Eur. Phys. J. A (2017) **53**: 96
DOI 10.1140/epja/i2017-12297-2

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Microscopic study of $^1S_0$ superfluidity in dilute neutron matter

G.E. Pavlou[1,a], E. Mavrommatis[1], Ch. Moustakidis[2], and J.W. Clark[3,4]

[1] Physics Department, Division of Nuclear and Particle Physics, National and Kapodistrian University of Athens, GR-15771 Athens, Greece
[2] Department of Theoretical Physics, Aristotelian University of Thessaloniki, GR-54124 Thessaloniki, Greece
[3] McDonnell Center for the Space Sciences and Department of Physics, Washington University, St. Louis, MO 63130, USA
[4] Center for Mathematical Sciences, University of Madeira, Funchal, 9000-390 Portugal

Received: 6 December 2016 / Revised: 22 March 2017
Published online: 17 May 2017 –
Communicated by D. Blaschke

Abstract. Singlet S-wave superfluidity of dilute neutron matter is studied within the correlated BCS method, which takes into account both pairing and short-range correlations. First, the equation of state (EOS) of normal neutron matter is calculated within the Correlated Basis Function (CBF) method in the lowest cluster order using the $^1S_0$ and ^3P components of the Argonne V_{18} potential, assuming trial Jastrow-type correlation functions. The $^1S_0$ superfluid gap is then calculated with the corresponding component of the Argonne V_{18} potential and the optimally determined correlation functions. The dependence of our results on the chosen forms for the correlation functions is studied, and the role of the P-wave channel is investigated. Where comparison is meaningful, the values obtained for the $^1S_0$ gap within this simplified scheme are consistent with the results of similar and more elaborate microscopic methods.

1 Introduction

Theoretical study of dilute neutron matter and its superfluid phase continues to be an active subfield of nuclear theory [1]. As a model system of strongly interacting fermions, it has been a testing ground for advances in *ab initio* microscopic approaches to quantum many-body problems. As an essential component of the inner crust of neutron stars, interpenetrating a lattice of neutron-rich nuclei, dilute neutron matter at baryon densities in the range $0.2\,\mathrm{fm}^{-1} \lesssim k_F \lesssim 1\,\mathrm{fm}^{-1}$ is of considerable importance in dense-matter astrophysics. Moreover, low-density neutron matter may also be found in the skins or outer envelopes of some exotic nuclei. Additionally, analogies with ultracold fermionic atomic gases and their theoretical treatment can be fruitfully developed.

The main aim is to study the superfluid dilute neutron matter and calculate the pairing gap in the $^1S_0$ channel. The existence of such phase in the inner crust of neutron stars has direct consequences for post-glitch relaxation, neutrino emission and cooling of neutron stars [2–5] and an accurate value for the gap is required. Many calculations of the $^1S_0$ superfluid gap have been carried out since the 1970s, using various microscopic many-body theories (for a review see ref. [1]). Among them we mention direct calculations based on the original Bardeen, Cooper, Schrieffer (BCS) theory using bare two-nucleon potentials [6,7] and three-nucleon potentials [8,9], application of the polarization potential model of Pines and coworkers [10], inclusion of medium-polarization within a G-matrix formulation [11,12], application of Dirac-Brueckner-Hartree-Fock-Bogoliubov (DBHFB) theory [13], calculations using the Correlated Basis Function Method (CBF) [14–16], applications of the Self-Consistent Green's Function Method [17,18], a Renormalization Group (RG) treatment [7], and pursuit of various Monte Carlo techniques [19–23]. In spite of these many efforts, there is still ambiguity in the value of the $^1S_0$ gap as a function of the density (or k_F), owing to the strong sensitivity of the gap to inputs for the pairing interaction and self-energies.

The results reported here were obtained by implementing a generalization of BCS theory within the CBF framework, giving explicit consideration to the role of short-range geometrical correlations induced by the strong nuclear force [24,25]. These results supplement previous work carried out with the same method [14], although estimates of perturbation corrections within the CBF formulation are not included. The correlation functions and single-particle energies that enter are determined in a straightforward manner from a lowest cluster order calculation of the equation of state (EOS) of normal neutron matter including the S and P partial-wave components of the realistic Argonne V_{18} potential [26].

It turns out that our results for the energy per neutron are, in some cases, quite similar to those obtained

[a] e-mail: gepavlou@phys.uoa.gr

including higher-order cluster contributions and additional components of the Argonne V_{18} potential. The EOS of mainly dilute neutron matter has of course been calculated by a broad range of many-body techniques (for a review see ref. [4]). Among these are: virial expansion [27], the lowest-order constrained variational (LOCV) method [28–30], the operator-chain variational techniques [31,32], CBF calculations [33–35], the Dirac-Brueckner-Hartree-Fock (DBHF) approach [36], Bethe-Brueckner-Goldstone (BBG) methods [37], mean field calculations with interactions derived by Renormalization Group (RG) techniques [7,38], calculations using inputs from effective field theory (EFT) [39,40] and Lattice Chiral EFT [41]. Additionally there have been major efforts within the general framework of Monte Carlo algorithms, especially variational Monte Carlo (VMC), Green's Function Monte Carlo (GFMC) [22,23,42] and Auxiliary-Field Diffusion Monte Carlo (AFDMC) [15,19,20,40,43,44]. It is expected that the correlation functions that we have determined can be used with some reliability in the calculation of other observables of low-density neutron matter besides the $^1S_0$ pairing gap. Allowing for the simplifications made in our treatment of the $^1S_0$ gap, the results obtained are generally compatible with the estimates reported by other authors. As it is well known, at low densities only the S-wave interaction is required, whereas at somewhat higher densities of neutron matter, it becomes necessary to include the P-wave component. In the density regime considered, higher partial waves have negligible impact, and corrections from higher cluster orders and three-nucleon forces are expected to be minimal.

This paper is organized as follows: in sect. 2 we outline our method of calculation within the framework of non-orthogonal CBF theory, for both the equation of state and the $^1S_0$ superfluid gap. In sect. 3 we present our results and compare them to those of other authors. Section 4 is devoted to a brief summary of our results and to some concluding remarks.

2 Methods of calculation

2.1 Equation of state

Our calculations are carried out within the method of correlated basis functions (CBF) [33], in which a correlation operator $F_N(1,\ldots,N)$ for N fermions generates not only a trial ground state wave function but also a complete set of non-orthogonal basis states constructed as follows:

$$|m\rangle \equiv |\Psi_m\rangle = I_{mm}^{-1/2} F_N(1,\ldots,N)|\Phi_m\rangle. \tag{1}$$

Here $\{|\Phi_m\rangle\}$ is a complete orthonormal set of states of a suitable independent-particle model and I_{mm} is the normalization constant given by

$$I_{mm} \equiv \langle\Phi_m|F_N^\dagger(1,\ldots,N)F_N(1,\ldots,N)|\Phi_m\rangle. \tag{2}$$

To describe normal neutron matter we adopt as model states $|\Phi_m\rangle$ a complete orthonormal set of wave functions of an ideal Fermi gas of non-interacting neutrons. The Hamiltonian matrix elements $H_{mn} = \langle m|\hat{H}|n\rangle$ in the correlated basis may be used to generate perturbative expansions [33] for the ground-state energy and other quantities. For the interaction we will consider essential components of the Argonne V_{18} two-nucleon potential [26].

