Eur. Phys. J. B (2019) 92: 103
https://doi.org/10.1140/epjb/e2019-90722-9

THE EUROPEAN PHYSICAL JOURNAL B

Regular Article

Tax evasion as a contagion game: evidences from an agent-based model

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Received 10 December 2018 / Received in final form 23 February 2019
Published online 20 May 2019

Abstract. This paper presents an agent-based model of a simple economic system where the personal satisfaction gained from public services and the perceived opinion of neighbors are shown to drive the individual decision about tax compliance. Results of simulations, consistent with existing literature on the topic, suggest a peculiar approach to face the plague of tax evasion.

1 Introduction

Tax evasion is the "illegal and intentional actions taken by individuals to reduce their legally due tax obligations" ([1], p. 55). Its main effect is a severe damage to the socio-economical environment that deprives governments of their fiscal resources and plays an important role in reducing well-being of societies. It is quite an age-old phenomenon that has been studied for decades, both theoretically and empirically. The well-known *free rider* problem rises when a selfish citizen consumes public goods and services without properly contributing to related costs [2]. This causes inefficiency and bad allocations of governments expenditures for healthcare, education, defence, social security, transportation, infrastructure, science and technology, as widely documented in a vast literature, among which Andreoni et al. [3], Slemrod and Yitzhaki [4], Torgler [5], Kirchler [6] and Slemrod [7]. Tax evasion and the so-called black economy are, also, related to social inequality, as underlined by part of the literature, among which Alstadsaeter et al. [8], Bertotti and Modanese [9,10], dealing with the differentiation of the propensity to evade with respect to income and with redistributive aspects. Finally, it matters in terms of social justice, since it specially afflicts poorer people, who do not have the possibility to substitute public services with private ones with higher prices.

In his *An Essay on the Nature and Significance of Economic Science*, Robbins [11] wrote that "economics is the science which studies human behavior as a relationship between ends and scarce means which have alternative uses". Thus, human decision on public goods can be interpreted as a behavioral choice in terms of cooperation vs. competition in the society. The model here presented aims to show the collective relevance of such behavioral elements in driving the decision of each citizen, which reflects also the perceived quality of the public good and the relational feedback received by her surrounding social environment.

The initial stages of the formal analysis of tax evasion can be dated back to the 70s, with contributions by Allingham and Sandmo [12] and Srinivasan [13]. Despite many similarities, such contributions, which are a propagation of an earlier approach advanced by Becker [14], differ from each other with respect to optimization procedures, taxpayer's risk attitudes (which affect second order conditions of chosen objective functions), decision variables, audit probabilities, tax tariffs and fine functions. In particular, Allingham and Sandmo [12] find that income understatement is decreasing in audit probabilities or in the fine, whereas the dependence on tax rate is more controversial, reflecting income and substitution effects. Yitzhaki [15] obtained a conter-intuitive result by modelling fines computed on the basis of evaded taxes (instead of the understated income): as the tax rate increases, the evasion decreases, differently from the empirical evidence shown in [16–18]. Many other studies have been done in the attempt to find a positive relationship between tax rate and evasion (see for example [19–22]).

Such a standard theoretical framework inspired several contributions in related literature, concerning tax evasion and related issues, such as the shadow economy, as in [23], psychological perception and society (social norms and moral sentiments like guilt or shame), as in [6,24–26] and many others. Also literature from statistical physics and network science is deeply connected to such topics, as shown in recent reviews by Perc and Szolnoki [27], Perc et al. [28,29] and Capraro and Perc [30].

More generally, a growing stream of literature presenting agent-based models dealing with tax evasion exists.

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A survey of such papers could be gained by the joint reading of Bloomquist [31], Alm [1], Hokamp [32], Pickhardt and Prinz [33], Oates [34] and Bazart et al. [35]. The advantage of agent-based models is that they are prone to describe the complexity of aggregate contexts, as documented in previous studies of socio-economic analysis, Pluchino et al. [36–38], Biondo et al. [39–44]. Simulative models (as in [45]) can help investigating relevant questions, as the correspondence between the provision of the public good and tax evasion, as in [32], the importance of social norms and auditing, as in [46], and the effect of social networks on the tax compliance, as in [47]. Such aspects, like many others, can be explained in terms of behavioral attributes, seeking for the roots of decisions in the evolution of personal traits, influenced by the surrounding environment.

As reported by the IRS [48], given the extent of the tax evasion, the expenditures paid by governmental authorities to induce virtuous behaviors are significant. Nonetheless, in many cases, free riders remain unpunished. Honest citizens considering the participation to social costs as a moral imperative are the sole fully compliant taxpayers. We rephrase the provoking question asked by Alm et al. [49]: why should people pay taxes? Different reasons can be reported: first of all, because of altruism, as recalled, for just some examples, in [50–52]; secondly, because of imitation, as in [53–55]; finally, because of an assessment of the quality of the public good, as in [56–59].

The main motivation of this paper is to combine the agent-based approach to the prisoner's dilemma perspective of the standard *public goods game*, in order to show a very simple way in which the conflict between *individual* and *collective* rationality [60] can be enriched by imitation coming from the social interaction. Such a social dilemma motivated a vast amount of literature, regarding the production of public goods, as in [61], the emergence of social norms and social interaction, as in [62] and [63], the reasons behind cooperative behavior, as in [64], the social preference models, as in [65–68] and with specific reference to moral preferences in [69] and [70]. The effect of the observation within social structures can be determinant to drive individual decisions, as shown in previous contributions, among which, [71] and [72]. Our model contributes to the stream of literature dealing with collective behavior (both in terms of triggering threshold and spreading) in the dynamic perspective of a public good game, in the case in which players are aware of the behavior of their neighbors within a realistic social network. A very recent contribution on the same line is [73].

The present model describes a community in which agents decide whether to pay taxes or not, according to their personal satisfaction and to the imitation of neighbors' behavior. In Section 2, we, firstly, present the simplest setting of the model, showing that it is able to reproduce results obtained by Elster [74] in terms of multi-person prisoner's dilemma; then, we highlight the role of taxpayers in contrast with that of evaders for the dynamics of the system and, finally, introduce the dynamic decisional rule based on personal satisfaction and imitation; in Section 3, we study the effect of three policy parameters regulating the tax rate, the fine and the audit probability; in Section 4 we present some conclusive insights.

