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Maximum entropy economics

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Abstract. A coherent statistical methodology is necessary for analyzing and understanding complex economic systems characterized by large degrees of freedom with non-trivial patterns of interaction and aggregation across individual components. Such a methodology was arguably present in Classical Political Economy, but was abandoned in the late nineteenth century with a theoretical turn towards a purely mechanical approach to understanding social and economic phenomena. Recent advances in economic theory that draw from information theory and statistical mechanics offers a compelling statistically based approach to understanding economic systems based on a general principle of maximum entropy for doing inference. We offer a brief overview of what we consider the state of maximum entropy reasoning in economic research.

1 Introduction

Economic theory considers systems with large numbers of degrees of freedom and non-trivial patterns of interaction and aggregation across their individual members. Successful capitalist economies are characterized by a complex self-organizing division of labor around commodity production and exchange, and tend to produce stable, recurrent observed macroscopic patterns over long periods despite observed disorder at the microscopic scale. Organization of phenomenal observations in economic theory inherently deals with problems of inductive inference due to the non-experimental and incomplete nature of economic data. The maximum entropy principle (MEP) offers a general, logical approach to inference for problems of incomplete information in systems with many degrees of freedom. As such, it provides a useful tool in economic analysis.

While a statistical understanding of the nature of economic processes and regularities was central to 18th and 19th century Classical Political Economy, the modern core of economic theory, embodied in Walrasian general equilibrium theory, is conceptually anchored in deterministic market equilibria. Modern econophysics has made important empirical and methodological contributions in an effort at reconceptualizing economic systems along the lines of statistical physics, but has fallen short in its aims and influence due to weak theoretical grounding and interpretive dependence on physical analogies. In this paper, we briefly survey the importance of a statistical methodology in the social sciences, discuss some of the modern applications of the

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MEP in economics, and define Maximum Entropy Economics as a broader field of economic research based in the general principle of maximum entropy inference.

2 Background

2.1 The submerged and forgotten approach

Adam Smith was the first philosopher to systematically consider how economic systems comprised of many individual, decentralized decision makers can self-organize a division of labor. Smith's central abstraction was to consider the social and economic outcomes that would emerge from the complex interactions among and between capitalist producers and wage-laborers when they were free to move between different lines of production in pursuit of the greatest remuneration. Smith predicted that due to the decentralized nature of capitalist production the migration of capital and labor between lines of production would be a ceaseless and turbulent process. One obvious reason for this persistent variation is that incomplete information is an unavoidable consequence of large-scale decentralized decision making. Smith envisioned a negative feedback mechanism in the changes in the supply and price of commodities that would give rise to centers of gravity for prices and the distribution of labor and capital among different lines of production. The central tendencies to which prices, profit rates, and wages would continuously gravitate were an equally important component to his theory as were the endogenous fluctuations around these tendencies.

While Karl Marx is famous for his critique of Adam Smith and Classical Political Economy in general, he retained an essentially statistical understanding of price, profit, and wage formation. As he argues in Volume III of Capital, the macroscopic properties of the system, such as the average rate of profit, wages, and prices of production, are statistical in nature:

> "[The] sphere [of circulation] is the sphere of competition, which is subject to accident in each individual case; i.e. where the inner law that prevails through the accidents and governs them is visible only when these accidents are combined in large numbers, so that it remains invisible and incomprehensible to the individual agents of production themselves." [46, Chap. XLVIII]

Marx emphasized that it was the competitive disposition of capital to seek out the highest rate of profit and in doing so would perpetually re-distribute capital and labor across various lines of production. These redistributions could be understood to give rise to systems of "prices of production", but only statistically. Dynamic, statistical fluctuations around prices of production were understood as the very form in which those prices were defined in a complex, self-organizing division of labor. A capitalist economy in absence of these endogenous fluctuations would cease to function altogether. Criticizing and synthesizing Classical thought, Marx offered a far more developed theory of value based on the dynamic and statistical nature of the rate of profit and it is regulatory role for the distribution of capital and social labor.