The correlation operator $F_N(1,\ldots,N)$ in eq. (1) is taken as a Jastrow product [33,34] of central two-body correlation functions $F_2(ij) = f(r_{ij})$

$$F_N(1,\ldots,N) = \prod_{i<j} F_2(ij), \tag{3}$$

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. Denoting the ground state of the non-interacting Fermi gas by $|\Phi_0\rangle$, the Hamiltonian expectation value $E_0 = \langle\Psi_0|H|\Psi_0\rangle$ with respect to the "ground" correlated basis state

$$|\Psi_0\rangle = \prod_{i<j} F_2(r_{ij})|\Phi_0\rangle \tag{4}$$

is developed in a cluster expansion [33] in orders of the smallness parameter

$$\xi = N^{-1}\sum_{ij} h_{m_i m_j, m_i m_j}, \tag{5}$$

where

$$h_{m_1m_2,m_1m_2} = \langle m_1m_2|f^2(r) - 1|m_1m_2 - m_2m_1\rangle, \tag{6}$$

with m_i and m_j representing the orbitals of two particles in the non-interacting Fermi sea, and r denoting their separation.

For the diagonal matrix elements of the Hamiltonian in the correlated basis we find

$$H_{mm} = \sum_i \varepsilon_{m_i}^{(0)} + \sum_{i<j} w_{m_i m_j, m_i m_j} + 0(\xi), \tag{7}$$

where the terms $\varepsilon_{m_i}^{(0)}$ are the single-particle energies of the orbitals composing the independent-particle model state $|\Phi_m\rangle$ and

$$w_{m_1m_2,m_1m_2} = \langle m_1m_2 - m_2m_1|w_2(12)|m_1m_2\rangle \tag{8}$$

are matrix elements of the effective two-body potential which equals

$$w_2(r) = \frac{\hbar^2}{M}\left(\nabla f(r)\right)^2 + v(r)f^2(r). \tag{9}$$

with the Jastrow correlations. Here M is the neutron mass and $v(r)$ is an appropriate central component of the two-body nucleon-nucleon potential. With Fermi-gas energy eigenstates taken for the model states $|\Phi_m\rangle$, it is straightforward to derive the formula

$$\frac{E}{N} = E_N = E_F + 2\pi\rho\sum_{\mathrm{S}}\int_0^\infty w_2^{\mathrm{S}}(r)G_{\mathrm{S}}(k_F r)r^2\mathrm{d}r \tag{10}$$

for the ground-state energy per particle of neutron matter. In this expression, i) $E_F = 3\hbar^2 k_F^2/(10M)$ is the energy per particle for a Fermi gas of non-interacting particles with M the bare neutron mass, ii) the sum runs over the two possible states of two-nucleon spin (singlet S = 0 and triplet S = 1), and iii) $w_2^{\rm S}$ is obtained from eq. (9) by inserting for $v(r)$ the central potential acting in the corresponding two-neutron state of total spin S. The factors $G_{\rm S}$ are the spatial pair distribution functions in singlet and triplet spin states, given by

$$G_{\rm S=0}(k_F r) = \frac{1}{4}(1 + l^2(k_F r)) \tag{11}$$

and

$$G_{\rm S=1}(k_F r) = \frac{3}{4}(1 - l^2(k_F r)), \tag{12}$$

where $l(x)$ is the Slater function

$$l(x) = 3x^{-3}(\sin x - x\cos x). \tag{13}$$

Our numerical calculations of the EOS of neutron matter were based on eq. (10) along with (9), (11), and (12). Ideally, the correlation function $f(r)$ would be determined by Euler-Lagrange minimization of the ground-state energy expectation value E_N. Following other authors, we approximate E_N by its leading cluster order and minimize it with respect to the parameters of a suitable analytic form for $f(r)$. This practice has proven satisfactory at the low densities involved in the S-wave pairing problem. In our study, we have considered two forms for $f(r)$, i) the so-called Davé type of ref. [14], *i.e.*,

$$f_{\rm S}(r) = \exp\left\{-\frac{1}{2}\left(\frac{b}{r}\right)^m \exp\left[-\left(\frac{r}{b}\right)^n\right]\right\}, \tag{14}$$

having parameters b, m, and n, and ii) the Benhar type [45]

$$f_{\rm S}(r) = \left[1 - \exp\left(-\frac{r^2}{b^2}\right)\right]^2 + gr\exp\left(-\frac{r^2}{c^2}\right), \tag{15}$$

with parameters b, c, and g. The parameter g is determined by applying the orthogonality condition [34]

$$\rho\int {\rm d}\boldsymbol{r}\,[1 - f_{\rm S}(r)]\,G_{\rm S}(k_F r) = 0 \tag{16}$$

to each spin state where $f(r)$ is written as $f_{\rm S}(r)$ to emphasize that the parameters in the correlation choices (14) and (15) can depend on the spin state involved.

Two sets of calculations of the EOS were performed based on the Argonne V_{18} interaction. In the more general case (called "spin-dependent"), contributions from both singlet and triplet components of the interaction are included. The singlet component is given by its S-wave part, *i.e.*, the interaction acting in the $^1S_0$ partial wave. The spin-triplet component, having no S-wave part in neutron matter, is taken as the interaction acting in the ^3P partial wave, averaged over the three substates involved, thus

$$v_{3P} = \frac{1}{9}v_{^3P_0} + \frac{3}{9}v_{^3P_1} + \frac{5}{9}v_{^3P_2}. \tag{17}$$

In the simpler case, only the $^1S_0$ component of the interaction is included ("S-wave only").

The same two forms of correlation function (Davé and Benhar types) are assumed for "S-wave only" and "spin-dependent" cases, but with individually determined parameter values. These optimal parameters are obtained by numerical minimization of the corresponding energy expectation value (10) at zeroth order in the small parameter ξ (leading cluster order), at each density considered.