2 Tax evasion model

Consider a community of N citizens (players), $\{P_i\}_{i=1,2,...,N}$, who have to pay their personal contributes $d_{i,t}$ to produce a public good, e.g. a public service, assumed to be perfectly non-excludable and only partly non-rival. We assume that time $t = 0, 1, 2, \dots, T$ is measured by a discrete variable indexing simulative steps (i.e., turns of the game). At each turn, a randomly number of citizens are assumed to consume the public good/service, from which they receive a utility in terms of units of a dimensionless reward, which we shall denominate $G_{i,t}$. Of course, the total available amount of the public good is limited to the cumulated contributes paid by tax-payers, i.e. $\sum_{i,t} G_{i,t} = \sum_{i,t} d_{i,t}$, $\forall t$. Initially, at $t = 0$, all community members are endowed with the same initial amount of capital $C_{i,0}$, which will be increased at each time step, by $\Delta C_{i,t} = \Delta C = 1$ unit, $\forall i$. Thus, the monetary value of individual utility is $U_{i,t} = C_{i,t} + G_{i,t}$. In the simplest setting, the topological configuration of the social network is irrelevant because each player is assumed to decide independently of other players' decisions. Later we will remove such an initial simplification and specify how people is reciprocally linked to each other.

2.1 The basic setting

At each turn, each player P_i chooses how to behave from the following two alternative strategies:

- *strategy A*: to pay $d_{i,t}$ units of capital to contribute to the public good production;
- *strategy B*: to evade the tax and possibly incur in the fine, equal to $h_{i,t}$ units of capital (which will *not* be redistributed to other players), with probability p.

Let us consider for simplicity a *lump-sum* taxation/fine system only, i.e., $d_{i,t} = d$ and $h_{i,t} = h$, $\forall i$, and assume that $h > d$. As in every other *prisoner's dilemma*, from the individual point of view, the best choice is to play the non-cooperative strategy B, since its payoff is greater than the one associated to strategy A for every possible decision assumed by other players. Consider that the partial non-rivalry of the public good is modeled by assuming that at each time step t, a random number of players is extracted to divide among them the total amount of resources collected from tax payment of the whole community. From the standpoint of the single citizen/player, the ex-ante probability to be selected, i.e. to use the public good, and receive any amount $G_{i,t}$ is the same in both cases (equal to γ, say) and is not affected by the fact that she is a cooperator or not. For the sake of simplicity, let us assume that $\gamma G_{i,t} = \Gamma$, $\forall i$. The individual payoff associated to the strategy A is $\pi^A_{i,t} = \Delta C + \Gamma - d$, whereas $\pi^B_{i,t} = \Delta C + \Gamma - ph$. Thus, for each player, the strategy B Nash-dominates the strategy A if $p < d/h$.

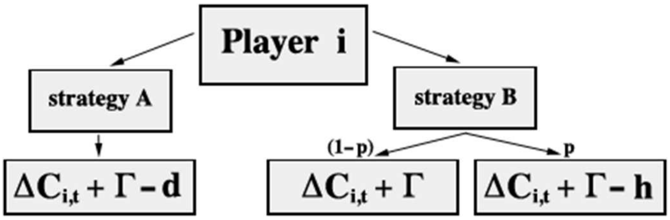

Fig. 1. Rules of the basic setting: at each turn, after receiving $\Delta C_{i,t}$, each player decides what to do: taxpayers play only strategy A; tax evaders play only strategy B.

Figure 1 depicts the basic setting of the game at each time step. Agents receive the exogenous ΔC and, according to their behavior, are partitioned in two groups: *taxpayers*, who altruistically always play strategy A, indicated as $\{A_i\}_{i=1,2,\ldots,N_{\mathrm{a}}}$; and *tax evaders*, who selfishly always play strategy B, indicated as $\{S_i\}_{i=1,2,\ldots,N_{\mathrm{s}}}$. Of course, $N = N_{\mathrm{a}} + N_{\mathrm{s}}$. The basic settings for the first set of simulations are: $p = 0.4$ (audit probability), $d = 2$ units (tax payment), $h = 3$ units (fine) and $\Delta C = 1$ units (external gain).

2.1.1 Effects of a change of N_{a}

Let us investigate the asymptotic behavior of the average social capital $\bar{C}(T) = \frac{1}{N}\sum_{i=1}^{N} C_i(T)$, calculated at the end of a set of single-run simulations, with $T = 100$ turns per players, when the percentage $f = N_{\mathrm{a}}/N$ of taxpayers changes by steps of 1% in $[0, 100]$. In Figure 2, the average final capital $\bar{C}(T)$ is reported (red line) as function of f, confronted with its two components, i.e., $\bar{C}_a(T) = \frac{1}{N_a}\sum_{i=1}^{N_a} C_i(T)$ (green line) and $\bar{C}_s(T) = \frac{1}{N_s}\sum_{i=1}^{N_s} C_i(T)$ (blue line) calculated separately for the altruistic taxpayers and selfish evaders (a horizontal black line at $\bar{C} = 0$ is also drawn for comparison). The three values of social capital have been rescaled in order to have $\bar{C}(T) = 0$ when $f = 0\%$, since – by definition – if no one pays taxes the collectivity must have zero benefits. A confirmation for a very elementary intuition, i.e., that the average capital $\bar{C}_s(T)$ of the evaders is a positive function of f. This happens because an always smaller number of evaders is surrounded by an increasing number of taxpayers. Furthermore, even the capital of taxpayers ($\bar{C}_a(T)$) is positively linked with f since, as long as the number of altruists grows, the amount of available public good increases for all citizens and, meanwhile, evaders are becoming numerically less relevant.

When the fraction f of taxpayers goes below a threshold f_{th}, which in Figure 2 is almost 40% (given the chosen parameters), the average capital of taxpayers becomes negative, while the collective capital $\bar{C}(T)$ is still positive. This means that – on average – they pay more than they receive. The average social capital of taxpayers is always lower than the average social capital of evaders, i.e. $\bar{C}_a(T) < \bar{C}_s(T)\ \forall f$. This is the graphical evidence of Nash-dominance of the non-cooperative behavior and reproduces the same result described by Elster [74] and shown in Figure 3. The two heavy lines in the Elster's diagram show the expected benefits, of both cooperators and free riders, as functions of the number of cooperators (altruists). The strong similitude is evident: as in our Figure 2, the line representing the reward to free riders is constantly above the other one. At the same time, it is better for everybody if all agents cooperate than if nobody does: in fact, $B > 0$. Free riders get the best result C, whereas the worst one (A) is reserved for the cooperators. If cooperators are, at least D in number, their benefits become positive. The thin line shows that the average benefits for the collectivity is a positive function of the number of cooperators (normalized, as usual for $f = 0$). The distance between the two heavy lines can be read as the cost (per altruist) of cooperation. It is very natural to conclude that the capital of the collectivity increases only thanks to the contribution of altruistic players. Thus, from the collective point of view, groups with more cooperators are favored compared to groups with few cooperators. Indeed, the more numerous altruists are, the smaller the cost of free riders (in both absolute and relative terms).

