numerical calculation of the stop lifetime. In the following we need to consider the Feynman diagrams shown in Figs. 1, 2, and 3, which contribute to the decay amplitude for $\tilde{t}_1(p) \to \widetilde{G}(p_1)\, W(k)\, b(p_2)$, with the momenta assignment shown in parentheses.

The total amplitude is given by

$$\mathcal{M} = \mathcal{M}_t + \mathcal{M}_{\tilde{b}_i} + \mathcal{M}^C_{\chi_i^+}, \tag{14}$$

where $\mathcal{M}_t$, $\mathcal{M}_{\tilde{b}_i}$, $\mathcal{M}^C_{\chi_i^+}$ denotes the amplitudes for top, sbottom, and chargino mediated diagrams, respectively. In the calculation of Ref. [30], the chargino-mediated diagram included only part of the vertex $V_6(\chi_i^+\, W\, \widetilde{G})$. Here, in order to keep control of the vertex V_6 and therefore $\mathcal{M}^c_{\chi_i^+}$, we shall split $\mathcal{M}^c_{\chi_i^+}$ into two terms as follows:

$$\mathcal{M}^c_{\chi_i^+} = \mathcal{M}^0_{\chi_i^+} + \widetilde{\mathcal{M}}_{\chi_i^+}, \tag{15}$$

where $\mathcal{M}^0_{\chi_i^+}$ denotes the amplitude considered in Ref. [30], which only includes the second term of (10) (with two gamma matrices), while $\widetilde{\mathcal{M}}_{\chi_i^+}$ includes the first term (with 3 gamma matrices). Then the averaged squared amplitude (14) becomes

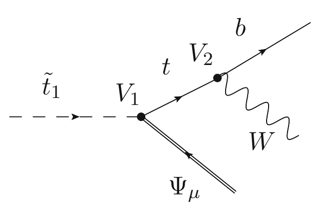

Fig. 1 Top mediated diagram

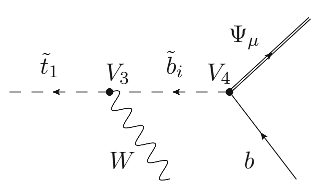

Fig. 2 Sbottom mediated diagram

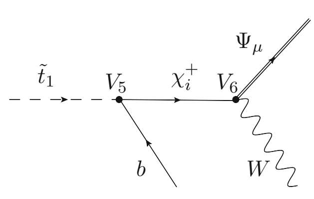

Fig. 3 Chargino mediated diagram

$$\begin{aligned} |\, \overline{\mathcal{M}}\, |^2 &= |\, \mathcal{M}_t\, |^2 + |\, \mathcal{M}_{\tilde{b}_i}\, |^2 + |\, \mathcal{M}^0_{\chi_i^+}\, |^2 + |\, \widetilde{\mathcal{M}}_{\chi_i^+}\, |^2 \\ &\quad + 2\,\mathbf{Re}\Big(\mathcal{M}^{0\,\dagger}_{\chi_i^+} \widetilde{\mathcal{M}}_{\chi_i^+} + \mathcal{M}^\dagger_t \mathcal{M}_{\tilde{b}_i} + \mathcal{M}^\dagger_t \mathcal{M}^0_{\chi_i^+} \\ &\quad + \mathcal{M}^\dagger_t \widetilde{\mathcal{M}}_{\chi_i^+} + \mathcal{M}^\dagger_{\tilde{b}_i} \mathcal{M}^0_{\chi_i^+} + \mathcal{M}^\dagger_{\tilde{b}_i} \widetilde{\mathcal{M}}_{\chi_i^+} \Big). \end{aligned} \tag{16}$$

From the inclusion of the vertices V_i from each graph, we can build each amplitude as follows:

$$\begin{aligned} \mathcal{M}_t &= C_t P_t(q_1)\, \overline{\Psi}_\mu p^\mu (A_t + B_t \gamma_5)(\not{q}_1 + m_t) \\ &\quad \times \gamma^\rho \epsilon_\rho(k) P_L u(p_2), \end{aligned} \tag{17}$$

$$\mathcal{M}_{\tilde{b}_i} = C_{\tilde{b}_i} P_{\tilde{b}_i}(q_2)\, \overline{\Psi}_\mu q_2^\mu (a_i P_l + b_i P_R) p^\rho \epsilon_\rho(k) P_L u(p_2), \tag{18}$$

$$\begin{aligned} \mathcal{M}^0_{\chi_i^+} &= C^0_{\chi_i^+} P_{\chi_i^+}(q_3)\, \overline{\Psi}_\mu \gamma^\rho \epsilon_\rho(k) \gamma^\mu (V_i + \Lambda_i \gamma_5)(\not{q}_3 + m_\chi) \\ &\quad \times (S_i + P_i \gamma_5) u(p_2), \end{aligned} \tag{19}$$

$$\begin{aligned} \widetilde{\mathcal{M}}_{\chi_i^+} &= C_{\chi_i^+} P_{\chi_i^+} \overline{\Psi}_\mu \not{p} \gamma^\rho \gamma^\mu (T_i + Q_i \gamma_5) \epsilon_\rho(k) (\not{q}_3 + m_\chi) \\ &\quad \times (S_i + P_i \gamma_5) u(p_2). \end{aligned} \tag{20}$$

Here $C_t = \frac{g_2}{2M}$, $C_{\tilde{b}_i} = \frac{g_2 \kappa_i}{M}$, $C^0_{\chi_i^+} = \frac{m_W}{M}$, and $C_{\chi_i^+} = \frac{1}{8M}$. We have defined $q_1 \equiv p - p_1$, $q_2 \equiv p - k$, and $q_3 \equiv p - p_2$, and $\epsilon_\rho(k)$ denotes the W polarization vector. Expressions for $A_{\tilde{t}}$, $B_{\tilde{t}}$, a_i, b_i, κ_i, V_i, A_i, S_i, and P_i are presented in Appendices A and B. Then, after performing the evaluation of each expression, we find it convenient to express each squared amplitude as follows:

$$|\, \mathcal{M}_{\psi_a}\, |^2 = C^2_{\psi_a}\, |\, P_{\psi_a}(q_a)\, |^2\, W_{\psi_a \psi_a}, \tag{21}$$

where $\psi_a = (t, \tilde{b}_j, \chi_k^+)$. The functions $P_{\psi_a}(q_a)$ correspond to the propagators factors, thus, for the chargino, $\psi_a = \chi_i^+$, we have

$$P_{\chi_i^+}(q_3) = \frac{1}{q_3^2 - m^2_{\chi_i^+} + i\epsilon}. \tag{22}$$

Similar expressions hold for the sbottom and the top contributions, $P_{\tilde{b}}(q_2)$ and $P_t(q_1)$, respectively. The terms $W_{\psi_a \psi_a}$ include the traces involved in each of the squared amplitudes,

$$\begin{aligned} W_{tt} &= \mathbf{Tr}\Big[M_{\rho\sigma} D_{\mu\nu} p^\mu p^\nu (A_{\tilde{t}} + B_{\tilde{t}} \gamma_5)(\not{q}_1 + m_t) \gamma^\rho \\ &\quad \times P_L \not{p}_2 P_R \gamma^\sigma (\not{q}_1 + m_t)(A_{\tilde{t}} - B_{\tilde{t}} \gamma_5) \Big], \end{aligned} \tag{23}$$

$$\begin{aligned} W_{\tilde{b}_i \tilde{b}_i} &= \mathbf{Tr}\Big[p^\rho p^\sigma M_{\rho\sigma} D_{\mu\nu} q_2^\mu q_2^\nu (R_i + Z_i \gamma_5) \\ &\quad \times \not{p}_2 (R_j - Z_j \gamma_5) \Big], \end{aligned} \tag{24}$$

$$\begin{aligned} W^0_{\chi_i^+ \chi_i^+} &= \mathbf{Tr}\Big[M_{\rho\sigma} D^{\rho\sigma} (V_i + \Lambda_i \gamma_5)(\not{q}_3 + m_\chi) \\ &\quad \times (S_i + P_i \gamma_5) \not{p}_2 \\ &\quad \times (S_j - P_j \gamma_5)(\not{q}_3 + m_\chi)(V_j - \Lambda_j \gamma_5) \Big], \end{aligned} \tag{25}$$

$$W_{\chi_i^+\chi_i^+} = \mathbf{Tr}\Big[M_{\rho\sigma} D_{\mu\nu} \not{p}\gamma^\rho\gamma^\mu(T_i + Q_i\gamma_5)(\not{q}_3 + m_\chi) \times(S_i + P_i\gamma_5)\not{p}_2(S_j - P_j\gamma_5) \times(\not{q}_3 + m_\chi)(T_j - Q_j\gamma_5)\gamma^\nu\gamma^\sigma\not{p}\Big]. \quad (26)$$

