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Review

No axion solution to strong CP using parity and supersymmetry

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Abstract. A major theoretical problem of the otherwise successful standard model (SM) is the presence of an arbitrary amount of CP violation induced by the periodic vacuum structure of Quantum Chromodynamics (known as the strong CP problem). While the most popular solution to this problem is the Peccei-Quinn mechanism, it predicts a new superllight particle, the axion, which has not been found despite extensive experimental searches. An alternative solution to this problem that does not predict an axion is one based on a parity symmetric extension of SM, which also provides a framework for understanding the neutrino masses via the seesaw mechanism. In this mini-review, I describe how minimal versions of the parity solution to strong CP require supersymmetry and how a class of SO(10) theories provide a natural grand unified (GUT) embedding of these models. These approaches have the advantage that the observed CKM CP violation emerges in a simple way in contrast to some other non-axion approaches. We discuss the importance of a search for electric dipole moment of the neutron as a way to probe these solutions.

1 Introduction

One of the major theoretical problems of the standard model is the arbitrary amount of CP violation induced by the periodic vacuum structure of Quantum Chromodynamics. This CP violation which conserves strangeness but violates parity and CP (thereby leading to a large electric dipole moment of the neutron) is characterized by a parameter θ which is bounded to be $\leq 10^{-10}$ from the electric dipole moment limits on the neutron. The problem then is to understand why the θ parameter is so small. A popular solution to this puzzle is the Peccei-Quinn theory [1] which assumes that there be an extra global $U(1)$ symmetry which is broken by the vacuum. Such theories predict the existence of a light pseudo-scalar particle, the axion [2,3], with model dependent mass and couplings. The most widely discussed example is the so-called invisible axion suggested in [4–10]. There have been numerous attempts to experimentally discover the axion, and many new techniques have been proposed recently to search for it. So far such attempts have been unsuccessful.

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Several alternative solutions to the θ problem that do not predict an axion were discussed in the literature starting 1978, a year after the PQ solution was proposed. The first set of models of this kind were based on parity symmetric extensions of the standard model [11,12], which is the focus of this review. Another approach suggested around 1984 is to assume that the desired theory is CP invariant [13,14] leading to vanishing θ with CP invariance being spontaneously broken. However a major challenge for the latter class of theories is to understand why θ so tiny whereas the CKM phase responsible for CP violation in K-mesons and B-mesons is so large [15,16]. As we discuss below, the former parity based no axion solutions do not suffer from this difficulty. In the remainder of this review, we focus on parity based solutions and the role of supersymmetry in them.

The parity based no axion solutions to strong CP [11,12] are based on the left-right symmetric extensions of SM (called LRSM here) [17–20], which were suggested around 1974–75 to provide a framework for understanding the origin of parity violation in weak interactions. The point was to explore whether nature is intrinsically mirror asymmetric or the observed parity symmetry breaking of weak interactions is an artifact of the energy at which all parity violating effects in weak processes have been observed. Relevant to our discussion is the fact that the conservation of parity (P) prior to symmetry breaking, implies that the quark Yukawa coupling matrices are hermitian when parity transformation P is defined in the obvious way i.e. as $Q_L \to Q_R$. This hermiticity implies that if after symmetry breaking, the vacuum expectation values (vevs) of the Higgs fields that break the standard model gauge symmetry are real, the resulting quark mass matrices will be hermitian, leading to $\arg\det M_u M_d \equiv \theta = 0$ at tree level. This is the first requirement of having a solution to the strong CP problem while the CKM phase remains unconstrained. These models however face the following challenges: (i) are the vacuum expectations of the Higgs fields naturally real to enable the solution to work and (ii) secondly, since parity symmetry must be broken at a scale higher than v_{wk} to explain observed parity violation in weak interactions, does a large θ reappear after parity breaking or to put it another way, how large are the quantum loop corrections to θ? As we show below, if the minimality of the matter fermion sector of SM is to be maintained, a natural way to resolve both these challenges is to use supersymmetry [21–23]. Of course Supersymmetry is not needed, if we allow the matter fermion sector to be extended by the addition of vector like fermions [24]. (For further discussion of models with vector-like fermions, see [25–28].) In this review, in view of length restriction, we concentrate only on models with minimal matter content i.e. SM fermions plus a right handed neutrino per generation for neutrino masses. The appearance of the right handed neutrino is required by anomaly cancellation of the theory and hence not an adhoc addition.