2.2 ^{1}S$_0$ superfluid gap

We adopt Correlated BCS theory [24,25] to study $^1S_0$ superfluidity in neutron matter. In this theoretical approach, the non-orthogonal CBF method is used to generalize BCS theory to treat strongly correlated Fermi systems. The correlated BCS ground state is constructed as

$$|{\rm CBCS}\rangle = \sum_N \sum_m \left(I_{mm}^{(N)}\right)^{-1/2} F_N |\Phi_m^{(N)}\rangle\langle\Phi_m^{(N)}|{\rm BCS}\rangle, \tag{18}$$

where the kets $|\Phi_m^{(N)}\rangle$ form a complete orthonormal set of independent-particle (Fermi gas) eigenstates and $|{\rm BCS}\rangle$ is the BCS ground state

$$|{\rm BCS}\rangle = \prod_{\boldsymbol{k}}\left(u_{\boldsymbol{k}} + v_{\boldsymbol{k}}\alpha_{\boldsymbol{k}\uparrow}^\dagger \alpha_{-\boldsymbol{k}\downarrow}^\dagger\right)|0\rangle. \tag{19}$$

Here $\boldsymbol{k}$ is the usual wave vector, $u_{\boldsymbol{k}}$ and $v_{\boldsymbol{k}}$ are Bogoliubov amplitudes, and $a_{\boldsymbol{k}\uparrow}^\dagger$, $\alpha_{-\boldsymbol{k}\downarrow}^\dagger$ are fermion creation operators, with the arrow subscripts indicating spin projections. The conventional normalization of (19) to unity implies the condition

$$u_{\boldsymbol{k}}^2 + v_{\boldsymbol{k}}^2 = 1 \tag{20}$$

on the Bogoliubov amplitudes $u_{\boldsymbol{k}}$ and $v_{\boldsymbol{k}}$. In the normal state, they reduce respectively to $\mathring{u}_{\boldsymbol{k}} = 1 - \theta(k_F - k)$ and $\mathring{v}_{\boldsymbol{k}} = \theta(k_F - k)$, where $\theta(x)$ is a step function, unity for $k_F > k$, zero otherwise and where $k = |\boldsymbol{k}|$.

In the correlated BCS state (18), the expectation value of an operator $\hat{O}_N$ that conserves the particle number is given by

$$\left\langle\hat{O}\right\rangle_s = \frac{\langle{\rm CBCS}|\hat{O}_N|{\rm CBCS}\rangle}{\langle{\rm CBCS}\mid{\rm CBCS}\rangle}, \tag{21}$$

with the subscript s standing for "superfluid".

In analyzing the properties of the correlated BCS state, we employ the commonly assumed decoupling approximation [6], which amounts to treating one Cooper pair at a time. Formally, the ratio (21) is expanded in a Taylor series around the normal correlated ground state, retaining terms of first order in the deviation of the quantity $v_{\boldsymbol{k}}^2$ from its normal-state counterpart $\mathring{v}_{\boldsymbol{k}}^2$ and of second order in $u_{\boldsymbol{k}}v_{\boldsymbol{k}}$. After some algebra [24], one may obtain the following result for the expectation value of a number-conserving operator $\hat{O}_N$:

see eq. (22) *on the next page*

$$\begin{aligned}\left\langle \hat{O} \right\rangle_s &= O_{oo}^{(A)} \\ &+ \sum_{\boldsymbol{l}}^{m} \left(\upsilon_{\boldsymbol{l}}^2 - \mathring{\upsilon}_{\boldsymbol{l}}^2 \right) \sum_N \sum_m \left(I_{mm}^{(N)} \right)^{-1} \prod_{\boldsymbol{k}' \neq \boldsymbol{l}}^{m} \mathring{\upsilon}_{\boldsymbol{k}'}^2 \prod_{\boldsymbol{k}''}^{\bar{m}} \left(1 - \mathring{\upsilon}_{\boldsymbol{k}''}^2 \right) \langle \Phi_m^{(N)} | F_N^\dagger \left(\hat{O}_N - O_{oo}^{(A)} \right) F_N | \Phi_m^{(N)} \rangle \\ &- \sum_{l}^{\bar{m}} \left(\upsilon_{\boldsymbol{k}}^2 - \mathring{\upsilon}_{\boldsymbol{k}}^2 \right) \sum_N \sum_m \left(I_{mm}^{(N)} \right)^{-1} \prod_{\boldsymbol{k}'}^{m} \mathring{\upsilon}_{\boldsymbol{k}'}^2 \prod_{\boldsymbol{k}' \neq \boldsymbol{k}}^{\bar{m}} \left(1 - \mathring{\upsilon}_{\boldsymbol{k}''}^2 \right) \langle \Phi_m^{(N)} | F_N^\dagger \left(\hat{O}_N - O_{oo}^{(A)} \right) F_N | \Phi_m^{(N)} \rangle \\ &+ \sum_{\boldsymbol{l}}^{m} \sum_{\boldsymbol{k}}^{\bar{m}} u_{\boldsymbol{l}} \upsilon_{\boldsymbol{l}} u_{\boldsymbol{k}} \upsilon_{\boldsymbol{k}} \sum_N \sum_m \left(I_{mm}^{(N)} \right)^{-1} \prod_{\boldsymbol{k}' \neq \boldsymbol{l}}^{m} \mathring{\upsilon}_{\boldsymbol{k}'}^2 \prod_{\boldsymbol{k}'' \neq \boldsymbol{k}}^{\bar{m}} \mathring{u}_{\boldsymbol{k}''}^2 \langle \Phi_m^{(N)} | F_N^\dagger \left(\hat{O}_N - O_{oo}^{(A)} \right) F_N a_{\boldsymbol{k}\uparrow}^\dagger a_{-\boldsymbol{k}\downarrow}^\dagger a_{-\boldsymbol{l}\downarrow} a_{\boldsymbol{l}\uparrow} | \Phi_m^{(N)} \rangle, \end{aligned} \tag{22}$$

where $\bar{m}$ stands for the set of single-particle orbitals complementary to m, the subscript "oo" refers to the normal correlated state, and A denotes the actual number of fermions. Substituting the number operator

$$\hat{N} = \sum_{\boldsymbol{k}} \left(\alpha_{\boldsymbol{k}\uparrow}^\dagger \alpha_{\boldsymbol{k}\uparrow} + \alpha_{-\boldsymbol{k}\downarrow}^\dagger \alpha_{-\boldsymbol{k}\downarrow} \right) \tag{23}$$

into this general formula, we find that the number of particles is not conserved by the correlated BCS state, as anticipated. However, number conservation in the mean is imposed by introduction of a Lagrange multiplier μ, identified in general with the chemical potential, and coincident with the single particle energy of the Fermi level ε_F at zero temperature in the absence of interactions. Thus, instead of the expectation value of $\hat{H}$ itself, one calculates [24,25]

$$\begin{aligned}\left\langle \hat{H} - \mu \hat{N} \right\rangle_s &= H_{oo}^{(N)} - \mu N + 2 \sum_{\boldsymbol{k}' > \boldsymbol{k}_F} \upsilon_{\boldsymbol{k}'}^2 [\varepsilon(\boldsymbol{k}') - \mu] \\ &\quad -2 \sum_{\boldsymbol{k} < \boldsymbol{k}_F} u_{\boldsymbol{k}}^2 [\varepsilon(\boldsymbol{k}) - \mu] \\ &\quad + \sum_{\boldsymbol{k}'} \sum_{\boldsymbol{k} \neq \boldsymbol{k}'} u_{\boldsymbol{k}'} \upsilon_{\boldsymbol{k}'} u_{\boldsymbol{k}} \upsilon_{\boldsymbol{k}} V_{\boldsymbol{k}\boldsymbol{k}'} \end{aligned} \tag{24}$$

in terms of the energy $H_{oo}^{(N)}$ of the correlated normal state of N neutrons, the single-particle energies $\varepsilon(\boldsymbol{k})$, and the correlated (or effective) pairing matrix elements $V_{\boldsymbol{k}\boldsymbol{k}'}$.