While unequipped with the modern methods of statistical physics, Classical and Marxian theory was essentially a statistical account of the formation of equilibrium prices, profit rates, and wages. All saw the importance of the centers of gravity as revealing the underlying mechanisms of price formation and distribution, but also emphasized the statistical variations around this center as a reflection of the underlying system dynamics that gave rise to natural prices, wages, and profit rates. While twentieth century research in Classical Political Economy generally adopted a "long-period" interpretation of Smith, Ricardo, and Marx, where profit rates and wages are treated as equalized in the abstract long-run it did so by jettisoning any ambition of reconciling the Classical vision with a coherent statistical methodology [1]. It did, of course, illuminate many important implications of Classical thought particularly with respect to production [2] and value theory [3], but it may have had the unfortunate effect of putting statistical thinking on the back burner of theory development.

It is easy to read Classical Political Economy anachronistically with the benefit of two centuries of theoretical development in physics and at least half a century of complex systems theory [4]. But, while it is fanciful to think Smith and Marx conceptualized economic phenomena in the same terms of modern statistical physics, the abstract vision of economic processes presented by them appears to integrate many of the same insights. Classical Political Economy tended to argue in terms of economic variables that did not scale with the size of the system, such as the rate of profit and prices rather than variables like endowments, capital stock, or land, which do scale with the size of the system. Though these "intensive" variables were subject to continual change arising from the reorganization of production through technological change and class conflict, the dynamics that governed the formation of natural prices and average profit rates made the evolution of these variables an essentially "quasi-static" process. Underlying the complex evolutionary and dynamic process of production and exchange was a common organizing logic of competition that keep these intensive variables in a "statistical" equilibrium.

Marx's theory of historical materialism which concerns the laws of social reproduction as well as the contradictions that inevitably give rise to change emphasizes the importance of institutional constraints that shape social reality over long periods of time. The elemental institution of competition may lead to the statistical equilibration of profit rates and the ratio of labor effort to wages so long as social reproduction is organized around capitalist production. But class conflict, technical change, and the dynamic adaptive nature of capitalism may only change the parameters of the equilibrium distributions and not the functional form, which arises from the overarching institutional constraints. For example, a falling rate of profit leads only to a translation of the profit rate distribution [5,6], or growing inequality through tax legislation, dismantling of labor unions, financialization, etc. may lead only to a change in the shape parameter of an otherwise stable distribution of incomes [7–9]. While it may at times be difficult to distinguish between a parametric and functional change in an evolutionary adaptive system like capitalism, there is good reason to believe that the underlying organizing logic of competition will lead to robust statistical regularities over significant periods of time.

2.2 The neoclassical utopian vision

The essentially statistical nature of Classical Political Economic thought was entirely abandoned in the late 19th and early 20th century when the marginalist vision of Stanley Jevons, Carl Menger, and Leon Walras, pushed forward the subjectivist theory that commodities sell at whatever price someone is willing to pay for them on the market. The marginalists, ignoring the deeper forces that tended to organize and regulate prices, stressed the arbitrary and subjective nature of value where prices determined at the market level are decoupled from any determination at the level of production. Theoretical focus shifted to market prices as reflecting an efficient allocation of privately owned scare resources where Say's law holds by virtue of barter exchange.

The principles of marginalist economics maintained an explicit mathematical vision equivalent to the mid-18th century physics of Hermann von Helmholtz [10].

The historical curiosity being that neoclassical economists embraced the theoretical isomorphism of economics and physics in the late 19th century, but despite the revolutionary statistical thinking of Maxwell, Boltzmann, and Gibbs, neoclassical theory espoused an antiquated theory of mechanics. Couched in a completely specified mechanical system, neoclassical theory was liberated from the theoretical difficulties of fully conceptualizing the complex and statistical nature of commodity production and exchange [11].

