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New fixed points of the renormalisation group for two-body scattering

M.C. Birse[1], E. Epelbaum[2], and J. Gegelia[3,4,a]

[1] Theoretical Physics Division, School of Physics and Astronomy, The University of Manchester, Manchester, M13 9PL, UK
[2] Institut für Theoretische Physik II, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany
[3] Institute for Advanced Simulation, Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany
[4] Tbilisi State University, 0186 Tbilisi, Georgia

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Abstract. We outline a separable matrix *ansatz* for the potentials in effective field theories of non-relativistic two-body systems with short-range interactions. We use this ansatz to construct new fixed points of the renormalisation-group equation for these potentials. New fixed points indicate a much richer structure than previously recognized in the RG flows of simple short-range potentials.

1 Introduction

The renormalisation group (RG) has proved to be a powerful tool for elucidating the scale dependences of systems in many areas of physics [1]. (For more recent reviews, see refs. [2,3].) These systems include ones consisting of two or three particles at low enough energies that the motion can be treated as non-relativistic. In the case of two-body scattering by short-range forces, the existence of a nontrivial fixed point of the RG was first noted by Weinberg [4], although he did not go on explore the RG flow in its vicinity. This idea was further developed in refs. [5–7], and a complete RG analysis of the flow around this fixed point was carried out in ref. [8].

This fixed point describes a system in the "unitary limit", where the scattering length is infinite. A system with a large scattering length compared to the scales of the underlying physics can be described in terms of perturbations around this point. The resulting expression for the scattering amplitude is just given by the effective-range expansion [9]. This provides a systematic organizing scheme, or "power counting" for an effective field theory (EFT) that has been applied to nucleon-nucleon scattering and to ultracold atoms in traps. A review of the RG approach in nuclear physics can be found in ref. [10].

The RG for two-body scattering is expected to have other fixed points. For example, there is a trivial one, corresponding to weakly interacting systems where the scattering can be treated perturbatively [8]. The existence of further, nontrivial fixed points has been conjectured [11] and hints of them were seen in a functional RG analysis by Harada and Kubo [12,13], but explicit forms for them were not found.

Here we present a systematic method for constructing an infinite number of possible fixed points of the RG for two-body scattering by short-range forces. For selected examples, we study the flows close to them, which determine the power counting rules for EFTs expanded around these points. We also construct some of the renormalised trajectories that flow from one fixed point to another. Each of the new fixed points has at least two unstable directions and so two or more parameters would need to be "fine tuned" for a physical system to be described by it.

2 RG flow

Following the approach of ref. [8], a convenient starting point is the Lippmann-Schwinger integral equation for the K-matrix for S-wave two-body scattering,

$$K(k',k,p) = V(k',k,p,\Lambda) + 2M\,\mathcal{P} \int \frac{\mathrm{d}^3 l\,\theta(\Lambda - l)}{(2\pi)^3}\,\frac{V(k',l,p,\Lambda)\,K(l,k,p)}{p^2 - l^2}, \quad (1)$$

where $\mathcal{P}$ stands for the principal value, M is the reduced mass, and $p = \sqrt{2ME}$ is the on-shell relative momentum. The integral over the momentum l of the intermediate state has been regulated by cutting it off at $l = \Lambda$.

[a] e-mail: jgegelia@hotmail.com

The solution to eq. (1) is the fully off-shell K-matrix, whose matrix elements depend on the initial and final off-shell momenta, k and k'. On-shell observables can be obtained from it by setting $k = k' = p$. For example, the on-shell K-matrix, $\mathcal{K}(p) = K(p,p,p)$, is related to the phase shift $\delta(p)$, and hence to the effective-range expansion, by

$$\frac{1}{\mathcal{K}(p)} = -\frac{M}{2\pi}\,p\cot\delta(p) = -\frac{M}{2\pi}\left(-\frac{1}{a} + \frac{1}{2}r_e p^2 + v_2 p^4 + \dots\right). \qquad (2)$$