The gap function, defined as

$$\Delta_{\boldsymbol{k}} = -\sum_{\boldsymbol{k}'} V_{\boldsymbol{k}\boldsymbol{k}'} u_{\boldsymbol{k}'} \upsilon_{\boldsymbol{k}'}, \tag{25}$$

measures the in-medium binding energy of the Cooper pair. Applying the Euler-Lagrange variational principle to the form (24) while observing the constraint (20) on the variational Bogoliubov amplitudes, one is led to the following equation for determination of the gap function in the case of singlet S-wave pairing under consideration: [24, 25]:

$$\Delta(k) = -\frac{1}{\pi} \int_0^\infty \frac{V(k,k')}{\sqrt{(\varepsilon(k') - \mu)^2 + \Delta^2(k')}} \Delta(k') k'^2 \mathrm{d}k', \tag{26}$$

where

$$V(k,k') = \frac{1}{kk'} \int_0^\infty w_2^{\mathrm{S}=0}(r) \sin(kr) \sin(k'r) \mathrm{d}r \tag{27}$$

and $w_2^{\mathrm{S}=0}(r)$ is the effective potential defined by eq. (9) for the state ${}^1S_0$. The same correlation functions are used for both superfluid and normal states.

Equation (26) is a nonlinear integral equation in which the denominator on the right side becomes vanishingly small as k' approaches the Fermi wave number k_F. In spite of this impending singularity, straightforward integration of the equation can be practical if a suitable starting value of the gap is available [14]. We choose instead to implement the more efficient and accurate separation method proposed in ref. [6]. Let $\varphi(k) = V(k,k_F)/V_F$ and assume that $V_F \equiv V(k_F,k_F) \neq 0$. The matrix elements of the pairing potential are then decomposed identically as follows:

$$V(k,k') = V_F \varphi(k) \varphi(k') + W(k,k') \tag{28}$$

into a separable term and a remainder $W(k,k')$ that vanishes when either argument is on the Fermi surface. Substitution of eq. (28) into the original gap equation (26) leads to an equivalent system of two coupled equations for the factors $\chi(k)$ and Δ_F of the product $\Delta(k) = \Delta_F \chi(k)$ [6]. The first equation is a quasi-linear integral equation for the shape $\chi(k) = \frac{\Delta(k)}{\Delta_F}$ of the gap function

$$\chi(k) = \varphi(k) - \frac{1}{\pi} \int_0^\infty \frac{W(k,k') \chi(k') k'^2 \mathrm{d}k'}{\sqrt{(\varepsilon(k') - \mu)^2 + (\Delta_F \chi(k'))^2}}. \tag{29}$$

The second equation, which embodies the log singularity, is a nonlinear integral equation for a number, namely the gap amplitude $\Delta_F \equiv \Delta(k_F)$

$$1 + \frac{V_F}{\pi} \int_0^\infty \frac{\varphi(k') \chi(k') k'^2 \mathrm{d}k'}{\sqrt{(\varepsilon(k') - \mu)^2 + (\Delta_F \chi(k'))^2}} = 0. \tag{30}$$

The first step in finding the gap is evaluation of the pairing matrix elements of the potential from eq. (27), using the optimal correlation functions determined in the EOS calculation outlined in sect. 2.1. For the single-particle energies, we adopt an effective-mass approximation, *i.e.*, $\varepsilon(k) = \hbar^2 k^2 / 2M^* + \text{const.}$, which should be

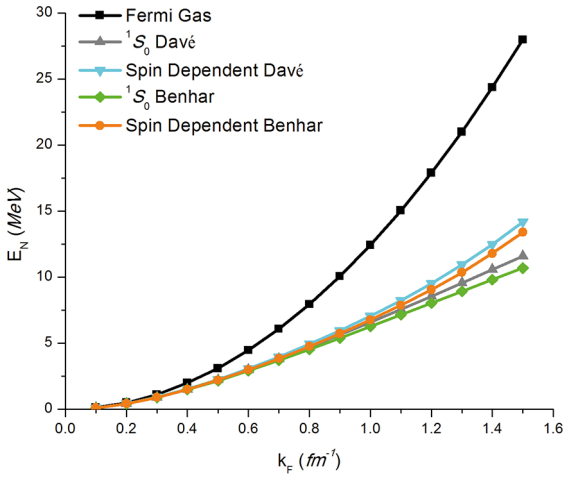

Fig. 1. (Color online) Energy per neutron E_N as a function of the wave number k_F based on the Argonne V_{18} interaction in the four calculational treatments as described in the text. The energy per neutron of the corresponding ideal Fermi gas is plotted for comparison.

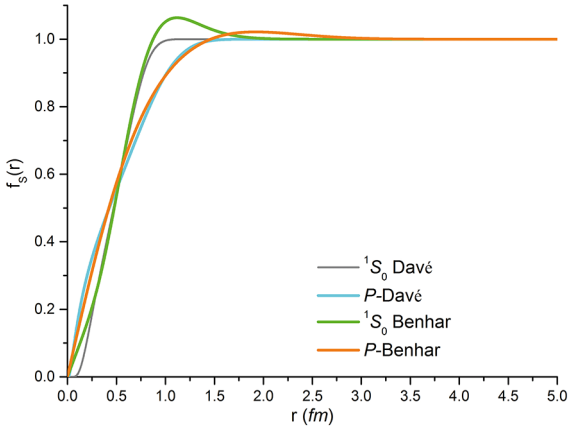

Fig. 2. (Color online) Correlation functions of the Jastrow type $f_S(r)$ (Davé and Benhar versions), determined optimally at Fermi wave number $k_F = 0.9\,\mathrm{fm}^{-1}$ and plotted *versus* the separation r of a pair of neutrons interacting i) via the S-wave component of the Argonne V_{18} potential ("S-wave only"), and ii) also via the P component of this potential.