The development of the marginalist vision into modern general equilibrium theory [12] maintains that many rational households which are simultaneously consumers and producers interact through universal perfectly competitive markets as price takers with costless and perfect information. The equilibrium concept is a simultaneous determination of all equilibrium prices (such that aggregate excess demand is zero) via a fictional "Auctioneer" (an analog to Maxwell's demon) that can generate all relevant market information without expending any resources. The Walrasian conceptualization of economic equilibrium is a balance of the gradients of utilities, much like force balance in mechanics, leading to a unique, stable fixed-point equilibrium price vector [11]. Walrasian general equilibrium theory requires every individual agent to be in equilibrium in the sense that consumers maximize their utility subject to a budget constraint and an equilibrium price system and firms maximize their profit subject to a convex technology constraint and the same equilibrium price system.

A competitive market equilibrium implies that no exchange takes place before equilibrium prices are discovered and "announced." Because no out-of-equilibrium exchange takes place individuals never move among different states and endogenous statistical fluctuations are precluded from the theory altogether. The result is that agents of the same type (same preferences and endowments) end up with the same consumption bundle in the post-exchange allocation and the Walrasian competitive equilibrium sustains a system of zero entropy. The welfare implications follow directly from the construction of equilibrium prices that implement a Pareto-efficient allocation.

Though modern general equilibrium theory is unable to account for such basic phenomena like the division of labor and the social nature of production, its failures to account for real economic phenomena run far deeper. The constraints that general equilibrium theory places on the state space configurations that maintain zero entropy end up being too restrictive to ever predict the type of economic data we actually see. In contrast to Classical Political Economy, neoclassical economic theory does not identify any law-like regularities in prices, profits, or wages since outcomes are predetermined by the model parameters. The statistical content of general-equilibrium analysis is imposed ad hoc and ex post, by assuming stochastic variation in the unobserved underlying preferences, technology, and endowments, which have the (un)fortunate effect of making general equilibrium theory compatible with virtually any empirical observation[1].

The absence of a statistical methodology in general equilibrium theory is partly behind the poor articulation between theory and measurement in economics. Dynamic stochastic general equilibrium (DSGE) models attempted to make neoclassical theory immune to such criticisms by reorienting macroeconomics towards solving and calibrating models rather than evaluating them against data with formal methods of statistical inference [14]. These models introduced statistical variation through the back door as exogenous shocks to an otherwise stable system and unsurprisingly tended to perform poorly when confronting empirical data generated by a complex system.

[1] This point is discussed by dos Santos [13] in this special issue.

Interestingly, even within the narrowly defined Walrasian pure exchange economies we can find important qualitatively different conclusions by introducing a statistical methodology. As Foley [15] demonstrates, by restructuring the Walrasian system in terms of statistical mechanics and conceptualizing the market as a probability field over transactions, exchange at non-equilibrium prices become possible and the maximum entropy exchange equilibrium results in identical agents with different final consumption bundles. Markets endogenously generate horizontal inequality among identical agents and the standard welfare implications no longer hold.

Walrasian equilibrium has had an unfortunately long-lasting impact on the way modern economists conceptualize equilibrium. Any concept of non-Walrasian equilibrium is typically understood as modeling disequilibrium dynamics. But, there is good reason to believe as did Smith and Marx, that studying the statistical effects of the collection of individual components in a system is fundamental to revealing regularities in the data. The concept of statistical equilibrium requires thinking along the lines of Classical Political Economy, that is, considering central tendencies as well as endogenous variation as indivisible elements of equilibrium arising from specific institutional constraints.

3 Statistical mechanics

Mechanics concerns systems that can be completely specified with respect to the number and behavior of the (possibly interacting) components, including the state of system at time t_0 and its time evolution to t_1 specified by Hamiltonian dynamics. *Statistical mechanics* concerns systems with a large number of degrees of freedom for which complete specification of the system is impractical due to incomplete information. Predictions in incompletely specified systems are necessarily probabilistic. Such systems may be a gas with a large number of particles, economic systems with many competitive firms and households, or biological or ecological systems with many interacting species. In any case, detailed dynamic predictions of individual trajectories are infeasible due to the size and complexity of the system. The statistical mechanical method addresses the indeterminacy of incompletely specified systems by substituting models based on probabilistic descriptions of a system constrained by whatever information is known about the system for detailed dynamic predictions.

