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Dynamics of charged viscous dissipative cylindrical collapse with full causal approach

S.M. Shah and G. Abbas[a]

Department of Mathematics, The Islamia University of Bahawalpur, Bahawalpur, Pakistan

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Abstract. The aim of this paper is to investigate the dynamical aspects of a charged viscous cylindrical source by using the Misner approach. To this end, we have considered the more general charged dissipative fluid enclosed by the cylindrical symmetric spacetime. The dissipative nature of the source is due to the presence of dissipative variables in the stress-energy tensor. The dynamical equations resulting from such charged cylindrical dissipative source have been coupled with the causal transport equations for heat flux, shear and bulk viscosity, in the context of the Israel-Steward theory. In this case, we have the considered Israel-Steward transportation equations without excluding the thermodynamics viscous/heat coupling coefficients. The results are compared with the previous works in which such coefficients were excluded and viscosity variables do not satisfy the casual transportation equations.

1 Introduction

An important and renowned aspect of general relativity can be observed in the gravitational collapse of sufficiently heavy stars, having mass greater than the mass of the Sun. This phenomenon occurs due to the positive difference of gravity and pressure, in the internal nuclear forces of the massive stars. Oppenheimer and Snyder [1] are innovators of this field of research. They took an initiative step in the field of gravitational collapse and discussed a detailed note on the subject of "On continued gravitational contraction". On the other hand an intellectual and substantial investigation was contributed by Misner and Sharp [2] in 1964. They considered a perfect fluid inside stars and formulated dynamical equations for adiabatic relativistic collapse. Vaidya [3] found the exact model of collapse for the radiating star and some researchers [4–16] investigated the gravitational collapse in different situations. According to Rosseland [17] ions are conversion of atoms having large strength and he explained that in between free particles, the law of cental force can be detected; he further addressed that electrical forces have a great effect for a star which has the mass $1.5M_{\odot}$ (where $M_{\odot}$ is mass of the Sun) and its molecular cloud has a weight of 2.8 units.

Mitra [18] observed that the formation and evolution of stars is a highly dissipative process, which can be divided into two phases. Phase one is free streaming approximation while the second phase is streaming approximation. Tewari [19–21] found the solutions of Einstein field equations for different models. A lot of work for diffusion approximation with electromagnetic field, anisotropy, inhomogeneity and viscosity has been discovered by many well-known relativists [22–25]. In 1987, it was investigated from a supernova that the regime of radiation is further from streaming out limit than the diffusion approximation [26]. Arnett and Kazanas [27,28] also detected that the amount of vigor of radiation is directly related to the temperature gradient. Generally, this detection is strongly reasonable because the mean free path of particles being efficient causes for energy transfer is smaller than the typical objects. Hence for an important progression star as the Sun, the mean free path of photons at the center is of the order of 2 cm. Also the mean free path of trapped neutrinos in the compact core of densities about $10^{12}\,\mathrm{g\,cm^{-3}}$ becomes smaller than the size of the stellar core. Eckart [29] and Landau [30] discussed the transport equation for shear viscosity and theory of relativistic irreversible thermodynamics. As effects of viscosity play a vital role in the development of neutron stars, therefore the coefficient of shear viscosity may attain a value up to $10^{20}\,\mathrm{g\,cm^{-1}\,s^{-1}}$ [31], while the coefficient of bulk viscosity has a maximum value of $10^{30}\,\mathrm{g\,cm^{-1}\,s^{-1}}$ [32] due to the Urca reaction in neutron stars and white dwarfs.

Alford and Blaschve [33,34] assumed that the two color super conducting quark matter may acquire the same or larger values for crossbreed stars [35]. Recently, Sharif and his collaborators [36–38] attempted to discuss the dynamics of a collapsing cylindrical source. They considered cylindrical spacetimes which resemble to spherical spacetime. Also, Herrera *et al.* [39] used the full casual approach

[a] e-mail: abbasg91@yahoo.com

to investigate the dynamics of a spherical dissipative fluid with the radial heat flux. This work is very interesting, informative and has a great impact on the study of many realistic astrophysical phenomena. Our present work stands out in the sense that we have taken the geometry of stars. In addition to this, we have introduced electromagnetic field in the energy momentum tensor and discussed its effects. This work opens a new direction and has different attractive and meaningful results in the field of dissipative gravitational collapse.