In this paper, we first discuss on parity-based models that solve the strong CP problem and why supersymmetry is needed and we then discuss embedding of these models into renormalizable SO(10) GUT models, which have been discussed in the literature in connection with neutrino mass predictions [31–45] and point out their implications and tests. The SUSY SO(10) uses a class of recently discussed renormalizable SUSY SO(10) models with Yukawa-generating Higgs superfields belonging to **10**, $\overline{\mathbf{126}} \oplus \mathbf{126}$ and **120** representations. The fact that models of this type have the potential to solve the strong CP problem was noted in [46] where it was pointed out that if CP symmetry is imposed on the model, the quark mass matrices in this model become hermitian like in the left-right models. This is because SO(10) contains a C-gauge symmetry as its subgroup, so additionally imposing CP makes the model P-symmetric. A complete analysis of the model was presented recently in [47]. We show that in these models, θ parameter is zero at the tree level and is calculable in the loops. Again, in this model, the value of CKM phase is unconstrained as in the minimal left-right SUSY and non-susy models.

2 Minimal TeV-scale Left-Right model without SUSY

To show how parity symmetry leads to vanishing tree level θ without an axion, we first present the field content of the model. The LR model [17–20] extends the SM gauge group $\mathcal{G}_{\rm SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ to $\mathcal{G}_{\rm LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The quarks and leptons are assigned to the following irreducible representations of $\mathcal{G}_{\rm LR}$:

$$Q_{L,i} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_i : \left(\mathbf{3},\mathbf{2},\mathbf{1},\frac{1}{3}\right), \qquad Q_{R,i} = \begin{pmatrix} u_R \\ d_R \end{pmatrix}_i : \left(\mathbf{3},\mathbf{1},\mathbf{2},\frac{1}{3}\right), \quad (1)$$

$$\psi_{L,i} = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_i : (\mathbf{1},\mathbf{2},\mathbf{1},-1), \qquad \psi_{R,i} = \begin{pmatrix} N_R \\ e_R \end{pmatrix}_i : (\mathbf{1},\mathbf{1},\mathbf{2},-1), \quad (2)$$

where $i = 1, 2, 3$ represents the family index, and the subscripts L, R denote the left- and right-handed chiral projection operators $P_{L,R} = (1 \mp \gamma_5)/2$, respectively.

In the minimal version of LR model, the Higgs sector consists of the following multiplets:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} : (\mathbf{1},\mathbf{2},\mathbf{2},0), \qquad \Delta_R = \begin{pmatrix} \Delta_R^+/\sqrt{2} & \Delta_R^{++} \\ \Delta_R^0 & -\Delta_R^+/\sqrt{2} \end{pmatrix} : (\mathbf{1},\mathbf{1},\mathbf{3},2). \quad (3)$$

Under parity, the fields transform as follows: $Q_L \leftrightarrow Q_R$, $\psi_L \leftrightarrow \psi_R$; As for the Higgs fields, $\phi \leftrightarrow \phi^\dagger$ and $\Delta_L \leftrightarrow \Delta_R$. The gauge symmetry $SU(2)_R \times U(1)_{B-L}$ is broken by the VEV of the neutral component of the $SU(2)_R$ triplet, $\langle \Delta_R^0 \rangle \equiv v_R$, to the SM group $U(1)_Y$.

2.1 Fermion masses

To see how the fermions get their masses and how seesaw mechanism arises in this model, we write down the Yukawa Lagrangian:

$$\begin{aligned} \mathcal{L}_Y &= h^a_{q,ij}\bar{Q}_{L,i}\Phi_a Q_{R,j} + \tilde{h}^a_{q,ij}\bar{Q}_{L,i}\tilde{\Phi}_a Q_{R,j} + h^a_{\ell,ij}\bar{\psi}_{L,i}\Phi_a\psi_{R,j} + \tilde{h}^a_{\ell,ij}\bar{\psi}_{L,i}\tilde{\Phi}_a\psi_{R,j} \\ &\quad + f_{ij}\psi^{\mathsf{T}}_{R,i} C i\tau_2 \Delta_R \psi_{R,j} \ + \ \text{H.c.} \end{aligned} \quad (4)$$