3.1 Entropy

As an example, consider a system with N individual components each with ν states such that for the ith component a complete state space description would be $X^i = \{x_i^1, x_i^2, \ldots, x_i^\nu\}$ for $i = 1, \ldots, N$. In a social context we might think of N as the population of an economy where each individual is characterized by their state of income, education, age, sex, race, etc. or as a firm characterized by their capital stock, operating income, number of employees, etc. We can characterize the distribution of a particular state j across the system $f(x^j)$ by "coarse graining" or partitioning the state space of x^j. We can describe $f(x^j)$ as a histogram vector $\{n_1^j, n_2^j, \ldots, n_K^j\}$ where K is the number of bins, n_k is the number of individual components in bin k, and $\sum_k n_k^j = N$ is the total number of individual components distributed across state j. The precise state j and identity of each individual component within each bin is a description of the microstate of the system. The histogram $f(x^j)$ describes the macrostate which is the distribution of state j over the N individual components. The macrostate tells us nothing about the exact microscopic state of the system. It is easy to see that many different configurations of individual components in state j will lead to the same distribution of individuals over K bins so that any macrostate will correspond to many microstates. The number of ways a particular macrostate can be realized is the multiplicity of the system which is measured by the multinomial coefficient:

$$W = \binom{N}{n_K^j} = \frac{N!}{n_1^j! n_2^j! \cdots n_K^j!}.$$

Using Stirling's approximation $\log(N!) \approx N \log(N) - N$ for $N >> 1$, the logarithm of the multiplicity is expressed as the entropy of the system:

$$H = \log(W) = -N \sum_{k=1}^{K} p_k^j \log(p_k^j),$$

where $p_k^j = \frac{n_k^j}{N}$. If all N components had identical state j the macrostate of the system can only be realized in one way since all components share the same bin. In this case the multiplicity is $W = 1$ and the entropy is minimized at $H = 0$. When $n_1^j = n_2^j = \ldots = n_K^j$ individual components are partitioned equally across all levels of state j and $f(x^j) = \frac{1}{K}$ for all k and entropy is maximized at $H = \log(K)$. As the system's components become more spread out or evenly distributed across the K bins the ways in which a particular histogram (macrostate) can be realized or configured increases dramatically. In this sense entropy is a measure of the dispersion of components and is bounded by the degenerate distribution (minimum entropy) and the uniform distribution (maximum entropy). Because entropy measures the dispersion of components across a state j, it has also been interpreted as a measure of disorder. A highly organized system puts more individual components in fewer bins. Disorder in this sense is equivalent to uncertainty about the system. This combinatorial approach to entropy was first articulated by Boltzmann [16], but endowed with the interpretation of uncertainty by Claude Shannon [17,18]. Conceptualizing and modeling incompletely specified systems has been done in at least three ways: Gibbs' macroscopic approach, Maxwell and Boltzman's ergodic approach, and Jaynes' information theoretic approach.

The macroscopic approach to statistical mechanics [19] uses known microscopic properties to study system level characteristics in systems with many (strictly speaking infinite) degrees of freedom. In this approach well-specified mechanics at the microscopic level are applied to many-body systems in order to derive the thermodynamic and material properties of the system.

The ergodic view of statistical mechanics [16,20], studies systems by focusing on infinite time averages of the system's observable state variables. The evolution of a system's state space configuration through time is represented by a phase function $\phi(x^j)$. The time average is:

$$\overline{\phi(x^j)} = \lim_{t\to\infty} \frac{1}{t} \int_0^t \phi(x^j(t))dt.$$

The ergodic theorems state that these infinite time averages will be equal (under certain assumptions) to the ensemble averages:

$$\overline{\phi(x^j)} = \langle \phi(x^j) \rangle = \frac{1}{N} \sum_{i=1}^{N} \phi(x_i^j(t)).$$

The implication of ergodic theory is that if the experiment on a (thermodynamic) measurement coincides with ensemble averages derived from statistical equilibrium, this is because the individual system on which the experiment performs has been drawn randomly from the infinite number of different systems in the ensembles.