For simplicity, we have written the completeness relations for the gravitino field and the vector polarization sum of the boson W as follows:

$$\sum_{\lambda=1}^{3}\epsilon_\rho(\mathbf{k},\lambda)\epsilon_\sigma^*(\mathbf{k},\lambda) = -g_{\rho\sigma} + \frac{k_\rho k_\sigma}{m_W^2} = M_{\rho\sigma} \quad (27)$$

$$\sum_{\tilde\lambda=1}^{3}\Psi_\mu(\mathbf{p}_1,\tilde\lambda)\overline{\Psi}_\nu(\mathbf{p}_1,\tilde\lambda) = -(\not{p}_1 + m_{\tilde G}) \times \left\{\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_{\tilde G}^2}\right)\right.$$

$$\left. -\frac{1}{3}\left(g_{\mu\sigma} - \frac{p_\mu p_\sigma}{m_{\tilde G}^2}\right)\left(g_{\nu\lambda} - \frac{p_\nu p_\lambda}{m_{\tilde G}^2}\right)\gamma^\sigma\gamma^\lambda\right\} = D_{\mu\nu}. \quad (28)$$

The functions $W_{\psi_a\psi_a}$ depend on the scalar products of the momenta p, p_1, p_2, k, q_1, q_2, and q_3. After carefully analyzing the resulting traces (handed with FeynCalc[3] [38,39]) we find that these functions can be written as powers of the intermediate state masses, as follows:

$$W_{\psi_a\psi_a} = \mathrm{w}_{1\psi_a\psi_a} + m_{\psi_a}\mathrm{w}_{2\psi_a\psi_a} + m_{\psi_a}^2\mathrm{w}_{3\psi_a\psi_a}. \quad (29)$$

Full expressions for each function $\mathrm{w}_{i\psi_a\psi_a}$, $\forall\, i = 1, 2, 3$, are included in Appendix A. Furthermore, we also find that the interference terms can be written in a similar form, namely:

$$\mathcal{M}_{\psi_a}^\dagger\mathcal{M}_{\psi_b} = C_{\psi_a}C_{\psi_b}P_{\psi_a}^*(q_a)P_{\psi_b}(q_b)W_{\psi_a\psi_b}. \quad (30)$$

Again, as in the previous case, the function $W_{\psi_a\psi_b}$ includes the traces appearing in the interferences, specifically we have

$$\widetilde{W}_{\chi_{i+}\chi_{i+}} = \mathbf{Tr}\Big[M_{\rho\sigma}D_{\mu\nu}\not{p}\gamma^\rho\gamma^\mu(T_i + Q_i\gamma_5)(\not{q}_3 + m_\chi) \times\not{p}_2(S_i - P_i\gamma_5)(S_j - P_j\gamma_5) \times(\not{q}_3 + m_\chi)(V_j - \Lambda_j\gamma_5)\gamma^\nu g^\sigma\Big], \quad (31)$$

$$W_{t\tilde b_i} = \mathbf{Tr}\Big[M_{\rho\sigma}p^\rho\not{p}_2P_R\gamma^\sigma(\not{q}_1 + m_t)(A_{\tilde t} - B_{\tilde t}\gamma_5) \times p^\mu D_{\mu\nu}q_2^\nu(R_i + Z_i\gamma_5)\Big], \quad (32)$$

$$W_{t\chi_i^+} = \mathbf{Tr}\Big[M_{\rho\sigma}\not{p}_2P_R\gamma^\sigma(\not{q}_1 + m_t)(A_{\tilde t} - B_{\tilde t}\gamma_5) \times p_\mu D^{\mu\rho}(\Lambda_i + V_i\gamma_5)(\not{q}_3 + m_\chi) \times(S_i + P_i\gamma_5)\Big], \quad (33)$$

$$\widetilde{W}_{t\chi_i^+} = \mathbf{Tr}\Big[M_{\rho\sigma}D_{\mu\nu}\not{p}\gamma^\rho\gamma^\mu p^\nu(T_i + Q_i\gamma_5)(\not{q}_3 + m_\chi) \times(S_i + P_i\gamma_5)\not{p}_2P_R\gamma^\sigma \times(\not{q}_1 + m_t)(A_{\tilde t} - B_{\tilde t}\gamma_5)\Big], \quad (34)$$

$$W_{\chi_i^+\tilde b_i} = \mathbf{Tr}\Big[M_{\rho\sigma}p^\rho(p_\nu - k_\nu)\not{p}_2(S_i - P_i\gamma_5) \times(\not{q}_3 + m_\chi)(\Lambda_i - V_i\gamma_5) \times D^{\nu\sigma}(R_j + Z_j\gamma_5)\Big], \quad (35)$$

$$\widetilde{W}_{\chi_i^+\tilde b_i} = \mathbf{Tr}\Big[M_{\rho\sigma}D_{\mu\nu}(p^\nu - k^\nu)(R_i + Z_i\gamma_5) \times\not{p}_2(S_i - P_i\gamma_5)(\not{q}_3 + m_\chi) \times(T_i - Q_i\gamma_5)\gamma^\mu\gamma^\rho\not{p}p^\sigma\Big]. \quad (36)$$

It turns out that the functions $W_{\psi_a\psi_b}$ can be expressed also in powers of the intermediate masses:

$$W_{\psi_a\psi_b} = \mathrm{w}_{1\psi_a\psi_b} + m_{\psi_a}(\mathrm{w}_{2\psi_a\psi_b} + m_{\psi_b}\mathrm{w}_{3\psi_a\psi_b}) + m_{\psi_b}\mathrm{w}_{4\psi_a\psi_b}. \quad (37)$$

The $\mathrm{w}_{j\psi_a\psi_b}$, $\forall\, j = 1, 2, 3, 4$, are as the $\mathrm{w}_{i\psi_a\psi_a}$ 4-momentum's scalar products functions completely determined by the kinematics of our decay. We consider (29) and (37) to be a useful way to present our results as well an easy manner to compute complicated and messy traces. Then the decay width can be obtained after integration of the 3-body phase-space

$$\frac{d\Gamma}{dx\,dy} = \frac{m_{\tilde t_1}^2}{256\,\pi^3}\mid\overline{\mathcal{M}}\mid^2. \quad (38)$$

The variables x and y are defined as $x = 2\frac{E_{\tilde G}}{m_{\tilde t_1}}$ and $y = 2\frac{E_W}{m_{\tilde t_1}}$. Numerical results for the lifetime $\tau = \frac{1}{\Gamma}$ will be presented and discussed in Sect. 3.

2.2 The amplitudes $\tilde t_1 \to G\,W\,b$ with the goldstino approximation

In this section we shall present the calculation of the stop decay using the gravitino–goldstino high energy equivalence theorem [32]. In the high energy limit ($m_{\tilde G} \ll m_{\tilde t_1}$) we could consider the gravitino field (spin $\frac{3}{2}$ particle) as the derivative of the goldstino field (spin $\left(\frac{1}{2}\right)$ particle). We shall consider in this section the same Feynman diagrams, Figs. 1, 2, and 3, as we used in Sect. 2.1, but with the proviso that the gravitino field shall be described by the goldstino fields. Making the replacement $\Psi_{\tilde G} \to i\sqrt{\frac{2}{3}}\frac{1}{m_{\tilde G}}\,\partial_\mu\Psi$ in the gravitino interaction lagrangian, one obtains the effective interaction lagrangian for the goldstino as is shown in [35]. The averaged squared amplitude for the Goldstino is then written as

[3] Progress in automatic calculation of MSSM processes with gravitino has appeared recently [36], some of our results have been checked by the authors of Ref. [37] and they found agreement in the results (private communication).

$$| \overline{\mathcal{M}^G} |^2 = | \mathcal{M}_t^G |^2 + | \mathcal{M}_{\tilde{b}_i}^G |^2 + | \mathcal{M}_{\chi_i^+}^G |^2 + 2\,\mathbf{Re}\left(\mathcal{M}_t^{G\,\dagger}\mathcal{M}_{\tilde{b}_i}^G + \mathcal{M}_t^{G\,\dagger}\mathcal{M}_{\chi_i^+}^G + \mathcal{M}_{\tilde{b}_i}^{G\,\dagger}\mathcal{M}_{\chi_i^+}^G\right). \quad (39)$$

As in Sect. 2.1, we can build the amplitudes from the inclusion of all the vertices into the expressions from each graph, namely:

$$\mathcal{M}_t^G = \widetilde{C}_t P_t(q_1)\overline{\Psi}(A_{\tilde{t}} + B_{\tilde{t}}\gamma_5)(\not{q}_1 + m_t)\gamma^\rho P_L \epsilon_\rho(k) u(p_2), \quad (40)$$

$$\mathcal{M}_{\tilde{b}_i}^G = \widetilde{C}_{\tilde{b}_i} P_{\tilde{b}_i}(q_2)\overline{\Psi}(R_i + Z_i\gamma^5)u(p_2)p^\sigma\epsilon_\sigma(k), \quad (41)$$

$$\mathcal{M}_{\chi_i^+}^G = \widetilde{C}_{\chi_i^+} P_{\chi_i^+}(q_3)\not{p}\gamma^\rho(T_i + Q_i\gamma_5)\overline{\Psi}\epsilon_\rho(k)(\not{q}_3 + m_\chi) \times (S_i + P_i\gamma_5)u(p_2). \quad (42)$$

