

# Relativistic quark-diquark model of baryons with a spin-isospin transition interaction: Non-strange baryon spectrum and nucleon magnetic moments

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**Abstract.** The relativistic interacting quark-diquark model of baryons, recently developed, is here extended introducing in the mass operator a spin-isospin transition interaction. This refined version of the model is used to calculate the non-strange baryon spectrum. The results are compared to the present experimental data. A preliminary calculation of the magnetic moments of the proton and neutron is also presented.

## 1 Introduction

According to standard quark models (QMs) [1–16], baryons are described as bound states of three constituent quarks. These are effective degrees of freedom that mimic the three valence quarks inside baryons, with virtual gluons and  $q\bar{q}$  sea pairs. The light baryons can then be ordered according to the approximate  $SU_f(3)$  symmetry into the multiplets  $[1]_A \oplus [8]_M \oplus [8]_M \oplus [10]_S$ . QMs explain quite well several properties of baryons, such as the strong decays, the magnetic moments and the electromagnetic elastic form factors. Nevertheless, they predict a larger number of states than the experimentally observed ones, that is known as *the missing resonance problem*. Furthermore, some states with certain quantum numbers appear in the spectrum at excitation energies much lower than predicted [17]. The problem of the missing resonances [17–19] has motivated the realization of several experiments, such as CB-ELSA [20], CBELSA/TAPS [21], TAPS [22–24], GRAAL [25, 26], SAPHIR [27, 28] and CLAS [29–31], which only provided a few weak indications about some states. Indeed, even if several experiments have been dedicated to the search of missing states, just a small number of new resonances has been included into the PDG [17].

There are two possible explanations to the puzzle of the missing resonances:

- 1) There may be resonances very weakly coupled to the single pion, but with higher probabilities of decaying into two or more pions or into other mesons [17–19]. The detection of such states is further complicated by

the problem of the separation of the experimental data from the background and by the expansion of the differential cross sections into many partial waves.

- 2) Alternately, it is possible to consider models that are characterized by a smaller number of effective degrees of freedom with respect to the three quarks QMs and to assume that the majority of the missing states, not yet experimentally observed, simply may not exist. This is the case of quark-diquark models (QDMs) [32–45] where two quarks are strongly correlated and thus the state space is heavily reduced.

In QDMs, the effective degrees of freedom of diquarks are introduced to describe baryons as bound states of a constituent diquark and quark [32, 33]. The notion of diquark dates back to 1964, when its possibility was mentioned by Gell-Mann [46] in his original paper on quarks. Since then, many papers have been written on this topic (for a review see ref. [34]) and, more recently, the diquark concept has been applied to various calculations [35–45, 47–56].

Important phenomenological indications for diquark-like correlations have been collected [35, 37, 57–59] and indications for diquark confinement have also been provided [60].

Theoretically, a fully Poincaré covariant three-quark solution of the nucleon's Faddeev equation [61] has been shown to agree with a consistently obtained quark-diquark calculation. These results have been also developed in a more recent work [62] concerning the quark-quark interactions in baryons.

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All these investigations make it plausible enough to use diquarks as a part of the baryon's wave function.

In ref. [38], one of us developed a non-relativistic interacting quark-diquark model, *i.e.* a potential model based on the effective degrees of freedom of a constituent quark and diquark. In refs. [42, 43], it was "relativized" and reformulated within the point form formalism [63–67]. In ref. [44], we used the wave functions of refs. [42, 43] to compute the nucleon electromagnetic form factors.

The aim of the present work is to improve the "relativized" quark-diquark constituent model [42–44] and compute the non-strange baryon spectrum within point form dynamics.

Even if our previous results for the non-strange baryon spectrum [42, 43] were in general quite good, here we intend to show that the introduction of a spin-isospin transition interaction, inducing the mixing between quark-scalar diquark and quark-axial-vector diquark states can further improve the spectrum, as already suggested in ref. [38]. We recall that scalar and axial-vector diquarks can be considered as two correlated quarks in  $S$  wave with spin 0 or 1, respectively [35, 36]. As our previous studies [42, 43], the present model is essentially phenomenological and no attempt is made to investigate the fundamental structure of the diquark.

The wave functions of the model are used here to calculate, by means of a preliminary approach, the magnetic moments of the proton and neutron. The quality of our results for the baryon magnetic moments is comparable to that obtained in QMs calculations, showing that the present model has a wide range of applications, in particular for the electromagnetic observables of the baryons.

In a following study we will use the new wave functions, obtained by solving the eigenvalue problem of the mass operator of the present work, to calculate also the nucleon electromagnetic form factors and the elicity transition amplitudes.