One difficulty of the ergodic assumption, as Jaynes [21] pointed out, is that the ergodic theorem is valid only when the time average is defined over *infinite* time. Because the observed data from (natural) experiments are always measured over finite times the ergodic approach substitutes the mathematical concept of the limit for the non-existent infinite sample of mechanical observations. Thus, it is impossible to determine whether or not the finite time averages in the experiment have actually "sufficiently approximated their limits for infinite times." The ergodic viewpoint thus puts forward an unnecessary claim because what is at stake is not why ensemble averages are equal to the time averages, but why ensemble averages are equal to observed experimental values.

3.2 The information theoretic approach

Claude Shannon [17,18] working on problems of communication at Bell Labs in 1948 wanted certain intuitive conditions to be satisfied in constructing a consistent measure of the amount of information (uncertainty) H associated with a random variable X using only the probabilities $p_i(x_i)$, $x \in X$. Shannon faced the practical problem of efficiently engineering communication channels over which information (messages) would travel. Because the communication system must be capable of operating for any possible message, we must regard any specific message as a realization from a set of all possible messages. Communication channels can be efficiently engineered if messages are transmitted by encoding them in such a way that the more probable a message is the fewer bits per second are required for transmission. In order to reduce the average transmission time it is necessary to have a quantitative measure of the uncertainty associated with the ensemble of messages. The conditions Shannon specified for such a measure were (1) uncertainty should be a continuous function of the probabilities p_i in order to avoid large changes in the measure of uncertainty from small changes in the probability distribution, (2) for a random variable with uniform probabilities, uncertainty should be a monotonically increasing function of the number of outcomes for the random variable X, which posits an association between the measure of uncertainty and real numbers, and (3) If one splits an outcome category into a hierarchy of functional equations then the uncertainty of the new extended system should be the sum of the uncertainty of the old system plus the uncertainty of the new sub-systems weighted by its probability. Shannon proved that the only H satisfying these conditions was

$$H = -K \sum_{i=1}^{K} p_i \log(p_i)$$

where K determines the unit of measurement, e.g. bits, nats, or digits. One discernible obstacle was how to derive the probabilities of messages based on frequency measurements.

E.T. Jaynes [22] recognized the generality of Shannon's result in information theory and discovered Shannon's situation for assigning probabilities to messages was not that different from statistical mechanics, where the physicist must assign probabilities to various quantum states due to the incomplete specification of the system under analysis. He argued that the number of possible quantum states (messages) in either case was so great that the frequency interpretation of probability would clearly be absurd as a means for assigning probabilities due to the impossibility of ever realizing those states (messages) in a finite amount of time. Instead, he argued that the equation for entropy is a measure of information that is to be understood as a description of a state of knowledge about the system[2]. Jaynes argued that imposing constraints on systems that change the maximum entropy distribution are just instances of using information for inference. Different probability assignments describe different states of knowledge independent of the type of system (physical, biological, social, etc.). From this perspective the connection between Shannon's information theory and statistical mechanics was that the former justifies the latter. As Jaynes argues, "We can have our justification for the rules of statistical mechanics, in a way that is incomparably simpler than anyone had thought possible, if we are willing to pay the price. The price is simply that we must loosen the connections between probability and frequency." [26]. The information theoretic approach to statistical mechanics emphasizes the inferential nature of studying any incompletely specified system with *any* number of degrees of freedom. For this reason, the information theoretic viewpoint is firmly in the tradition of Laplace.