Here the superindex "G" that appears in the amplitudes (40)–(42) refers to the goldstino amplitudes. The constants appearing in front of the amplitudes are $\widetilde{C}_t = -g_2\left(\frac{m_t^2 - m_{\tilde{t}_1}^2}{4\sqrt{6}Mm_{\tilde{G}}}\right)$, $\widetilde{C}_{\tilde{b}_i} = g_2\kappa_i\left(\frac{m_b^2 - m_{\tilde{b}_i}^2}{4\sqrt{6}Mm_{\tilde{G}}}\right)$ and $\widetilde{C}_{\chi_i^+} = -\frac{m_{\chi_i^+}}{\sqrt{6}Mm_{\tilde{G}}}$. We obtain similar expressions to (21) for the squared amplitudes of the goldstino case, namely:

$$| \mathcal{M}_{\psi_a}^G |^2 = \widetilde{C}_{\psi_a}^2 \,| P_{\psi_a}(q_a) |^2\, W_{\psi_a\psi_a}^G, \quad (43)$$

where the function $W_{\psi_a\psi_a}^G$ includes traces corresponding to the goldstino squared amplitudes, which are given as follows:

$$W_{tt}^G = \mathbf{Tr}\Big[(\not{p}_1 + m_{\tilde{G}})(A_{\tilde{t}} + B_{\tilde{t}}\gamma_5)(\not{q}_1 + m_t)\gamma^\rho P_L M_{\rho\sigma}\not{p}_2 \times P_R\gamma^\sigma(\not{q}_1 + m_t)(A_{\tilde{t}} - B_{\tilde{t}}\gamma_5)\Big], \quad (44)$$

$$W_{\tilde{b}_i\tilde{b}_i}^G = \mathbf{Tr}\Big[p^\rho p^\sigma M_{\rho\sigma} \times (\not{p}_1 + m_{\tilde{G}})(B_i + Z_i\gamma_5)\not{p}_2(B_j - Z_j\gamma_5)\Big], \quad (45)$$

$$W_{\chi_i^+\chi_i^+}^G = \mathbf{Tr}\Big[M_{\rho\sigma}(\not{p}_1 + m_{\tilde{G}})\not{p}\gamma^\rho(T_i + Q_i\gamma_5)(\not{q}_3 + m_\chi) \times (S_i + P_i\gamma_5)\not{p}_2(S_j - P_j\gamma_5)(\not{q}_3 + m_\chi) \times (T_j - Q_j\gamma_5)\gamma^\sigma\not{p}\Big], \quad (46)$$

the functions $W_{\psi_a\psi_a}^G$ depend on the scalar products of the momenta p, p_1, p_2, k, q_1, q_2, and q_3, these functions will also be written as powers of the intermediate state masses, namely:

$$W_{\psi_a\psi_a}^G = \mathrm{w}_{1\psi_a\psi_a}^G + m_{\psi_a}\mathrm{w}_{2\psi_a\psi_a}^G + m_{\psi_a}^2\mathrm{w}_{3\psi_a\psi_a}^G. \quad (47)$$

All the full expressions for each function $\mathrm{w}_{i\psi_a\psi_a}^G$, $\forall\, i = 1, 2, 3$, can be found in Appendix B. Again, the interference terms for the goldstino are also written in the form

$$\mathcal{M}_{\psi_a}^{G\,\dagger}\mathcal{M}_{\psi_b}^G = \widetilde{C}_{\psi_a}\widetilde{C}_{\psi_b}P_{\psi_a}^*(q_a)P_{\psi_b}(q_b)W_{\psi_a\psi_b}^G. \quad (48)$$

The functions $W_{\psi_a\psi_b}$ correspond to the traces involved in the interference terms, i.e.

$$W_{t\tilde{b}_i}^G = \mathbf{Tr}\Big[M_{\rho\sigma}\not{p}_2 P_R\gamma^\sigma(\not{q}_1 + m_t)(A_{\tilde{t}} - B_{\tilde{t}}\gamma_5) \times (\not{p}_1 + m_{\tilde{G}})(B_i + Z_i\gamma_5)p^\rho\Big], \quad (49)$$

$$W_{t\chi_i^+}^G = \mathbf{Tr}\Big[M_{\rho\sigma}(\not{p}_1 + m_{\tilde{G}})\not{p}\gamma^\rho(T_i + Q_i\gamma_5)(\not{q}_3 + m_\chi) \times (S_i + P_I\gamma_5) \times \not{p}_2 P_R\gamma^\sigma(\not{q}_1 + m_t)(A_{\tilde{t}} - B_{\tilde{t}}\gamma_5)\Big], \quad (50)$$

$$W_{\chi_i^+\tilde{b}_i}^G = \mathbf{Tr}\Big[M_{\rho\sigma}\not{p}_2(S_i - P_i\gamma_5)(\not{q}_3 + m_\chi)(T_i - Q_i\gamma_5)\gamma^\rho\not{p} \times (\not{p}_1 + m_{\tilde{G}})(R_j + Z_j\gamma_5)p^\sigma\Big]. \quad (51)$$

The $W_{\psi_a\psi_b}^G$ functions also are expressed as powers of the intermediate masses:

$$W_{\psi_a\psi_b} = \mathrm{w}_{1\psi_a\psi_b}^G + m_{\psi_a}(\mathrm{w}_{2\psi_a\psi_b}^G + m_{\psi_b}\mathrm{w}_{3\psi_a\psi_b}^G) + m_{\psi_b}\mathrm{w}_{4\psi_a\psi_b}^G. \quad (52)$$

The full expressions for $\mathrm{w}_{j\psi_a\psi_b}^G$, $\forall\, j = 1, 2, 3, 4$, can be found in Appendix B.

3 Numerical results

The decay width is obtained by integrating the differential decay width over the dimensionless variables x, y which have limits given by $2\mu_G < x < 1 + \mu_{\tilde{G}} - \mu_W$ with $\mu_i = \frac{m_i^2}{m_{\tilde{t}_1}^2}$ and

$$y_\pm = \frac{(2 - x)\left(\mu_{\tilde{G}} + \mu_W - x + 1\right) \pm \sqrt{x^2 - 4\mu_{\tilde{G}}}\left(\mu_{\tilde{G}} - \mu_W - x + 1\right)}{2\left(\mu_{\tilde{G}} - x + 1\right)}, \quad (53)$$

$$\Gamma = \int_{2\mu_G}^{1+\mu_G-\mu_W}\int_{y_-}^{y_+}\frac{m_{\tilde{t}_1}^2}{256\pi^3}\,| \overline{\mathcal{M}} |^2\,\mathrm{d}y\,\mathrm{d}x. \quad (54)$$

After integrating numerically the expressions for the differential decay width, we obtain the values for the decay width, for a given set of parameters. We consider two values for the stop mass, $m_{\tilde{t}_1} = 200$ GeV and $m_{\tilde{t}_1} = 350$ GeV, we also fix the chargino mass to be $m_{\chi_i^+} = 200, 500$ GeV, while the sbottom mass is fixed to be $m_{\tilde{b}_i} = 300, 500$ GeV.

In Figs. 4 and 5 we show the lifetime of the stop, as a function of the gravitino mass, within the ranges 200–250 GeV for the case with $m_{\tilde{t}_1} = 350$ GeV, and 50–100 GeV for $m_{\tilde{t}_1} = 100$ GeV. We show the results for the case when one uses the full expression for chargino–gravitino–W vertex (circles), as well as the case when the partial inclusion of such vertex, as was done in [30] (triangles) and in the limit of the

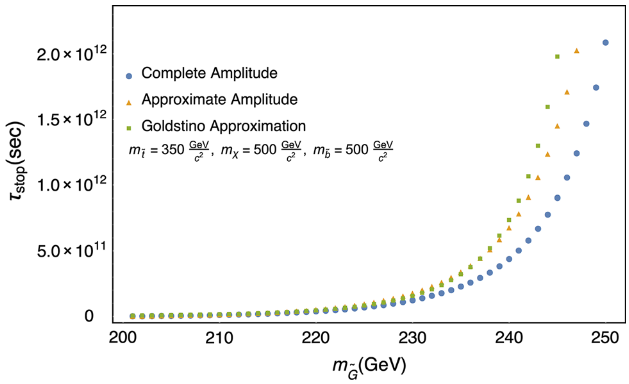

Fig. 4 Stop lifetime 1

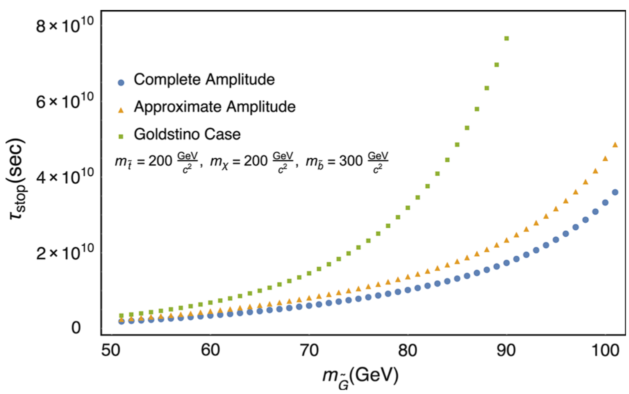

Fig. 5 Stop lifetime 2

goldstino approximation (squares). We noticed that for low gravitino masses ($m_{\tilde{G}} \to 0$) the full gravitino result becomes almost indistinguishable from the goldstino case, while the partial gravitino result has also similar behavior. For large gravitino masses ($m_{\tilde{G}} \cong m_{\tilde{t}_1}$) the results for the stop lifetime using the full gravitino and goldstino approximation could be very different, up to $O(50\ \%)$ different.