## 2 The mass operator

We represent the baryon as a quark-diquark system. In the baryon rest frame, we introduce the operator  $\mathbf{r}$  as the relative distance between the two constituents and  $\mathbf{q}$  as the corresponding conjugate momentum. We propose a relativistic quark-diquark model, based on the following mass operator:

$$M = E_0 + \sqrt{\mathbf{q}^2 + m_1^2} + \sqrt{\mathbf{q}^2 + m_2^2} + M_{\text{dir}}(r) + M_c(q, r) + M_{\text{ex}}(r) + M_{\text{tr}}(r), \quad (1)$$

where  $E_0$  is a constant,  $m_1$  and  $m_2$  represent the diquark and quark mass, respectively;  $M_{\text{dir}}(r)$ ,  $M_c(q, r)$ ,  $M_{\text{ex}}(r)$  and  $M_{\text{tr}}(r)$ , respectively, represent the direct, contact, exchange and transition quark-diquark interaction terms.

We now discuss in more detail the various interaction terms of the previous equation. The direct term is a Coulomb-like interaction with a cut-off plus a linear confinement term:

$$M_{\text{dir}}(r) = -\frac{\tau}{r} (1 - e^{-\mu r}) + \beta r. \quad (2)$$

One also needs an exchange interaction [38, 68], that is a crucial ingredient for an accurate quark-diquark description of baryons. We have

$$M_{\text{ex}}(r) = (-1)^{L+1} e^{-\sigma r} [A_S \mathbf{s}_1 \cdot \mathbf{s}_2 + A_I \mathbf{t}_1 \cdot \mathbf{t}_2 + A_{SI} (\mathbf{s}_1 \cdot \mathbf{s}_2)(\mathbf{t}_1 \cdot \mathbf{t}_2)], \quad (3)$$

where  $\mathbf{s}_i$  and  $\mathbf{t}_i$  ( $i = 1, 2$ ) are the spin and the isospin operators of the constituents.

Moreover, we consider a contact interaction similar to that introduced by Godfrey and Isgur [69] for the study of the mesonic spectroscopy:

$$M_c(q, r) = \left( \frac{m_1 m_2}{E_1 E_2} \right)^{1/2} \frac{\eta^3 D}{\pi^{3/2}} e^{-\eta^2 r^2} \delta_{L,0} \delta_{s_1,1} \left( \frac{m_1 m_2}{E_1 E_2} \right)^{1/2}; \quad (4)$$

note that this interaction also depends on the constituent energy operators  $E_i = \sqrt{\mathbf{q}^2 + m_i^2}$  ( $i = 1, 2$ ).

All the terms of the mass operator analyzed up to this point (*i.e.*, excluding  $M_{\text{tr}}(r)$ ) have (almost) the same form as the corresponding terms of the mass operator used in the previous version of the relativistic quark-diquark model [42, 43]. The only small difference is that the parameter  $\epsilon$  of the contact interaction of refs. [42, 43] is set as  $\epsilon = 0$  in the present model. This choice has been done to keep the number of free parameters as small as possible.

As a specific aspect of the present model, we point out that the diquark mass  $m_1$ , that appears in the kinetic energy operators and in the contact operator  $M_c(q, r)$ , takes the values  $m_S$  or  $m_{AV}$  when the mass operator is acting on a scalar ( $S$ ) or an axial-vector ( $AV$ ) diquark state, respectively [35, 36, 70–78].

Finally, in this model we have introduced a spin-isospin transition interaction,  $M_{\text{tr}}(r)$ , in order to mix quark-scalar diquark and quark-axial-vector diquark states.  $M_{\text{tr}}(r)$  is chosen in the form:

$$M_{\text{tr}}(r) = V_0 e^{-\frac{1}{2}\nu^2 r^2} (\mathbf{s}_2 \cdot \mathcal{S})(\mathbf{t}_2 \cdot \mathcal{T}), \quad (5)$$

where  $V_0$  and  $\nu$  are free parameters.

The matrix elements of the spin transition operator,  $\mathcal{S}$  (of spherical components  $\mathcal{S}_{1,\mu}$ ) are easily obtained by means of the Wigner-Eckart theorem from the following reduced matrix elements:

$$\langle 1 \| \mathcal{S}_1 \| 1 \rangle = \langle 0 \| \mathcal{S}_1 \| 0 \rangle = 0, \quad (6a)$$

$$\langle 1 \| \mathcal{S}_1 \| 0 \rangle = 1 \quad (6b)$$

and

$$\langle 0 \| \mathcal{S}_1 \| 1 \rangle = -1. \quad (6c)$$

The same expressions hold for the matrix elements of the isospin transition operator,  $\mathcal{T}$ .

