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Review

First an idol, then an outcast: both for wrong reasons?

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Abstract. We present a critique on the present status of supersymmetry, based on the viewpoint that its most important motivation in the realm of observable particle phenomenology is the existence of dark matter in the universe. This motivation may as well override the issue of naturalness of the electroweak scale, and it sets up different yardsticks for the supersymmetric particle spectrum, out of which the dark matter candidate emerges. A few investigations in the context of slightly 'unusual' scenarios are briefly reported in this context.

1 Introduction

The reception of supersymmetry (SUSY) in the sphere of particle physics has taken rather unexpected turns in recent times. The community developed interest in SUSY initially for its mathematical/aesthetic appeal, which was converted, the early 1980's onwards, into adulation due to its perceived phenomenological potential [1,2]. More than three decades were spent thereafter in intense pursuit at colliders, in low-energy studies and also in the context of astrophysical and cosmological questions. Model-building and bids for unification of forces as well as parameters went on in parallel with equal intensity; SUSY became the template for physics beyond the standard model (BSM), all the way till the Large Hadron Collider (LHC) started running. Then, soon after a Higgs-like scalar (if not 'the' Higgs boson) with mass around 125 GeV had been identified [3,4], interest in SUSY started dwindling, making such a nosedive that it is considered hardly a hot subject now. And it is so in spite of the persistent expectation that BSM physics lurks around the corner.

A number of factors have contributed to this. On the one hand, the upward-moving mass limits threaten the solution to the naturalness problem. At the same time, difficulties are faced in reconciling the mass of the 125-GeV scalar with SUSY parameters in the minimal scenario unless they are on the high side according to the traditional view on naturalness [5]. These have undeniably served to take the spotlight away from SUSY. Of course, it is invoked in some 'emergent' forms (though mostly in quantum-mechanical versions), to explain phenomena ranging from the occurrence of magnetic monopoles [6] to observed properties of topological insulators [7]. Nevertheless, the rapid onset of coldness in shoulders tuned towards testable SUSY at the fundamental level is indeed enigmatic.

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As hinted above, there are definite reasons behind the idolatry lasting over a certain period, and the subsequent lack of warmth. Recounting them is certainly instructive. In addition, however, one may like to examine whether things have been a little overdone in both directions, emphasizing issues that might not have been absolutely compelling. We will start with such an examination in the following pages. Then we point out how SUSY may still provide guiding principles for uncovering new physics in the world of elementary particles.

In Section 2, we make a comparative, and slightly critical, assessment of the various factors that made physicists enthusiastic about SUSY. The relative stress on the various motivations are occasionally questioned during our analysis, at the end of which our conclusion is that the issue of dark matter of the universe is perhaps the biggest motivation for studying SUSY, even in forms where the question of naturalness is at least temporarily shelved. Two classes of SUSY scenarios, not conventionally talked about, are discussed in this light in Sections 3 and 4. We conclude in Section 5.

Many works relevant to this article could not be cited because of space constraint. The essential papers cited here, including those by the present author, contain more exhaustive references.

2 A perspective: pros, cons and the reality

Let us start by stating some reasons that strengthened the interest of the particle physics community in SUSY as an ingredient of fundamental physics. Listed neither chronologically nor in order of importance, the major factors influencing such interest are:

- The sheer elegance of a boson-fermion symmetry.
- The excitement of capturing the only possible extension of the Poincare algebra, albeit in a 'graded' form, incorporating anticommutators in addition to commutators [8].
- The potential of a spectacular UV completion, leading, on the one hand, to a drastic reduction of the minimal SUSY standard model (MSSM), and, on the other, to connections with gravity in particular and string theories in general.
- Improving the prospect of Grand Unification and combine the strong, weak and electromagnetic forces, via the convergence of the three gauge couplings at about 2×10^{16} GeV.
- The fact that SUSY offered a convincing way of stabilising the Higgs mass and the electroweak scale.
- Having a candidate particle for cold dark matter (CDM) of the universe.

Let us examine these motivations one by one, assess how compelling each one is, and ask ourselves whether they constitute grounds to believe that SUSY should lurk within the reach of the LHC, especially within the event samples accumulated so far. *The purpose here is to neither eulogise nor downplay the motivations. What is enquired is whether they imply that SUSY, if at all there, should have already left its mark in experiments.* We are of course aware that there is scope for divergent views on this.

Theoretical elegance of a boson-fermion symmetry: It is of course a beautiful thing to have such a symmetry. However, here one is talking about an action of the fundamental quantum theory of nature, which is invariant when the bosonic and fermionic fields are mixed up according to a specific prescription, in a space spanned by Grassmann variables. Frankly, there is no observable indication yet of a symmetry of this kind, however appealing it is aesthetically. There is no evidence of the existence of a boson for every known fermion and *vice versa*. Besides, since exactly degenerate superpartners are not observed anyway, sheer theoretical elegance is insufficient for implying that their mass splitting should be within the energy scale probed in terrestrial experiments so far, or within one's reach in the near future.

Encapsulating an extension of the Poincare algebra: The judgement is the same as in the previous case. The Coleman-Mandula theorem [9], itself a deep observation linking the saturatedness of a Poincare-type commutator algebra with Lorentz invariance of scattering matrices, does not necessitate an anticommutator extension *per se*.

UV completion and connection with gravity: The prospect of reducing the proliferation of free parameters through an UV completion with just a few fundamental parameters is certainly appealing. It is also rewarding to have such frameworks as the mSUGRA-based constrained MSSM (cMSSM) [10] or gauge-mediated SUSY breaking (GMSB) [11], which connect low-energy physics to the gravitational force. A justification of electroweak symmetry breaking at the right scale is also a wonderful bonus. On the other hand, it is somewhat artificial to imagine that there remains a grand desert across some 12–16 orders in energy, with SUSY breaking and the associated phenomenology at our accessible scale determined by such an UV design. Nature has so far shown, in physics ranging from eV-range energies to the GeV-range, that new physics looms up as one goes down by every two or three orders of smallness in distance. If one has to take any lesson from this, then the idea of parameter economy as well as new physics around the electroweak scale, dictated by a UV design, appears to be a *non sequitur*.