On the other hand, the values for the stop lifetime using the full gravitino and partial gravitino limit are very similar for low gravitino masses, while for the largest allowed masses the difference in the results is at most of order $O(50\ \%)$. The value of the lifetime obtained in all theses cases turns out to be of order 10^7–10^{12} s, which results in a scenario with large stop lifetime that has very special signatures both at colliders and has also important implications for cosmology, as discussed in Ref. [30].

For instance, regarding the effect on BBN, the stop $\tilde{t}_1$ has to form quasi-stable sbaryons ($\tilde{t}_1 qq$) and mesinos ($\tilde{t}_1 \bar{q}$), whose late decays could have affected the light element abundance obtained in BBN, while negatively charged stop sbaryons and mesinos could contribute to lower the Coulomb barrier for nuclear fusion process occurring in the BBN epoch. However, as argued in [30] the great majority of stop antisbaryons would have annihilated with ordinary baryons to make stop antimesinos and most stop mesinos and antimesinos would have annihilated. The only remnant would have been neutral mesinos which would be relatively innocuous, despite their long lifetime because they would not have important bound state effects. Further discussion of BBN issues of Ref. [33] divide the stop lifetime into regions that could have an effect, but the larges ones (which represent our results) do not pose problems for the success of BBN. Then, regarding the effect of late stop decay on the Cosmic Microwave Background (CMB), we have included some comments in the text, to estimate the main effects. The arguments read as follows. Very long lifetimes ($\tau > 10^{12}$ s) would have been excluded if one uses the approximate results of Ref. [40], which present bounds on the lifetime τ (for the case when stau is the NLSP) using the constraint in the chemical potential $\mu < 9 \times 10^{-5}$. However, it was discussed in Ref. [41] that a more precise calculation reduces the excluded region for lifetimes, ending at about $\tau \sim 10^{11}$ s–10^{12} s. Thus, the region with very large stop lifetimes could also survive. Specific details that change from the stop decay (3-body) as compared with stau decays (2-body), such as the energy release or stop hadronization, will affect the calculation, but the numerical evaluation of such an effect is beyond the scope of our paper.

4 Conclusions

In this paper we have calculated the stop $\tilde{t}_1$ lifetime in MSSM scenarios where the massive gravitino is the lightest supersymmetric particle (LSP), and therefore is a viable dark matter candidate. The lightest stop $\tilde{t}_1$ corresponds to the next-to-lightest supersymmetric particle (NLSP). We have focused on the kinematical domain $m_{\tilde{G}} + m_t > m_{\tilde{t}_1} > m_{\tilde{G}} + m_b + m_W$, where the stop decay width is dominated by the mode $\tilde{t}_1 \to b\, W\, \tilde{G}$.

The amplitude for the full calculation of the stop 3-body decay width includes contributions from top, sbottom, and chargino as intermediate states. We have considered the full chargino–gravitino vertex, which improves the calculation presented in Ref. [30]. Besides performing the full calculation with a massive gravitino, we have also evaluated the stop decay lifetime for the limit when the gravitino can be approximated by the goldstino state. Our analytical results are conveniently expressed, in both cases, using an expansion in terms of the intermediate state mass, which helps in order to identify the massless limit.

We find that the results obtained with the full chargino vertex are not very different from the approximation used in Ref. [30], in fact they only differ approximately by 50 %. The comparison of the full numerical results with the ones obtained for the goldstino approximation, show that in the limit of low gravitino mass ($m_{\tilde{G}} \ll m_{\tilde{t}_1}$) there is not a significant difference in values of the stop lifetime obtained from each method. However, for $m_{\widetilde{G}} \lesssim m_{\tilde{t}_1}$ the difference in lifetime could be as high as 50 %. Numerical results for the stop lifetime give a value of order 10^7–10^{12} s, which makes the stop behave like a quasi-stable state, which leaves special imprints for LHC search. Our calculation shows that the inclusion of the neglected term somehow gives a decrease in the lifetime of the stop. However, it should be pointed out that the region of parameter space corresponds to the NUHM model.

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A Analytical expressions for amplitudes with gravitino in the final state

In this appendix we present explicitly the full results for the 10 $\mathrm{w}_{\psi_a\psi_a}$ functions that arose from a convenient way to express the large traces that appear in the squared amplitudes (21), as well as the 18 $\mathrm{w}_{\psi_a\psi_b}$ functions in the interferences (30) of the 3-body stop $\tilde{t}_1$ decay with gravitino in the final state. First, we shall present the contributions for the squared amplitudes, then we shall present the interferences.

A.1 Top contribution

For the averaged squared amplitude of the top quark contribution, the functions w_{1tt}, w_{2tt}, and w_{3tt} are

$$\begin{aligned}\mathrm{w}_{1tt} =& \frac{4a_1h_1}{3m_W^2 m_{\widetilde{G}}^2}\Big(f_2(m_W^2(6m_{\widetilde{G}}^2+2h_5m_{\tilde{t}_1}^2-q_1^2)\\&+6f_3m_W^2+4f_3^2)-2f_1\big(m_{\widetilde{G}}^2\big(-\big(4f_3+3m_W^2\big)\big)\\&+f_2(4f_3+3m_W^2)+f_3q_1^2\big)+(q_1^2-2m_{\widetilde{G}}^2)\\&\times(m_W^2m_{\widetilde{G}}^2+3f_3m_W^2+2f_3^2)\\&+4f_1^2(f_2-m_{\widetilde{G}}^2)-2m_W^2m_{\widetilde{G}}^2h_5m_{\tilde{t}_1}^2-4f_2^2m_W^2\Big), \quad (55)\\ \mathrm{w}_{2tt} =& \frac{8a_2h_1}{3m_W^2 m_{\widetilde{G}}}(m_W^2(m_{\widetilde{G}}^2+m_{\tilde{t}}^2-2f_2)-f_1(4f_3+3m_W^2)\\&+3f_3m_W^2+2f_1^2+2f_3^2), \quad (56)\\ \mathrm{w}_{3tt} =& \frac{4a_3h_1}{3m_W^2 m_{\widetilde{G}}^2}(m_W^2(f_2-m_{\widetilde{G}}^2)-3f_3m_W^2-2f_3^2+2f_1f_3). \\& \quad (57)\end{aligned}$$

The functions f_1, f_2, and f_3 are functions of the variables x and y that were defined previously in Sect. 3, they are $f_1=\frac{m_{\tilde{t}_1}^2}{2}y$, $f_2=\frac{m_{\tilde{t}_1}^2}{2}x$, $f_3=\frac{m_{\tilde{t}_1}^2}{2}(-1-\mu_{\widetilde{G}}-\mu_W+x+y)$, with $\mu_{\widetilde{G}}=\frac{m_{\widetilde{G}}^2}{m_{\tilde{t}_1}^2}$ and $\mu_W=\frac{m_W^2}{m_{\tilde{t}_1}^2}$. We have also used in (55)–(57) the substitutions $h_1=(f_2^2-m_{\widetilde{G}}^2m_{\tilde{t}_1}^2)$, $a_1=(A_{\tilde{t}}-B_{\tilde{t}})^2$, $a_2=A_{\tilde{t}}^2-B_{\tilde{t}}^2$, and $a_3=(A_{\tilde{t}}+B_{\tilde{t}})^2$, with $A_{\tilde{t}}=\cos\theta_{\tilde{t}}+\sin\theta_{\tilde{t}}$ and $B_{\tilde{t}}=\cos\theta_{\tilde{t}}-\sin\theta_{\tilde{t}}$.

A.2 Sbottom contribution

For the averaged squared amplitude of the squark sbottom contribution, the function $\mathrm{w}_{1\tilde{b}_i\tilde{b}_i}$ is

$$\mathrm{w}_{1\tilde{b}_i\tilde{b}_i}=\frac{8D_{ij1}h_2h_3((f_2-f_3)^2-q_2^2m_{\widetilde{G}}^2)}{3m_W^2m_{\widetilde{G}}^2}. \quad (58)$$

Here $h_2=f_2-f_3-m_{\widetilde{G}}^2$ and $h_3=f_1^2-m_W^2m_{\tilde{t}_1}^2$. We have made in (58) the following substitution: $a_iP_R+b_iP_L=\frac{1}{2}(R_i+Z_i\gamma_5)$ such that $D_{ij1}=R_iR_j+Z_iZ_j$, with $R_i=a_i+b_i$, $Z_i=a_i-b_i$, $R_j=a_j+b_j$, and $Z_j=a_j-b_j$, and with $a_i=(\sin\theta_{\tilde{b}},\cos\theta_{\tilde{b}})$, $b_i=(\cos\theta_{\tilde{b}},-\sin\theta_{\tilde{b}})$ and $\kappa_i=(\cos\theta_{\tilde{t}}\cos\theta_{\tilde{b}},-\cos\theta_{\tilde{t}}\cos\theta_{\tilde{b}})$.