2.1.2 Degrees of necessity, degrees of failure...

The basic setting showed the known market failure due to the presence of free riders in action. Economic theory says that in this case the intervention of the policy maker is the only solution to create a remedy, which hopefully can represent a *second best* solution, since the social optimum cannot be reached by private market forces only. Despite such a result is quite definitive, we try to advance the analysis collaterally, by questioning whether the degree of importance (or the difficulty of substitution) of the public good at hand can play a role and become an endogenous mechanism to partly amend the disequilibrium. If, for example, a public good were a primary good, then the low quality caused by insufficient funds could force people (free riders included) to buy substitutes on the private market. In other words, if a public pool is always crowded or dirty is one thing; if a hospital is unable to aid people even for immediate emergency services, is another. Both cases respond to the logic of the model with free riders and the set of consequences is theoretically identical. Nonetheless, a citizen would suffer from two qualitatively different losses. In particular, for the case of the hospital, the primary need for health services pushes the citizen to buy them on the private market. This dramatically creates a strong form of social injustice, since poor people will be hit much harder than rich ones. For the sake of simplicity, we neglect such redistributive issues (being developed in a forthcoming paper, in which tax evasion is a function of personal income as well) and focus on the case of a primary public good/service.

If the number of taxpayers is not big enough, tax evaders may end up with a negative final capital value, since they will have to buy on the market what has not been produced because of their evasion. Thus, when the public good is a primary good, a loss suffered by free riders emerges and the percentage of taxpayers turns out to be a fundamental ingredient for the tax evasion be a convenient strategy, as shown in Figure 4. As a consequence, we can recognize no longer one but three thresholds, namely a, b, c.

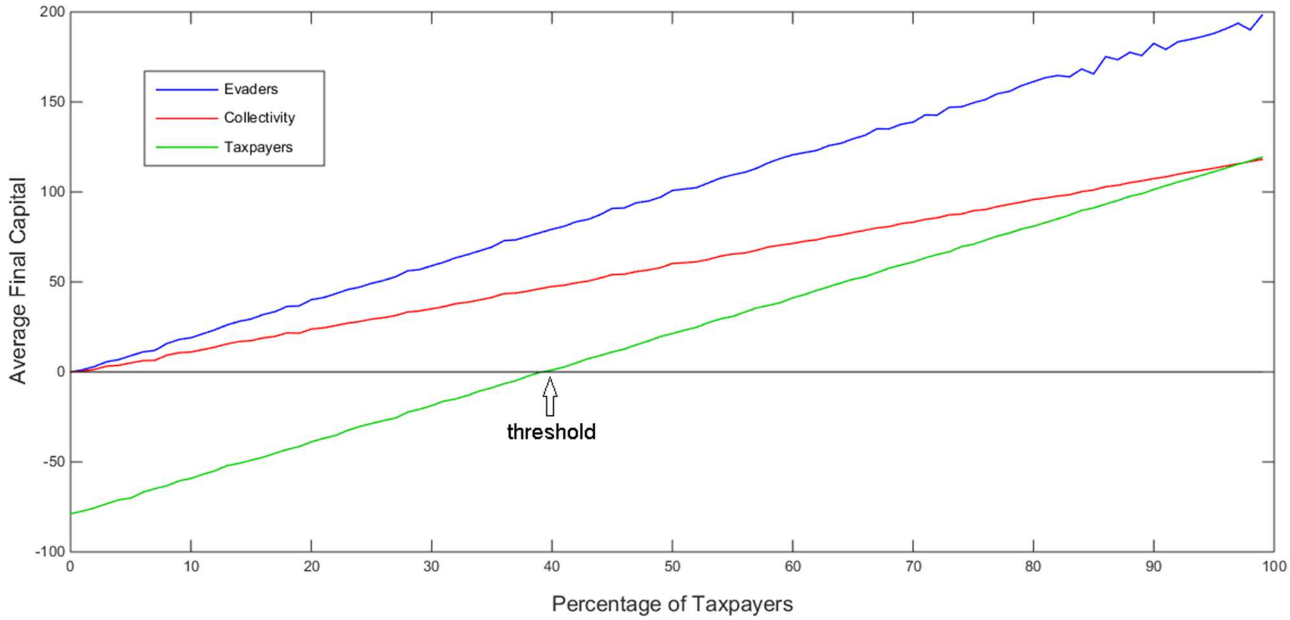

Fig. 2. The final average social capital $\bar{C}(T)$ (red), over $T = 100$ turns, as a function of the percentage f of taxpayers. Average final capitals $\bar{C}_a(T)$ (green) and $\bar{C}_s(T)$ (blue), for taxpayers and tax evaders, respectively, are also reported and compared with the first one.

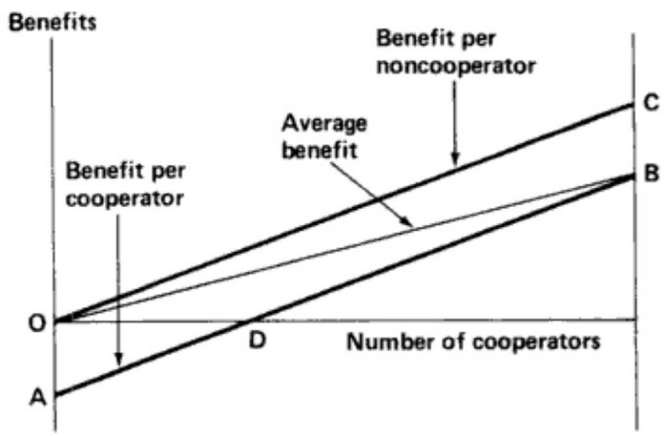

Fig. 3. Many-person prisoner's dilemma. Expected benefits as a function of the number of cooperators for the collective group, for the cooperators and for the free riders [74].

(1) When $f < a$, social capital is negative for all of the three groups because paid taxes cannot grant the public services for everybody. It is worth noting that this is a very bad situation, in which tax evaders damage both the community and also themselves, and tax payers suffer twice because they pay taxes and may also need to buy private substitutes of the public good.
(2) When $a \leq f < b$, the number of altruists is sufficient to cover the expenses of the public good production that make, on average, tax evasion convenient for tax evaders (the break-even point is reached in point a). The average capital of tax payers and that of the whole society remain negative.
(3) When $b \leq f < c$, the negative average capital of tax payers is more than compensated, at the social level, by the gain of evaders.
(4) When $f \geq c$, average capital for taxpayers becomes finally positive and public services are sufficient for the community (even if the collected resources are less than they should be). Tax evaders are still present in the community (unless $f = 100\%$) and continue to be better off than altruists. In particular, the reduction of tax evaders would improve the life conditions of the community either by increasing (if possible) the quality/quantity of the public good, or by reducing the tax pressure.