Improved prospect of Grand Unification: The low-energy measurement of α_s, the $SU(3)$ gauge coupling constant, indicates rather pleasantly that SUSY broken around a TeV supplies the right threshold effect for the three couplings to converge at the right scale [12–14]. However, the specified mass scale is a consequence of new physics intervening effectively via a single parameter, without any intermediate scale all the way up to 10^{16} GeV.

Naturalness: This is till now a very strong motivation for SUSY within the reach of the LHC. The issue here is explaining the stability of the electroweak scale, related to the standard model (SM) Higgs boson mass which is not protected by any symmetry from large radiative corrections. Thus one expects the radiative correction to be

$$\Delta m_h^2 \sim \Lambda^2, \tag{1}$$

where Λ marks the upper limit of validity of the SM, and may be set at M_P, the Planck scale, if one assumes that the SM is valid all the way till quantum gravity takes over. The resulting need for order-by-order fine-tuning of the SM parameters to $\gtrsim 30$ places of decimal to reproduce the observed Higgs boson mass makes the theory 'unnatural'.

How SUSY solves this problem in arguably the most satisfactory way need not be repeated here [15]. It is sufficient to recall the following relation relating the Z-boson mass to the MSSM parameters via the electroweak symmetric breaking (EWSB) conditions:

$$\frac{m_Z^2}{2} = -\mu^2 + \frac{m_2^2 \tan^2\beta - m_1^2}{1 - \tan^2\beta}, \tag{2}$$

where m_1, m_2 are respectively the radiatively corrected mass terms for the two Higgs doublets that couple to the down-and up-type quarks, and $\tan\beta$ is the ratio of their vacuum expectation values (vev), μ being the SUSY-invariant Higgsino mass parameter,

The LHC lower limits on superparticle masses, especially those of coloured particles, are gradually moving upwards. In addition, measured properties of the 125-GeV scalar (agreed by most to be the lighter neutral CP-even Higgs) implies high values of MSSM parameters, including the stop mass eigenvalues. This is rendering the scenario uncomfortable from the angle of naturalness. One is thus haunted by the spectre of a 'little hierarchy' between MSSM mass parameters ($\mu, m_{\tilde{t_{1,2}}}$, the trilinear SUSY-breaking parameter A_t) and the EWSB scale.

Several ways of parametrizing the level of fine-tuning involved here have been suggested, as sumarised in [15]. To mention a couple of them, one is

$$\Delta_{BG} = Max(c_i), \quad c_i = \frac{p_i^2}{m_Z^2}|\frac{\partial m_Z^2}{\partial p_i^2}|, \tag{3}$$

where p_i is the ith parameter in the SUSY scenario. It represents the degree to which the electroweak scale is sensitive to large variations in SUSY parameters.

Another oft-quoted quantifier is obtained by separating the various tree-and loop-level contributions on the right-hand side of equation (2). In the first category come terms proportional to μ^2, $m_{1(tree)}^2$ and $m_{2(tree)}^2$, while the various terms contributing to radiative corrections to m_2^1 and m_2^2 belong to the second category. Out of them, one defines

$$\Delta_{EW} = \frac{Max(d_i)}{m_Z^2}, \tag{4}$$

where d_i is the magnitude of the largest of the terms in equation (2), as classified above. Thus Δ_{EW} is essentially a measure of the extent to which various contributions undergo mutual cancellations to yield the electroweak scale.

The (in)compatibility between LHC data and naturalness, is frequently studied in terms of the above quantifiers, using both the MSSM and its extensions. Current data constrain various models, in terms of Δ_{BG}, to minimum values of a few tens to nearly 1000. The last situation corresponds to fine-tuning at the level of 0.1%. In terms of Δ_{EW}. Values upto about 30 is considered acceptable, since $\log \frac{M_P^2}{M_{EW}^2}$ amounts to such a number approximately.

One important reason why SUSY has slipped from the position of an idol to that of an outcast is the fact that the level of fine-tuning is on its way up according to the benchmarks quoted above. However, one may wonder if a line is being drawn here rather arbitrarily. The coloured superparticle mass limits going up by a factor of 3 or so with respect to where it will stand in the near future (that is, to $\simeq$10 TeV) enhances Δ_{BG} approximately by one order. But the resulting lack of faith in SUSY as a BSM candidate is probably a trifle hasty for the following reasons.

First of all, the fine-tuning criteria do not take into account any yet unknown UV completion of SUSY, which may reveal a correlation of parameters, justifying cancellations amounting to even $\lesssim$0.1%. Therefore, such possibilities cannot be rejected outright in a bottom-up approach. Secondly, large hierarchies or 'fine-tuned' values are especially unnatural when it comes to scalar masses, not, for example, fermion masses whose such values pass off as 'technically natural' [16]. This keeps the path open for some UV completion where the mass parameters in, say, equation (2) have their origin in some fermion masses whose justification in an overseeing theory. In cMSSM, for example, gaugino masses largely dictate the evolution of sfermion masses to their low-energy values [17]. Thirdly, even with a SUSY breaking scale high enough for LHC detection, one cannot rule out additional new physics, either at that itself or slightly below it. The independent unfolding of new physics in the strong and electroweak sectors, which finally led to the $SU(3)_c \times SU(2)_L \times U(1)_Y$ theory, bears testimony to this. Such a complementary framewrok, in conjunction with SUSY, may serve to smoothen the rough edges of naturalness. And finally, the naturalness problem to some extent depends on one's outlook, since its perfect quantisation is impossible. Thus it is not altogether unreasonable to live with it at least temporarily, if SUSY exhibits some independent, empirically justifiable motivation.[1]

Cold dark matter: As against the somewhat philosophy-dependent character of unification or naturalness, the existence of cold dark matter (DM) of the universe is a concrete and unremitting issue. Observations such as gravitational lensing effects seen beyond visible tails of bullet clusters point towards the particle nature of DM [18,19]. There is no available candidate in the existing spectrum of the SM, thus making BSM a necessity. Till now no clear evidence for DM is found in direct searches or collider experiments, while mass-to-light ratios in, for example, the Coma cluster, and the anisotropy of the cosmic microwave background radiation (CMBR) make the DM hypothesis compelling [20–23]. The single and most important missing component in the whole game, however, is any clue on the mass(es) and interactions of the DM candidate particle(s).