satisfactory at the low densities in question. Specifically, with $\varrho(k) \equiv \mathrm{d}\varepsilon(k)/\mathrm{d}k$, M^* is estimated as

$$M^* = \hbar^2 k_F \varrho^{-1}(k_F). \tag{31}$$

In the general case where the singlet-S and triplet-P components of the potential are included, the single-particle energies are expressed more explicitly as

$$\varepsilon(k) = \frac{\hbar^2 k^2}{2M} + \pi\rho \int_0^\infty r^2 w_2^{\mathrm{S}=0}(r) \left[1 + \frac{\sin(kr)}{kr} l(kr)\right] \mathrm{d}r + 3\pi\rho \int_0^\infty r^2 w_2^{\mathrm{S}=1}(r) \left[1 - \frac{\sin(kr)}{kr} l(kr)\right] \mathrm{d}r. \tag{32}$$

Given these preparations, eqs. (29) and (30) are solved by iteration starting from a constant value for $\Delta_F \chi(k')$ in eq. (30), continuing until satisfactory convergence is achieved for Δ_F [6] —typically in very few steps.

3 Results and discussion

3.1 Equation of state

We first present the results for the EOS of normal dilute neutron matter. In fig. 1, the optimized energy per neutron is shown as a function of the Fermi wave number k_F, for each of the four cases considered for the Argonne V_{18} two-nucleon interaction (S-waves only; S and P waves; Davé and Benhar correlation functions). Figure 2 features plots of the optimized S and P correlation functions $f_S(r)$ at the typical value $k_F = 0.9\,\mathrm{fm}^{-1}$, for each of the two types considered. Almost identical results [46] were obtained with the corresponding components of the Argonne $V_{4'}$ potential [47]. Even though k_F values only up to about $1\,\mathrm{fm}^{-1}$ are needed to describe the inner crust of neutron stars, the EOS has been plotted for k_F up to $1.5\,\mathrm{fm}^{-1}$, in order to show the influence of the P-state contribution as the density increases.

Individually for the "S-wave only" and "spin-dependent" cases, the energetic results obtained for the optimal Davé and Benhar correlation functions are found to be rather close, but with a discrepancy that increases with density. This is quite as expected, since the differences between these functions at short range is better resolved at higher densities. Further, it is to be noted that the Davé form has three free parameters, compared to two in the Benhar case. Another distinction between these two correlation functions, seen in fig. 2, is that the Benhar form allows $f_S(r)$ to overshoot unity at small r, whereas the Davé form does not. Comparing the energetic results obtained in the "S-wave only" and "spin-dependent" cases for a given correlation form, it is seen that inclusion of the positive P-wave contribution begins to play a role with increasing density, such that its incorporation becomes necessary for k_F values beyond about $0.8\,\mathrm{fm}^{-1}$.

Of special interest is the magnitude of the "smallness parameter" ξ defined in eq. (5), which governs the convergence of the cluster expansion of the energy. Over the densities and correlation functions considered, this parameter rises monotonically with the density and ranges from 5.66×10^{-6} for $k_F = 0.1\,\mathrm{fm}^{-1}$ to 0.006 for $k_F = 1\,\mathrm{fm}^{-1}$ and to 0.026 for $k_F = 1.5\,\mathrm{fm}^{-1}$ for the Benhar case and from 5.36×10^{-6} for $k_F = 0.1\,\mathrm{fm}^{-1}$ to 0.006 for $k_F = 1\,\mathrm{fm}^{-1}$ and to 0.027 for $k_F = 1.5\,\mathrm{fm}^{-1}$ for the Davé case. On this basis, the higher cluster corrections neglected in the present study should not be important below about $k_F = 1.0\,\mathrm{fm}^{-1}$, as far as the EOS itself is concerned.

Results obtained for E_N with the Benhar correlation function for the Argonne V_{18} interaction in the "spin-dependent" case are plotted in fig. 3 along with results from other computational many-body methods. In choosing between the Davé or Benhar forms of the correlation function $f_S(r)$ to show results on fig. 3 (and later on fig. 6)

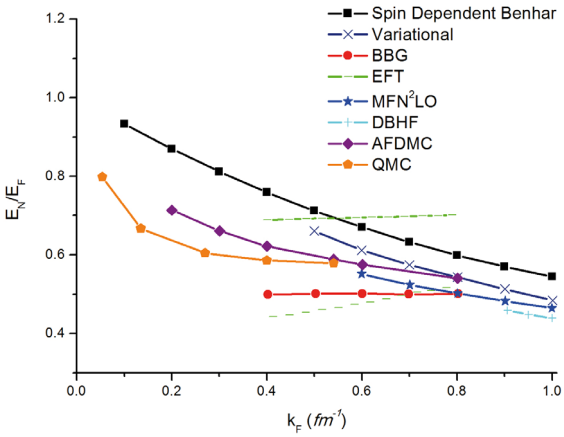

Fig. 3. (Color online) Comparison of results for the ratio of the energy per neutron E_N of neutron matter (*i.e.*, its EOS) to the energy per particle E_F of a Fermi gas of non-interacting particles of the same mass, obtained by different microscopic many-body methods, generally with differing input potentials, considered to be realistic. See text for details. The curve marked with black squares traces the EOS obtained here for the Argonne V_{18} interaction when the Benhar correlation function is adopted in the "spin-dependent" case.

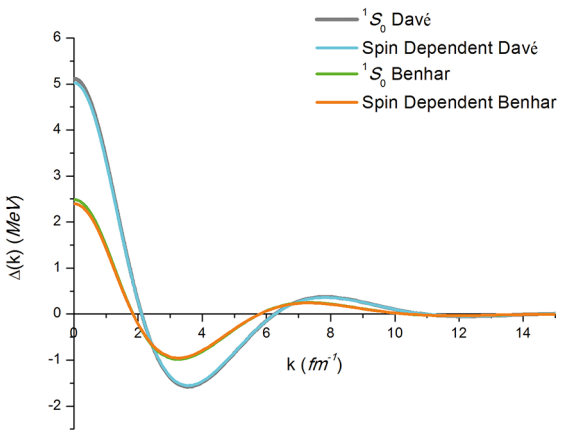

Fig. 4. (Color online) Gap function $\Delta(k)$ at $k_F = 0.9\,\mathrm{fm}^{-1}$ as a function of wave number k, obtained with an effective pairing interaction determined from eq. (9), based on the Argonne V_{18} interaction. The four curves correspond to the four different choices for the Jastrow correlation function $f_{S=0}(r)$, as identified in the main text.