The purpose of the present study is to discuss the dynamics of charged viscous cylindrical gravitational collapse in the framework of the Misner formalism. The dissipative nature of the source is prescribed by dissipative variables. The dynamical equations resulting from such charged cylindrical dissipative source are then coupled with causal transport equations for heat flux, shear and bulk viscosity, in the context of the Israel-Steward theory by including the thermodynamics viscous/heat coupling coefficients. Many regarded the inclusion of these coefficients as much important in the non-uniform stellar models [40] and viscosity variables satisfy the casual transportation equations. The plan of the paper is as follows: in the next section, we have presented the dynamical equations for a charged viscous cylindrical source. In sect. 3, the causal transport equations are coupled with dynamical ones. Finally, the results of the paper have been summarized in the last section.

2 Viscous dissipative fluid enclosed by a cylindrical stellar object

This section deals with the geometry of the stellar object, charged dissipative source of matter and the equations of motion. The interior metric in cylindrical coordinates is given by

$$\mathrm{d}s_-^2 = -A^2\mathrm{d}t^2 + B^2\mathrm{d}r^2 + R^2\mathrm{d}\theta^2 + \mathrm{d}z^2, \tag{1}$$

where the constraints on coordinates are the following: $-\infty \leq t \leq \infty$, $-\infty \leq z \leq \infty$, $0 \leq r$, $0 \leq \theta \leq 2\pi$ and $A = A(r,t)$, $B = B(r,t)$ and $R = R(r,t)$.

The energy momentum tensor for a charged viscous dissipative fluid is

$$\begin{aligned}T_{\alpha\beta} = {} & (\mu + P + \Pi)V_\alpha V_\beta + (P+\Pi)g_{\alpha\beta} + q_\alpha V_\beta \\ & + q_\beta V_\alpha + \epsilon l_\alpha l_\beta + \pi_{\alpha\beta} \\ & + \frac{1}{4\pi}\left(F^\gamma_\alpha F_{\beta\gamma} - \frac{1}{4}F^{\gamma\delta}F_{\gamma\delta}g_{\alpha\beta}\right),\end{aligned} \tag{2}$$

where μ, p, V_α, $\pi_{\alpha\beta}$, Π, ϵ and q_α are energy density, isotropic pressure, four velocity, shear viscosity, bulk viscosity, radiation density and radial heat flux, respectively. Also, $F_{\alpha\beta} = -\phi_{\alpha,\beta} + \phi_{\beta,\alpha}$ is the electromagnetic tensor with potential ϕ_α. Moreover, l^α is a radial null four vector. The above quantities satisfy the following relations:

$$\begin{aligned} l^\alpha V_\alpha = -1, \qquad V^\alpha q_\alpha = 0, \qquad l^\alpha l_\alpha = 0, \\ V^\alpha V_\alpha = -1, \qquad V^\alpha q_\alpha = 0, \qquad \pi_{\alpha\beta}V^\alpha = 0.\end{aligned}$$

Further,

$$\begin{aligned} & V^\alpha = A^{-1}\delta^\alpha_0, \qquad q^\alpha = qB^{-1}\delta^\alpha_1, \qquad V_\alpha = -X\delta^0_\alpha, \\ & l^\alpha = A^{-1}\delta^\alpha_0 + B^{-1}\delta^\alpha_1 \quad \text{and} \\ & \pi_{\alpha\beta} = \Omega\left(\chi_\alpha\chi_\beta - \frac{1}{3}h_{\alpha\beta}\right).\end{aligned} \tag{3}$$

The Maxwell field equations are

$$F^{\alpha\beta}_{\ ;\beta} = 4\pi J^\alpha, \qquad F_{[\alpha\beta;\gamma]} = 0, \tag{4}$$

where J_α is the four-current. We assume the following form of electromagnetic potential and current:

$$\phi_\alpha = \phi\delta^0_\alpha, \qquad J^\beta = \zeta V^\beta.$$

Here $\zeta(r,t)$ and $\phi(r,t)$ are charge density and scalar potential, respectively.