To obtain the RG equation, we first demand that the solution to eq. (1) be independent of the cutoff Λ. From the physical point of view it is not necessary to have a cutoff-independent off-shell K-matrix. However, it is always possible to find an on-shell equivalent modification of the potential such that the off-shell K-matrix is cutoff independent. The obtained RG equation contains only the potential, in contrast to the potential $V_{\text{low-}k}$ of Bogner *et al.* [14], whose evolution equation involves the scattering matrix as well. The resulting potential $V(k',k,p,\Lambda)$ then has a well-defined evolution with Λ [8],

$$\frac{\partial V}{\partial \Lambda} = \frac{M}{\pi^2}\,V(k',\Lambda,p,\Lambda)\,\frac{\Lambda^2}{\Lambda^2 - p^2}\,V(\Lambda,k,p,\Lambda). \qquad (3)$$

Next, we express all low-energy scales in units of Λ, $\hat{p} = p/\Lambda$, etc., and we define the rescaled potential

$$\hat{V}(\hat{k}',\hat{k},\hat{p},\Lambda) = \frac{M\Lambda}{\pi^2}\,V(\Lambda\hat{k}',\Lambda\hat{k},\Lambda\hat{p},\Lambda). \qquad (4)$$

This converts eq. (3) into the form of an RG equation,

$$\Lambda\frac{\partial\hat{V}}{\partial\Lambda} = \hat{k}'\frac{\partial\hat{V}}{\partial\hat{k}'} + \hat{k}\frac{\partial\hat{V}}{\partial\hat{k}} + \hat{p}\frac{\partial\hat{V}}{\partial\hat{p}} + \hat{V}(\hat{k}',\hat{k},\hat{p},\Lambda) + \hat{V}(\hat{k}',1,\hat{p},\Lambda)\,\frac{1}{1-\hat{p}^2}\,\hat{V}(1,\hat{k},\hat{p},\Lambda). \qquad (5)$$

This has a similar structure to the RG equations that govern the evolution of interactions in other areas of physics [1]. In this rescaled equation, the cutoff Λ, which can be thought of as the highest acceptable low-energy scale, is the only dimensioned quantity. The scaling of the potential with Λ is directly related to the dependence on the original low-energy variables, p, k and k', as can be seen from the logarithmic derivatives on the right-hand side of eq. (5).

As $\Lambda \to 0$, the solutions to eq. (5) tend to fixed points, independent of Λ[1]. This is because, for low enough values of Λ, all memory of the scales of the underlying physics is lost and Λ becomes the only scale controlling the dependence of the potential on energy and momentum. Expressed in units of Λ, the corresponding rescaled potential becomes a constant.

[1] In some cases the flows can drive the potential to infinity at a finite value of Λ. In such cases it is better to follow the flow of the inverse of the potential, which simply passes through zero and continues towards a fixed point.

These fixed points describe scale-invariant systems. For a system that lies close to one of these points, we can use the RG flow near that point to define a systematic expansion of the potential in powers of the low-energy scales. The resulting power counting can be used to organise the terms in a low-energy effective theory. In ref. [8], two fixed points were identified. One is just the trivial point, $\hat{V} = 0$. This is a stable point since all the perturbations around it are irrelevant, that is, they flow towards it as $\Lambda \to 0$. In fact, their scaling with Λ follows from naive dimensional analysis and the resulting expansion can be used to describe weakly interacting systems.

The second fixed point is the momentum-independent one that describes scattering in the unitary limit. As described in ref. [8], this is an unstable point, with one relevant perturbation which corresponds to the scattering length a. The expansion around this point can be used to describe systems where the scattering length is much larger than the range of the forces. In the power counting that controls this expansion, the terms are promoted by two orders relative to naive dimensional analysis [8]. The coefficients of these terms are directly related to those of the effective-range expansion, and the counting reflects the enhancement of the corresponding terms in scattering amplitude by a factor of $1/a^2$.