Since it is expected, though not in an absolutely compelling manner, that a DM candidate is stable, most theoretical scenarios resort to some symmetry to prevent its decay. In SUSY, the ready availability of such a symmetry in the form of R-parity, with $R = (-)^{3B+L+2J}$, makes it a candidate theory for DM. R-parity arises as the surviving symmetry of the MSSM; while the continuous R-symmetry of the underlying graded Lie algebra is broken via Majorana masses, it remains intact in two directions, namely, 0 and π, thus leading to a residual reflection symmetry, or Z_2, which ensures the stability of the lightest SUSY particle (LSP).

In a sense. this Z_2 arises in MSSM in the least artificial manner. Of course, R-parity violation is possible, since lepton or baryon number is carried by scalars (sleptons and squarks). However, till date there is no experimental evidence of lepton- or baryon-number violation by odd units. This statement will remain valid even if $\Delta L = 2$ Majorana masses for neutrinos are discovered. In contrast, practically all other theories of dark matter have Z_2 or some other symmetry introduced without independent empirical support. Besides, the gauge invariance of the SM extends to the superparticle sector, implying a spectrum that opens up a number of annihilation channels in general. This makes SUSY a robust enough DM model. In fact, as we shall se below, one has options beyond the lightest neutralino LSP,[2] with interesting implications in the context of dark matter. Thus the existence of cold dark matter (CDM) may well be regarded as the strongest motivation for SUSY, without having any had-and-fast requirement about the LSP mass or the SUSY breaking scale.

As it is well-known, the reason people have lost much of their interest in SUSY after several decades of hot pursuit is largely that experimental limits from collider

[1] It should perhaps be remembered that we have decided to live with the cosmological constant which in the context of SUSY requires a much more severe fine-tuning.

[2] In principle, the existence of DM decaying slowly enough is allowed in R-parity violating SUSY as well. We shall briefly comment on this later.

data are showing an ever-upward trend [24], coming into conflict with the prevalent criteria of naturalness. We have already pointed out that such criteria contain a subjective element. At the same time, it is also argued that the very large number of free parameters in any low-energy SUSY scenario is not only uneconomical but also allows endless manoeuvres around experimental constraints, thus missing the Popperian hallmark of falsifiability. This has often led to derisive comments on attempts to find out regions of the low-energy parameter space, which have not been 'ruled out' so far.

We thus suggest that the quest for SUSY continues to be meaningful, if offering a DM candidate is taken instead as its main motivation. This motivation is largely robust against upward revision of experimental limits on superparticle masses. The proliferation of parameters can do us a favour here. We already have, and are likely to have in the coming years, a wide variety of mutually unrelated terrestrial, cosmological and astrophysical observations on dark matter. Notwithstanding their current lack of unanimity, these will ultimately serve to guide one even to unlikely corners, out of the large number of available choices.

Dark matter and 'unusual' SUSY scenarios: If an explanation of dark matter is accepted as the most concrete motivation for SUSY at a fundamental level, then it becomes important to examine scenarios beyond the commonest ones, simply because we know so little about the origin of DM. One may even feel emboldened to break free, at least partially, of the shackles of naturalness. DM detection, direct as well as indirect, faces new challenges in such cases. Furthermore, SUSY can open up not a single option but multiple possibilities of DM candidates, by virtue of the unique nature of the particle spectrum it yields, thus guiding us to hitherto unexplored aspects of dark matter physics. Some of these scenarios even suggest a paradigm shift in the collider search strategies for SUSY. While reporting on some recent studies in such scenarios, it is emphasized that in each case the spectrum and its various features are guided by the requirement of consistency with all issues pertaining to dark matter.

Such scenarios may lie within the ambit of the MSSM, or may in some cases require going beyond it in terms of either the spectrum or the Lagrangian. Examples of both categories will be discussed in the next two sections. We shall also emphasize how the characteristic features of specific SUSY spectra may lend detectability to these scenarios.

3 MSSM with a trans-TeV neutralino LSP

Rates for events with missing-E_T (MET) at colliders as well as for inelastic scattering in direct search experiments depend on the mass as well as the interactions of the DM particle. Either form of detection becomes rather difficult if the mass of the DM candidate, in our context a weakly interacting massive particle (WIMP), approaches a TeV [25]. For a general WIMP DM, for example, production at the LHC depends mostly on Drell-Yan (DY) processes where production gets suppressed by $\hat{s}$, the square of the subprocess centre-of-mass energy, and also by the parton distribution function at high x. The MSSM by its very construction leads to a different situation. There the pair-production of coloured superparticles (squarks/gluions) at the LHC via strong interaction is followed by decays in cascade leading to DM pair-production. Once more, in spite of the advantage of strong production, the event rates go down significantly when one looks at the current limits on coloured superparticle masses and various other constraints on the MSSM spectrum[...]. On the whole, a near-TeV neutralino LSP falls mostly on the wrong side of the borderline of detection at the high-luminosity LHC in the MSSM [26,27], while the reach is considerably lower for most other scenarios where the 'dark sector' is at most weakly interacting [28]. The direct detection limits are also rather weak for such a massive WIMP. Probing trans-TeV DM is therefore a challenge. One obviously has to depend on indirect evidence here, and think of as many different avenues as possible, since indirect searches from astrophysical phenomena are fraught with uncertainties. Here we specifically discuss one such indirect signal, namely, radio synchrotron flux from dwarf spheroidal galaxies (dSph).