The behavioral characterization of each citizen i can be described by adapting a dated, yet interesting, diagram proposed by Cipolla [75] and showed in Figure 5. Let us indicate on the horizontal axis the quantity/quality of the public good Γ_i and, on the vertical axis, the individual contribution C_i of i, in order to identify four types of players:

Smart: those who consume the public good and participate to its production for the whole community by their personal contribution, namely, taxpayers when $f \geq c$;
Naive: those who contribute to the public good production for the whole community even when their consumption is insufficient or absent, namely, taxpayers when $f < c$;
Stupid: those who do not contribute to the production of the public good but suffer because their consumption of it is insufficient or absent, namely, tax evaders when $f < a$;

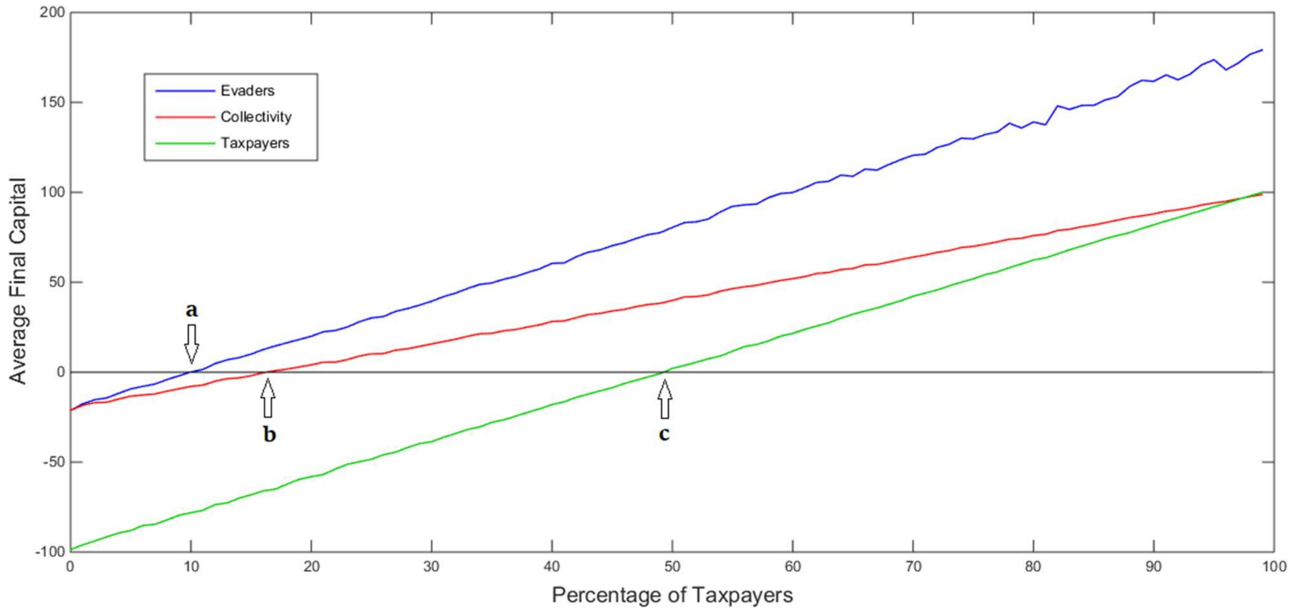

Fig. 4. When the public good is a primary good, the loss suffered by free riders matters. Average final capital functions presented in Figure 2, i.e., $\bar{C}(t)$, $\bar{C_a}(t)$ and $\bar{C_s}(t)$, cannot be rescaled and the percentage of taxpayers turns out to be a fundamental ingredient for the tax evasion be a convenient strategy. Two new thresholds appear: see text for details.

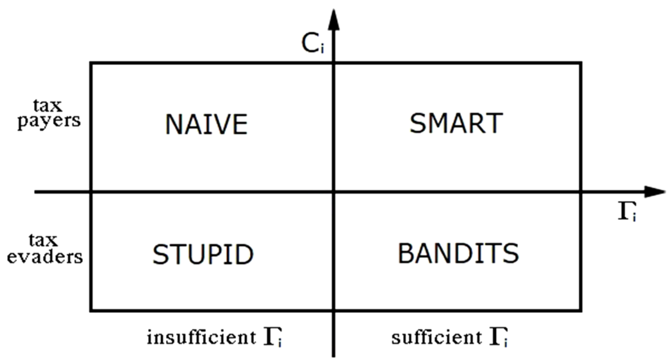

Fig. 5. A possible taxonomy of citizens can be derived by considering how each of them combine individual contribution (C_i) to the production of the public good and consumption of it (Γ_i).

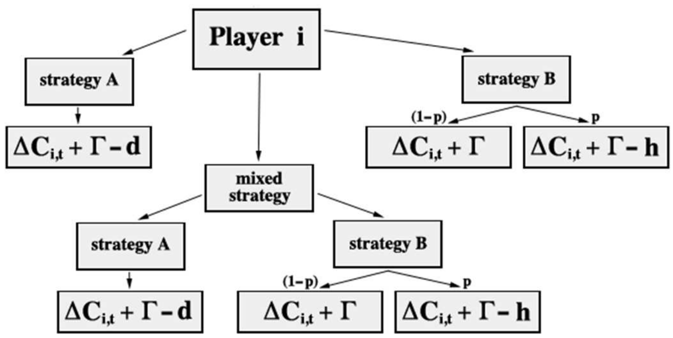

Fig. 6. New rules for a community of N players: taxpayers play only strategy A, tax evaders play only strategy B and mixed players randomly alternate between strategy A and strategy B.

Bandits: those who do not contribute to the production of the public good and enjoy the a satisfactory consumption of it at expenses of the rest of the community, namely, tax evaders when $f \geq a$.

2.2 Satisfaction and contagion on complex network

In this section, we update the set of possible behaviors that each player can choose. Differently from the basic setting, we add the possibility, at each time step, to randomly alternate (with probability 0.5) between strategy A and strategy B. As we named taxpayer players always choosing strategy A and tax evaders those ones always choosing strategy B, we label "mixed players" all citizens choosing such a third strategy, as explained in Figure 6. As described before, regarding the basic setting, chosen strategies now determine univocally three groups in the population.