Potentials corresponding to fine-tuned systems with $1/a = 0$ lie on a critical surface [1,2] and flow towards the unitary fixed point as $\Lambda \to 0$. If $1/a$ is nonzero, then the relevant perturbation drives the flow away from the unitary fixed point for $\Lambda \lesssim 1/a$ and towards the trivial point as $\Lambda \to 0$. This reflects the fact that at very low energies, scattering can be treated perturbatively, at least so long as the scattering length is finite. The RG flow line linking the two fixed points is known as a "renormalised trajectory" [1,2]. The theory corresponding to this trajectory is renormalisable, both perturbatively and non-perturbatively, in terms of a single coupling constant (in this case, the coefficient of the energy- and momentum-independent contact interaction).

3 More fixed points

To find further fixed points of the two-body system, we consider here potentials that can be expressed as bivariate polynomials in the off-shell momenta k and k', whose coefficients are functions of the on-shell energy p^2. The structure of the nonlinear term in the RG equation (5) means that no approximation is involved in choosing this ansatz. It allows us not only to identify the fixed points but also to follow the RG flow lines in their vicinity. These include the renormalised trajectories which run from one fixed point to another.

If we restrict our potentials to be Hermitian, as these are of most interest, then our ansatz for them is conveniently written in a separable matrix form,

$$V(k',k,p,\Lambda) = \chi^{\mathrm{T}}(k')\,\omega(p,\Lambda)\,\chi(k), \qquad (6)$$

where ω is an $N \times N$, matrix. Here $\chi(k)$ is defined as a column vector of powers of momentum and $\chi^{\mathrm{T}}(k)$ is its transpose,

$$\chi^{\mathrm{T}}(k) = \left(k^{2n_1}, \ldots, k^{2n_N}\right), \tag{7}$$

where $\bar{S}_N = \{n_1, \ldots, n_N\}$ is a set of N non-negative integers. The K-matrix for this potential has a similar separable form,

$$K(k', k, p) = \chi^{\mathrm{T}}(k')\, \kappa(p)\, \chi(k). \tag{8}$$

From the Lippmann-Schwinger equation (1), we find that $\kappa(p)$ can be related to $\omega(p, \Lambda)$ by

$$\kappa(p)^{-1} = \omega(p, \Lambda)^{-1} - \mathcal{G}(p, \Lambda), \tag{9}$$

where $\mathcal{G}(p, \Lambda)$ is the matrix

$$\mathcal{G}(p, \Lambda) = 2M\, \mathcal{P}\!\int \frac{\mathrm{d}^3 l\, \theta(\Lambda - l)}{(2\pi)^3}\, \frac{\chi(l)\, \chi^{\mathrm{T}}(l)}{p^2 - l^2}. \tag{10}$$

The elements of this can be written as

$$\mathcal{G}_{ij}(p, \Lambda) = \frac{M}{\pi^2}\, I_{n_i + n_j}(p, \Lambda), \tag{11}$$

where the regularised loop integrals are

$$I_n(p, \Lambda) = -\sum_{m=0}^{n} \frac{\Lambda^{2m+1} p^{2(n-m)}}{2m+1} + \frac{p^{2n+1}}{2} \ln \frac{\Lambda + p}{\Lambda - p}. \tag{12}$$

The rescaled version of the potential (6) can be written analogously as

$$\hat{V}(\hat{k}', \hat{k}, \hat{p}, \Lambda) = \chi^{\mathrm{T}}(\hat{k}')\, \hat{\omega}(\hat{p}, \Lambda)\, \chi(\hat{k}), \tag{13}$$

where the elements of $\hat{\omega}(\hat{p}, \Lambda)$ are

$$\hat{\omega}_{ij}(\hat{p}, \Lambda) = \frac{M}{\pi^2}\, \Lambda^{2(n_i + n_j)+1} \omega_{ij}(\Lambda \hat{p}, \Lambda). \tag{14}$$