we have decided in favor of the former for two reasons. First, it gives a slightly lower upper bound for the energy. The second reason concerns the question whether $f_S(r)$ should overshoot unity before going to zero to suppress the core region of the potential. There are some studies that support the presence of such an overshoot: i) LOCV calculations of the energy per particle (see for example ref. [30]), which give results close to those of FHNC/SOC and MC calculations upon solving Euler-Lagrange equations at the two-body cluster level, produce correlation functions that seem to overshoot unity. ii) Recently, optimized FHNC indicate a small overshoot in neutron matter for the Argonne V_{18} interaction [48]. The methods shown in fig. 3 include variational approaches that introduce state-dependent correlations [29,28,30,31], Brueckner-Bethe-Goldstone (BBG) theory [37], difermion effective field theory (EFT) [39], a Mean Field calculation using chiral N^{2}LO three-nucleon forces (MFN^{2}LO) (showing error bounds) [38], Dirac-Brueckner Hartree-Fock theory (DBHF) [36], Auxiliary Field Diffusion Monte Carlo (AFDMC) [43] (including a three-nucleon interaction), and Quantum Monte Carlo (QMC) [22,23]. The most meaningful comparison of the present results would be with the variational calculations of Friedman and Pandharipande (FP) [31] and of Modarres *et al.* [29,28,30]. It is understandable that the "spin-dependent" Benhar curve would lie somewhat above that of FP, since the latter calculation involves a more flexible variational ansatz and includes essentially all components of the assumed two-nucleon interaction. The same is true for the results obtained with the LOCV method which lie very near the ones of the FP paper, but are available for k_F's $\gtrsim 1\,\mathrm{fm}^{-1}$. Results of ref. [30] are below those of refs. [42] and [44] obtained with GFMC and FHNC methods respectively.

It is worth noting at this point that at similar densities, precise calculation of the EOS of pure neutron matter is less demanding than that for symmetrical nuclear matter, where large-scale cancellations occur between kinetic and potential contributions to E_N. An additional consideration is that in the density regime of neutron matter of interest for $^1S_0$ pairing, three-nucleon interactions are not expected to play a very significant role [8,9,49–51].

3.2 $^1S_0$ superfluid gap

As explained above, we solve the gap equation (26) using the separation method of Khodel, Khodel, and Clark [6] and the optimal correlation functions $f_{S=0}(r)$ determined here for the normal neutron matter. In fig. 4, the resulting gap function $\Delta(k)$ is plotted as a function of k for $k_F = 0.9\,\mathrm{fm}^{-1}$ for all four correlation choices $f_{S=0}(r)$ ("S-wave only", "Spin Dependent" Davé and Benhar forms) based on the Argonne V_{18} interaction. An interesting feature prominent in these plots is the occurrence of a node in the gap function at $k \simeq 2\,\mathrm{fm}^{-1}$, which is generic to pairing interactions that possess a substantial inner repulsion in coordinate space, along with the outer attraction required by the experimental $^1S_0$ phase shift. Due to the non-monotic behavior of the interaction, the negative excursion of the gap function is generally necessary for the existence of a solution of the gap equation, as emphasized in ref. [6]. A feature specific to the present calculation is the tiny discrepancy between the gap functions generated in the S-wave only and "spin-dependent" cases. The effect of the P-wave component of the two-nucleon potential on the optimal correlation function $f_{S=0}(r)$ of either type is minuscule in the gap function.

The resulting energy gap on the Fermi surface, $\Delta(k_F) \equiv \Delta_F$, is plotted in fig. 5 as a function of k_F for the case of the Argonne V_{18} interaction. As a check, we have also solved the gap equation by straightforward iteration,

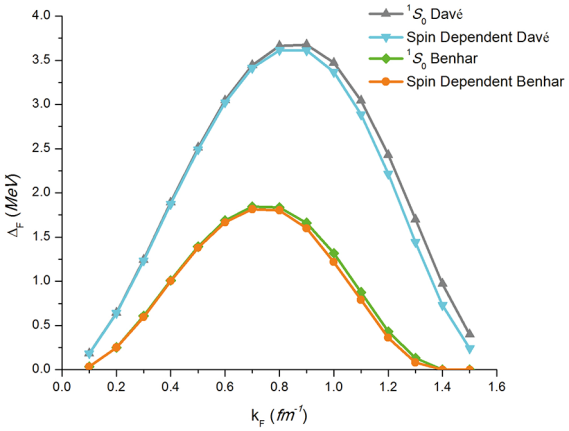

Fig. 5. (Color online) Pairing gap Δ_F on the Fermi surface for the $^1S_0$ superfluid state of neutron matter as a function of Fermi wave number k_F, as obtained with an effective pairing interaction determined from eq. (9) based on the Argonne V_{18} interaction. The four curves correspond to the four different choices for the Jastrow correlation function $f_{\mathrm{S}=0}(r)$, as identified in the main text.

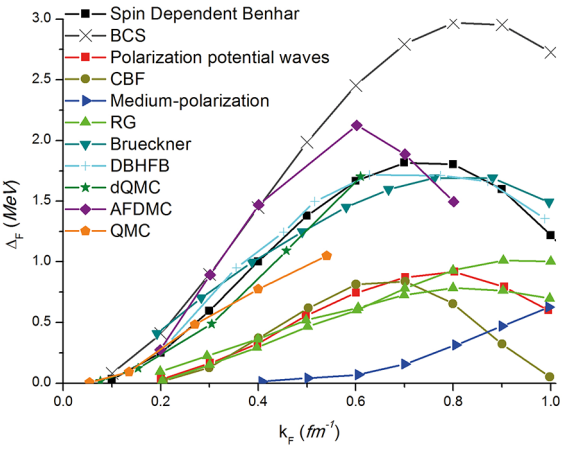

Fig. 6. (Color online) Results for the $^1S_0$ neutron gap Δ_F *versus* Fermi wave number k_F, as obtained for the Argonne V_{18} interaction using the Benhar-type correlation function in the "spin-dependent" case (curve marked with black squares). Results for Δ_F calculated by other microscopic methods are displayed for comparison (see text for details).

taking proper care in dealing with the small values of the denominator around the Fermi surface. Good agreement was found [46], the essential point being that the Jastrow correlations act to "tame" the extreme non-monotonicity of the bare interaction.