In order to select the transition rules from a group to another, each player has been given a new variable, i.e., the "believeness" $B_{i,t} \in [0, 1]$, which measures the level of commitment in choosing and maintaining a given strategy. Values assumed by this variable change in time and affect possible transitions. Values of $B_{i,t}$ close to 1 mean that the player is a sort of zealot and that most probably she will not change behavior; on the contrary, values of $B_{i,t}$ close to 0 mean that the agent is easily influenced and that her behavioral change is highly probable. For both taxpayers and tax evaders, the transition leads to become a mixed player (and the value of $B_{i,t}$ is re-set at random again). For mixed players, instead, the believeness operates as a sort of *reservoir*, whose level affects the successive transformation: values lower than 0.5 reveal a propensity to become

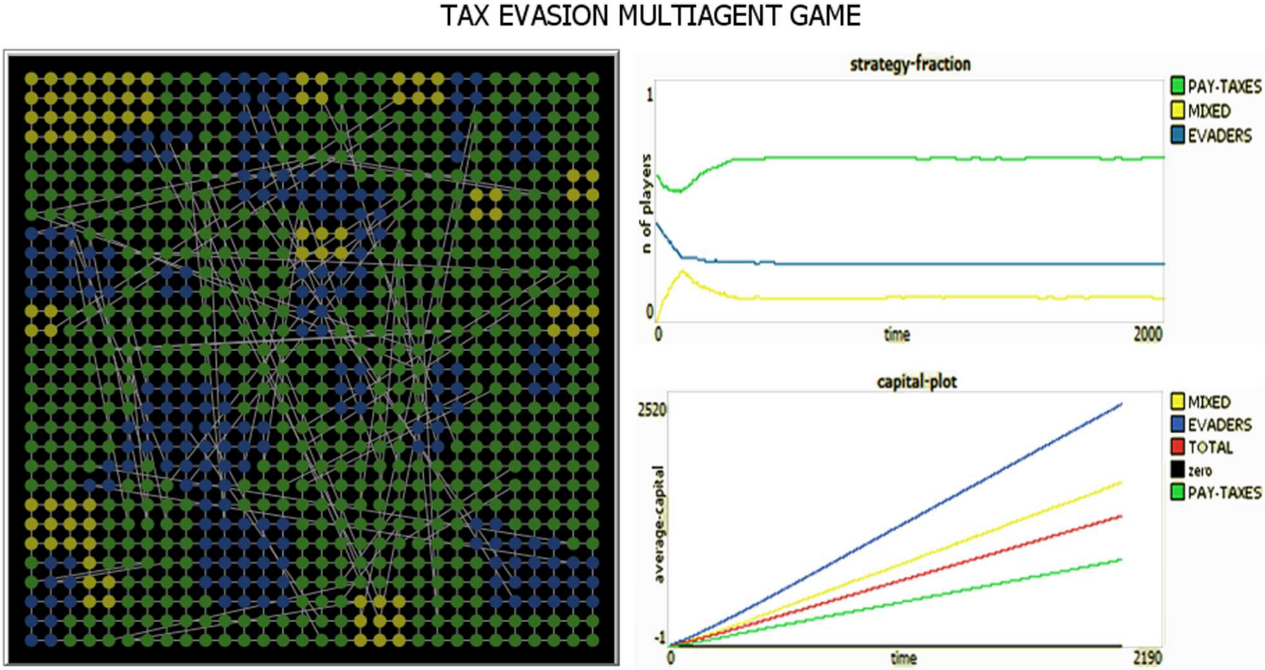

Fig. 7. Left panel: the small-world configuration of the community, with taxpayers, tax evaders and mixed players (respectively green, blue and yellow nodes). Right panel: dynamics of groups (top) and respective average social capital (bottom). Initial percentage of taxpayers is 60% (above the critical value for both IF and CF equal to 1).

an evader, if $B_i = 0.5$, the agent is and remains undecided, and values greater than 0.5 show the tendency to become taxpayers. The value of $B_{i,t}$ is based on both the imitation of neighbors and on the individual evaluation of the public good.

2.2.1 Imitation and social ties

In order to account the first point, we need to introduce a topological structure for our community. We assume a realistic social structure configured as a small-world [76], where each player is a node connected with short-range ties (to mimic strong social relationships) to four neighbors, and with a small rewiring probability ($r = 0.02$) of substituting one of those ties with a long-range one (representing a weak social relationship). In the left panel of Figure 7 a small-world 2D-lattice is depicted, with taxpayers, tax evaders and mixed players (respectively, green, blue and yellow nodes).

For taxpayers and evaders, if at a given time step the number of nearest neighbors belonging to their same group is smaller (greater) than the sum of the players of other groups, included the mixed one, the believeness value B_i decreases (increases) of a quantity $IF \times \delta B$, where IF is the "Imitation Factor" and $\delta B = 0.01$. When $B_i \leq 0$, player i becomes a mixed player and a new value of $B_i \in [0, 1]$ is randomly assigned to her.

For mixed players, if the number of nearest neighbors belonging to their same group is smaller than the sum of players of other two groups, B_i decreases of the quantity $IF \times \delta B$ if the evaders are more than taxpayers, otherwise it increases of the same quantity. Instead, if the number of mixed players is greater than (or equal to) the sum of the players belonging to the other two groups, B_i moves towards 0.5 of a quantity $IF \times \delta B$ and the agent maintains her mixed behavior. When, for a given mixed player i, $B_i \leq 0$ (respectively $B_i \geq 1$), that player becomes a tax evader (respectively, a taxpayer).

2.2.2 Satisfaction and behavioral reactions

The second mechanism influencing behavioral transitions concerns the economic situation of the players. If the social capital of a given player is negative, the agent will be disappointed because of her experience and so more prone to change her strategy. Thus, for both evaders and taxpayers, when their capital is negative the believeness value decreases of a quantity $CF \times \delta B$, where CF is the "Capital Factor". Instead, for mixed players, B_i varies by the quantity $CF \times \delta B$, depending on their actual state: if $B_i \geq 0.5$ it will increase, if $B_i < 0.5$ it will decrease.

Setting the initial percentage of mixed players to zero, if taxpayers are more numerous than tax evaders, our simulations show that there exist a critical value for the initial percentage of taxpayers (depending on both IF and CF), below which the global situation gets always worse. Similarly, above that threshold, it gets always better. In Figures 7 and 8, results of two typical single-run simulations are reported, with $IF = CF = 1$ and two different initial percentages of taxpayers, respectively, above (60%) and below (50%) the critical value (that, for these values of IF and CF turns out to be around 55%). In the first one, the final economic condition appears to be good for all citizens and a majority of taxpayers emerges; instead, in the second case, the final economic situation is good only for tax evaders, who become the majority. Threshold