Inserting this into the RG equation (5), we find that the evolution of $\hat{\omega}(\hat{p}, \Lambda)$ with Λ is governed by the equation

$$\begin{aligned}\Lambda \frac{\partial \hat{\omega}}{\partial \Lambda} &= \hat{p}\, \frac{\partial \hat{\omega}}{\partial \hat{p}} + 2\, D(S_N)\, \hat{\omega} + 2\, \hat{\omega}\, D(S_N) \\ &\quad + \hat{\omega} + \hat{\omega}\, \frac{\chi(1)\chi(1)^{\mathrm{T}}}{1 - \hat{p}^2}\, \hat{\omega},\end{aligned} \tag{15}$$

where $D(S_N)$ is the diagonal matrix of the elements of S_N.

This equation can be more easily solved if, following ref. [15], it is rewritten as a linear equation for $\hat{\omega}^{-1}$ (see footnote[2]),

$$\begin{aligned}\Lambda \frac{\partial \hat{\omega}^{-1}}{\partial \Lambda} &= \hat{p}\, \frac{\partial \hat{\omega}^{-1}}{\partial \hat{p}} - 2\, \hat{\omega}^{-1} D(S_N) \\ &\quad - 2\, D(S_N)\, \hat{\omega}^{-1} - \hat{\omega}^{-1} - \frac{\chi(1)\chi(1)^{\mathrm{T}}}{1 - \hat{p}^2}.\end{aligned} \tag{16}$$

[2] Further fixed points are obtained by considering non-invertible ω matrices. We do not deal with that case here.

In this form, we can see that each of the elements of $\hat{\omega}^{-1}$ satisfies an uncoupled RG equation,

$$\begin{aligned}\Lambda \frac{\partial \left[\hat{\omega}^{-1}\right]_{ij}}{\partial \Lambda} &= \hat{p}\, \frac{\partial \left[\hat{\omega}^{-1}\right]_{ij}}{\partial \hat{p}} \\ &\quad - (2n_i + 2n_j + 1) \left[\hat{\omega}^{-1}\right]_{ij} - \frac{1}{1 - \hat{p}^2},\end{aligned} \tag{17}$$

which can be integrated straightforwardly.

For any set of numbers S_N, we can find a nontrivial fixed-point solution, $\omega_0(\hat{p})$, to eq. (16), whose elements satisfy the ODEs

$$\hat{p}\, \frac{\partial \left[\hat{\omega}_0^{-1}\right]_{ij}}{\partial \hat{p}} = (2n_i + 2n_j + 1) \left[\hat{\omega}_0^{-1}\right]_{ij} + \frac{1}{1 - \hat{p}^2}. \tag{18}$$

This should satisfy the boundary condition that the matrix ω be analytic in $\hat{p}^2$ as $\hat{p} \to 0$ (or, in other words, it should be analytic in the energy). Taking into account eq. (9) we obtain for the elements of the resulting matrix

$$\left[\hat{\omega}_0^{-1}\right]_{ij} = C_{ij}\, \hat{p}^{2n_i + 2n_j + 1} + \hat{I}_{n_i + n_j}(\hat{p}), \tag{19}$$

where C_{ij} are arbitrary and we have introduced rescaled versions of the loop integrals (12),

$$\begin{aligned}\hat{I}_n(\hat{p}) &= \frac{1}{\Lambda^{2n+1}}\, I_n(\Lambda \hat{p}, \Lambda) \\ &= -\sum_{m=0}^{n} \frac{\hat{p}^{2(n-m)}}{2m+1} + \frac{\hat{p}^{2n+1}}{2} \ln \frac{1 + \hat{p}}{1 - \hat{p}}.\end{aligned} \tag{20}$$