A.3 Partial chargino contribution ($\mathcal{M}^0_{\chi_i^+}$)

For the averaged squared amplitude of the chargino contribution, the functions $\mathrm{w}^0_{k\chi_i^+\chi_i^+}$, $\forall k=1,2,3$, are as follows:

$$\begin{aligned}\mathrm{w}^0_{1\chi_i^+\chi_i^+} =& -\frac{8\Sigma_{\mathrm{ij1}}h_4}{3m_W^2m_{\widetilde{G}}^2}\Big((m_{\widetilde{G}}^2+f_3)(2(m_{\widetilde{G}}^2+m_W^2)+4f_3-q_3^2)\\&+f_2(-2m_{\widetilde{G}}^2-2f_3+q_3^2)-2f_1(m_{\widetilde{G}}^2+f_3)\Big), \quad (59)\\ \mathrm{w}^0_{2\chi_i^+\chi_i^+} =& -\frac{8h_4(\Sigma_{\mathrm{ij1}}+\Sigma_{\mathrm{ij2}})(h_5-f_1-f_2)}{3m_W^2m_{\widetilde{G}}}, \quad (60)\\ \mathrm{w}^0_{3\chi_i^+\chi_i^+} =& \frac{8\Sigma_{\mathrm{ij3}}h_4h_2}{3m_W^2m_{\widetilde{G}}^2}, \quad (61)\end{aligned}$$

with $h_4=2m_W^2m_{\widetilde{G}}^2+f_3^2$ and $h_5=m_{\widetilde{G}}^2+2f_3+m_W^2$, we have also used the following substitutions: $\Sigma_{ij1}=(S_iS_j+P_iP_j)(V_iV_j-\Lambda_i\Lambda_j)-(S_iP_j+P_iS_j)(\Lambda_iV_j-V_i\Lambda_j)$, $\Sigma_{ij2}=(S_iS_j+P_iP_j)(V_iV_j-\Lambda_i\Lambda_j)+(S_iP_j+P_iS_j)(\Lambda_iV_j-V_i\Lambda_j)$, $\Sigma_{ij3}=(S_iS_j+P_iP_j)(V_iV_j+\Lambda_i\Lambda_j)+(S_iP_j+P_iS_j)(\Lambda_iV_j+V_i\Lambda_j)$, with $V_i=V_{i2}\sin\beta+U_{i2}\cos\beta$ and $\Lambda_i=V_{i2}\sin\beta-U_{i2}\cos\beta$. For the low-to-moderate range of $\tan\beta$ we have

$$S_1=\frac{1}{2}\left(-g_2\cos\phi_L+\frac{g_2m_t\sin\phi_L\sin\theta_{\tilde{t}}}{\sqrt{2}m_W\sin\beta}\right), \quad (62)$$

$$P_1=\frac{1}{2}\left(-g_2\cos\phi_L-\frac{g_2m_t\sin\phi_L\sin\theta_{\tilde{t}}}{\sqrt{2}m_W\sin\beta}\right), \quad (63)$$

where $\cos\phi_L, \pm\sin\phi_L$ are elements of the matrix V that diagonalize the chargino mass matrix, and expressions for S_2 and P_2 may be obtained by replacing $\cos\phi_L \to -\sin\phi_L$ and $\sin\phi_L \to \cos\phi_L$ in (62) and (63).

A.4 Full chargino contribution ($\widetilde{\mathcal{M}}_{\chi_i^+}$)

For the averaged squared amplitude $\widetilde{\mathcal{M}}_{\chi_i^+}$ of the chargino contribution, the functions $\mathrm{w}_{k\chi_i^+\chi_i^+}, \forall k = 1, 2, 3$, are:

$$\mathrm{w}_{1\chi_i^+\chi_i^+} = q_3^2 P_{\mathrm{ij}1} h_7, \tag{64}$$

$$\mathrm{w}_{2\chi_i^+\chi_i^+} = \frac{16 m_{\tilde{G}}(P_{\mathrm{ij}1} + P_{\mathrm{ij}2})}{3m_W^2} \times (h_5 - f_1 - f_2)(2f_1^2 - 5m_W^2 m_{\tilde{t}_1}^2), \tag{65}$$

$$\mathrm{w}_{3\chi_i^+\chi_i^+} = P_{\mathrm{ij}2} h_7, \tag{66}$$

where we have defined

$$h_7 = \frac{16}{3m_W^2 m_{\tilde{G}}^2}\Big(2f_1(f_2(2(f_2 - f_3)f_3 - m_{\tilde{G}}^2(2f_3 + m_W^2)) - f_3 m_{\tilde{G}}^2 m_{\tilde{t}_1}^2) + h_2(2f_2^2 m_W^2 - m_{\tilde{t}_1}^2 h_6) + f_1^2(4f_2 m_{\tilde{G}}^2 - 2m_{\tilde{G}}^4)\Big). \tag{67}$$

With $h_6 = 3m_W^2 m_{\tilde{G}}^2 + 2f_3^2$, we have used the substitution $V_{i1}P_R - U_{i1}P_L = T_i + Q_i\gamma_5$ in the first term of the interaction vertex $V_6(\chi_i^+ W \tilde{G})$, we have also made the following substitutions in the functions (64)–(66):

$$P_{ij1} = (S_i S_j + P_i P_j)(T_i T_j + Q_i Q_j) - (S_i P_j + P_i S_j) \times (T_i Q_j + Q_i T_j), \tag{68}$$

$$P_{ij2} = (S_i S_j + P_i P_j)(T_i T_j + Q_i Q_j) + (S_i P_j + P_i S_j) \times (T_i Q_j + Q_i T_j). \tag{69}$$

A.5 Interference terms

$\mathcal{M}_{\chi_i^+}^{0\dagger}\widetilde{\mathcal{M}}_{\chi_i^+}$ interference

For the interference term $\mathcal{M}_{\chi_i^+}^{0\dagger}\widetilde{\mathcal{M}}_{\chi_i^+}$, the $\widetilde{\mathrm{w}}_{k\chi_i^+\chi_i^+}$ functions, $\forall k = 1, 2, 3, 4$, are

$$\widetilde{\mathrm{w}}_{1\chi_i+\chi_i+} = \frac{16 S_{\mathrm{ij}1}}{3m_W^2 m_{\tilde{G}}}\Big(f_1^2(m_{\tilde{G}}^2(8f_3 + 2m_W^2 - q_3^2) + 4f_3^2) + f_1\big(4f_2 f_3(2m_{\tilde{G}}^2 + f_3 - q_3^2) - (m_{\tilde{G}}^2(4f_3 + m_W^2) + 2f_3^2)(2(m_{\tilde{G}}^2 + m_W^2) + 4f_3 - q_3^2)\big) + f_2 m_W^2(-m_{\tilde{G}}^2(2f_2 - 4f_3 - 2m_W^2 + q_3^2) + 2m_{\tilde{G}}^4 + f_2 q_3^2) + f_3^2 q_3^2 m_{\tilde{t}_1}^2\Big), \tag{70}$$

$$\widetilde{\mathrm{w}}_{2\chi_i+\chi_i+} = \frac{16(S_{\mathrm{ij}2} + S_{\mathrm{ij}3})}{3m_W^2 m_{\tilde{G}}^2}\Big(m_{\tilde{t}}^2(-f_3 m_{\tilde{G}}^2(f_3 - 3m_W^2) + 2m_W^2 m_{\tilde{G}}^4 - 2f_3^3) + 2f_1\big(f_3 m_{\tilde{G}}^2 h_5 + 2f_2(m_W^2 m_{\tilde{G}}^2 + f_3^2)\big) - f_2 m_W^2(5m_{\tilde{G}}^2 h_5 + f_2(2f_3 - 3m_{\tilde{G}}^2)) - f_1^2(4f_3 m_{\tilde{G}}^2 + m_{\tilde{G}}^4)\Big), \tag{71}$$

$$\widetilde{\mathrm{w}}_{3\chi_i+\chi_i+} = -\frac{16 S_{\mathrm{ij}4}}{3m_W^2 m_{\tilde{G}}}\Big(f_2 m_W^2(f_2 - m_{\tilde{G}}^2) + f_1(m_{\tilde{G}}^2(4f_3 + m_W^2) + 2f_3(f_3 - 2f_2)) - f_1^2 m_{\tilde{G}}^2 + f_3^2 m_{\tilde{t}_1}^2\Big), \tag{72}$$

In order to have control in the calculations with huge expressions, we have made the following substitutions in the functions (70)–(72):

$$S_{ij1} = (S_i S_j + P_i P_j)(T_i V_j + Q_i \Lambda_j) - (S_i P_j + P_i S_j) \times (Q_i V_j + T_i \Lambda_j), \tag{73}$$

$$S_{ij2} = (S_i S_j + P_i P_j)(T_i \Lambda_j + Q_i V_j) - (S_i P_j + P_i S_j) \times (Q_i \Lambda_j + T_i V_j), \tag{74}$$

$$S_{ij3} = (S_i S_j + P_i P_j)(T_i \Lambda_j + Q_i V_j) + (S_i P_j + P_i S_j) \times (Q_i V_j + T_i \Lambda_j), \tag{75}$$

$$S_{ij4} = (S_i S_j + P_i P_j)(T_i V_j + Q_i \Lambda_j) + (S_i P_j + P_i S_j) \times (Q_i V_j + T_i \Lambda_j). \tag{76}$$