The mass operator given in eq. (1) has the following *good* quantum numbers: the total spin  $S$ , being  $\mathcal{S} = \mathbf{s}_1 + \mathbf{s}_2$ , the total isospin quantum numbers  $T$  and  $T_3$ , being  $\mathcal{T} = \mathbf{t}_1 + \mathbf{t}_2$ , the orbital angular momentum  $L$ , the parity  $P = (-1)^L$  and the total angular momentum  $J$ , being  $\mathcal{J} = \mathcal{L} + \mathcal{S}$ . Due to the spin-isospin transition operator

**Table 1.** Resulting values for the model parameters.

$E_0 = 826 \text{ MeV}$	$m_q = 140 \text{ MeV}$
$m_S = 150 \text{ MeV}$	$m_{AV} = 360 \text{ MeV}$
$\tau = 1.23$	$\mu = 125 \text{ fm}^{-1}$
$\beta = 1.57 \text{ fm}^{-2}$	$\sigma = 0.60 \text{ fm}^{-1}$
$A_S = 125 \text{ MeV}$	$A_I = 85 \text{ MeV}$
$A_{SI} = 350 \text{ MeV}$	$D = 2.00 \text{ fm}^{-2}$
$\eta = 10.0 \text{ fm}^{-1}$	$V_0 = 1450 \text{ MeV}$
$\nu = 0.35 \text{ fm}^{-1}$	

$M_{\text{tr}}(r)$ ,  $s_1$  and  $t_1$  are no more *good* quantum numbers: in the eigenstates of the total mass operator, states having  $s_1 = t_1 = 0$  are mixed with states having  $s_1 = t_1 = 1$ . The presence of the states with  $s_1 = t_1 = 0$  implies that the mixed eigenstates can only have  $S = T = \frac{1}{2}$ .

Considering all the properties of  $M_{\text{tr}}(r)$ , its matrix elements can be conveniently written as

$$\begin{aligned} \langle \Psi' | M_{\text{tr}}(r) | \Psi \rangle &= \frac{1}{4} V_0 \delta_{s'_1, s_1 \pm 1} \delta_{S, \frac{1}{2}} \delta_{t'_1, t_1 \pm 1} \delta_{T, \frac{1}{2}} \\ &\times \langle \Phi' | e^{-\frac{1}{2}\nu^2 r^2} | \Phi \rangle, \end{aligned} \quad (7)$$

where  $\Phi(\mathbf{r}) = \langle \mathbf{r} | \Phi \rangle$  represents the spatial part of the wave function of a generic state  $|\Psi\rangle$ .

In this work  $M_{\text{tr}}(r)$  is introduced to improve the description of the non-strange baryon spectrum [42, 43] (see the results of fig. 1). Furthermore, it makes possible to have a nucleon wave function with a quark-axial-vector diquark component in addition to the quark-scalar diquark one of the standard QDMs. In this way one obtains a description of the nucleon that is more similar to that given by the QMs in which two quarks, (denoted, here and in the following, by the indices  $a$  and  $b$ ) are coupled in a state with  $s_{ab} = t_{ab} = 0$  and also in a state with  $s_{ab} = t_{ab} = 1$ . In those models, the two states are necessary to construct a completely symmetric spin-isospin function for the three quarks. Furthermore, the two states appear in the wave function with the same amplitude, that is  $a_{ab}^0 = a_{ab}^1 = 1/\sqrt{2}$ . In this way, the magnetic properties of the nucleon can be described with good accuracy. Here, the quark-axial-vector diquark component, carrying an intrinsic angular momentum and electric charge, is also expected to improve the reproduction of the magnetic response of the nucleon with respect to the standard quark-diquark model, in which only the  $S$  diquark is present.

The free parameters of the present model are:  $E_0$ ,  $m_q$ ,  $m_S$ ,  $m_{AV}$ ,  $\tau$ ,  $\mu$ ,  $\beta$ ,  $\sigma$ ,  $A_S$ ,  $A_I$ ,  $A_{SI}$ ,  $D$ ,  $\eta$ ,  $V_0$  and  $\nu$ . It is worth noting that the number of model parameters increases only by one with respect to our previous model of refs. [42, 43], since there are two new parameters,  $V_0$  and  $\nu$ , but, as anticipated, the parameter  $\epsilon$  of the contact interaction of refs. [42, 43] has been removed.

The numerical values of the parameters that are used to fit the baryon spectrum are shown in table 1. One can notice that the values of the model parameters are changed with respect to those of refs. [42–44] due to the introduc-

tion of the transition interaction of eq. (5). In particular, the string tension goes from  $2.15 \text{ fm}^{-2}$  to  $1.57 \text{ fm}^{-2}$ . Furthermore, one can see that the masses of the two constituents (the quark and the diquark) are now smaller than before, which is consistent with a relativistic description of the baryon system in terms of constituents. Also, the mass difference between the scalar and the axial-vector diquark is smaller too (it goes from  $350 \text{ MeV}$  to  $210 \text{ MeV}$ ).

Finally, it has to be noted that in the present work all the calculations are performed without any perturbative approximation, with the same technique used in refs. [42, 43].