where a is for labeling the Higgs bi-doublets, $\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2$ (σ_2 being the second Pauli matrix) and $C = i\gamma_2\gamma_0$ is the charge conjugation operator (γ_μ being the Dirac matrices). After symmetry breaking, we have $diag\langle\phi_a\rangle = (\kappa_a, \kappa'_a)$ and $\langle\Delta_R^0\rangle = v_R$, $\langle\Delta_L^0\rangle = v_L$, with κ, κ' complex in general. By a gauge rotation, one can make κ real leaving $\kappa' = |\kappa'|e^{i\alpha}$ as the only complex vev.The quark and charged lepton masses are given by the generic formulae $M_u = h_q^a\kappa_a + \tilde{h}_q^a\kappa_a'^*$ for up-type quarks, $M_d = h_q^a\kappa'_a + \tilde{h}_q^a\kappa^*$ for down-type quarks, (dropping the family index) and similarly for the charged leptons. The above Yukawa Lagrangian leads to the Dirac mass matrix for neutrinos $m_D = h_\ell\kappa + \tilde{h}_\ell\kappa_a'^*$ and the Majorana mass matrix $M_N = fv_R$ for the RH neutrinos, which via the seesaw mechanism lead to neutrino masses and mixings. Note that if κ, κ' are real, the quark mass matrices are hermitian and lead to $\theta_{tree} = 0$, providing the first requirement for solving the strong CP problem without the axion. A full solution requires that the quantum corrections to θ are small enough to satisfy the neutron edm bound, a challenge which can be met, as shown below. This model therefore provide an unified approach to the neutrino masses as well as strong CP problem.

3 Higgs potential and Higgs vev in the minimal LR model

In order to discuss whether the Higgs vevs are real, we need to write down the Higgs potential of the parity-symmetric theory involving the bi-doublet and triplet Higgs fields. The most general gauge invariant renormalizable scalar potential for the Φ and Δ_R fields, is given by

$$\begin{aligned}\mathcal{V} \;=\; & -\mu_1^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \mu_2^2\left[\mathrm{Tr}(\tilde\Phi\Phi^\dagger) + \mathrm{Tr}(\tilde\Phi^\dagger\Phi)\right] - \mu_3^2\,\mathrm{Tr}(\Delta_R\Delta_R^\dagger) \\ & +\lambda_1\left[\mathrm{Tr}(\Phi^\dagger\Phi)\right]^2 + \lambda_2\left\{\left[\mathrm{Tr}(\tilde\Phi\Phi^\dagger)\right]^2 + \left[\mathrm{Tr}(\tilde\Phi^\dagger\Phi)\right]^2\right\} \\ & +\lambda_3\,\mathrm{Tr}(\tilde\Phi\Phi^\dagger)\mathrm{Tr}(\tilde\Phi^\dagger\Phi) + \lambda_4\,\mathrm{Tr}(\Phi^\dagger\Phi)\left[\mathrm{Tr}(\tilde\Phi\Phi^\dagger) + \mathrm{Tr}(\tilde\Phi^\dagger\Phi)\right] \\ & +\rho_1\left[\mathrm{Tr}(\Delta_R\Delta_R^\dagger)\right]^2 + \rho_2\,\mathrm{Tr}(\Delta_R\Delta_R)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger) \\ & +\alpha_1\,\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \left[\alpha_2 e^{i\delta_2}\mathrm{Tr}(\tilde\Phi^\dagger\Phi)\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{H.c.}\right] \\ & +\alpha_3\,\mathrm{Tr}(\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger)\,.\end{aligned} \tag{5}$$

Note that due to parity symmetry, all 12 parameters $\mu^2_{1,2,3}$, $\lambda_{1,2,3,4}$, $\rho_{1,2}$, $\alpha_{1,2,3}$ are real. However, note the CP-violating phase δ_2 associated with the coupling α_2, Due to the presence of this phase, minimizing the potential with respect to the three VEVs κ, κ', v_R and the phase α associated with the VEV κ', we see that the phase parameter $\alpha \neq 0$ at the potential minimum. Substituting this in the Yukawa Lagrangian shows that even though the Yukawa couplings matrices are hermitian, the resulting quark mass matrices are not and therefore they lead to an arbitrary non-vanishing θ and do not solve the strong CP problem. It is then clear that we must go beyond this minimal non-supersymmetric LR model.

At this stage, one can either add extra global or gauge symmetries on the model. We however found in 1994 [21–23] that supersymmetry can also make the vevs at the potential minimum real, as required to solve strong CP. We follow this path in this review.