Maximum entropy distributions use information in the form of constraints. A constraint is anything that restricts the set of possible probability distributions. In an economic context, examples of information we use to impose constraints are really no different than the typical model closures necessary to make a theory well-determined. They include, for example, a budget constraint, market clearing conditions, the non-negativity of prices, stock-flow consistency, savings equal to investment, behavioral constraints such as utility maximization, conservation of value in exchange, full employment, functional distribution of income, equalization of the rate of profit, etc. Mathematically, the maximum entropy formalism with m constraints on the expectations of functions $f_h(x_i)$ can be expressed as a constrained optimization problem:

$$\begin{aligned} \max \quad & -\sum_{i=1}^{K} p_i \log(p_i), \\ \text{s.t.} \quad & \sum_{i=1}^{K} p_i = 1, \\ & \sum_{i=1}^{K} p_i f_h(x_i) = F_h \qquad h = 1, \dots, m. \end{aligned}$$

The resulting maximum entropy probability is:

$$p_i = \frac{1}{Z(\lambda_1, \dots, \lambda_m)} \exp\left[-\lambda_1 f_1(x_i) - \dots - \lambda_m f_m(x_i)\right],$$

where

$$Z(\lambda_1, \dots, \lambda_m) = \sum_{i=1}^{K} \exp\left[-\lambda_1 f_1(x_i) - \dots - \lambda_h f_h(x_i)\right]$$

and λ_m is the Lagrangian multiplier for mth constraint. In this light, the Maximum Entropy Principle is a framework for drawing rational inferences when faced with incomplete information [27]. In principle any constraint can be incorporated in the maximum entropy program. Formulating the relevant constraints in economic and social systems, however, can be far from trivial.

[2] See [23–25] for an explicit information theoretic treatment of concepts in statistical mechanics.

4 Econophysics

The influence of the physical sciences on economics appears in the genesis of Classical economic theory [28]. Newton's *Principia* and *Opticks* had a deep impact on European philosophies at the time and Adam Smith's moral philosophy was no exception. The formal mathematical influence of Newton's system in economics, however, remained shelved for a century until its awkward appearance in the marginalist school of thought [10]. While stochastic models, and concepts of scalability and self-organization appear scattered throughout the mid-twentieth century [29–34], modern econophysics [35] really developed in the last decade of the century and is defined by "the activities of physicists who are working on economics problems to test a variety of new conceptual approaches deriving from the physical sciences." [35]. Note, this definition is based on *who* is doing the work and not what problems and methods define econophysics irrespective of disciplinary background.

Because of the conflictual history of physics in economics, modern econophysics has had a mixed reception by economists [36–38]. Criticisms of conventional neoclassical economic doctrine by physicists may be well founded, but claims of scientific superiority and such statements as "The only scientific alternative [to economics] is to approach markets as a physicist, and ask the market data what are the underlying unstable dynamics" [39] will inevitably fall on economists' deaf ears. Economists embedded in the neoclassical "Citadel" tend to find a deep methodological disconnect with the statistical methods of modern physics and rationally prefer not to overhaul the canonical microfoundations of modern economic theory. Heterodox economists working in the tradition of Keynes, Veblen, or the Classical Political Economy of Smith, Ricardo, and Marx, share many of the same criticisms of neoclassical theory as econophysicists, but will often make the opposite claim of [39], preferring more inter-disciplinarity with history and the humanities[3]. The analogy of social and physical systems here is often seen as the original sin of modern economics and the idea of fixing the physics and not the economics is sensibly met with opposition [38]. Though it is claimed that "Econophysics does not mean lifting tools and models from statistical physics and then applying them directly to economics" it can be difficult for many economists to understand what exactly *is* meant by "Econophysics, simply stated, means following the example of physics in observing and modeling markets" [39].

5 Economists react

Perhaps the best known skeptic of "physicism" in economics was Paul A. Samuelson who often lamented the "perpetual attempts to fabricate for economics concepts of 'entropy' imported from the physical sciences or constructed by analogy to Classius-Boltzmann magnitudes... [or] grandiose schemes to replace the dollar as a unit of value by energy or entropy units." [40]. Though Samuelson was rightfully skeptical of the alliance of "superficial knowledge of thermodynamics" and "ignorance of economics" he is careful to narrow his reproof to exclude the indisputable "physicism" of the marginalist vision that bridles his own *Foundations of Economic Analysis*. For Samuelson, the origins of neoclassical economics in rational mechanics and the energy concept were irrelevant and obsolete. For economic historians like Philip Mirowski,

[3] The interesting exception being Farjoun and Machover's [1] plea for introducing the methodology of statistical mechanics in classical political economy.

adopting and abusing potential isomorphisms between social sciences and other scientific disciplines is the calling card of economists [10,41].