Demanding analyticity of the potential in p^2 at $p^2 = 0$ means that we have to take $C_{ij} = 0$. When we undo the rescaling of eq. (14), we find that, in physical units,

$$\omega_0^{-1}(p, \Lambda) = \mathcal{G}(p, \Lambda). \tag{21}$$

From eqs. (9) and (21) we see that each fixed point corresponds to a K-matrix with vanishing $\kappa(p)^{-1}$ or, provided $n_1 = 0$, infinite scattering length. For example, the case $N = 1$ and $S_1 = \{0\}$ gives the unitary fixed point as in ref. [8],

$$\hat{V}_U(\hat{p}) = \frac{1}{\hat{I}_0(\hat{p})} = -\left[1 - \frac{\hat{p}}{2} \ln \frac{1 + \hat{p}}{1 - \hat{p}}\right]^{-1}. \tag{22}$$

Adding an energy-independent perturbation to $\left[\hat{\omega}_0^{-1}\right]_{11}$ leads to a solution to eq. (17) with the form

$$\left[\hat{\omega}^{-1}\right]_{11} = \frac{\alpha}{\Lambda} + \hat{I}_0(\hat{p}). \tag{23}$$

The corresponding K-matrix is

$$\mathcal{K}(p) = \frac{\pi^2}{M\alpha}, \tag{24}$$

and so the parameter α is related to the physical scattering length by

$$\alpha = \frac{\pi}{2a}. \tag{25}$$

This perturbation grows as Λ is lowered and so it is a relevant one. It defines a renormalised trajectory which consists of the potentials

$$\hat{V}(\hat{p},\Lambda)=\left[\frac{\alpha}{\Lambda}-1+\frac{\hat{p}}{2}\ln\frac{1+\hat{p}}{1-\hat{p}}\right]^{-1}. \tag{26}$$

This flows from unitary fixed point for $\Lambda \gg 1/a$ to the trivial one as $\Lambda \to 0$. All other perturbations around the unitary point are irrelevant [8] and so for $\alpha = 0$ the other perturbations define the critical surface of potentials that flow into this point as $\Lambda \to 0$.

The more general ansatz described above allows us to construct an infinite number of other fixed points using different sets S_N of powers of the off-shell momenta. The simplest of these points have a one-term-separable structure. For example, the one with $N = 1$ and $S_1 = \{1\}$ is

$$\hat{V}_S(\hat{k}',\hat{k},\hat{p})=\frac{\hat{k}'^2\hat{k}^2}{\hat{I}_2(\hat{p})} = -\hat{k}'^2\left[\frac{1}{5}+\frac{\hat{p}^2}{3}+\hat{p}^4-\frac{\hat{p}^5}{2}\ln\frac{1+\hat{p}}{1-\hat{p}}\right]^{-1}\hat{k}^2. \tag{27}$$

The solution to the RG equation (16),

$$\hat{V}(\hat{k}',\hat{k},\hat{p},\Lambda)=\hat{k}'^2\left[\frac{\gamma}{\Lambda^5}+\frac{\eta\,\hat{p}^2}{\Lambda^3}+\frac{\sigma\,\hat{p}^4}{\Lambda}+\hat{I}_2(\hat{p})\right]^{-1}\hat{k}^2, \tag{28}$$

shows that this fixed point has three relevant perturbations. This means that it would describe triply fine-tuned systems, which makes it very unlikely to be realised in practice. The potentials (28) that contain only these perturbations form a three-parameter family of renormalised trajectories that run from $\hat{V}_S$ to the trivial point.

At the same order in the off-shell momenta, there is also a fixed point with $N = 2$ and $S_2 = \{0, 1\}$:

$$\hat{V}_L(\hat{k}',\hat{k},\hat{p})=\begin{pmatrix}1, & \hat{k}'^2\end{pmatrix}\begin{pmatrix}\hat{I}_0(\hat{p}) & \hat{I}_1(\hat{p})\\ \hat{I}_1(\hat{p}) & \hat{I}_2(\hat{p})\end{pmatrix}^{-1}\begin{pmatrix}1\\ \hat{k}^2\end{pmatrix}. \tag{29}$$