The non-zero components of the expansion scalar are

$$\Theta = \frac{1}{A}\left(\frac{2\dot{B}}{B} + \frac{\dot{R}}{R}\right), \tag{5}$$

where $\partial_t = \dot{}$. The field equations for the given source are

$$8\pi\left(\mu + \epsilon + \frac{\pi}{2}E^2\right)A^2 = \frac{\dot{B}\dot{R}}{BR} + \left(\frac{A}{B}\right)^2\left(\frac{A'R'}{AR} - \frac{R''}{R}\right), \tag{6}$$

$$8\pi(q+\epsilon)AB = \frac{\dot{R}'}{R} - \frac{\dot{B}R'}{BR} - \frac{\dot{R}A'}{AR}, \tag{7}$$

$$\begin{aligned} & 8\pi\left(p + \Pi + \epsilon + \frac{2}{3}\Omega + \frac{\pi}{2}E^2\right)B^2 = \frac{A'R'}{AR} \\ & + \left(\frac{B}{A}\right)^2\left(-\frac{\ddot{R}}{R} + \frac{\dot{A}\dot{R}}{AR}\right),\end{aligned} \tag{8}$$

$$\begin{aligned} & 8\pi\left(p + \Pi - \frac{\Omega}{3} - \frac{\pi}{2}E^2\right)R^2 = \\ & \left(\frac{1}{AB}\right)\left(\frac{\dot{A}\dot{B}}{A^2} - \frac{A'B'}{B^2} - \frac{\ddot{B}}{A} + \frac{A''}{B}\right),\end{aligned} \tag{9}$$

where $\partial_r = {}'$, $E = \frac{\hat{Q}(r)}{2\pi R}$ and $\hat{Q}(r) = 4\pi\int_0^r \zeta BR\,\mathrm{d}r$.

The gravitational energy per specific length (also, known as C-energy) for cylindrical symmetric spacetime is given as follows [41,42]:

$$\hat{E}(r,t) = \frac{(1 - l^{-2}\nabla^\alpha\tilde{r}\,\nabla_\alpha\tilde{r})}{8}.$$

For a cylindrically symmetric model with killing vectors, the circumference radius ρ and specific length l and areal radius $\tilde{r}$ are defined as follows [41,42]:

$$\rho^2 = \xi_{(1)\alpha}\xi^\alpha_{(1)}, \qquad l^2 = \xi_{(2)\alpha}\xi^\alpha_{(2)}, \qquad \tilde{r} = l\rho.$$

The C-energy in the total interior region with the electromagnetic field [43] is given by

$$m(r,t) = l\hat{E}(r,t) = \frac{l}{8}\left[1 + \left(\frac{\dot{R}}{A}\right)^2 - \left(\frac{R'}{B}\right)^2\right] + \frac{l^2\hat{Q}^2}{2R}. \tag{10}$$

The collapsing fluid resides inside the non-static metric (1), therefore it must be matched to a suitable exterior. If heat leaves the fluid across the boundary surface then the exterior region of the collapsing star will not be vacuum, but the outgoing Vaidya-like spacetime which models the radiation and has metric [44]

$$\mathrm{d}s_+^2 = -\left(-\frac{2M(\nu)}{R}+\frac{\tilde{q}^2(\nu)}{R^2}\right)\mathrm{d}\nu^2 - 2\,\mathrm{d}\nu\,\mathrm{d}R \\ +R^2(\mathrm{d}\theta^2+\lambda^2\mathrm{d}\phi^2), \tag{11}$$

where $M(\nu)$ and $\tilde{q}(\nu)$ are mass and charge, respectively; these both are measured in units of length (as we have used the relativistic units in our calculations). Also, λ is an arbitrary constant having the units of a length. It has been introduced to balance the units in the metric. Using the continuity of extrinsic curvature of the spacetimes given in eq. (1) and eq. (11), we get

$$M(\nu) = \frac{R}{2}\left[\left(\frac{\dot{R}}{A}\right)-\left(\frac{R'}{B}\right)\right]+\frac{\tilde{q}}{2R}, \tag{12}$$