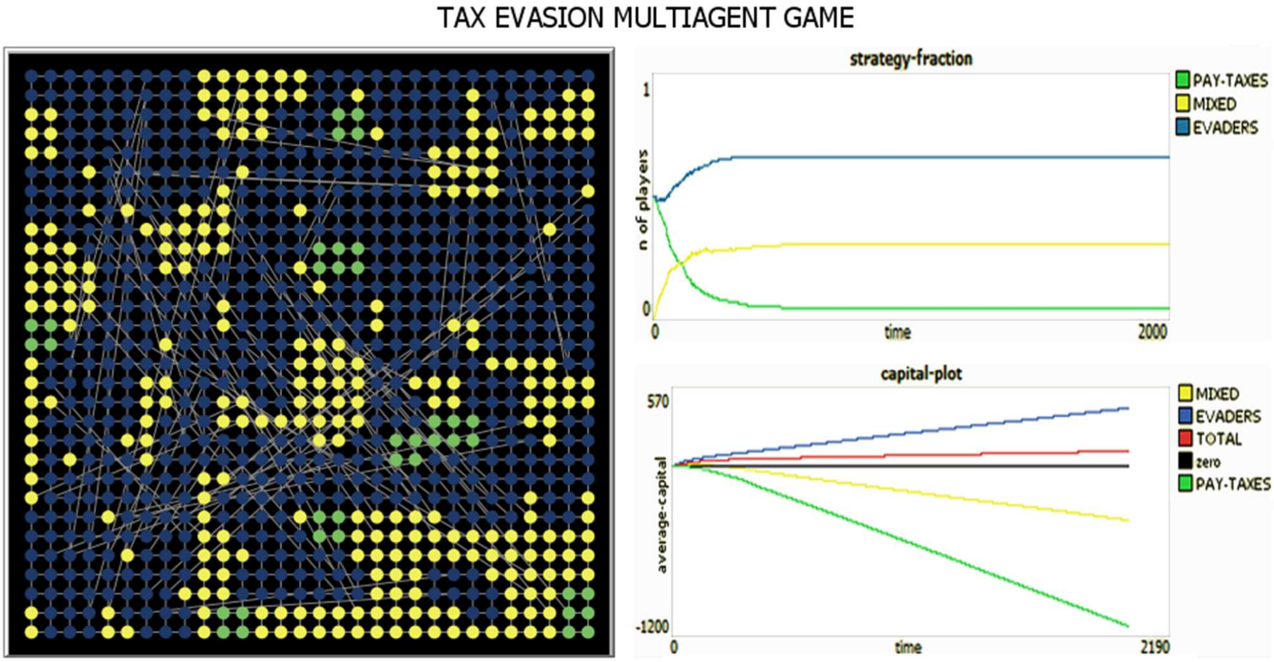

Fig. 8. The same setting of Figure 7, with the same plots, but in the case of an initial percentage of taxpayers equal to 50% (below the critical value for both IF and CF equal to 1).

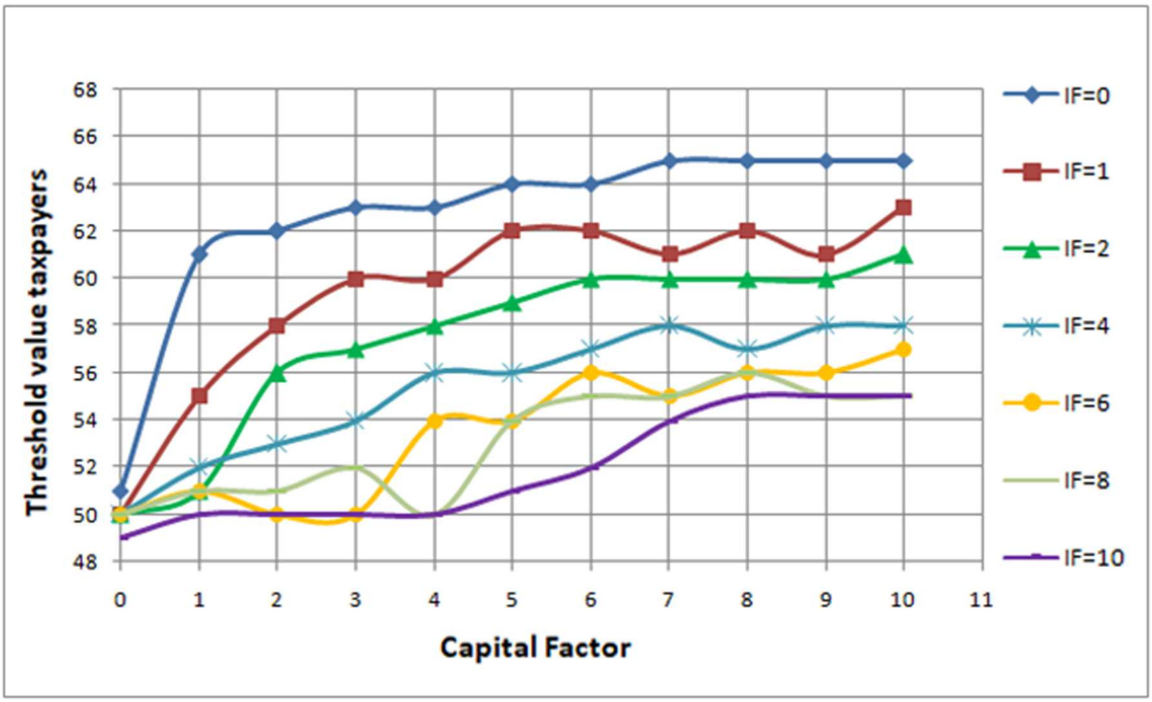

Fig. 9. Threshold values for the initial percentage of taxpayers as a function of the capital factor CF and for different values of the imitation factor IF.

values for the initial percentage of taxpayers as a function of CF are showed in Figure 9 for different values of IF. As we can see, for a given value of IF, generally the critical value of the initial percentage of pay taxes rapidly increases with CF, then it tends to oscillate around a stationary asymptotic value, which decreases with IF. For $IF = 0$, i.e. without imitation, a change in strategy is due to CF only. Thus, when CF is low, i.e., when the dissatisfaction for a negative economic situation is not significant, a small initial percentage of taxpayers is enough to induce a final positive trend for the whole community; on the contrary, when CF is high, i.e., when a negative capital heavily acts on the personal disappointment, the initial percentage of taxpayers has to be more consistent in order to counterbalance the proliferation of tax evaders. Such an effect is reduced by the effect of the imitation, which on average helps: however, the initial fraction of taxpayers has to be always greater than 50%, for the altruism to spread around the community.

3 Tax rate, fines and audit probability

In this section, we present results of simulations with different values of the three main free parameters of our model, namely, the tax rate, the fine and the probability of an audit, set hitherto to: $d = 2$, $h = 3$ and $p = 0.4$. Changes in ΔC will be neglected because they would only produce a symmetric rescaling on the final capital of all groups. Values for IF and CF have been set equal to 1, and two different initial percentages of taxpayers have been used, above and below the 55% threshold (which has already been shown in Fig. 9).