But many contemporary economists, particularly those already critical of neoclassical theory, seem genuinely open to and interested in mathematical models that can account for the complex reality of economic systems. The tremendous success of physics in dealing with problems of large hypothesis and state spaces and large degrees of freedom certainly offers promise in this direction, but communicating the ideas cogently to economists embedded in a considerably different tradition is no trivial task. This difficulty is partly due to an unavoidable dissonance in theoretical economics.

There may be notable disagreement in theoretical physics, but there is a generally agreed-upon core of methodological principles, experimental protocols, and measurable fundamental physical quantities. While there is little disagreement on the principles of conservation, the formulation of force as a field, or the wave-particle duality of quantum entities, there is considerable disagreement in economics about such elemental concepts as how to represent human behavior in an economic model, the role of money in an exchange economy, or even how deal with economies of scale in production.

Unlike physics, there is an inherently self-referential nature of economics that makes the goal of an objective value-free explanation of social phenomena unobtainable. The same economic data are generally consistent with multiple models and subject to multiple interpretations. Social scientists are always forced to combine the objective information in the data with their own subjective judgment or communality of beliefs in order to reach substantive conclusions [42]. Physicists may well recognize this point, but without recognizing the alternative traditions in economic thought they have overlooked critical features of economic systems. For example, by focusing solely on exchange relations physicists are implicitly adopting the marginalist tradition[4]. Because the economic questions econophysics raises tend to be narrowly defined by mathematical analogies to those that arise in physics the interpretations of the models are often unconvincing. For example, the idea that a dollar can be viewed as energy and that observed Boltzmann-Gibbs exponential distributions for income may reflect an underlying "conservation of money" [44] rather than the conservation of value in exchange as was stressed by Marx [45,46].

Econophysics has the greatest potential to influence economists already outside of the mainstream traditions who share the same fundamental criticisms of the neoclassical paradigm [47]. Because physicists engaging with economics tend to have little exposure to heterodox approaches the field has primarily engaged with the doctrines of neoclassical economic theory. The unfortunate effect is that econophysics has struggled to make much of impact with mainstream economists due to their fidelity to "The Citadel" or with the heterodoxy due to the limited engagement. As Gallegati [48] has already emphasized in this journal "econophysics should avoid the deadly kiss of mainstream economics and, at the same time, go beyond the boundaries of physics to become a social discipline in which the non-ergodicity is the 'norm'." We would add that econophysics can realize this potential by recognizing the inferential nature of physical and social sciences and adopting the more general methods of maximum entropy inference.

[4] McCauley [43] argues "neo-classical models of production are no better than neo-classical exchange models: there is inadequate or nonexisting empirical basis for any neo-classical assumption." This disregard to non-neoclassical theories of production reinforces the first concern of [36] of "a lack of awareness of work which has been done within economics itself."

6 Maximum entropy economics

Entropy maximization enters naturally in the econophysics approach through the study and application of stochastic processes as a means of modeling and understanding economic phenomena. However, there is an alternative tradition in economics that employs the principle of maximum entropy as a general principle for doing rational inference when faced with any incomplete, underdetermined problem. A considerable part of this work has been associated in some way with the Info-Metrics Institute at American University, which offers a general interdisciplinary framework for doing rational inference [27] of any kind. The wide range of applications of the MEP to economic and econometric problems include information theoretic approaches to modeling economies [49], econometrics [50–52], the foundations of asset pricing [53,54], modeling income inequality [9,55,56] and industrial dynamics [5,57–61], and the deep philosophical background of theory choice [62–64]. Maximum entropy methods have also led to new insights in our understanding of human behavior and strategic interactions [65–67] and have provided behavioral foundations for the type of aggregate statistical fluctuations envisioned by Smith and Marx [6,68].