$\mathcal{M}_{\chi_i^+}^{0\dagger}\mathcal{M}_{\tilde{b}_i}$ interference

For the interference term $\mathcal{M}_{\chi_i^+}^{0\dagger}\mathcal{M}_{\tilde{b}_i}$, the functions $\mathrm{w}_{j\chi_i^+\tilde{b}_i}$, $\forall j = 1, 2$, are

$$\mathrm{w}_{1\chi_i^+\tilde{b}_i} = -\frac{4\eta_{\mathrm{ij}1}}{3m_W^2 m_{\tilde{G}}}\Big(-f_1\big(m_{\tilde{t}}^2(m_W^2 m_{\tilde{G}}^2 + f_3^2) - 2f_3 f_2(m_{\tilde{G}}^2 + 3f_3 - m_W^2) + 2f_3^2 h_5 + f_2^2(2f_3 - m_W^2)\big) + m_W^2\big(m_{\tilde{t}}^2 bigl(m_{\tilde{G}}^2(-2f_2 + 4f_3 + m_W^2) + 2m_{\tilde{G}}^4 + f_3^2) + f_2(-f_2(2m_{\tilde{G}}^2 + 6f_3 + m_W^2) + 2f_3 h_5 + 2f_2^2)\big) + f_1^2(-m_{\tilde{G}}^2(-2f_2 + 4f_3 + m_W^2) - 2m_{\tilde{G}}^4 + 2f_3^2) + f_1^3 m_{\tilde{G}}^2\Big), \tag{77}$$

$$\mathrm{w}_{2\chi_i^+\tilde{b}_i} = \frac{8\eta_{\mathrm{ij}2} h_2}{3m_W^2 m_{\tilde{G}}^2}(m_W^2(m_{\tilde{G}}^2 h_5 m_{\tilde{t}_1}^2 + f_2(f_3 - f_2)) - f_1^2 m_{\tilde{G}}^2 + f_1(f_2 - f_3)f_3). \tag{78}$$

In the functions (77) and (78), we have made the following substitutions:

$$\eta_{ij1} = R_j(\Lambda_i S_i - V_i P_i) + Z_j(\Lambda_i P_i - V_i S_i), \quad (79)$$
$$\eta_{ij2} = R_j(\Lambda_i S_i + V_i P_i) + Z_j(\Lambda_i P_i + V_i S_i). \quad (80)$$

$\mathcal{M}_t^\dagger \mathcal{M}^0_{\chi_i^+}$ interference

For the interference term $\mathcal{M}_t^\dagger \mathcal{M}^0_{\chi_i^+}$ the functions $\mathrm{w}_{jt\chi_i^+}$, $\forall j = 1, 2, 3, 4$, are

$$\begin{aligned}
\mathrm{w}_{1t\chi_i^+} &= \frac{2\Omega_{\mathrm{i}1}}{3m_W^2 m_{\tilde G}}\Big(2f_1\big(m_{\tilde t}^2(m_{\tilde G}^2(f_3+2m_W^2)-f_3^2)\\
&-f_2(m_{\tilde G}^2 h_8 + 2f_3(3f_3+m_W^2))\\
&-f_3 m_{\tilde G}^2 h_5 + f_2^2(2f_3-3m_W^2)\big)\\
&+m_{\tilde t}^2\big(-m_W^2 m_{\tilde G}^2(-4f_2+6f_3+m_W^2)\\
&-4m_W^2 m_{\tilde G}^4 + f_3((2f_2+f_3)m_W^2+2f_3(f_3-f_2))\big)\\
&+f_1^2(m_{\tilde G}^2(-4f_2+10f_3+3m_W^2)+4m_{\tilde G}^4+8f_2f_3)\\
&+f_2\big(2f_3^2(m_{\tilde G}^2+m_W^2)\\
&+f_2m_W^2(4m_{\tilde G}^2-4f_2+m_W^2)+4f_2f_3m_W^2+4f_3^3\big)\\
&-6f_1^3 m_{\tilde G}^2\Big), \qquad (81)\\
\mathrm{w}_{2t\chi_i^+} &= \frac{4\Omega_{\mathrm{i}2}}{3m_W^2 m_{\tilde G}^2}\Big(-m_{\tilde t}^2(f_2(f_3^2-2m_W^2 m_{\tilde G}^2)+2m_W^2 m_{\tilde G}^4)\\
&+f_1\big(f_3 m_{\tilde G}^2 m_{\tilde t_1}^2 + m_{\tilde G}^4(m_W^2-f_3)\\
&-f_2 m_{\tilde G}^2(2f_3+m_W^2)+2f_2^2 f_3\big)\\
&+f_2(m_{\tilde G}^2((2f_2-f_3)m_W^2+f_3^2)+f_2(f_3-2f_2)m_W^2)\\
&+2f_1^2 m_{\tilde G}^2(m_{\tilde G}^2-f_2)\Big), \qquad (82)\\
\mathrm{w}_{3t\chi_i^+} &= \frac{2\Omega_{\mathrm{i}3}}{3m_W^2 m_{\tilde G}}\Big(m_{\tilde t}^2 h_9 - f_1 m_{\tilde G}^2(f_1-2f_3+2m_W^2)\\
&-3f_2^2 m_W^2 + 2f_2f_3\left(m_W^2-f_3\right)\Big), \qquad (83)\\
\mathrm{w}_{4t\chi_i^+} &= -\frac{2\Omega_{\mathrm{i}4}}{3m_W^2 m_{\tilde G}^2}\Big(-m_{\tilde G}^2 m_{\tilde t_1}^2(4f_3m_W^2+h_9)\\
&-2f_1\big(f_3 m_{\tilde G}^2 h_5 + f_2(2f_3^2-m_W^2 m_{\tilde G}^2)\big)\\
&+2f_2f_3^2 h_5 + f_2^2 m_W^2(3m_{\tilde G}^2+2f_3)+f_1^2(4f_3m_{\tilde G}^2+m_{\tilde G}^4)\Big). \qquad (84)
\end{aligned}$$

Here $h_8 = 2f_3 - m_W^2$ and $h_9 = 3m_W^2 m_{\tilde G}^2 + f_3^2$. We have made the following substitutions in the functions (81)–(84): $\Omega_{i1} = (A_{\tilde t} - B_{\tilde t})(S_i - P_i)(\Lambda_i + V_i)$, $\Omega_{i2} = (A_{\tilde t} - B_{\tilde t})(S_i - P_i)(\Lambda_i - V_i)$, $\Omega_{i3} = (A_{\tilde t} + B_{\tilde t})(S_i - P_i)(\Lambda_i - V_i)$, and $\Omega_{i4} = (A_{\tilde t} + B_{\tilde t})(S_i - P_i)(\Lambda_i + V_i)$.

$\mathcal{M}_t^\dagger \mathcal{M}_{\tilde b_i}$ interference

For the interference term $\mathcal{M}_t^\dagger \mathcal{M}_{\tilde b_i}$, the functions $\mathrm{w}_{jt\tilde b_i}$, $\forall j = 1, 2$, are

$$\begin{aligned}
\mathrm{w}_{1t\tilde b_i} &= \frac{2\,(\Delta_{\mathrm{i}1}+\Delta_{\mathrm{i}2})}{3m_W^2 m_{\tilde G}^2}\Big(f_1^2\big(2f_2m_{\tilde G}^2(-2m_{\tilde t_1}^2+h_8)\\
&-2m_{\tilde G}^2 m_{\tilde t_1}^2(f_3-2m_{\tilde G}^2)+m_{\tilde G}^4 h_8\\
&-4f_2^2(m_{\tilde G}^2+f_3)+4f_2^3\big)+2f_1\big(m_{\tilde t_1}^2\big(-m_{\tilde G}^4(f_3-2m_W^2)\\
&+f_3m_W^2m_{\tilde G}^2+f_2f_3(f_3-f_2)\big)+f_3m_{\tilde G}^2m_{\tilde t_1}^4\\
&+f_2\big(f_3m_{\tilde G}^2(f_2-f_3+m_W^2)\\
&-m_W^2m_{\tilde G}^4+f_2(f_3-f_2)m_W^2\big)\big)\\
&+m_W^2\big(m_{\tilde t_1}^2\big(m_{\tilde G}^4(2f_2+m_W^2)\\
&+(4f_2^2-4f_3f_2-f_3^2)m_{\tilde G}^2\\
&-2f_2(f_2-f_3)^2\big)-2m_{\tilde G}^2m_{\tilde t_1}^4(2m_{\tilde G}^2-f_2+f_3)\\
&-f_2^2m_{\tilde G}^2(2f_2-h_8)\big)\\
&+4f_1^3m_{\tilde G}^2(f_2-m_{\tilde G}^2)\Big), \qquad (85)\\
\mathrm{w}_{2t\tilde b_i} &= \frac{2\,(\Delta_{\mathrm{i}1}-\Delta_{\mathrm{i}2})}{3m_W^2 m_{\tilde G}}\Big(f_1^2(2f_2^2-m_{\tilde G}^2(2m_{\tilde t_1}^2+h_8))\\
&+f_1\big(m_{\tilde t}^2(m_{\tilde G}^2h_8-f_3^2)\\
&+2f_2(m_W^2m_{\tilde G}^2-f_3m_W^2+f_3^2)-f_2^2(2f_3+m_W^2)\big)\\
&+m_W^2\big(m_{\tilde t}^2(-m_{\tilde G}^2\big(2f_2\\
&+m_W^2)-2f_2^2+2f_3f_2+f_3^2)+2m_{\tilde G}^2m_{\tilde t_1}^4\\
&+f_2^2(2f_2-h_8)\big)+f_1^3m_{\tilde G}^2\Big). \qquad (86)
\end{aligned}$$

with $\Delta_{i1} = (R_i - Z_i)A_{\tilde t}$ and $\Delta_{i2} = (Z_i - R_i)B_{\tilde t}$.