Comparing the two parametrized forms for the correlation function we realize sensitivity. In the Davé case we found a somewhat larger range of k_F values over which a non-zero gap exists, than for the Benhar form, as well as a somewhat larger range of gap values. Comparing the gap values in the S-wave–only case with those for which the P-wave contribution affects the optimal choice of the correlation function $f_{\mathrm{S}=0}(r)$, we find that the latter are slightly smaller than the former for $k_F \gtrsim 0.7\,\mathrm{fm}^{-1}$. Accordingly, we again affirm that inclusion of the P channel in our procedure has a small negative effect on the gap. This is true for either form chosen for the correlation function. Concerning the Fermi wave number (effectively the density) at which the peak value Δ_F is reached, it is located between $k_F = 0.8\,\mathrm{fm}^{-1}$ and $k_F = 0.9\,\mathrm{fm}^{-1}$, for the Davé correlations, while for Benhar correlations it lies between $k_F = 0.7\,\mathrm{fm}^{-1}$ and $k_F = 0.8\,\mathrm{fm}^{-1}$. With respect to the (upper) density at which the gap closes, our approach gives a values close to $k_F = 1.5\,\mathrm{fm}^{-1}$ and $1.3\,\mathrm{fm}^{-1}$ for Davé and Benhar choices, respectively. While differences between Benhar and Davé forms may not matter much at all for the energy, the different balance between the positive and negative parts of the effective pairing interactions for these two choices may matter a great deal in determining the gap, the more so when compounded with the quasi-exponential dependence of this quantity on the inputs for the pairing interaction and self-energy. Here it is again prudent to mention that although k_F does not exceed about $1\,\mathrm{fm}^{-1}$ in the neutron-star inner crust, we have chosen to plot the gap results up to $1.5\,\mathrm{fm}^{-1}$ in order to show how the P-wave component of the two-nucleon interaction might affect, through the two-body correlations it influences, the density at which the gap closes. Analogous calculations have been performed for the Argonne $V_{4'}$ potential, with results [46] that show very little difference from those reported here for the Argonne V_{18} interaction.

Our results for the S-wave gap may be compared with those of refs. [6,46] that were obtained from ordinary BCS theory using the bare $^1S_0$ component of the AV_{18} potential as pairing interaction (thus including no corrections for short-range geometrical correlations or medium polarization). The corresponding curve of Δ_F *versus* k_F is marked with crosses in fig. 6. The gap values calculated with Benhar-type correlations in the "spin-dependent" case are seen to be suppressed relative to the pure BCS gap by a factor of about 2/3 in region of the peak, which occurs at a slightly lower density in our calculation.

Since the Davé form for the Jastrow correlation function was employed in the non-orthogonal CBF approach applied by Chen *et al.* [14] to the problem of $^1S_0$ neutron pairing, it is of special interest to compare the results obtained here with the Davé form ("spin-dependent" case) with those from this earlier CBF calculation (plotted as "CBF" in fig. 6). Other than in the methods used to solve the gap equation, both of which are sufficiently accurate, the difference between the two studies lies primarily in the inclusion, by Chen *et al.*, of a correction of second order in CBF perturbation theory to account for in-medium modification of the pairing interaction ("polarization effects"). However this correction is very approximately estimated and somewhat questionable, as discussed in refs. [14] and, more recently, in [52]. Absent that correction, the results for the gap are found to be very similar, as expected. The impact of differences in the pairing interactions assumed (Reid V_4 in ref. [14] and Argonne V_{18} herein) is minimal.

Also reproduced in fig. 6 are gap results obtained through a number of other microscopic many-body methods, diverse in their inclusion (or not) of various physical

effects influencing the pairing gap. These methods include the polarization-potential model of Pines *et al.* [10], a medium-polarization calculation [11], an application of RG theory [7], approaches grounded in Brueckner theory [12], DBHFB [13], determinantal lattice QMC (dQMC) [21], AFDMC [19], and QMC [23]. Other noteworthy calculations not represented in fig. 6 are those of refs. [8,9,15–18].

Qualitatively, our results for Δ_F using Benhar correlations are closer to those of refs. [12,13,21–23], while differing substantially from those of refs. [7,10,11,14]. The density at which our predicted gap reaches a maximum is similar to what found in refs. [10–13]. However, useful conclusions cannot be drawn from such commonality or disparity, due to differences in the pairing interactions assumed and in calculational methods, especially the treatment (or not) of in-medium modification of the bare interaction.

It is not surprising that considerable uncertainty remains in the quantitative determination of the behavior of the $^1S_0$ gap Δ_F in neutron matter, in view of the inherent strong sensitivity to the inputs for the pairing interaction and the self-energies (or density of states).

4 Summary and conclusions

The $^1S_0$ superfluid gap Δ_F of dilute neutron matter in the density range $0.2\,\mathrm{fm}^{-1} \lesssim k_F \lesssim 1\,\mathrm{fm}^{-1}$ has been calculated in the framework of correlated BCS theory at lowest cluster order, an approximation suitable in the low-density regime where pairing occurs in the singlet S-wave state of two neutrons. Inputs to this theory consist of the bare Argonne V_{18} nucleon-nucleon interaction and corresponding optimized Jastrow-type correlation functions that modify it to create an effective pairing interaction that takes account of the effects of strong short-range correlations present in nuclear systems. The CBF gap equation has been solved for this effective pairing interaction using the robust and accurate separation method introduced in ref. [6].

A many-body approach limited to inclusion of the effects of S- and P-wave components of the AV_{18} interaction is adequate at the low densities relevant to pairing in the $^1S_0$ state. The parameters of the chosen forms of Jastrow correlation function $f(r)$ have been determined by minimization of the expectation value of the system Hamiltonian with respect to the Jastrow trial function

$$|\Psi_0\rangle = \prod_{i<j} f(r_{ij})|\Phi_0\rangle, \tag{33}$$

where $|\Phi_0\rangle$ is the ground state of the neutron system with interactions turned off. In practice, this expected energy is evaluated to leading order in a small parameter ξ. Roughly speaking, this parameter is given by the ratio of the volume per particle in which $f^2(r) - 1$ is appreciable, to the total volume per particle. Thus, the many-body description adopted rests on a low-density approximation, presumed to be adequate for description of the low-density neutron matter in the inner crust of a neutron star, where, roughly, $\xi \sim 0.1$. As a by-product, execution of this optimization process yields a corresponding approximation to the ground-state energy per particle of neutron matter, *i.e.*, its equation of state (EOS), in the relevant density regime ranging up to about one-fourth the saturation density of symmetrical nuclear matter. These EOS results have been contrasted with those generated by other many-body methods, generally with other choices of basic interactions. In our calculation, the contributions of the P-wave and higher partial-wave channels become important for the EOS, and indirectly to the $^1S_0$ pairing gap, only for $k_F \gtrsim 0.8\,\mathrm{fm}^{-1}$, as must also be the case in other studies of neutron matter in this density regime. Similarly, fundamental three-nucleon forces should have only modest impact on the EOS and $^1S_0$ superfluid gap in this regime [8,9,49–51].

Comparison of the results for the $^1S_0$ gap obtained here with the results of the numerous antecedent calculations is obscured by the wide range of both many-body methods employed and types of input interactions assumed. One significant finding is a quenching of the gap obtained with the effective pairing interaction generated by the CBF approach, relative to the gap predicted by pure BCS theory for the bare Argonne V_{18} interaction. It may also be noted that the gaps Δ_F obtained with our approach are found to be rather close to those calculated by Gezerlis and Carlson [23] using quantum Monte Carlo techniques (also for the Argonne V_{18} interaction), and similar to results from a determinantal lattice QMC approach [21].