The criticisms of Samuelson [40] and Georgescu-Roegen [69] on the limits of the entropy concept in economics appear relevant only to the extent that we understand economic systems are conceptually different from physical systems in some important ways. Unlike particles, humans are purposive, complex, and adaptive, in their individual and collective behavior. The existence of social institutions and historical contingencies shape economic systems in complicated ways with hard to find parallels in physics. Yet, upon closer examination the popular distinctions between natural and social sciences become less convincing. The reciprocal influence of research and methods across the disciplines reveals more similarities than differences[5]. Though questions may be distinct to the disciplines, the increasing complexity of the problems addressed in physical and social sciences unveils the common features of natural and social systems. Both systems can be configured in an astronomical number of ways leading to large hypothesis and state spaces, but certain constraints and regularities, such as principles of conservation in physics, or social institutions in economics, make some configurations far more likely than others. Jaynes' fundamental insight was that Shannon's information theory reveals the inferential nature of such problems independent of the nature of the system. The physicist, like the economist, must reason as best they can about highly complex systems with limited information.

The use of Shannon's information theory and Jaynes' Principle of Maximum Entropy Inference in economics has led to considerable advances in economics conceptually distinct from those problems addressed by physicists. The current scope of economic research applying the MEP is too broad to be reduced to econophysics. Thus, we consider *Maximum Entropy Economics* as a more encompassing term defined not by *who* is doing the research, but by *what* research is being done. Maximum Entropy Economics includes the work of economists across the theoretical spectrum as well as those in the physical and mathematical sciences. The MEP can serve as theoretical foundation for econophysics, as suggested by Rosser Jr. [70], and any economic problem that requires drawing inferences from incomplete information.

7 Conclusion

Statistical thinking is necessary in economics as it is in physics. Classical Political Economy evaluated the complex process of production and exchange in terms of the

[5] As Foley [42] argues, perhaps their differences largely lie in the specific roles they play in social reproduction.

statistical regularities that emerge and are sustained at the system level. Statistical and combinatorial reasoning was conceptually fundamental in the determination of natural prices, profit rates, and wages. Neoclassical economics abandoned the methodological approach of the Classicals instead advocating a mechanical idealization of capitalism by studying the pricing implications and abstract properties of individuals as utility maximizers solving a resource allocation problem. The culmination of late nineteenth and early twentieth century economics in general equilibrium theory codified and obscured this mechanic ideology in Bourbakian mathematics. The associated problems in the articulation of theory and measurement eventually led to a late introduction of statistical reasoning in twentieth century economics and a reorientation of the discipline towards applied econometric research. This turn towards econometrics and "applied theory" largely dispensed with general equilibrium theory and its heroic ambitions of a unified economic theory in favor of understanding particular social and economic phenomena in an essentially fragmented system [71]. While applied econometric work has put statistics at the center of a significant part of contemporary economic research it tends to do so on classical frequentist grounds and absent of a statistically based theory, or independent of theory altogether. Thus, the predictive and explanatory success of econometrics has often been in substantiating primitive intuitions in specific areas rather than identifying or explaining social and economic phenomena in general. Rodrik [72] perfectly captures this view in arguing that, "the strength of economics lay precisely in small-scale theorizing... A modest science practiced with humility... is more likely to be useful than a search for universal theories about how capitalist systems function or what determines wealth and poverty."

Econophysics has illuminated the potential for statistical thinking in economics as a basis of theory with clear implications for the articulation with empirical measurement. But it has failed to inspire economists in general due to the narrow use of the entropy principle and tendency to adopt a neoclassical tradition for understanding economic phenomena. The maximum entropy principle is so powerful precisely because it is not limited to a pure physical and thermodynamic interpretation. It is a basis for doing rational inference of any kind. Maximum entropy economics considers a statistical viewpoint to be fundamental to theory development and the conceptual link to empirical measurement.

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