From a formal point of view, we notice that our model satisfies the condition of relativistic covariance, exactly as explained in refs. [42–44]. We recall that, the eigenfunctions of the mass operator of eq. (1) can be thought as eigenstates of the mass operator with interaction in a Bakamjian-Thomas construction [79]. The interaction is introduced adding an interaction term to the free mass operator  $M_0 = \sqrt{\mathbf{q}^2 + m_1^2} + \sqrt{\mathbf{q}^2 + m_2^2}$ , in such a way that the interaction commutes with the non-interacting Lorentz generators and with the non-interacting four-velocity [80].

The dynamics is given by a point form Bakamjian-Thomas construction. Point form means that the Lorentz group is kinematic. Furthermore, since we are doing a point form Bakamjian-Thomas construction, here  $P = Mv_0$ , where  $v_0$  is the non-interacting four-velocity (whose eigenvalue is  $v$ ).

The general quark-diquark state, defined on the product space  $H_1 \otimes H_2$  of the one-particle spin  $s_1$  (0 or 1) and spin  $s_2$  ( $\frac{1}{2}$ ) positive energy representations  $H_1 = L^2(R^3) \otimes S_1^0$  or  $H_1 = L^2(R^3) \otimes S_1^1$  and  $H_2 = L^2(R^3) \otimes S_2^{1/2}$  of the Poincaré group, can be written as [42, 43]:

$$|p_1, p_2, \lambda_1, \lambda_2\rangle, \quad (8)$$

where  $p_1$  and  $p_2$  are the four-momenta of the diquark and the quark, respectively, while  $\lambda_1$  and  $\lambda_2$  are, respectively, the  $z$ -projections of their spins.

We introduce the velocity states as [42, 43, 63–66]:

$$|v, \mathbf{k}_1, \lambda_1, \mathbf{k}_2, \lambda_2\rangle = U_{B(v)} |k_1, s_1, \lambda_1, k_2, s_2, \lambda_2\rangle_0, \quad (9)$$

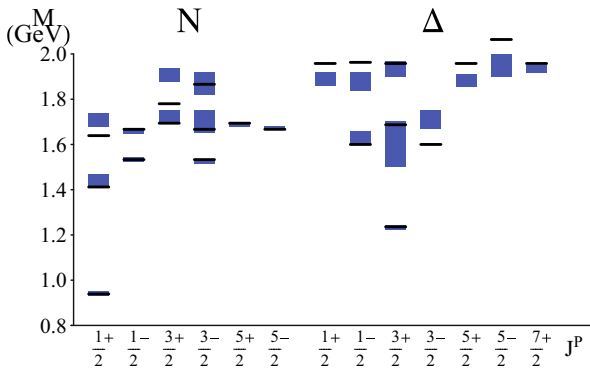
where the suffix 0 means that the diquark and the quark three-momenta  $\mathbf{k}_1$  and  $\mathbf{k}_2$ , called internal momenta, satisfy

$$\mathbf{k}_1 + \mathbf{k}_2 = 0. \quad (10)$$

Following the standard rules of the point form approach, the boost operator  $U_{B(v)}$  is taken as a canonical one, obtaining that the transformed four-momenta are given by  $p_{1,2} = B(v)k_{1,2}$  and satisfy the point form relation

$$p_1^\mu + p_2^\mu = \frac{P_N^\mu}{M_N} \left( \sqrt{\mathbf{q}^2 + m_1^2} + \sqrt{\mathbf{q}^2 + m_2^2} \right), \quad (11)$$

where  $P_N^\mu$  is the observed baryon four-momentum and  $M_N$  is its mass. It is worthwhile noting that eq. (9) redefines the single-particle spins. Having applied canonical boosts, the conditions for a point form approach [63–66, 81] are



**Fig. 1.** (Color online) Comparison between the calculated masses (black lines) of the  $3^*$  and  $4^*$  non-strange baryon resonances (up to 2 GeV) and the experimental masses from PDG [17] (boxes).

satisfied. Therefore, the spins on the left-hand state of eq. (9) perform the same Wigner rotations as  $\mathbf{k}_1$  and  $\mathbf{k}_2$ , allowing to couple the spin and the orbital angular momentum as in the non-relativistic case [63–66], while the spins in the ket on the right-hand side of eq. (9) undergo the single-particle Wigner rotations.

In point form dynamics, eq. (1) corresponds to a good mass operator since it commutes with the Lorentz generators and with the four-velocity. We diagonalize eq. (1) in the Hilbert space spanned by the velocity states. Finally, instead of the internal momenta  $\mathbf{k}_1$  and  $\mathbf{k}_2$  we use the relative momentum  $\mathbf{q}$ , conjugate to the relative coordinate  $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ , thus considering the following velocity basis states:

$$|v, \mathbf{q}, \lambda_1, \lambda_2\rangle = U_{B(v)} |k_1, s_1, \lambda_1, k_2, s_2, \lambda_2\rangle_0. \quad (12)$$

### 3 Results and discussion

Figure 1 and table 2 show the comparison between the experimental data [17, 82] and the results of our quark-diquark model calculation, obtained with the set of parameters of table 1. In addition to the experimental data from PDG [17], we also consider the latest multi-channel Bonn-Gatchina partial wave analysis results, including data from Crystal Barrel/TAPS at ELSA and other laboratories [82]. In particular, these data differ from those of the PDG [17] in the case of the  $\Delta(1940)D_{33}$ .