$\widetilde{\mathcal{M}}^\dagger_{\chi_i^+} \mathcal{M}_{\tilde b_i}$ interference

For the interference term $\widetilde{\mathcal{M}}^\dagger_{\chi_i^+} \mathcal{M}_{\tilde b_i}$, the functions $\widetilde{\mathrm{w}}_{j\chi_i^+\tilde b_i}$, $\forall j = 1, 2$, are

$$\begin{aligned}
\widetilde{\mathrm{w}}_{1\chi_i^+\tilde b_i} &= \frac{8f_1C_{\mathrm{ij}1}}{3m_W^2 m_{\tilde G}^2}\Big(2f_1\big(f_3m_{\tilde G}^2(m_{\tilde t}^2+h_5)\\
&+f_2(2m_W^2m_{\tilde G}^2+m_{\tilde G}^4-2f_3^2)\\
&+f_2^2(2f_3-m_{\tilde G}^2)\big)-m_{\tilde t}^2\big(m_{\tilde G}^4(2f_3+m_W^2)\\
&+m_{\tilde G}^2\big(f_3(3f_3+4m_W^2)\\
&-2f_2(f_3+m_W^2)\big)+2(f_2-f_3)f_3^2\big)\\
&-f_2m_W^2\big(m_{\tilde G}^2(-3f_2+4f_3+2m_W^2)\\
&+2m_{\tilde G}^4+2f_2(f_2-f_3)\big)+f_1^2(m_{\tilde G}^4-4f_2m_{\tilde G}^2)\Big), \qquad (87)
\end{aligned}$$

$$\widetilde{w}_{2\chi_i^+\tilde{b}_i} = -\frac{16 f_1 C_{ij2} h_2(-f_3 h_5 m_{\tilde{t}_1}^2 - f_2 m_W^2 + f_1(f_2+f_3))}{3 m_W^2 m_{\tilde{G}}}. \quad (88)$$

We have made the following substitutions in the functions (87) and (88):

$$C_{ij1} = T_i(R_j S_i + Z_j P_i) - Q_i(R_j P_i + Z_j S_i),$$
$$C_{ij2} = T_i(R_j S_i + Z_j P_i) + Q_i(R_j P_i + Z_j S_i). \quad (89)$$

$\mathcal{M}_t^\dagger \widetilde{\mathcal{M}}_{\chi_i^+}$ interference

For the interference term $\mathcal{M}_t^\dagger \widetilde{\mathcal{M}}_{\chi_i^+}$, the functions $\widetilde{w}_{jt\chi_i^+}$, $\forall\, j = 1,2,3,4$, are:

$$\begin{aligned}\widetilde{w}_{1t\chi_i^+} = \frac{8R_{i1}}{3m_W^2 m_{\tilde{G}}^2}\Big(&-m_{\tilde{G}}^2 m_{\tilde{t}_1}^4(4f_3 m_W^2 + h_9)\\ &+m_{\tilde{t}}^2\big(f_2^2 m_W^2(3m_{\tilde{G}}^2 + 4f_3) + m_{\tilde{G}}^2\big(m_W^2 m_{\tilde{G}}^2\\ &-f_3^2\big)h_5 + 2f_2 h_5(m_W^2 m_{\tilde{G}}^2 + f_3^2)\big)\\ &+f_1^2\big(m_{\tilde{G}}^2 m_{\tilde{t}_1}^2(m_{\tilde{G}}^2 + 4f_3) - 3m_{\tilde{G}}^4 h_5\\ &+f_2(4m_W^2 m_{\tilde{G}}^2 + 6m_{\tilde{G}}^4) + f_2^2(8f_3 - 4m_{\tilde{G}}^2)\big)\\ &+2f_1\big(m_{\tilde{t}}^2\big(-(2m_{\tilde{G}}^4(f_3 + m_W^2)\\ &+(f_3 - 2f_2)m_{\tilde{G}}^2(f_3 + m_W^2) + 2f_2 f_3^2\big)\big)\\ &-2f_2(f_2 - m_{\tilde{G}}^2)(f_3 h_5 + f_2 m_W^2)\big)\\ &-f_2^2 m_W^2(m_{\tilde{G}}^2 + 2f_2)h_5 + f_1^3(6m_{\tilde{G}}^4 - 8f_2 m_{\tilde{G}}^2)\Big),\end{aligned} \quad (90)$$

$$\begin{aligned}\widetilde{w}_{2t\chi_i^+} = \frac{8R_{i2}}{3m_W^2 m_{\tilde{G}}}\Big(&2f_1(m_{\tilde{t}}^2(m_{\tilde{G}}^2(f_3 + 2m_W^2) - f_3^2)\\ &-2f_2(f_3 h_5 + f_2 m_W^2))\\ &+h_5(m_{\tilde{t}}^2(f_3^2 - m_W^2 m_{\tilde{G}}^2) + f_2^2 m_W^2)\\ &+f_1^2\big(m_{\tilde{G}}^2(-2f_2 + 6f_3 + 3m_W^2)\\ &+3m_{\tilde{G}}^4 + 8f_2 f_3\big) - 6f_1^3 m_{\tilde{G}}^2\Big),\end{aligned} \quad (91)$$

$$\begin{aligned}\widetilde{w}_{3t\chi_i^+} = \frac{8R_{i3}}{3m_W^2 m_{\tilde{G}}^2}\Big(&2f_1(-f_3 m_{\tilde{G}}^2 h_5 m_{\tilde{t}_1}^2\\ &+2f_2 m_{\tilde{G}}^2(f_3 - m_W^2) - 2f_3 f_2^2)\\ &+m_{\tilde{t}}^2(2f_2(m_W^2 m_{\tilde{G}}^2 + f_3^2) - f_3^2 m_{\tilde{G}}^2 + m_W^2 m_{\tilde{G}}^4)\\ &-f_2^2 m_W^2(m_{\tilde{G}}^2 + 2f_2 - 4f_3)\\ &+f_1^2(4f_2 m_{\tilde{G}}^2 - 3m_{\tilde{G}}^4)\Big),\end{aligned} \quad (92)$$

$$\begin{aligned}\widetilde{w}_{4t\chi_i^+} = \frac{8R_{i4}}{3m_W^2 m_{\tilde{G}}}\Big(&m_{\tilde{t}}^4 h_9 - m_{\tilde{t}}^2\big(-m_{\tilde{G}}^2(f_3^2 - 2f_2 m_W^2)\\ &+m_W^2 m_{\tilde{G}}^4 + f_2((3f_2 - 4f_3)m_W^2\\ &+2f_3^2)\big) + 2f_1\big(f_2(2m_{\tilde{G}}^2(m_W^2 - f_3)\\ &+f_2(2f_3 - m_W^2)) - m_{\tilde{t}}^2(m_{\tilde{G}}^2(m_W^2 - 2f_3) + 2f_2 f_3)\big)\\ &+f_1^2(-6f_2 m_{\tilde{G}}^2 - m_{\tilde{G}}^2 m_{\tilde{t}_1}^2 + 3m_{\tilde{G}}^4 + 4f_2^2)\\ &+f_2^2 m_W^2(m_{\tilde{G}}^2 + 2f_2 - 4f_3)\Big).\end{aligned} \quad (93)$$

We have made the following substitutions in the functions (90)–(93): $R_{i1} = (A_{\tilde{t}} - B_{\tilde{t}})(S_i + P_i)(T_i - Q_i)$, $R_{i2} = (A_{\tilde{t}} + B_{\tilde{t}})(S_i - P_i)(T_i + Q_i)$, $R_{i3} = (A_{\tilde{t}} - B_{\tilde{t}})(S_i + P_i)(T_i + Q_i)$, and $R_{i4} = (A_{\tilde{t}} + B_{\tilde{t}})(S_i - P_i)(T_i - Q_i)$.

B Analytical expressions for the amplitudes for the goldstino approximation

In this appendix we present explicitly the full results for the seven $w^G_{\psi_a\psi_a}$ functions that arose from the squared amplitudes (43), as well as the eight $w^G_{\psi_a\psi_b}$ functions that appear in the interference terms (48) of the 3-body stop $\tilde{t}_1$ decay with goldstino in the final state. First, we shall present the contribution for the squared amplitudes, then we shall present the interferences. We shall shown that the $w^G_{\psi_a\psi_a}$ and $w^G_{\psi_a\psi_b}$ functions are very compacts expressions, opposed to the resulting functions in the gravitino case that we have presented in Appendix A. The approximation of the gravitino field by the derivative of the goldstino field is good in the high energy limit ($m_{\tilde{G}} \ll m_{\tilde{t}_1}$), in the sense that in this limit they behave similar and also in the simplification of the computations.