This is the simplest example of a fixed point that does not have a one-term separable form. The existence of such a point had been hinted at previously [11,12] but no explicit expression for it was found. It has six relevant perturbations, which can be seen in the potential,

$$\hat{V}(\hat{k}',\hat{k},\hat{p},\Lambda)=\begin{pmatrix}1, & \hat{k}'^2\end{pmatrix}\times\begin{pmatrix}\frac{\alpha}{\Lambda}+\hat{I}_0(\hat{p}) & \frac{\beta}{\Lambda^3}+\frac{\delta\,\hat{p}^2}{\Lambda}+\hat{I}_1(\hat{p})\\ \frac{\beta}{\Lambda^3}+\frac{\delta\,\hat{p}^2}{\Lambda}+\hat{I}_1(\hat{p}) & \frac{\gamma}{\Lambda^5}+\frac{\zeta\,\hat{p}^2}{\Lambda^3}+\frac{\eta\,\hat{p}^4}{\Lambda}+\hat{I}_2(\hat{p})\end{pmatrix}^{-1}\times\begin{pmatrix}1\\ \hat{k}^2\end{pmatrix}, \tag{30}$$

which satisfies the RG equation (16). The on-shell K-matrix for this is

$$\mathcal{K}(p)=\frac{\pi^2}{M}\,\frac{\gamma+\zeta\,p^2+\eta\,p^4-2(\beta+\delta\,p^2)p^2+\alpha\,p^4}{\alpha(\gamma+\zeta\,p^2+\eta\,p^4)-(\beta+\delta\,p^2)^2}. \tag{31}$$

The fixed point describes the scale-free limit where all of the parameters $\alpha, \ldots, \eta$ vanish. However the corresponding scattering amplitude is not uniquely defined until one specifies how this limit is taken.

The renormalised trajectories that flow out of this fixed point can be followed more easily by rewriting eq. (30) in the form

$$\hat{V}(\hat{k}',\hat{k},\hat{p},\Lambda)=\begin{pmatrix}1, & \hat{k}'^2\end{pmatrix}\det\left[\omega(\hat{p},\Lambda)\right]\times\begin{pmatrix}\frac{\gamma}{\Lambda^5}+\frac{\zeta\,\hat{p}^2}{\Lambda^3}+\frac{\eta\,\hat{p}^4}{\Lambda}+\hat{I}_2(\hat{p}) & -\frac{\beta}{\Lambda^3}-\frac{\delta\,\hat{p}^2}{\Lambda}-\hat{I}_1(\hat{p})\\ -\frac{\beta}{\Lambda^3}-\frac{\delta\,\hat{p}^2}{\Lambda}-\hat{I}_1(\hat{p}) & \frac{\alpha}{\Lambda}+\hat{I}_0(\hat{p})\end{pmatrix}\times\begin{pmatrix}1\\ \hat{k}^2\end{pmatrix}, \tag{32}$$

where

$$\det\left[\omega(\hat{p},\Lambda)\right]=\left[\left(\frac{\alpha}{\Lambda}+\hat{I}_0(\hat{p})\right)\left(\frac{\gamma}{\Lambda^5}+\frac{\zeta\,\hat{p}^2}{\Lambda^3}+\frac{\eta\,\hat{p}^4}{\Lambda}+\hat{I}_2(\hat{p})\right)-\left(\frac{\beta}{\Lambda^3}+\frac{\delta\,\hat{p}^2}{\Lambda}+\hat{I}_1(\hat{p})\right)^2\right]^{-1}. \tag{33}$$

In general these potentials run to the trivial fixed point. For example, in the case that $\alpha\gamma-\beta^2$ is nonzero, the determinant behaves for small Λ as

$$\det\left[\omega(\hat{p})\right]=\frac{\Lambda^6}{\alpha\gamma-\beta^2}+\mathcal{O}(\Lambda^7). \tag{34}$$

As a result, all elements of the potential (32) vanish at least linearly in Λ as $\Lambda \to 0$.