Considering a population with 60% taxpayers, 40% tax evaders and 0% mixed players. Figure 10 presents results of simulations with different tax rates after 2000 time-iterations: the final percentage of individuals (Fig. 10a) and the final average capital (Fig. 10b) are confronted with increasing values of the tax rate d. It clearly appears that for $d < 3$ units (i.e. when the tax is lower than the fine h) the final composition of the population is dominated by taxpayers, whose percentage grows above 70%. This implies a good situation for all citizens. For higher values of the tax (i.e. for $d \geq h$), the evolution of the system leads to a majority of tax evaders and this situation fits good only to evaders. This conclusion is strongly consistent with the initial setting of the model (without the mixed strategy) in which tax evasion Nash-dominates the other strategy if the probability to incur in a fine is lower than the ratio between the tax rate and the fine amount.

Figure 10 presents, instead, results of simulations performed increasing values of the fine h (Figs. 10c and 10d) and of the audit probability p (Figs. 10e and 10f), respectively. Looking at the average capital in Figures 10d and 10f, we can see that for values of the fine $h > 6$ and for the audit probability $p > 0.8$, tax evaders are worse off than taxpayers and mixed players. This does not reduce of tax evasion: while the number of mixed players increases at expenses of taxpayers, Figures 10c and 10e show that the final percentage of evaders is not lower for $h > 6$ and $p > 0.8$. The reason is that in the present release of the model the behavioral update rules are constant and not adaptive with respect to the ex-post probability that a tax evader is discovered. Therefore, when required conditions occur (according to the dynamics induced by imitation and/or capital factors) group shifting happens, irrespective of the fact that simulations are running for higher values of fine and audit probability. This means that both the amount of the fine and the audit probability have an indirect effect. A higher probability and/or a more expensive fine cause more frequent fine payments and an overall stronger reduction of the capital of tax evaders. This can induce them, by means of the capital factor, to change group. It is worth to notice that, in the current setting, the number of taxpayer is sufficient to maintain the convenience to be tax evader even for high values of the audit probability and neutralize the consequent negative indirect effect on the capital. A small change in the percentage of tax payers can affect this conclusion.

Considering a population with 50% taxpayers, 50% tax evaders and 0% mixed players. Figure 11 shows results of simulation with different tax rates (Figs. 11a and 11b), fines (Figs. 11c and 11d) and audit probability (Figs. 11e and 11f). As expected, the final capital of the taxpayers is always lower than before. Looking at the final capital of tax evaders (right panels) a conclusion similar to the previous case emerges: small values for the tax and high values for both the fine and the audit probability do induce a worse economic situation for tax evaders. Further, the final composition of the community (left panels) is always dominated by tax evaders, with the exception of the case with $d = 1$. It is worth to notice that, differently from the previous case, an increase of either the fine or the audit probability, has now the beneficial effect of reducing the number of evaders, who tend to change strategy becoming mixed players. However, for maximum values $h = 10$ and $p = 1.0$, this trend is inverted and the percentage of evaders slightly increases.

4 Conclusions

In this paper we presented a simple model of tax evasion, which augments a prisoner's dilemma framework with an agent-based design, in order to characterize some elements of collective dynamics when altruism and egoism come to play with regards to the number of taxpayer of a community.

In the first part of the paper, the impact of a varying fraction of altruistic players on the final capital of a fully connected community has been shown. Results of simulations showed that, with its basic settings, the model replicated consolidated results of basilar game theory, as presented by Elster [74]. A further specification of the model, in which the public good has been designed as a primary good, identified a threshold level for the fraction of taxpayers in the community. Below such a threshold, not only do tax evaders create a damage for the collectivity (as usual), but they harm themselves as well.

In the second part of the paper, the model was enriched by the introduction of some extensions: a small-world network topology for the social community (driving the imitation), and a third group of "mixed" players (playing alternatively, at random, the two possible strategies). New interesting results have been obtained, showing the presence of a threshold, in the initial percentage of taxpayers, able to ensure an average economic advantage to altruists. Such a threshold is influenced by the individual propensity of agents to imitate and by their sensitivity with respect to their personal economic situation.

Finally, a brief parametric analysis has shown how the tax rate, the fine and the audit probability are able to influence both the final composition of the community and