To put things in the perspective, the annihilation of DM particles in our galaxy as well as in extra-galactic objects leads to gamma-ray signals as well as positrons, antiprotons etc. Constraints have been imposed on DM annihilation rates in various ways out of the (non)-observation of such signals [29,30]. Another option is to look for radio synchrotron emission from galaxies, arising out of electron-positron pairs produced in cascades from DM annihilation and accelerated by the galactic magnetic field. The potential of the upcoming Square Kilometre Array (SKA) radio telescope [31] has been found rather promising in this regard.

Radio flux as signal of DM annihilation has been explored in earlier works [32–36]. In particular, reference [37] demonstrates that the SKA opens up a striking possibility for tans-TeV DM annihilation, when it happens in dSph's not too far away from our galaxy. The galactic magnetic field and the DM distribution in the dSph have to be favourable for this possibility to open up. In general dSph's are useful for studying radio signals in our context, since, in addition to large DM concentration, star formation rates there are low, thus minimising fake signals of astrophysical origin.

As mentioned above, the generic faintness of a dSph suggests that it is better to concentrate on those among them, which are satellites of the Milky Way (at distances of about 15–20 kilopersec). The SKA promises sufficient sensitivity to detect the faint signal from them. Its dishes, spread over two continents, ensure high enough resolution to remove foregrounds. It was shown in [37] that about 100 h of observation at the SKA can accumulate flux at least three times above the detectability threshold, for radio signals from the annihilation of MSSM neutralinos in the 5–10 TeV range.

Of course, the compatibility of such massive WIMP with the observed relic density is less than obvious, though a WIMP DM with mass upto $\gtrsim 50$ TeV is consistent with theoretical requirements such as unitarity. It has to be remembered that the channels of DM annihilation in a dSph are not necessarily the only ones to contribute to the velocity-averaged cross-section $\langle \sigma v \rangle_0$ in the context of the early universe, which is responsible for keeping the relic density $\Omega_0 h^2$ within the current limits. What one basically requires is a 'dark sector spectrum' (consisting of, say, Z_2-odd fields) *with scope for co-annihilation* in the early universe. As was demonstrated in [37], the MSSM indeed admits of such a spectrum in the trans-TeV range, consistent with all phenomenological constraints, but having at least a partially compression, as required for chargino-neutralino or stau-neutralino co-annhilation.

The MSSM, at least in some ramifications, thus serves as a template of trans-TeV DM scenarios detectable via radio synchrotron signals from objects of sufficient DM density. In this way, the reach of the LHC for WIMP detection may be extended considerably through radio telescope exploration. Further, regions in the DM parameter space where SKA can observe radio signals from a dSph are consistent with limits from γ-ray observations as well as cosmic rays data.

We next show show some results, taken from reference [37]. After a reporting briefly on them we shall come back to the underlying mechanism of radio flux enhancement from the annihilation very massive DM.

Figure 1 shows the minimum value of $\langle \sigma v \rangle$, the velocity-averaged annihilation cross-section for different DM masses (m_χ), necessary to have the flux rise 3 times above the noise level at the SKA with 100 h of observation of the dSph

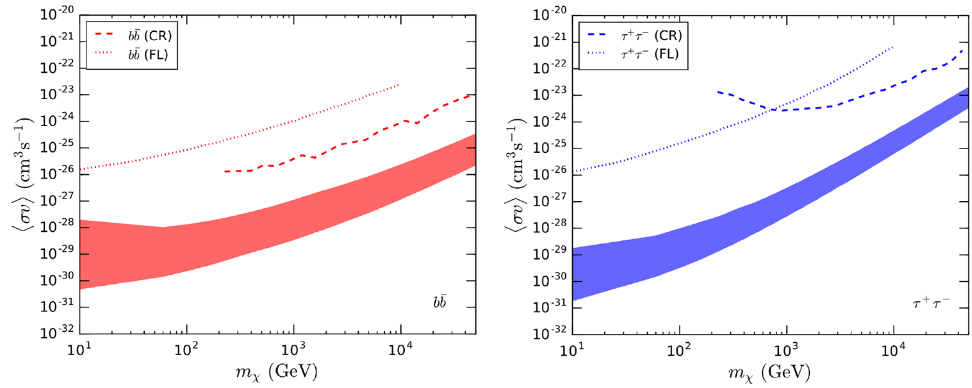

Fig. 1. The minimum velocity-averaged DM annihilation cross-section as function of m_χ, required for 3σ detection of radio flux from the dSph Draco, with 100 h of observation at the SKA. The maximum allowed values of $\langle\sigma v\rangle$ from Fermi-Lat (dotted lines) and cosmic ray (broken lines) data are also shown. The left panel corresponds to $b\bar{b}$ as the dominant annihilation channel, while the results with $\tau^+\tau^-$ as the dominant channel appear on the right. The bands correspond to the range 0.1–1.0 μG for the galactic magnetic field. The value of the diffusion coefficient D_0 is $3 \times 10^{28}\mathrm{cm}^2\mathrm{s}^{-1}$, as in the corresponding figure in [37].

Draco.[3] A flux rising 3 times above the projected noise level is taken somewhat conservatively as the detection criterion. At the same time the maximum value of $\langle\sigma v\rangle$ consistent with continuum γ-ray observations from Fermi-LAT as well as with cosmic-ray antiparticle data (which provide stronger upper limits for $m_\chi \gtrsim$ 200 GeV) are shown, The two sets of plots correspond to $b\bar{b}$ and $\tau^+\tau^-$ as the dominant annihilation channels. Similar curves for annihilation into W^+W^- and $t\bar{t}$ are found in reference [37]. It is clear from these figures that the SKA holds considerable promise of probing DM well above 10 TeV. One has similarly encouraging results for several other dSph's such as Seg1, Carina, Fornax, Sculptor etc.