$$E'-M \stackrel{\Sigma}{=} \frac{1}{8}, \qquad p+\Pi+\frac{2}{3}\Omega \stackrel{\Sigma}{=} q, \tag{13}$$

where $s \stackrel{\Sigma}{=} \tilde{q}$ has been used. These are the conditions for the smooth matching of two regions. Here, $p+\Pi+\frac{2}{3}\Omega \stackrel{\Sigma}{=} q$ implies that the effective pressure on the boundary of the cylinder is non-zero and it is equal to the radial heat flux which provides the possibility of gravitational radiation produced by the collapsing fluid.

3 Dynamical equations

By using the Misner and Sharp[2,45] concept, we may be able to observe the dynamical behavior of field equations. So, we define D_t as the proper time derivative of the following form:

$$D_t = \frac{1}{A}\frac{\partial}{\partial t}. \tag{14}$$

The velocity U is $U = D_tB < 0$ (collapse).

Also from eq. (10), we have

$$\frac{R'}{B} = \left(1+U^2-\frac{8m}{l}+\frac{4\hat{Q}^2l}{R}\right)^{\frac{1}{2}} = \hat{E}. \tag{15}$$

The proper time derivative of the mass function is

$$D_tm = l\left(\frac{\dot{R}\ddot{R}}{4A^3}-\frac{\dot{R}^2\dot{A}}{BA^4}-\frac{R'\dot{R}'}{4B^2A}+\frac{R'^2\dot{B}}{4AB^3}\right)-\frac{\dot{R}\hat{Q}^2l^2}{2AR^2}. \tag{16}$$

Using eq. (6), eq. (7) and the value of $E=\frac{\hat{Q}}{2\pi R}$ then the above equation takes the form

$$D_tm = -2\pi l\left(\hat{E}(q+\epsilon)B+U(p+\Pi+\epsilon+\frac{2}{3}\Omega)\right)R. \tag{17}$$

The above relation leads us to the variation rate of energy in the cylinder of radius R and the right-hand side of this equation describes the increment in energy in the interior region of radius R. The term $(p+\Pi+\frac{2}{3}\Omega)$ is the effective radial pressure while ϵ denotes the pressure of radiation. The standard thermodynamical relation is $\pi_{\alpha\beta}$ in the static phase and the first term in the R.H.S of the above equation shows the energy of the source, which is leaving the cylindrical surface.

The dynamics of the collapsing system can be observed through the proper radial derivative D_R, which is defined as follows:

$$D_R = \frac{1}{R'}\frac{\partial}{\partial r}. \tag{18}$$

Substituting eq. (17) in eq. (9), we have

$$D_Rm = \frac{l}{R'}\left[\frac{\dot{R}\dot{R}'}{4A^2}-\frac{\dot{R}^2A'}{4A^3}-\frac{R'R''}{4B^2}+\frac{B'R'^2}{4B^3}\right. \\ \left.+\frac{l\hat{Q}\hat{Q}'}{R}-\frac{l\hat{Q}^2R'}{2R^2}\right]. \tag{19}$$

Substituting eq. (5) and eq. (6) in eq. (17), we have

$$D_Rm = 2\pi Rl\left(4(\mu+\epsilon)+\frac{U}{\hat{E}}(q+\epsilon)B\right)+\frac{l^2\hat{Q}\hat{Q}'}{RR'}-\frac{l^2\hat{Q}^2}{R^2}. \tag{20}$$

The above equation on integration yields

$$m = \int_0^R \pi Rl\left(4(\mu+\epsilon)+\frac{U}{\hat{E}}(q+\epsilon)B\right)\mathrm{d}R+\frac{l^2\hat{Q}^2}{2R} \\ -\frac{l^2}{2}\int_0^R\frac{\hat{Q}^2}{R^2}\mathrm{d}R. \tag{21}$$

Here, $m(0)=0$ has been used. Now we obtain the acceleration D_tU as

$$D_tU = \frac{1}{A}\frac{\partial}{\partial t}\left(\frac{\dot{R}}{A}\right) \Rightarrow D_tU = \frac{\ddot{R}}{A^2}-\frac{\dot{R}\dot{A}}{A^3}. \tag{22}$$