As discussed in the previous sect. 2, the spin-isospin transition interaction of eq. (5) mixes quark-scalar diquark and quark-axial-vector diquark states whose total spin and isospin are  $S = T = \frac{1}{2}$ . Thus, in this version of the model the nucleon state, as well as states such as the  $D_{13}(1520)$ , the  $S_{11}(1535)$  and the  $P_{11}(1440)$ , all contain both a  $s_1 = 0$  and a  $s_1 = 1$  component. In particular, the nucleon state, obtained by solving the eigenvalue problem of eq. (1), in a schematic notation can be written as

$$|N\rangle = a_S |qD_S, L=0\rangle + a_{AV} |qD_{AV}, L=0\rangle, \quad (13)$$

where  $D_S$  and  $D_{AV}$  stand for the scalar and axial-vector diquarks, respectively, and  $q$  for the quark. The coefficients

$a_S$  and  $a_{AV}$ , obtained by solving the eigenvalue problem of eq. (1), are

$$a_S = 0.727, \quad (14a)$$

$$a_{AV} = 0.687. \quad (14b)$$

As anticipated in the previous sect. 2, these two values favourably compare with the corresponding amplitudes of the QMs, that are  $a_0^{ab} = a_1^{ab} = 1/\sqrt{2} = 0.7071$ .

The radial wave functions (in momentum space) of the quark-scalar diquark [ $\Phi_S(q)$ ] and quark-axial-vector diquark [ $\Phi_{AV}(q)$ ] systems of eq. (13) can be fitted by harmonic oscillator wave functions

$$\Phi_S(q) = \frac{2\alpha_S^{3/2}}{\pi^{1/4}} e^{-\frac{1}{2}\alpha_S^2 q^2}, \quad (15a)$$

$$\Phi_{AV}(q) = \frac{2\alpha_{AV}^{3/2}}{\pi^{1/4}} e^{-\frac{1}{2}\alpha_{AV}^2 q^2}, \quad (15b)$$

with  $\alpha_S = 3.29 \text{ GeV}^{-1}$  and  $\alpha_{AV} = 3.04 \text{ GeV}^{-1}$ . The same can be done for the  $\Delta(1232)$  radial wave function

$$\Phi_\Delta(q) = \frac{2\alpha_\Delta^{3/2}}{\pi^{1/4}} e^{-\frac{1}{2}\alpha_\Delta^2 q^2}, \quad (16)$$

where  $\alpha_\Delta = 3.14 \text{ GeV}^{-1}$ . This parametrization can then be used to compute observables, such as the nucleon electromagnetic form factors.

The introduction of the interaction of eq. (5) determines an improvement in the overall quality of the reproduction of the experimental data (considering only  $3^*$  and  $4^*$  resonances), with respect to that obtained with the previous version of this model [42, 43]. In particular, the Roper resonance,  $N(1440) P_{11}$ , is far better reproduced than before and the same holds for  $N(1680) F_{15}$ .

The present version of the relativistic quark-diquark model predicts only one missing state below the energy of 2 GeV (see table 2), while three quarks QMs give rise to several missing states [17]. For example, Capstick and Isgur's model [4] has 5 missing states up to 2 GeV, the hypercentral QM [83] has 8, Glozman and Riska's model [11, 84] has 4 and the  $U(7)$  model [5, 6] has 17. The only missing resonance of our model,  $N_{\frac{3}{2}^+}(1990)$ , lies at the same energy of the three-star state  $N(2000) F_{15}$ , which was previously a two-star state of the PDG [17]. Indeed the two resonances,  $N_{\frac{3}{2}^+}(1990)$  and  $N(2000) F_{15}$ , have the same quantum numbers, except for the total angular momentum, because their spin ( $S = \frac{1}{2}$ ) and orbital angular momentum ( $L = 2$ ) are coupled to  $J^P = \frac{3}{2}^+$  or  $\frac{5}{2}^+$ . Thus, to split the two resonances one should take a spin-orbit interaction into account.

The whole mass operator of eq. (1) is diagonalized by means of a numerical variational procedure, based on harmonic oscillator trial wave functions. With a variational basis made of  $N = 200$  harmonic oscillator shells, the results converge very well.