4 Strong CP with SUSY LRSM

In this section we show how embedding the minimal LRSM into a supersymmetric framework solves the vev phase problem. To see this, we first introduce the particle content of the model using the chiral field notation used in supersymmetry i.e. left-handed fields will denoted without any subscript L and right handed ones will have a superscript c instead of subscript R. The field content is: Q, Q^c; L, L^c, $\Phi_{1,2}$, $\Delta, \bar\Delta$ and $\Delta^c, \overline{\Delta^c}$. Under parity, we assume that $Q \leftrightarrow Q^{c*}$, $L \leftrightarrow L^{c*}$, $\Phi_a \leftrightarrow \Phi_a^\dagger$, $\Delta \leftrightarrow \Delta^{c*}$, $\overline{\Delta} \leftrightarrow \overline{\Delta^{c*}}$. We assume also that the supersymmetric coordinate θ_{susy} goes to its complex conjugate under parity helping to keep the superpotential part of the action parity invariant. The Yukawa superpotential of the model is given by (written symbolically without showing the needed Pauli matrices)

$$\begin{aligned}W = & Y_q^{(a)} Q^T\Phi_a Q^c + Y_L^{(a)} L^T\Phi_a L^c + (f L^T L\Delta + f_c L^{cT} L^c\Delta^c) \\ & +\mu_{ab} Tr(\Phi_a^T\Phi_b) + \mu_c Tr(\Delta\bar\Delta) + \mu^{c'} Tr(\Delta^c\bar\Delta^c).\end{aligned} \tag{6}$$

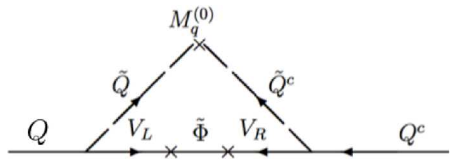

Fig. 1. Typical one loop diagram that contributes to θ at the one loop level.

Two relevant points for strong CP is that (i) the parameters $Y^{(a)}$ are hermitian due to parity invariance and (ii) The Higgs potential has all parameters real e.g. μ_{ab} which are real i.e. CP is an exact symmetry of the potential terms (including D-terms). In such cases there are results in the literature [29,30] showing that for MSSM with multiple Higgs doublets where CP is a good symmetry of the potential, there is no CP violating phases in the vevs. This then leads to the important result that all vevs (κ_a, κ'_a) are real without any adjustment of parameters. The quark mass matrices are then hermitian making tree level $\theta_{tree} = 0$. One then has to look at the one loop graphs which have to be computed with supersymmetry breaking terms included. That will involve new parameters such as the gaugino masses and SUSY breaking A terms. As has been shown in [21–23], the A terms are hermitian under the above definition of parity and the gluino and $B-L$ gaugino mass terms are real at the tree level. The first i.e. reality of gaugino mass is relevant to the strong CP parameter being zero at the tree level since in supersymmetric theories, θ is given by:

$$\theta = \text{Arg Det} M_u M_d - 3\text{ArgDet} M_{\tilde{g}}. \tag{7}$$

However the gluino phase is induced at the one loop level and has been estimated in [21–23] to be $\sim 10^{-8}$, which is close to the upper limit on θ from neutron edm. In this model, the $SU(2)_{L,R}$ gaugino masses are also not real and contribute to θ at the one-loop level (see Fig. 1). The magnitude of the dominant contribution depends on the Yukawa couplings of quarks, the A parameter of supersymmetry breaking as well as the SUSY breaking scale as A^3/M^3_{SUSY}. Adjusting all these parameters, it is possible to make this contribution to θ small enough.

There is however another way to make these contributions small: these weak gaugino mass terms to become naturally real, when the SUSY LR model is embedded into the SO(10) GUT since both of them become part of one multiplet under $SO(10)$ symmetry. This provides a motivation to explore the SUSY SO(10) solutions to strong CP using parity, which we do next.

4.1 Effects of higher dimensional terms and limit on $SU(2)_R$ scale

So far we have considered only renormalizable terms in the superpotential. One could however include higher dimensional terms coming from e.g. Planck scale effects:

$$W_{NR} = \frac{\alpha_L}{M_P} Tr(\Phi_1^T \tau_2 \Phi_2 \tau_2) Tr(\Delta\overline{\Delta}) + \frac{\alpha_R}{M_P} Tr(\Phi_1^T \tau_2 \Phi_2 \tau_2) Tr(\Delta^c \overline{\Delta^c}). \tag{8}$$

The couplings associated with these terms are not real due to parity. As a result they will induce phases in the $\langle \Phi_{1,2} \rangle$ of magnitude $\frac{v_R^2}{M_P}$. Keeping these phases to less than 10^{-10} implies $v_R \leq 10^9$ GeV.