Now from eqs. (8), (10) and (22), we get

$$D_tU = -\left[\frac{m}{R^2}+8\pi\left(p+\Pi+\epsilon+\frac{2}{3}\Omega\right)R\right]+\frac{A'\hat{E}}{AB} \\ +\frac{\hat{Q}^2}{R}\left(\frac{l^2}{2R^2}-1\right)+\frac{l}{8R^2}\left(1+U^2-\hat{E}^2\right). \tag{23}$$

Using the conservation law, we get the following dynamical equations:

$$T^{\mu\nu}_{;\nu}V_\mu = -\frac{1}{A}\left(\dot{\mu}+\dot{\epsilon}-\pi\dot{E}E\right)-\frac{1}{B}(q'+\epsilon') \\ -\frac{\dot{R}}{AR}\left(\mu+p+\Pi+\epsilon-\frac{\Omega}{3}-\frac{\pi}{2}E^2\right) \\ -\frac{\dot{B}}{AB}\left(\mu+p+\Pi+2\epsilon+\frac{2}{3}\Omega\right)-2\frac{(ABR)'}{AB^2R}(q+\epsilon) \tag{24}$$

and

$$
\begin{aligned}
T^{\mu\nu}_{;\nu}\chi_\mu &= \frac{1}{A}\left(\dot{q}+\dot{\epsilon}\right)+\frac{2}{A}\frac{(BR)^{\cdot}}{BR}(q+\epsilon)\\
&+\frac{1}{B}\left(p'+\Pi'+\epsilon'-\frac{(2\Omega)'}{3}-2\pi EE'\right)\\
&+\frac{1}{B}\frac{A'}{A}\left(\mu+p+\Pi+2\epsilon+\frac{2}{3}\Omega\right)+\frac{2}{B}\frac{R'}{R}(\epsilon+\Omega).
\end{aligned}\tag{25}
$$

Using the value of $\frac{A'}{A}$ from eq. (23) into eq. (25) and considering eqs. (6)–(9), we get the main dynamical equation

$$
\begin{aligned}
&\left(\mu+p+\Pi+2\epsilon+\frac{2}{3}\Omega\right)D_tU=\\
&-\left(\mu+p+\Pi+2\epsilon+\frac{2}{3}\Omega\right)\\
&\times\left[\frac{m}{R^2}+8\pi R\left(p+\Pi+\epsilon+\frac{2}{3}\Omega\right)\right.\\
&\left.-\frac{\hat{Q}^2}{R}\left(\frac{l^2}{2R^2}-1\right)-\frac{l}{8R^2}(1+U^2)\right]\\
&-\hat{E}^2\left[D_R\left(p+\Pi+2\epsilon+\frac{2}{3}\Omega+\frac{\hat{Q}\hat{Q}'}{4\pi^2R}\right)\right.\\
&\left.+\frac{\hat{Q}^2}{4\pi R^3}+\frac{2}{R}(\epsilon+\Omega)\right]\\
&-\hat{E}\left[D_tq+D_t\epsilon+2(q+\epsilon)\frac{U}{R}+\frac{2\dot{B}}{AB}(q+\epsilon)\right].
\end{aligned}\tag{26}
$$

Here, we can analyze that the factor $(\mu+p+\Pi+2\epsilon+\frac{2}{3}\Omega)$ appears on the left side and in the first term on the right side: this is the effective inertial mass, and according to the equivalence principle it is also known as passive gravitational mass. On the right side, in the first term the square bracket factor explains the effects of dissipative variables on the active gravitational mass of the collapsing cylinder; this fact has been notified firstly by Herrera *et al.*[39]. In the second square bracket there are the gradient of the total effective pressure which is influenced by dissipative variables, radiation density and electromagnetic field. The last square bracket contains different contributions due to the dissipation nature of the system. The third term in this factor is positive ($U<0$) implying that the outflow of $q>0$ and $\epsilon>0$ reduces the integrated energy of the contracting source, which decreases the rate of collapse.