The bands correspond to different values of B, the galactic magnetic field in the range $0.1 - 1.0$ μG. Higher values of B tends to produce more flux for the same $\langle\sigma v\rangle$ (see discussion below for explanation). Thus the lower ends of each band stands for higher B, implying that regions of the DM parameter space with lower annihilation rates are amenable to radio probe in such case. As we shall see below, another important parameter deciding the flux is the galactic diffusion coefficient D_0, for whom a conservative value has again been used. These predictions correspond to the Navarro-Frank-White (NFW) DM profile. The use of other profiles with reasonable values of profile parameters do not significantly change the predictions in [37].

The estimates in the above figures, although model-independent, are somewhat idealised, in the sense that overwhelming dominant branching ratio in one channel at a time has been assumed. In practice one needs to include a weighted sum over all annihilation channels in a given theoretical scenario. Such predictions for Draco, in terms of three benchmark MSSM points in reference [37] are shown in Figure 2. As Table 1 of [37] shows, these benchmarks correspond to m_χ in the range 1–8.5 TeV, with a variety of annihilation channels ($b\bar{b}, \tau^+\tau^-, t\bar{t}, W^+W^-, ZZ$) with different branching ratios, sums over which lead to the predictions in Figure 2. The benchmark points are consistent with existing cosmic-ray data. The MSSM points can clearly be probed with 100 h of observation.

[3] One should distinguish between $\langle\sigma v\rangle$ above, and $\langle\sigma v\rangle_0$ which refers to the total annihilation rate during thermal freeze-out. The former refers to DM pair-annihilation which alone can take place in a dSph, while the latter includes co-annihilation channels as well.

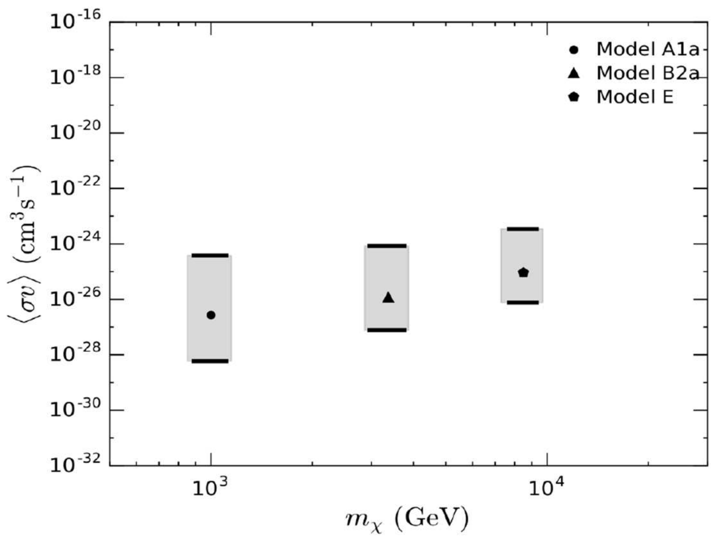

Fig. 2. The location of several MSSM benchmark points in the $\langle\sigma v\rangle - m_\chi$ plane, as in Figure 12 of [37]. The upper bars above the benchmark values of $\langle\sigma v\rangle$ correspond to the 95% C.L. upper limits from cosmic ray data. The lower bars show the minimum $\langle\sigma v\rangle$ required for these benchmark points for detection of radio flux from Draco with 100 h of observation at the SKA. The illustrative results correspond to $B = 1\ \mu G, D_0 = 3 \times 10^{28}\text{cm}^2\text{s}^{-1}$.

What favours the emission of detectable radio flux from trans-TeV DM? In order to understand this, one needs to examine the role of the most important astrophysical quantities involved in this mechanism. One of them, of course, is the DM profile in the particular galaxy, denoted by $\rho_\chi(r)$. on which we have commented above. In addition, the flux is decided by a variable called the source function, defined as

$$Q_e(E,r) = \langle\sigma v\rangle[\Sigma_f B_f \frac{dN_f^e}{dE}]N_{\text{pairs}}(r), \tag{5}$$

where $N_{\text{pairs}}(r) = \frac{\rho_\chi^2(r)}{2m_\chi^2}$ is the local number density of DM pairs inside the dSph, and B_f is the branching fraction for annihilation in the fth channel. $\frac{dN_f^e}{dE}$ is the energy distribution of e^+e^--pairs in the corresponding channel, when they are produced in cascade decays of the annihilation products.

The $e^\pm$ move along the galactic medium, by virtue of their inherited momenta, and also through diffusion, and are accelerated into spiral trajectories by the galactic magnetic field B. At the same time they lose energy via Coulomb loss, synchrotron radiation and also Inverse Compton effects (bremsstrahlung gives relatively small contribution in the energy range under consideration here). All these lead to a steady state energy distribution $\frac{dn}{dE}$, which yields the finally resulting flux after convolution with the synchrotron power spectrum and the J-factor pertaining to the particular dSph (see Eqs. (3)–(9) in [37]).

The dependence on B and D_0, and in general the issues responsible for generating sizeable flux for high-mass DM, can be outlined as follows:

– Large values of $\langle\sigma v\rangle$ are primarily helpful. It offsets the suppression $\sim\frac{1}{m_\chi^2}$ in $Q_e(E,r)$.