While the absolute values of the diquark masses are model dependent, their difference is not. Comparing our

**Table 2.** Comparison between the experimental values [17] of non-strange baryon resonances masses (up to 2 GeV) and the results of the model (all values are expressed in MeV). Tentative assignments of  $2^*$  and  $1^*$  resonances are shown in the second part of the table. The quantum numbers  $L$ ,  $S$  and  $J^P$  have been introduced in sect. 2. The  $N$  states with  $S = \frac{1}{2}$  are always given by a mixing of states with  $s_1 = 0$  and  $s_1 = 1$ ; the  $N$  states with  $S = \frac{3}{2}$  and all the  $\Delta$  states have only  $s_1 = 1$ ; finally  $n_r$  is the number of nodes in the radial wave function.

Resonance	Status	$M^{\text{exp.}}$ (MeV)	$J^P$	$L$	$S$	$n_r$	$M^{\text{calc.}}$ (MeV)
$N(939) P_{11}$	****	939	$\frac{1}{2}^+$	0	$\frac{1}{2}$	0	939
$N(1440) P_{11}$	****	1420–1470	$\frac{1}{2}^+$	0	$\frac{1}{2}$	1	1412
$N(1520) D_{13}$	****	1515–1525	$\frac{3}{2}^-$	1	$\frac{1}{2}$	0	1533
$N(1535) S_{11}$	****	1525–1545	$\frac{1}{2}^-$	1	$\frac{1}{2}$	0	1533
$N(1650) S_{11}$	****	1645–1670	$\frac{1}{2}^-$	1	$\frac{3}{2}$	0	1667
$N(1675) D_{15}$	****	1670–1680	$\frac{5}{2}^-$	1	$\frac{3}{2}$	0	1667
$N(1680) F_{15}$	****	1680–1690	$\frac{5}{2}^+$	2	$\frac{1}{2}$	0	1694
$N(1700) D_{13}$	***	1650–1750	$\frac{3}{2}^-$	1	$\frac{3}{2}$	0	1667
$N(1710) P_{11}$	***	1680–1740	$\frac{1}{2}^+$	0	$\frac{1}{2}$	2	1639
$N(1720) P_{13}$	****	1700–1750	$\frac{3}{2}^+$	2	$\frac{1}{2}$	0	1694
$N(1875) D_{13}$	***	1820–1920	$\frac{3}{2}^-$	1	$\frac{1}{2}$	1	1866
$N(1880) P_{11}$	**	1835–1905	$\frac{1}{2}^+$	0	$\frac{1}{2}$	3	1786
$N(1895) S_{11}$	**	1880–1910	$\frac{1}{2}^-$	1	$\frac{1}{2}$	1	1866
$N(1900) P_{13}$	***	1875–1935	$\frac{3}{2}^+$	0	$\frac{3}{2}$	0	1780
missing	–	–	$\frac{3}{2}^+$	2	$\frac{1}{2}$	1	1990
$N(2000) F_{15}$	**	1950–2150	$\frac{5}{2}^+$	2	$\frac{1}{2}$	1	1990
$\Delta(1232) P_{33}$	****	1230–1234	$\frac{3}{2}^+$	0	$\frac{3}{2}$	0	1236
$\Delta(1600) P_{33}$	***	1500–1700	$\frac{3}{2}^+$	0	$\frac{3}{2}$	1	1687
$\Delta(1620) S_{31}$	****	1600–1660	$\frac{1}{2}^-$	1	$\frac{1}{2}$	0	1600
$\Delta(1700) D_{33}$	****	1670–1750	$\frac{3}{2}^-$	1	$\frac{1}{2}$	0	1600
$\Delta(1750) P_{31}$	*	1708–1780	$\frac{1}{2}^+$	0	$\frac{1}{2}$	0	1857
$\Delta(1900) S_{31}$	**	1840–1920	$\frac{1}{2}^-$	1	$\frac{1}{2}$	1	1963
$\Delta(1905) F_{35}$	****	1855–1910	$\frac{5}{2}^+$	2	$\frac{3}{2}$	0	1958
$\Delta(1910) P_{31}$	****	1860–1920	$\frac{1}{2}^+$	2	$\frac{3}{2}$	0	1958
$\Delta(1920) P_{33}$	***	1900–1970	$\frac{3}{2}^+$	2	$\frac{3}{2}$	0	1958
$\Delta(1930) D_{35}$	***	1900–2000	$\frac{5}{2}^-$	1	$\frac{3}{2}$	0	2064
$\Delta(1940) D_{33}$	**	1940–2060	$\frac{3}{2}^-$	1	$\frac{1}{2}$	1	1963
$\Delta(1950) F_{37}$	****	1915–1950	$\frac{7}{2}^+$	2	$\frac{3}{2}$	0	1958