In the more fine-tuned case where γ is nonzero but $\alpha\gamma - \beta^2 = 0$, the determinant behaves as

$$\det\left[\omega(\hat{p})\right]=\frac{\Lambda^5}{\gamma\hat{I}_0(\hat{p})}+\mathcal{O}(\Lambda^6), \tag{35}$$

and all elements of the potential vanish except for $\hat{V}_{11}$. For small Λ this has the form

$$\hat{V}_{11}(\hat{p},\Lambda)=\frac{1}{\hat{I}_0(\hat{p})}+\mathcal{O}(\Lambda), \tag{36}$$

which is just the unitary fixed point in the limit $\Lambda \to 0$. Finally, in the case that the only nonzero relevant perturbation is α, we get a potential that runs to the separable fixed point $\hat{V}_S$.

The other perturbations around each of these fixed points are all irrelevant. They involve either higher powers of the energy (p^2) or different powers of the off-shell momenta. The scaling of the former can be found easily by adding additional energy-dependent terms to the potentials just discussed. For the latter, we can use a more general version of eq. (15) that contains all powers of k'^2 and k^2, not just the ones that appear in the fixed point. Adding these perturbations leads to a critical surface for each point, consisting of all potentials that flow to that point as $\Lambda \to 0$.

In cases such as S-wave nucleon-nucleon scattering where the coefficients of the relevant perturbations are unnaturally small, the potential lies close to the critical surface for large cutoffs. As Λ is lowered the potential initially runs towards the fixed point. Then, when Λ becomes comparable to the scales of the relevant perturbations (such as $1/a$), the flow deviates from the critical surface and heads towards a renormalised trajectory that leads to a different fixed point, generally the trivial one. (See, for example, fig. 1 of ref. [8].)

4 Summary

In this paper we have outlined a separable matrix ansatz for the potentials that arise in EFT descriptions of two-body systems with short-range interactions. This provides a tool for constructing new fixed points of the RG for these systems, as well as the renormalised trajectories connecting them. In particular we are able to construct a fixed point whose existence has previously been only conjectured. These new fixed points indicate a much richer structure than previously recognized in the RG flows of simple short-range potentials. Each of them is unstable in at least two directions and so a physical system described by one of them would need to have fine-tuned values for at least two parameters.

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References

1. K.G. Wilson, J.B. Kogut, Phys. Rep. **12**, 75 (1974).
2. T.R. Morris, Prog. Theor. Phys. Suppl. **131**, 395 (1998).
3. Y. Meurice, R. Perry, S.-W. Tsai (Editors), *New applications of the renormalization group in nuclear, particle and condensed matter physics*, in *Philosophuical Transactions of the Royal Society of London A*, Vol. **369** (The Royal Society, 2011).
4. S. Weinberg, Nucl. Phys. B **363**, 3 (1991).
5. J. Gegelia, Phys. Lett. B **429**, 227 (1998).
6. D.B. Kaplan, M.J. Savage, M.B. Wise, Phys. Lett. B **424**, 390 (1998).
7. U. van Kolck, Nucl. Phys. A **645**, 273 (1999).
8. M.C. Birse, J.A. McGovern, K.G. Richardson, Phys. Lett. B **464**, 169 (1999).
9. H.A. Bethe, Phys. Rev. **76**, 38 (1949).
10. M.C. Birse, Phil. Trans. R. Soc. London A **369**, 2662 (2011).
11. T.D. Cohen, private communication.
12. K. Harada, H. Kubo, Nucl. Phys. B **758**, 304 (2006).
13. K. Harada, H. Kubo, A. Ninomiya, Int. J. Mod. Phys. A **24**, 3191 (2009).
14. S.K. Bogner, T.T.S. Kuo, A. Schwenk, Phys. Rep. **386**, 1 (2003).
15. T. Barford, M.C. Birse, Phys. Rev. C **67**, 064006 (2003).