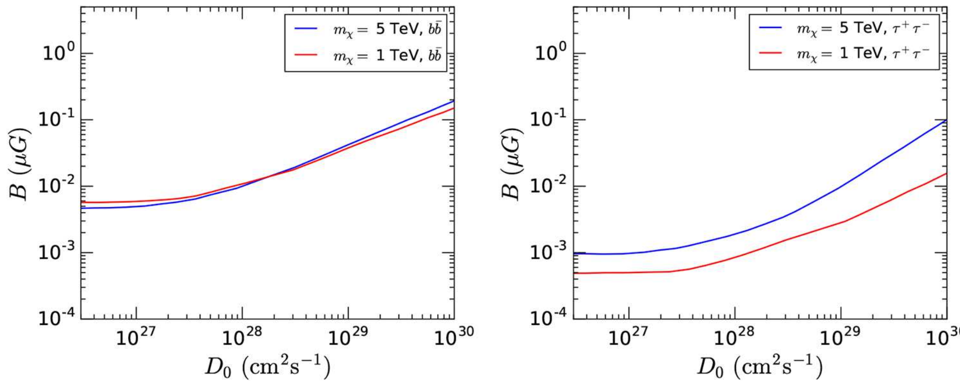

Fig. 3. Limits that can be imposed in the $B - D_0$ plane upon observation of radio signal from Draco dSph with 100 h of observation at SKA for $m_\chi = 5$ TeV (blue curves) and 1 TeV (red curves) [37]. Annihilation channels are $b\bar{b}$ (left panel) and $\tau^+\tau^-$ (right panel). The DM annihilation rate ($\langle\sigma v\rangle$) in each case has been taken at the 95% C.L. upper limit obtained from cosmic-ray antiproton observation [30].

- In addition to the DM particle mass, the overall spectrum of the BSM framework yielding DM has to be conducive to annihilation rates consistent with large DM. In particular, the possibility of a SUSY spectrum compressed enough to trigger co-annihilation can have a decisive role for DM masses approaching 10 TeV.
- The dSph under observation for radio signal should have DM profiles (accessible to independent determination), for which sufficiently high values of $N_{\rm pairs}(r) = \frac{\rho_\chi^2(r)}{2m_\chi^2}$ are obtained.
- Closer proximity to a resonance can drive copious annihilation even for high DM mass. It also helps in keeping the relic density within limit, over and above the provision for co-annihilation. A high-mass pseudoscalar Higgs in the MSSM plays a supportive role here.
- Large enough $\Sigma\frac{dN_f^e}{dE}B_f$ is reinforces the source function. Along with $\langle\sigma v\rangle$, it counters the suppression from $\sim\frac{1}{m_\chi^2}$ in $Q_e(E,r)$.
- One requires a copious supply of high-energy electron-positron pairs. A trans-TeV m_χ favours this because of the sheer enlargement of the appropriate phase space. A good fraction of such electrons, after various kinds of energy loss, end up in the right range to have radio synchrotron emission in the desired frequency range, namely, 300 MHz–50 GHz.
- A higher value of D_0, the diffusion coefficient, causes greater loss of $e^\pm$ beyond the dSph, and thus reduces the intensity of the flux. Thus a galaxy with lower D_0 holds more promise.
- Larger B, the magnetic field, not only enhances the synchrotron emission rate but also retain a larger fraction of the $e^\pm$ by causing spiralling motion within the dSph, before they get lost by diffusion.

Thus D_0 and B are two important quantities in this context. One should note here that the observation of radio flux from a dSph can help in identifying allowed regions in the $D_0 - B$ space corresponding to the galaxy. This is illustrated in Figure 3, for with $b\bar{b}$ and $\tau^+\tau^-$ respectively as the dominant annihilation channels. The regions above the red and blue lines in each case correspond to the highest $\langle \sigma v \rangle$ allowed by γ-ray and cosmic-ray data.

4 Other DM candidates in SUSY: new signals?

4.1 MSSM + right-chiral neutrino superfields

The SM or the MSSM contain left-handed neutrino superfields only, and there is no lepton-number violating inputs. Thus neutrinos are strictly massless there—something that is not supported by observations such as neutrino oscillation data. Incorporating ingredients for the generation of neutrino masses has thus a strong motivation.

Postponing the explanation of smallness of neutrino masses, the simplest option is to have Dirac masses arising via the addition of right-handed neutrinos. This can be done by augmenting the MSSM superpotential with the neutrino Yukawa terms:

$$W + W_{MSSM} + Y_{ij} \ell_i \nu_j^c H_2, \tag{6}$$

where i, j are lepton family indices, ℓ stands for left-handed Su(2) doublet lepton superfields, ν for right-handed neutrino superfields, and H_2, the Higgs doublet responsible for $T_3 = +1/2$ fermion masses. The neutrino Yukawa couplings Y_{ij} have to be as small as $\simeq 10^{-13}$ in order to fit oscillation data.

$\tilde{\nu}_R^i$, the superpartner of the ν_R for any i, is a gauge singlet scalar whose mass is a free soft SUSY-breaking parameter. Unlike the $\tilde{\nu}_L^i$, whose candidature for dark matter is strongly constrained by direct search results, one of the right sneutrinos can easily be the LSP and thus the DM candidate. The ultra-small Yukawa coupling is of course its only mode of interaction with the rest of the spectrum, and is inadequate for thermalisation, qualifying it as a non-thermal DM component. Its candidature is further supported by the way the right-chiral sneutrino mass parameters evolve down from any conceivable UV completion:

$$\frac{dM^2_{\tilde{\nu}_R}}{dt} = \frac{2}{16\pi^2} y_\nu^2 \ A_\nu^2, \tag{7}$$

where A_ν is the trilinear SUSY-breaking term for the sneutrino . Thus a sufficiently low-lying sneutrino mass term at high scale may stay frozen at the bottom of the spectrum, while the other sfermion masses are jacked up by gauge couplings as one comes down to the TeV-scale. A good example of the next-to-lightest SUSY particle (NLSP) is one of the sfermions that have substantial Yukawa couplings, such as the lighter stau-or stop-eigenstate [38–40].