B.1 Top contribution

For the averaged squared amplitude of the top quark contribution, the resulting functions $\widetilde{w}_{jtt}$, $\forall\, j = 1,2,3$, are

$$\begin{aligned}\widetilde{w}_{1tt} = 4\frac{2a_1}{m_W^2}\Big(&f_2(m_W^2(6m_{\tilde{G}}^2 + 2h_5 m_{\tilde{t}_1}^2 - q_1^2) + 6f_3 m_W^2\\ &+4f_3^2) - 2f_1(-m_{\tilde{G}}^2(4f_3 + 3m_W^2)\\ &+f_2(4f_3 + 3m_W^2) + f_3 q_1^2) + (q_1^2 - 2m_{\tilde{G}}^2)(m_W^2 m_{\tilde{G}}^2\\ &+3f_3 m_W^2 + 2f_3^2)\\ &+4f_1^2(f_2 - m_{\tilde{G}}^2) - 2m_W^2 m_{\tilde{G}}^2 h_5 m_{\tilde{t}_1}^2 - 4f_2^2 m_W^2\Big),\end{aligned} \quad (94)$$

$$\begin{aligned}\widetilde{w}_{2tt} = \frac{4a_2 m_{\tilde{G}}}{m_W^2}\big(&m_W^2(m_{\tilde{G}}^2 + m_{\tilde{t}}^2 - 2f_2) - f_1(4f_3 + 3m_W^2)\\ &+3f_3 m_W^2 + 2f_1^2 + 2f_3^2\big),\end{aligned} \quad (95)$$

$$\widetilde{w}_{3tt} = \frac{2a_3(m_W^2(f_2 - m_{\tilde{G}}^2) - 3f_3 m_W^2 - 2f_3^2 + 2f_1 f_3)}{m_W^2}. \quad (96)$$

With a_1, a_2, and a_3 defined previously in Appendix A.

B.2 Sbottom contribution

We have for the averaged squared amplitude of the sbottom squark contribution the $\widetilde{\mathrm{w}}_{1\tilde{b}_i\tilde{b}_i}$ function,

$$\widetilde{\mathrm{w}}_{1\tilde{b}_i\tilde{b}_i} = \frac{4D_{\mathrm{ij}1}h_2h_3}{m_W^2}, \tag{97}$$

with D_{ij1} defined previously in Appendix A.

B.3 Chargino contribution

For the averaged squared amplitude of the chargino contribution, the resulting functions $\widetilde{\mathrm{w}}_{j\chi_i^+\chi_i^+}$, $\forall\, j = 1, 2, 3$, are

$$\widetilde{\mathrm{w}}_{1\chi_i^+\chi_i^+} = \frac{4q_3^2P_{\mathrm{ij}1}}{m_W^2}\Big(m_{\tilde{t}}^2(m_W^2(m_{\tilde{G}}^2 + f_2) + 3f_3m_W^2 + 2f_3^2 - 2f_1f_3) + 2f_2(2f_1(f_1 - f_3) - (3f_1 + f_2)m_W^2)\Big), \tag{98}$$

$$\widetilde{\mathrm{w}}_{2\chi_i^+\chi_i^+} = 12m_{\tilde{G}}h_5m_{\tilde{t}_1}^2(P_{\mathrm{ij}1} + P_{\mathrm{ij}2})(h_5 - f_1 - f_2), \tag{99}$$

$$\widetilde{\mathrm{w}}_{3\chi_i^+\chi_i^+} = \frac{4P_{\mathrm{ij}2}}{m_W^2}\Big(m_{\tilde{t}}^2(m_W^2(m_{\tilde{G}}^2 + f_2) + 3f_3m_W^2 + 2f_3^2 - 2f_1f_3) + 2f_2(2f_1(f_1 - f_3) - (3f_1 + f_2)m_W^2)\Big), \tag{100}$$

where P_{ij1} and P_{ij2} are defined in Appendix A.

B.4 Interference terms

$\mathcal{M}_t^{G\,\dagger}\mathcal{M}_{\tilde{b}_i}^G$ interference

For the interference term $\mathcal{M}_t^{G\,\dagger}\mathcal{M}_{\tilde{b}_i}^G$, the functions $\mathrm{w}_{jt\tilde{b}_i}$, $\forall\, j = 1, 2$, are

$$\mathrm{w}_{1t\tilde{b}_i} = \frac{2(\Delta_{\mathrm{i}1} + \Delta_{\mathrm{i}2})}{m_W^2}\Big(-f_1(f_3(m_{\tilde{t}}^2 - m_{\tilde{G}}^2) + f_2m_W^2) + m_W^2\big(m_{\tilde{t}}^2(2m_{\tilde{G}}^2 + f_3) - f_2(m_{\tilde{G}}^2 + m_{\tilde{t}}^2)\big) + 2f_1^2(f_2 - m_{\tilde{G}}^2)\Big), \tag{101}$$

$$\mathrm{w}_{2t\tilde{b}_i} = \frac{2m_{\tilde{G}}(\Delta_{\mathrm{i}1} - \Delta_{\mathrm{i}2})(m_W^2(f_2 - m_{\tilde{t}}^2) + f_1^2 - f_3f_1)}{m_W^2}. \tag{102}$$

Here $\Delta_{\mathrm{i}1}$ and $\Delta_{\mathrm{i}2}$ are defined above in Appendix A.

$\mathcal{M}_{\chi_i^+}^{G\,\dagger}\mathcal{M}_{\tilde{b}_i}^G$ interference

For the interference term $\mathcal{M}_{\chi_i^+}^{G\,\dagger}\mathcal{M}_{\tilde{b}_i}^G$, the functions $\mathrm{w}_{j\chi_i^+\tilde{b}_i}^G$, $\forall\, j = 1, 2$, are:

$$\mathrm{w}_{1\chi_i^+\tilde{b}_i}^G = \frac{4C_{\mathrm{ij}1}m_{\tilde{G}}}{m_W^2}\Big(m_{\tilde{t}}^2(m_W^2(h_5 - f_2) + f_1f_3) - f_1^2h_5\Big), \tag{103}$$

$$\mathrm{w}_{2\chi_i^+\tilde{b}_i}^G = \frac{4C_{\mathrm{ij}2}}{m_W^2}\big(-m_W^2h_5m_{\tilde{t}_1}^2h_2 + f_1^2(2f_2 - m_{\tilde{G}}^2) - f_1(f_3h_5m_{\tilde{t}_1}^2 + f_2m_W^2)\big), \tag{104}$$

with $C_{\mathrm{ij}1}$ and $C_{\mathrm{ij}2}$ defined above in Appendix A.

$\mathcal{M}_t^{G\dagger}\mathcal{M}_{\chi_i^+}^G$ interference

For the interference term $\mathcal{M}_t^{G\dagger}\mathcal{M}_{\chi_i^+}^G$, the functions $\mathrm{w}_{jt\chi_i^+}^G$, $\forall\, j = 1, 2, 3, 4$, are

$$\mathrm{w}_{1t\chi_i^+}^G = \frac{4R_{\mathrm{i}1}\,m_{\tilde{G}}}{m_W^2}\Big(m_{\tilde{t}}^2(m_W^2(4m_{\tilde{G}}^2 - 3f_2 + 3m_W^2) + 5f_3m_W^2 - 2f_3^2) + f_2m_W^2(2f_2 - 3h_5) - 2f_1^2m_{\tilde{G}}^2 + f_1(4f_2(f_3 + m_W^2) - 3m_W^2h_5m_{\tilde{t}_1}^2)\Big),$$

$$\mathrm{w}_{2t\chi_i^+}^G = \frac{4R_{\mathrm{i}2}}{m_W^2}\Big(m_{\tilde{t}}^2(f_3(2f_3 + m_W^2) - m_W^2m_{\tilde{G}}^2) + f_2m_W^2(3h_5 - 2f_2) + 2f_1^2m_{\tilde{G}}^2 - 4f_1f_2(f_3 + m_W^2)\Big), \tag{105}$$

$$\mathrm{w}_{3t\chi_i^+}^G = \frac{4R_{\mathrm{i}3}\,m_{\tilde{G}}}{m_W^2}\Big(m_W^2(m_{\tilde{t}}^2 - f_2) - f_1(2f_3 + 3m_W^2) + 2f_1^2\Big), \tag{106}$$

$$\mathrm{w}_{4t\chi_i^+}^G = \frac{R_{\mathrm{i}4}}{m_W^2}\Big(4(2f_1(f_1 - f_3) - (3f_1 + f_2)m_W^2)(2f_2 - m_{\tilde{G}}^2) + 4h_5m_{\tilde{t}_1}^2((f_2 + 3f_3)m_W^2 + 2f_3(f_3 - f_1))\Big), \tag{107}$$

with $R_{\mathrm{i}1}$, $R_{\mathrm{i}2}$, $R_{\mathrm{i}3}$, and $R_{\mathrm{i}4}$ defined in Appendix A.

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