**Table 3.** Mass difference (in MeV) between scalar and axial-vector diquarks, according to some previous studies.

$m_S$ (MeV)	$m_{AV} - m_S$ (MeV)	Source
730	210	Bloch <i>et al.</i> [48]
750–860	10–170	Oettel <i>et al.</i> [51]
–	290	Wilczek [36]
–	210	Jaffe [35]
600	350	Ferretti <i>et al.</i> [43]
852	224	Galatà and Santopinto [45]
–	200–300	Lichtenberg <i>et al.</i> [70]
770	140	de Castro <i>et al.</i> [71]
420	520	Schäfer <i>et al.</i> [72]
692	330	Cahill <i>et al.</i> [73]
595	205	Lichtenberg <i>et al.</i> [74]
688	202	Maris [75]
–	360	Orginos [76]
750	100	Flambaum <i>et al.</i> [77]
–	162	Babich <i>et al.</i> [78]
–	135	Santopinto and Galatà [85]
710	199	Ebert <i>et al.</i> [86]
–	183	Chakrabarti <i>et al.</i> [87]
780	280	Roberts <i>et al.</i> [88]
150	210	This work

result for the mass difference  $m_{AV} - m_S$  between the axial-vector and the scalar diquark to those reported in table 3, it is interesting to note that our estimation is comparable with all the others. Such evaluations come from phenomenological observations [35, 36, 74], lattice QCD calculations [76, 78], instanton liquid model calculations [72], applications of Dyson-Schwinger, Bethe-Salpeter and quark-diquark Faddeev equations [48, 73, 75, 77] and calculations of QDMs [43, 70, 71, 85–88].

The main aim of the present work is to improve previous constituent QDMs [42–44] without changing completely their basic features. In particular, the mass operator has been modified only by introducing the spin-isospin transition interaction  $M_{tr}(r)$  of eq. (5). For this reason the character of the present model remains phenomenological, and no attempt is made to study the role of the quark and diquark masses at a fundamental level [54, 75]. In principle, other values (given by a more fundamental investigation) of  $m_q$ ,  $m_S$  and  $m_{AV}$  could be used to fit the spectrum but the interaction should be substantially modified. On the other hand, the set of numerical values of the parameters used in the present work is still consistent with the general picture of our previous QDMs.

The phenomenological character of this model also reflects on the relatively high number of parameters; in this context the possible contributions of diquarks with other quantum numbers (pseudoscalar, vector, etc.) [75], have been disregarded. However, these states, that should be not very important for positive-parity baryons, could pre-

sumably play a relevant role for negative parity resonances and should be studied in a comprehensive investigation about the quark-quark interaction in hadronic systems.

The relativistic quark-diquark model can also be used to compute other observables, in particular the electromagnetic ones. The main difficulty related to the present model (not found in the standard approach of QDMs [42, 43]) is given by the non-vanishing matrix elements between  $S$ - $AV$  diquark states that are *also* found when calculating electromagnetic observables. Here we preliminary study the baryon magnetic moments. To this aim, we re-write the nucleon wave function of eq. (13) formally representing the diquark as composed by two quarks, obtaining

$$\begin{aligned}
 |\Psi_{S,M_s;T,T_3}\rangle = & a_S \left| \chi_{0,0}^{ab} \chi_{\frac{1}{2},M_s}^2 \right\rangle \\
 & \times \left| \phi_{0,0}^{ab} \phi_{\frac{1}{2},T_3}^2 \right\rangle |\psi_S^{ab}\rangle |\Phi_S\rangle \\
 & + a_{AV} \left| \left[ \chi_1^{ab} \otimes \chi_{\frac{1}{2}}^2 \right]_{\frac{1}{2},M_s} \right\rangle \left| \left[ \phi_1^{ab} \otimes \phi_{\frac{1}{2}}^2 \right]_{\frac{1}{2},T_3} \right\rangle \\
 & \times |\psi_{AV}^{ab}\rangle |\Phi_{AV}\rangle, \quad (17)
 \end{aligned}$$

where the symbols  $\chi$  and  $\phi$  denote the spin and isospin wave functions, respectively; their lower indices give the corresponding quantum numbers and the upper indices represent the quarks to which they refer. In particular, the special notation  $a, b$  is used for the quarks of the diquark, while the standard upper index 2 refers to the quark not belonging to the diquark;  $|\Phi_S\rangle$  and  $|\Phi_{AV}\rangle$  represent the spatial wave functions of the quark-diquark system with the diquark in the  $S$  and  $AV$  state, respectively. The *internal* spatial wave functions of the diquark  $|\psi_S^{ab}\rangle$  and  $|\psi_{AV}^{ab}\rangle$  are unknown, because, in the present work, we do not develop a detailed model for the internal structure of the diquark. Finally, the amplitudes  $a_S$  and  $a_{AV}$  are given in eq. (14).