4 The transport equation

The objective of this article is to discuss a full causal approach for the viscous dissipative gravitational collapse of stellar objects along with heat conduction. This implies that all dissipative variables must satisfy the transport equations obtained from causal thermodynamics. Consequently, we use the transport equations for heat, bulk and shear viscosity from the Muller-Israel-Stewart theory [46–48] for a dissipative material. These transport equations for heat, bulk and shear viscosity [39] are

$$
\tau_0\Pi_{;\alpha}V^\alpha+\Pi=-\xi\Theta+\alpha_0\xi q^\alpha_{;\alpha}-\frac{1}{2}\xi T\left(\frac{\tau_0}{\xi T}V^\alpha\right)_{;\alpha}\Pi,\tag{27}
$$

$$
\begin{aligned}
&\tau_1h^\beta_\alpha q_{\beta;\mu}V^\mu+q_\alpha=-\kappa\Big[h^\beta_\alpha T_{,\beta}(1+\alpha_0\Pi)+\alpha_1\pi^\mu_\alpha\\
&\times h^\beta_\mu T_{,\beta}+T(\alpha_0-\alpha_0\Pi_{;\alpha}-\alpha_1\pi^\mu_{\alpha;\mu})\Big]\\
&-\frac{1}{2}\kappa T^2\left(\frac{\tau}{\kappa T^2}V^\beta\right)_{;\beta}q_\alpha,
\end{aligned}\tag{28}
$$

$$
\begin{aligned}
&\tau_2h^\mu{}_\alpha\pi_{\mu\nu;\rho}V^\rho+\pi_{\alpha\beta}=-2\eta\sigma_{\alpha\beta}+2\eta\alpha_1q_{\langle\beta;\alpha\rangle}\\
&-\eta T\left(\frac{\tau^2}{2\eta T}V^\nu\right)_{;\nu}\pi_{\alpha\beta},
\end{aligned}\tag{29}
$$

$$
q_{\langle\beta;\alpha\rangle}=h^\mu{}_\alpha h^\nu{}_\beta\left(\frac{1}{2}(q_{\mu;\nu}+q_{\nu;mu})-\frac{1}{3}q_{\sigma;\kappa}h^{\sigma\kappa}\right),\tag{30}
$$

where relaxation times have the following values:

$$
\tau_0=\xi\beta_0\qquad\tau_1=\kappa T\beta_1\qquad\tau_2=2\eta\beta_2,\tag{31}
$$

where β_1, β_2 are thermodynamic coefficients for different contributions to entropy density, α_0, α_1 are thermodynamics viscous/heat coupling coefficients, ξ and η are coefficients of bulk and shear viscosity. Equations (27)–(30), with the help of the given interior metric take the following form:

$$
\begin{aligned}
\tau_0\dot{\Pi}&=-\left(\xi+\frac{\tau_0\Pi}{2}\right)A\Theta+\frac{A}{B}\alpha_0\xi\left[q'+q\left(\frac{A'}{A}+\frac{2R'}{R}\right)\right]\\
&-\Pi\left[\frac{\xi T}{2}\left(\frac{\tau_0}{\xi T}\right)^{\cdot}+A\right],
\end{aligned}\tag{32}
$$

$$
\begin{aligned}
\tau_1\dot{q}&=\frac{A}{B}\kappa T'\left(1+\alpha_0\Pi+\frac{2}{3}\alpha_1\Omega\right)+T\left[\frac{A'}{A}-\alpha_0\Pi'\right.\\
&\left.-\frac{2}{3}\alpha_1\left(\Omega'+\left(\frac{A'}{A}+3\frac{R'}{R}\right)\Omega\right)\right]\\
&-q\left[\frac{\kappa T^2}{2}\left(\frac{\tau_0}{\kappa T^2}\right)^{\cdot}+\frac{\tau_1}{2}\Theta A+A\right],
\end{aligned}\tag{33}
$$

$$
\begin{aligned}
\tau_1\dot{\Omega}&=-2\eta\sigma+2\eta\alpha_1\frac{A}{B}\left(q'-q\frac{R'}{R}\right)\\
&-\Omega\left[\eta T\left(\frac{\tau_2}{2\eta T}\right)^{\cdot}+\frac{\tau_2}{2}\Theta A+A\right].
\end{aligned}\tag{34}
$$