The mass eigenstate answering to the DM candidate is obtained by diagonalising the sneutrino mass matrix:

$$m^2_{\tilde{\nu}} = \left[\begin{array}{cc} M^2_{\tilde{L}} + \frac{1}{2} m_Z^2 \cos 2\beta & y_\nu v (A_\nu \sin\beta - \mu \cos\beta) \\ y_\nu v (A_\nu \sin\beta - \mu\cos\beta) & M^2_{\tilde{\nu}_R} \end{array} \right], \tag{8}$$

where $M_{\tilde{L}}$ = soft mass for the left-handed sleptons, $M_{\tilde{\nu}_R}$ = soft mass for the right-handed sneutrinos, and μ is the Higgsino mass parameter.[4]

[4] $M_{\tilde{L}}$, $M_{\tilde{\nu}_R}$ and A_ν are 3×3 matrices in general.

SUSY cascades at colliders, following the production of coloured superparticles, lead to the production of NLSP pairs. The subsequent decay of the NLSP to the LSP, however, is extremely slow, since all interactions of the right-sneutrino-dominated mass eigenstates obtained from equation (8) are in general proportional to the ultra-small neutrino Yukawa couplings. Thus the NLSP is stable on the scale of collider detectors such as that of CMS or ATLAS. For reasons mentioned above, two frequently studied options are stau-or stop-NLSP. Either of these leads to *stable charged tracks* at the detector, and these, together with jets or leptons, are what distinguish SUSY processes, unlike the age-old signals where large missing transverse energy (MET) constitutes the archetypal signature of SUSY.

The signal of SUSY (and almost synonymously, of dark matter) in such a scenario is thus two stable, massive, highly ionising charged tracks along with hard jets and/or leptons. It may appear that such signals are largely background-free. This, however, is not the case. The very large number of events with hard muon pairs *prima facie* constitute a formidable background, along with cosmic-ray muons. A carefully formulated event selection strategy is therefore required. We illustrate this, and the remaining issues on this scenario, in the context of a stau-NLSP.

Among a number of proposed criteria, it was demonstrated in earlier studies [41–43] that a stiff cut on the track-p_T and also on $\Sigma|p_T^{vis}|$, the scalar sum over the transverse momenta of all visible tracks, was decidedly in favour of the signals. While this worked very well for NLSP masses $\lesssim$500 GeV, additional criteria are required for higher masses[5] [44,45]. The most effective one turns out to be the time-delay between the tracker and the muon chamber, translated into $\beta = v/c$ for each track [46–48]. It is found that, by demanding $\beta \leq 0.85-0.95$, one can achieve 5σ statistical significance for the signal with integrated luminosities of 1000–2000 fb^{-1}, for $m_{\text{NLSP}} \gtrsim$ 600–700 GeV [44].

Other than those from collider data, the major constraints on such a scenario come from the following considerations:

- **Light element abundance**: Te standard big-bang nucleosysthesis (BBN) model requires, for a stau-NLSP, an upper limit of about 100 s on the lifetime [49,50]. This in turn imposes limits on the (Dirac) neutrino Yukawa coupling, $\tilde{\tau}_L - \tilde{\tau}_R$ mixing and the μ-parameter.

- **Stau freeze-out**: Since the NLSP is long-lived, and stable on the timescale leading to its decoupling, it constitutes a 'dead mass' before it decays, whose resultant contribution to the relic density is obtained by scaling with $m_{\tilde{\nu}_R}/m_{\tilde{\tau}_R}$ [44].

- **Freeze-in rate from all heavier particles**: This occurs while all the SUSY particles other than right sneutrinos are in thermal equilibrium, leading often to a higher contribution to the relic density than that from stau freeze-out. An upper limit thus follows on $m_{\tilde{\nu}_R}\Sigma\frac{g_i\Gamma_i}{m_i^2}$, where g_i = no. of degrees of freedom for the ith decaying particle whose decay width is Γ_i. This implies non-trivial constraints [45] on $m_{\tilde{\nu}_R}$, depending on the MSSM spectrum.

An interesting conclusion emerges in this scenario, regarding the UV completion of MSSM. It was thought for quite some time that the constrained MSSM (cMSSM), based on minimal supergravity (mSUGRA), provided a glimpse of the much sought-after organising principle, leading to an economy of parameters. The entire low-energy

[5] Although strong lower limits have been inferred on stau-track masses, these are mostly derived with specific assumptions about the SUSY spectrum, such as that in a gauge mediate SUSY breaking (GMSB) model. Thus we go beyond such limits in a phenomenological study, where the coloured particle masses have greater flexibility.

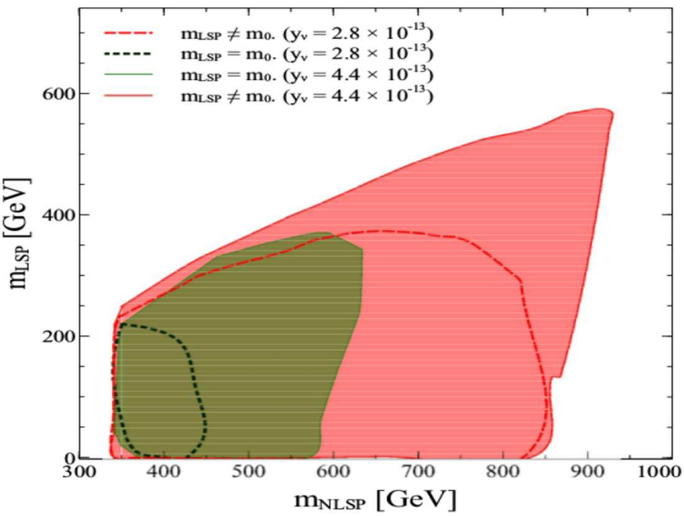

Fig. 4. The lowered allowed ranges in the $m_{NLSP} - m_{LSP}$ plane in the cMSSM + $\tilde{\nu}_R$ LSP scenario with a right-sneutrino LSP and stau NLSP . The results indicate, as exemplified in [44], that values of m_0 and $m_{1/2}$ considerably lower than those for usual cMSSM with a neutralino LSP are allowed now. The upper limits of the contours appear because of an artificial truncation of parameter scan.

spectrum could at least in principle be predicted in terms of the high-scale universal scalar and gaugino masses $(m_0, m_{1/2})$, the trilinear SUSY-breaking parameter A_0, the sign of μ, and $\tan\beta$. This scenario has also the virtue of predicting radiative electroweak symmetry breaking at the right scale. However, hopes in this direction have dwindled. On the one hand, the highly interconnected nature of the spectrum has been pushing up all the lower mass limits increasingly. At the same time, the constraints from the relic density are putting pressure on the annihilation channels of χ_1^0, the neutralino DM candidate. This happens since (a) higher masses for superparticles in the electroweak sectors make some annihiltion channels ineffective, and (b) the electroweak symmetry breaking condition (which decides μ upto a sign) makes it difficult to achieve the optimum gaugino-Higgsino admixture in χ_1^0.