The electromagnetic current operator of the two-state diquark is in principle unknown. As a starting point, for the magnetic dipole operator  $\boldsymbol{\mu}$  we take an expression of non-relativistic kind, that will be transformed with the aim of taking into account the main symmetries of the model. This expression is obtained by summing the contributions of all the quarks:

$$\boldsymbol{\mu} = \frac{f}{m} (e_a \mathbf{s}_a + e_b \mathbf{s}_b) + \frac{f_2}{m_2} e_2 \mathbf{s}_2, \quad (18)$$

where the indices  $a, b$  have the same meaning explained above; furthermore,  $e_i$  and  $\mathbf{s}_i$  ( $i = a, b$  and 2) respectively represent the quark charge and spin operators;  $m$  is the mass of the quarks belonging to the diquark; finally, the constants  $f, f_2$  have been introduced to take into account possible anomalous magnetic moments of the quarks (for Dirac point-like quarks one would have  $f = f_2 = 1$ ).

In order to use the permutational symmetry of the quarks  $a$  and  $b$  of the diquark, we re-write eq. (18) as

$$\boldsymbol{\mu} = \frac{f}{2m} e_D \mathbf{s}_D + \frac{f}{2m} \Delta e_D \Delta \mathbf{s}_D + \frac{f_2}{m_2} e_2 \mathbf{s}_2, \quad (19)$$

where

$$e_D = e_a + e_b \quad (20a)$$

is the charge operator of the diquark in the isospin space,

$$\mathbf{s}_D = \mathbf{s}_a + \mathbf{s}_b \quad (20b)$$

represents its spin,

$$\Delta e_D = e_a - e_b \quad (20c)$$

is the difference between the charges of quarks  $a$  and  $b$  and finally

$$\Delta \mathbf{s}_D = \mathbf{s}_a - \mathbf{s}_b \quad (20d)$$

is the difference between the spin operators of the two quarks. The first two operators in eqs. (20) are symmetric under the exchange of quarks  $a$  and  $b$ , while the operators  $\Delta e_D$  and  $\Delta \mathbf{s}_D$  are antisymmetric. It is worthwhile noting that: 1) the first term in eq. (19) has null matrix elements for scalar diquarks, with  $S_D = 0$ ; 2) the first and third terms have null matrix elements between diquark states with different spins (namely, between  $S$  and  $AV$  diquarks); 3) the second term has null matrix elements between  $S$ - $S$  and  $AV$ - $AV$  diquarks and non-null between  $S$ - $AV$  diquarks.

In eq. (19),  $f$  and  $m$  are unknown quantities given that the internal diquark structure is not determined. As a consequence, taking into account that the first term of eq. (19) only gives non-vanishing contributions for  $AV$  diquarks, we make, in this term, the following substitution:

$$\frac{f}{2m} \rightarrow \frac{f_{AV}}{m_{AV}}; \quad (21)$$

taking the axial-vector diquark mass of the model  $m_{AV} = 360$  MeV (see table 1),  $f_{AV}$  is considered as a free parameter, used to fit to the experimental data.

As for the second term of eq. (19), we recall that this term gives non-vanishing matrix elements only when it is calculated between  $S$  and  $AV$  diquark states. As a consequence, we make the substitution

$$\frac{f}{2m} \rightarrow \frac{f_\Delta}{\langle m \rangle}, \quad (22a)$$

where  $f_\Delta$  is a free parameter; furthermore we have formally introduced

$$\langle m \rangle = \frac{1}{2}(m_S + m_{AV}). \quad (22b)$$

The third term in eq. (19) does not require changes;  $f_2$  is the third free parameter for the calculation of the nucleon magnetic moments.

Thus, eq. (19) is finally re-written as

$$\boldsymbol{\mu} = \frac{f_{AV}}{m_{AV}} e_D \mathbf{s}_D + \frac{f_\Delta}{\langle m \rangle} \Delta e_D \Delta \mathbf{s}_D + \frac{f_2}{m_2} e_q \mathbf{s}_q \quad (23)$$

that represents the effective magnetic dipole operator of our model. The mean values of the effective dipole operator are easily calculated with the wave function of eq. (17), so that the numerical values of free parameters  $f_{AV}$ ,  $f_\Delta$  and  $f_2$  can be fitted to reproduce exactly

the proton and neutron magnetic moments [17], that are  $\mu_p = 2.793$  n.m.u. and  $\mu_n = -1.913$  n.m.u. Because the number of free parameters is larger than that of the experimental informations, we take

$$f_2 = 1; \quad (24)$$

the resulting values for the other parameters are

$$f_{AV} = 0.0462, \quad f_\Delta = 0.146. \quad (25)$$

This preliminary calculation shows that our model is able to reproduce the nucleon magnetic moments. A more detailed study of the electromagnetic current of the quark-diquark system will be undertaken in a subsequent work with the aim of calculating the nucleon electromagnetic form factors and the helicity amplitudes of baryon resonances [89]. We think that, in general, the present model can be helpful to the experimentalists in their analysis of the properties of the  $N$ - and  $\Delta$ -type resonances.

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