Now, to observe the influence of various dissipative variables on the cylindrical collapsing source, we substitute eq. (33) in eq. (26) and after some rearrangements, we obtain

$$
\begin{aligned}
&\left(\mu+p+2\epsilon+\frac{2}{3}\Omega\right)(1-\Lambda)D_tU=(1-\Lambda)F_{grav}+F_{hyd}\\
&+\frac{\kappa}{\tau_1}\hat{E}^2\left[D_RT\left(1+\alpha_0\Pi+\frac{2}{3}\alpha_1\Omega\right)\right]\\
&-\frac{\kappa}{\tau_1}\hat{E}^2T\left[\left(\alpha_0D_R\Pi+\frac{2}{3}\alpha_1+\left(D_R\Omega+\frac{3}{R}\Omega\right)\right)\right]\\
&-\hat{E}\left[\frac{2\dot{B}}{AB}(q+\epsilon)-\frac{q}{\tau_1}-2(q+\epsilon)\frac{U}{R}\right]\\
&+\hat{E}\left[\frac{\kappa,T}{2\tau_1D_t}\left(\frac{\tau_1}{\kappa T^2}-D_t\epsilon\right)+A\frac{\tau_1}{2}\Theta\right],
\end{aligned}
\tag{35}
$$

where F_{grav}, F_{hyd} and Λ are defined by

$$
\begin{aligned}
F_{grav}=&-\left(\mu+p+\Pi+2\epsilon+\frac{2}{3}\Omega\right)\left[\frac{m}{R^2}+8\pi R\left(p+\Pi\right.\right.\\
&\left.\left.+\epsilon+\frac{2}{3}\Omega\right)-\frac{\hat{Q}^2}{R}\left(\frac{l^2}{2R^2}-1\right)-\frac{l}{8R^2}(1+U^2)\right],\\
F_{hyd}=&-\hat{E}^2\left[D_R\left(p+\Pi+\epsilon+\frac{2}{3}\Omega+\frac{\hat{Q}^2}{4\pi^2R^2}-\frac{\hat{Q}}{2\pi^2R^2}\right)\right.\\
&\left.+\frac{\hat{Q}^2}{2\pi^2R^3}+\frac{2}{3R}(\epsilon+\Omega)\right],\\
\Lambda=&\frac{\kappa T}{\tau_1}\left(\mu+p+2\epsilon+\frac{2}{3}\Omega\right)^{-1}\left(1-\frac{2}{3}\alpha_1\Omega\right).
\end{aligned}
\tag{36}
$$

Taking the value of Θ from eq. (27) and using eq. (35), we have the following resulting equation:

$$
\begin{aligned}
&\left(\mu+p+2\epsilon+\frac{2}{3}\Omega\right)(1-\Lambda+\Delta)D_tU=\\
&(1-\Lambda+\Delta)F_{grav}+F_{hyd}\\
&+\frac{\kappa}{\tau_1}\hat{E}^2\left[D_RT\left(1+\alpha_0\Pi+\frac{2}{3}\alpha_1\Omega\right)\right]\\
&-\frac{\kappa}{\tau_1}\hat{E}^2T\left[\left(\alpha_0D_R\Pi+\frac{2}{3}\alpha_1\left(D_R\Omega+\frac{3}{R}\Omega\right)\right)\right]\\
&-\hat{E}^2\left(\mu+p+2\epsilon+\frac{2}{3}\Omega\right)\Delta\left(\frac{D_Rq}{q}-\frac{4q}{R}\right)\\
&-\hat{E}\left[\frac{2\dot{B}}{AB}(q+\epsilon)-\frac{q}{\tau_1}-2(q+\epsilon)\frac{U}{R}\right]\\
&+\hat{E}\left[\frac{\kappa T^2q}{2\tau_1}D_t\left(\frac{\tau_1}{\kappa T^2}\right)-D_t\epsilon\right]\\
&+\hat{E}\left(\mu+p+2\epsilon+\frac{2}{3}\Omega\right)\frac{\Delta}{2\alpha_0\kappa q}\\
&\times\left(1+2\xi TD_t\left(\frac{\tau_0}{\xi T}\right)\Pi+\frac{\tau_0}{A}D_t\Pi\right),
\end{aligned}
\tag{37}
$$

where

$$
\Delta=\alpha_0\xi q\left(\frac{3q+4\epsilon}{2\xi+\tau_0\Pi}\right)\left(\mu+p+\Pi+2\epsilon+\frac{2}{3}\Omega\right)^{-1}.
\tag{38}
$$