References

1. A. Gezerlis, C.J. Pethick, A. Schwenk, *Novel Superfluids*, Vol. **2** edited by K.H. Bennemann, J.B. Ketterson (Oxford University Press, 2014) chapt. 22.
2. W. Becker (Editor), *Neutron Stars and Pulsars*, in *Astrophysics and Space Science Library*, Vol. **357** (Springer-Verlag Berlin Heidelberg, 2009).
3. P. Haensel, A.Y. Potekhin, D.G. Yakovlev (Editors), *Neutron Stars 1: Equation of State and Structure*, in *Astrophysics and Space Science Library*, Vol. **326** (Springer-Verlag New York, 2007).
4. S. Gandolfi, A. Gezerlis, J. Carlson, Annu. Rev. Nucl. Part. Sci. **65**, 303 (2015).
5. W.C.G. Ho, K.G. Elshamouty, C.O. Heinke, A.Y. Potekhin, Phys. Rev. C **91**, 015806 (2015).
6. V.A. Khodel, V.V. Khodel, J.W. Clark, Nucl. Phys. A **598**, 390 (1996).
7. A. Schwenk, B. Friman, G.E. Brown, Nucl. Phys. A **713**, 191 (2003).
8. S. Maurizio, J.W. Holt, P. Finelli, Phys. Rev. C **90**, 044003 (2014).
9. C. Drischler, T. Krüger, K. Hebeler, A. Schwenk, Phys. Rev. C **95**, 024302 (2017).
10. J. Wambach, T.L. Ainsworth, D. Pines, Nucl. Phys. A **555**, 128 (1993).
11. H.J. Schulze, J. Cugnon, A. Lejeune, M. Baldo, U. Lombardo, Phys. Lett. B **375**, 1 (1996).

12. L.G. Cao, U. Lombardo, P. Schuck, Phys. Rev. C **74**, 064301 (2006).
13. J. Margueron, H. Sagawa, K. Hagino, Phys. Rev. C **77**, 054309 (2008).
14. J.M.C. Chen, J.W. Clark, R.D. Davé, V.V. Khodel, Nucl. Phys. A **555**, 59 (1993).
15. A. Fabrocini, S. Fantoni, A.Y. Illarionov, K.E. Schmidt, Phys. Rev. Lett. **95**, 192501 (2005).
16. A. Fabrocini, S. Fantoni, A.Y. Illarionov, K.E. Schmidt, Nucl. Phys. A **803**, 137 (2008).
17. H. Müther, W.H. Dickhoff, Phys. Rev. C **72**, 054313 (2005).
18. D. Ding, A. Rios, H. Dussan, W.H. Dickhoff, S.J. Witte, A. Carbone, A. Polls, Phys. Rev. C **94**, 025802 (2016).
19. S. Gandolfi, A.Y. Illarionov, S. Fantoni, F. Pederiva, K.E. Schmidt, Phys. Rev. Lett. **101**, 132501 (2008).
20. S. Gandolfi, A.Y. Illarionov, F. Pederiva, K.E. Schmidt, S. Fantoni, Phys. Rev. C **80**, 045802 (2009).
21. T. Abe, R. Seki, Phys. Rev. C **79**, 054003 (2009).
22. A. Gezerlis, J. Carlson, Phys. Rev. C **77**, 032801 (2008).
23. A. Gezerlis, J. Carlson, Phys. Rev. C **81**, 025803 (2010).
24. E. Krotscheck, J.W. Clark, Nucl. Phys. A **333**, 77 (1980).
25. E. Krotscheck, R.A. Smith, A.D. Jackson, Phys. Rev. B **24**, 6404 (1981).
26. R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
27. C.J. Horowitz, A. Schwenk, Phys. Lett. B **638**, 153 (2006).
28. G.H. Bordbar, M. Modarres, Phys. Rev. C **57**, 714 (1998) and references therein.
29. G.H. Bordbar, M. Bigdeli, Phys. Rev. C **75**, 045804 (2007).
30. M. Modarres, A. Tafrihi, Nucl. Phys. A **941**, 212 (2015).
31. B. Friedman, V.R. Pandharipande, Nucl. Phys. A **361**, 502 (1981).
32. A. Akmal, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C **58**, 1804 (1998).
33. J.W. Clark, L.R. Mead, E. Krotscheck, K.E. Kürten, M.L. Ristig, Nucl. Phys. A **328**, 45 (1979).
34. J.W. Clark, Prog. Part. Nucl. Phys. **2**, 89 (1979).
35. I. Bombaci, A. Fabrocini, A. Polls, I. Vidaña, Phys. Lett. B **609**, 232 (2005).
36. J. Margueron, E. van Dalen, C. Fuchs, Phys. Rev. C **76**, 034309 (2007).
37. M. Baldo, C. Maieron, Phys. Rev. C **77**, 015801 (2008).
38. K. Hebeler, A. Schwenk, Phys. Rev. C **82**, 014314 (2010).
39. A. Schwenk, C.J. Pethick, Phys. Rev. Lett. **95**, 160401 (2005).
40. I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C **93**, 024305 (2016).
41. E. Epelbaum, H. Krebs, D. Lee, U.G. Meißner, Eur. Phys. J. A **40**, 199 (2009).
42. J. Carlson, J. Morales, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C **68**, 025802 (2003).
43. S. Gandolfi, A.Y. Illarionov, K.E. Schmidt, F. Pederiva, S. Fantoni, Phys. Rev. C **79**, 054005 (2009).
44. A. Lovato, O. Benhar, S. Fantoni, A.Y. Illarionov, K.E. Schmidt, J. Phys. Conf. Ser. **336**, 012016 (2011).
45. O. Benhar, C.C. Degli Atti, A. Kallio, L. Lantto, P. Toropainen, Phys. Lett. B **60**, 129 (1976).
46. G. Pavlou, Master's Thesis, University of Athens (2009).
47. R.B. Wiringa, S.C. Pieper, Phys. Rev. Lett. **89**, 182501 (2002).
48. E. Krotscheck (2017), private communication.
49. H.W. Hammer, A. Nogga, A. Schwenk, Rev. Mod. Phys. **85**, 197 (2013).
50. W. Zuo, Z. Li, G. Lu, J. Li, W. Scheid, U. Lombardo, H.J. Schulze, C. Shen, Phys. Lett. B **595**, 44 (2004).
51. X.R. Zhou, G.F. Burgio, U. Lombardo, H.J. Schulze, W. Zuo, Phys. Rev. C **69**, 018801 (2004).
52. J.W. Clark, *Fifty Years of Nuclear BCS*, edited by R.A. Broglia, V. Zelevinsky (World Scientific Publishing, 2013) chapt. 27.