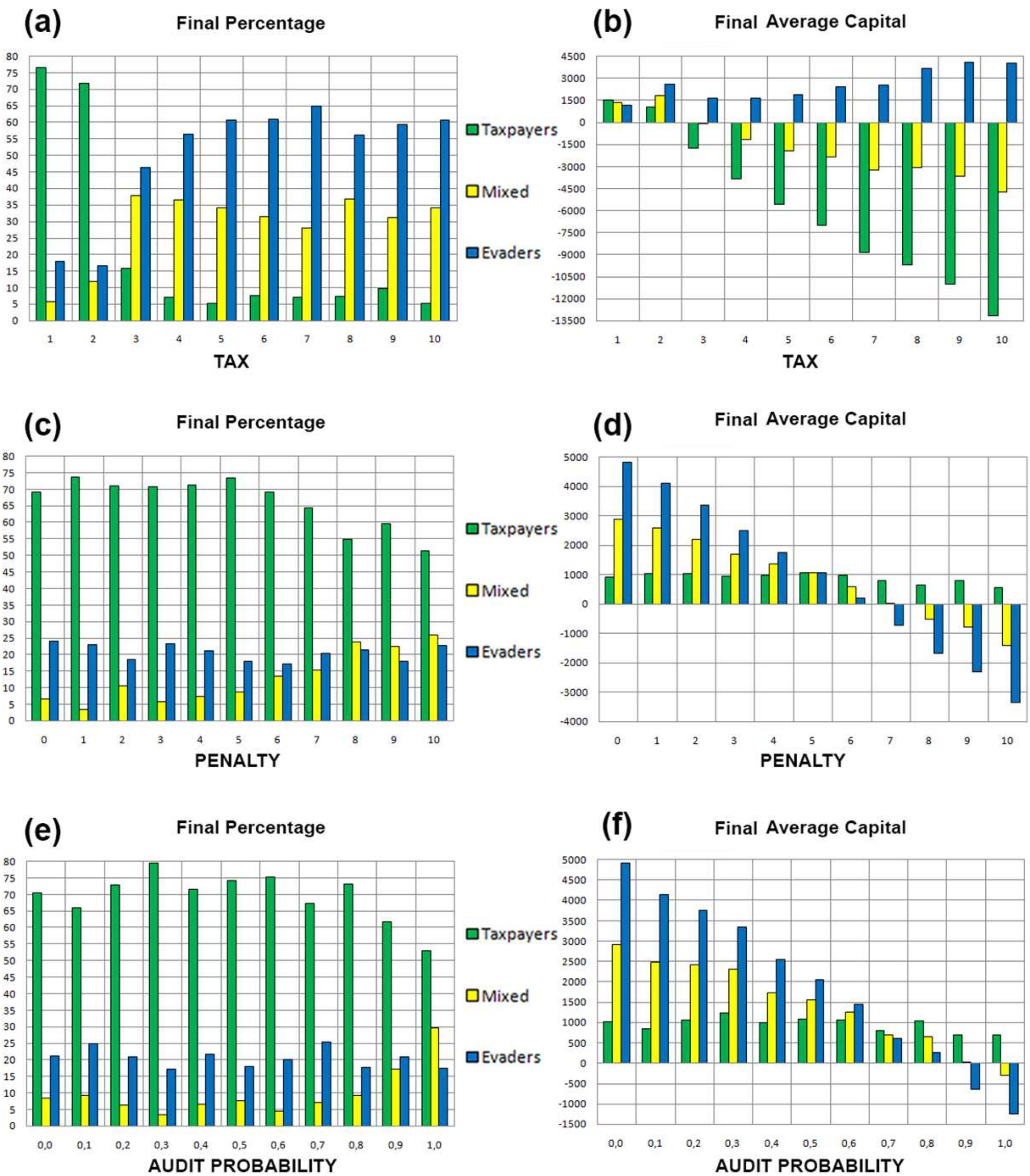

Fig. 10. Final percentage and final average capital after 2000 turns of the three groups for increasing values of, respectively, tax (a, b), fine (c, d) and audit probability (e, f). The initial percentage of taxpayers is always equal to 60%.

the final average capital of the three social groups (taxpayers, evaders and mixed players). As in real systems, the percentage of citizens paying taxes is crucial in sustaining a sufficient quality/quantity of the public good. Results of simulations show that values above such a threshold can paradoxically allow tax evaders to resist even to very high values of the audit probability.

Following such results, reasonable policy intuitions are devoted to induce a feeling of satisfaction in taxpayers, in order to reduce the temptation to evade, even when the personal economic situation is bad. Thus, if Government takes care of taxpayers more than tax evaders is better: e.g., an educational policy spreading a tax morale is expected to be more effective than a tax amnesty, because it operates in such a way that individuals feel themselves rewarded by institutions. Education can also impact on the number of taxpayers, which has been described as a key factor in determining the average social capital. There

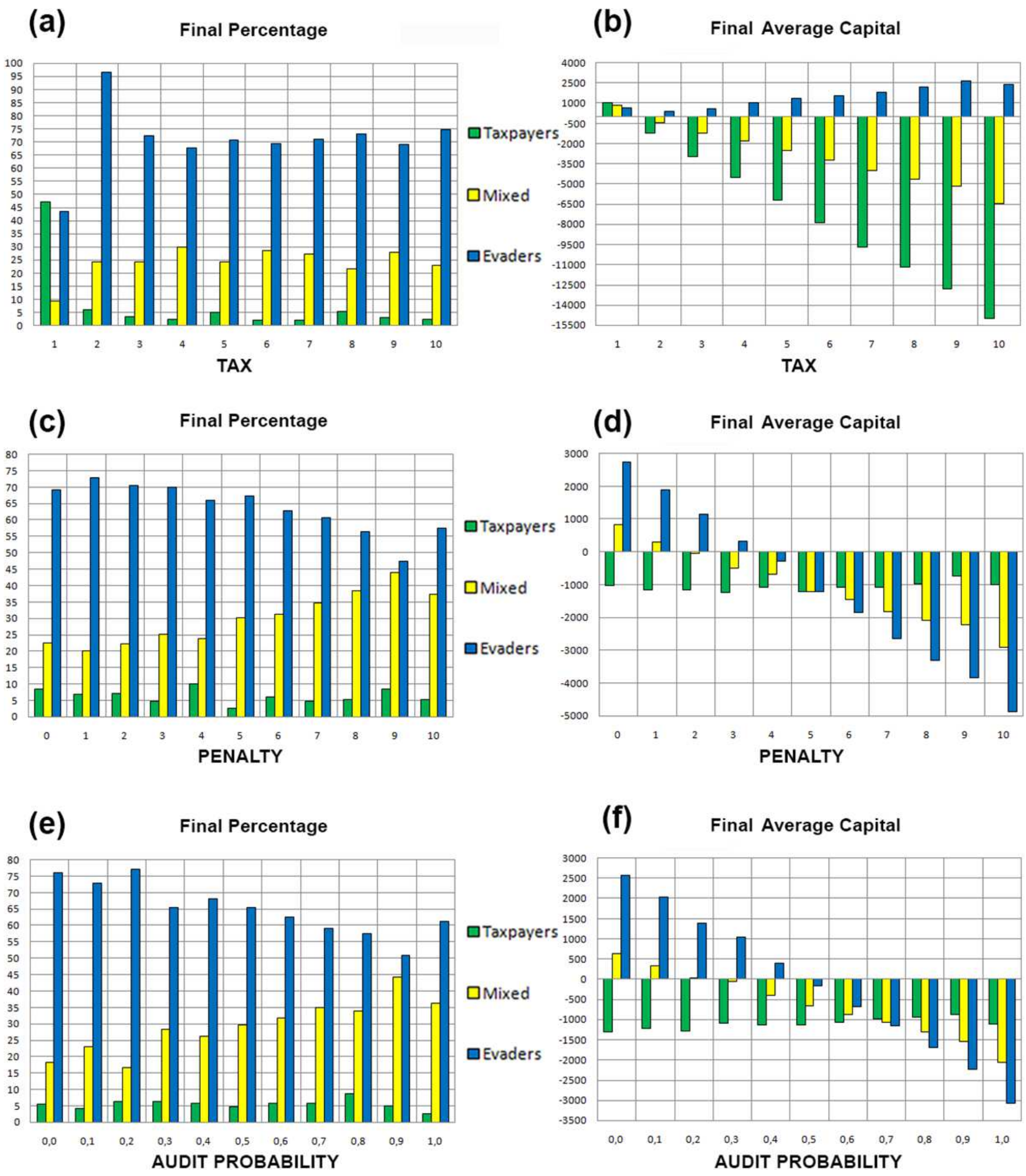

Fig. 11. Final percentage and final average capital after 2000 turns of the three groups for increasing values of, respectively, tax (a, b), fine (c, d) and audit probability (e, f). The initial percentage of taxpayers is always equal to 50%.

is also evidence that the amount of the fine should be far greater than the tax pressure, in order to induce tax payment as a strong economic convenience while an increase in the probability of an audit drastically reduces the value of tax evasion.

Further research will be devoted to deepen both the analytical and the computational elements of the model, after a more detailed design of individual decision process is added. The incentive to pay taxes will be shown to descend more directly from interactions among citizens, when reputation and transparency of personal conduct are added to the model.

Author contribution statement

L.D.M. worked in simulations, data collection, data analysis and reporting of results. A.P. worked in Netlogo model design, physics literature review and writing the paper. A.E.B. worked in economic model design, economics literature review and writing the paper.

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