Hence, by taking into account the casual transportation equations and their coupling with the dynamical equations, we find that the factor $(\Delta-\Lambda+1)$ affects significantly the internal energy and passive gravitational mass density. This result is in agreement with [39]. We would like to mention that we have considered the charged dissipative viscous fluid, but no terms appear in the relevant equations which couple the electromagnetic field and dissipative variables. Therefore the role of electromagnetic field on the dynamical process is the same as in the absence of shear viscosity already discussed in [49].

5 Conclusion

Immediately after the Einstein theory of gravity in the early 20th century, the study of cylindrically symmetric systems was started by Weyl [50] and Levi-Civita [51]. After the complete study of spherical objects, theoretical physicists were interested to explore the properties of astrophysical compact stars that have axially symmetry. The relativistic fluids involving heat flux and viscosity are very important for studying the evolution of compact stars. So, it is important to include the dissipative variables in gravitational collapse.

In this paper, we have constructed the dynamical equations which deal significantly with the structure and evolutionary phases of a charged gravitating viscous cylindrical source. In order to see the effects of dissipation on the dynamical evolution of a gravitating source, we have assumed the convenient form of the dissipative variables which satisfy the transportation equations of heat, bulk and shear viscosity resulting from the casual thermodynamics. Furthermore, the dissipative coefficients due to viscosity and heat flux have been included in the discussion of dynamical equations. In a very broad sense, we are mainly interested in time scales whose order may be smaller or equal to the radiation time. During the study of transport equations for dissipative variables, we have preferred to use the hyperbolic theory of dissipation because this theory is more reliable and has less difficulties than parabolic theory [8,48,52,53].

A full casual approach has been adopted in [39] to explore the effects of dissipative variables on the spherical collapse; this study provides the meaningful results which have significant implications in astronomy. The application of these results to some stellar system implies that in a pre-supernovae event, the thermal conductivity of the dissipative source might be large enough to produce an observable reduction in the gravitational force of the system that results in the expansion of the gravitating source instead of collapse. It is quite relevant to mention that thermodynamics viscous/heat coupling coefficients have been taken as non-vanishing because this assumption provides the significant basis for the modeling of a non-uniform stellar system. In a recent investigation [49], we have considered a non-casual (irreversible thermodynamics) approach to discuss the dynamics of the charged bulk viscous cylindrical collapse by neglecting the thermodynamics viscous/heat coupling coefficients in the transportation equations. So, our present analysis is the extension of our previous study with non-casual approach [49]. *But it is important to note that this analysis with cylindrical symmetry is analogous to the full casual approach adopted by Herrera* et al. [39] *for spherical stellar objects.*

As a consequence of a full casual approach to the dynamics of charged dissipative cylindrical collapse, we obtain a dynamical equation (37), which explains how the value of effective inertial mass is influenced by the dissipative variables and thermodynamics viscous/heat coupling coefficients. All the dissipative variables have a great effect in a pre-supernovae event, for example a large enough value of heat conductivity κ can produce a rapid decrease in the force of gravity, thereby resulting in the reversal of collapse [39]. A numerical model predicting this type of bouncing behavior has been presented Herrera *et al.* [11]. It is to be noted that such numerical estimations in the present case are beyond the scope of this work. Here we just want to ensure that during the dissipative gravitational collapse, thermodynamics viscous/heat coupling coefficients must not be excluded *a priori* in the transportation equations. In the future, we are interested to extend these results in modified $f(R)$, $f(R,T)$ and $f(G)$ theories of gravity.

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