A scenario with a $\tilde{\nu}_R$ DM frees one from some of the constraints mentioned above [44]. First of all, the lightest neutralino is not the DM candidate anymore, and therefore one need not worry about its annihilation channels. Moreover, one can envision possibilities with, for example, the lighter stau (and possibly other particles too) lighter than χ_1^0. Consequently, the interconnected cMSSM spectrum is granted some extra freedom, which works in favour of a (cMSSM + $\tilde{\nu}_R$) scenario with lower allowed values of $(m_0, m_{1/2})$ than is allowed in pure MSSM, provided that the freeze-in constraints are satisfied. An example of this can be found in Figure 4. Here the upper limits of $m_0, m_{1/2}$ are just because of a cut-off in the parameter space scan, and it is the lowest allowed values that illustrate our point. Two sets of contours are shown. For one case, the sneutrino LSP mass has its origin in the same m_0, just as in the case of the other sfermion masses, leading to rather restricted allowed regions. Another set corresponds to the situation where a different high-scale mass may evolve down to the LSP mass, thus allowing somewhat larger regions. Two illustrative values of the neutrino Yukawa coupling corresponding to the LSP state are used for each scenario.

Thus the mere addition of three right-handed neutrino superfields may turn out to be a game-changer in SUSY phenomenology. In addition to the novel discovery signals of such a scenario, issues such as reconstruction of masses in the electroweak superparticle spectrum have been addressed in recent investigations. Astrophysical signals of DM annihilation as well as radiative decays in the sneutrino sector have also come under scrutiny.

4.2 Two other examples of SUSY DM

Before we conclude, we mention for completeness a few other examples of SUSY DM other than a neutralino LSP.

– **A left-chiral sneutrino ($\tilde{\nu}_L$)**: This is normally disfavoured, because a $\tilde{\nu}_L$ DM candidate will have unsuppressed interaction with the Z-boson. A conflict thus arises with direct search data unless the $\tilde{\nu}_L$-mass is high up on the TeV-scale. This constraint, however, depends on the $\tilde{\nu^R}_L - \tilde{\nu^I}_R - Z$ interaction, where $\tilde{\nu}_L = \tilde{\nu^R}_L + i\tilde{\nu^I}_L$. There can in principle be a mass-splitting of $\gtrsim$ a few hundred keV between the real and imaginary parts. This can be engineered by having $\Delta L = 2$ sneutrino mass terms [51–54] via coupling to a scalar triplet, or with the help of higher-dimensional operators. If such a split occurs, then the lighter of the two physical states, being the DM candidate, cannot undergo scattering in detectors, since it has to bridge the mass gap with kinetic energy, thus requiring its speed to exceed the escape velocity in our galaxy.

  One characteristic signal of sneutrino DM at the LHC is an excess of same-sign trilepon events [55]. However, current direct detection data disfavour scattering mediated by the 125-GeV Higgs as well, the corresponding interactions arising from D-terms. The survival of this possibility for sub-TeV sneutrinos thus requires theoretical scenarios that can work around such problems.

– **Decaying DM in R-parity violating scenarios**: Gravitino and axino DM are of course widely explored options, mostly opening up horizons for 'warm dark matter'. We do not enter into discussions on them; what, however, can be emphasized is that they are consistent with R-parity breaking scenarios as well. It is well known that R-parity breaking enable neutrino mass generation mechanisms, via either radiative effects (as in trilinear R-breaking) or or at the tree-level (as happens with bilinear R-parity breaking in the superpotential). In the latter case, mixing takes place between neutralinos and neutrinos. Since the gravitino/axino has interaction with a neutralino (which contains a Bino component) and a photon, it can decay, albeit slowly, into the $\nu\gamma$ final state, consistently with observations on γ-rays, the CMBR as well antiparticles in cosmic-rays. The implications of such decay for X-ray telescopes, modulo all constraints on such scenarios, have been studied [56], where it is shown that an an axino DM particle has better detection prospect than a gravitino.

5 Concluding remarks

Taking a slightly unorthodox standpoint, we argue that the existence of dark matter makes the most appealing case for SUSY. The simplest (though not mandatory) picture there involves a stable DM particle, and the symmetry required to ensure its stability emerges in SUSY, in arguably the least artificial manner. Once we accept this as the main motivation, it seems a trifle over-restrictive to legitimise values of SUSY masses only within the somewhat nebulous borderline set down by naturalness.

Other than the MSSM neutralino as the DM candidate, a number of other scenarios remain alive in SUSY, either in its minimal version or in slightly augmented forms. These include thermal as well as non-thermal DM, and also cold as well as warm DM. Trans-TeV SUSY breaking scales, justifying its invisibility in present-day accelerators, is another possibility, consistent with all DM-related observations, admitting probes via indirect signals alone. And lastly, SUSY can be reconciled with even a decaying DM.

It is thus justifiable to continue with investigations on SUSY in its various ramifications. The non-fulfilment of naturalness criteria may, after all, be too flimsy a ground to abandon such probes. It is particularly so if fine-tuning to a few additional orders is required, as we have no clue yet on physics at higher energy scales.

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