5 SO(10) embedding

In this section, we discuss the SUSY SO(10) embedding of the left-right models where one can resolve the θ problem with the help an extra discrete symmetry CP and several extra discrete symmetries to prevent complex couplings appearing in the superpotential including higher dimensional terms. One would like to avoid these extra symmetries and this is currently under investigation. We believe the model with the extra discrete symmetries is still interesting enough that we present this here.

So(10) models come in many versions. We consider a class of SO(10) models, where Yukawa couplings of SM fermions are generated by renormalizable Yukawa couplings. These models are interesting due to their predictivity in the fermion sector. Two classes of such models have been explored in the literature: (i) one which uses only **10** and **126** Higgs multiplets [31–45] to generate fermion masses and (ii) a second one where, one uses Higgs multiplets belonging to **10**, $\overline{\mathbf{126}}$ and **120** representations (denoted in this case by $H, \bar{\Delta}, \Sigma$ respectively) [46]. The fermions of the model belong to the **16** dimensional spinor representation (denoted by ψ) in both cases. The first class of models do not allow for parity solution to strong CP problem and therefore we focus on the second class of models from now on.

The most general Yukawa superpotential in the second case can be written as

$$W_Y = h_{ij}\psi_i\psi_j H + f_{ij}\psi_i\psi_j\bar{\Delta} + g'_{ij}\frac{Z_\psi}{\Lambda}\psi_i\psi_j\Sigma, \tag{9}$$

where h, f are symmetric matrices, g' is an antisymmetric matrix, and Z_ψ is a spurion singlet field.

The two key requirements are to have hermitian Yukawa couplings and real vevs of Higgs fields. For this purpose, we require this theory to have an additional CP symmetry under which $\psi \to \psi^*$, $H, \bar{\Delta} \to H^*, \bar{\Delta}^*$, $Z_\psi \to Z_\psi^*$ and $\Sigma \to -\Sigma^*$. Requirement of CP invariance then implies that h, f are symmetric and real matrices and g' is an imaginary antisymmetric matrix, i.e. $g' = ig''$ with g'' real. We then define $g \equiv g''\langle Z_\psi\rangle/\Lambda$. This leads to the effective quark Yukawa couplings at GUT scale to be hermitian.

The symmetry breaking pattern is as follows: at GUT scale, both SO(10) and parity symmetry break but SUSY is unbroken. SUSY breaking appears as soft scalar and guino masses at the TeV scale. As we show below the vevs of fields that break this symmetry are real. The MSSM doublets appear below the GUT scale as linear combination of the ones contained in $H, \bar{\Delta}, \Delta, \Sigma$ and the two up and down type MSSM doublets will be kept at the weak scale by the fine tuning of the Higgs superpotential parameters as is done in usual SUSY GUT theories. After substituting the vevs of the resulting MSSM doublets, we have (see Ref. [46] for convention)

$$\begin{aligned}
\mathcal{M}_u &= \tilde{h} + r_2\tilde{f} + ir_3\tilde{g},\\
\mathcal{M}_d &= \frac{r_1}{\tan\beta}(\tilde{h} + \tilde{f} + i\tilde{g}),\\
\mathcal{M}_e &= \frac{r_1}{\tan\beta}(\tilde{h} - 3\tilde{f} + ic_e\tilde{g}),\\
\mathcal{M}_{\nu_D} &= \tilde{h} - 3r_2\tilde{f} + ic_\nu\tilde{g},\\
\mathcal{M}_\nu &= fv_L - \mathcal{M}_{\nu_D}(fv_R)^{-1}\mathcal{M}_{\nu_D}^T,
\end{aligned} \tag{10}$$

where $\tan\beta = v_u/v_d$ for vevs $v_{u,d}$ of the MSSM fields $H_{u,d}$. For $\lambda = h, f, g$, the couplings $\tilde{\lambda}_{ij}$ are related to λ_{ij} by [46]

$$\tilde{h} \equiv \mathcal{V}_{11} h\, v_u;\ \ \tilde{f} \equiv \frac{\mathcal{U}_{14} f v_u}{r_1\sqrt{3}};\ \ \tilde{g} \equiv \frac{\mathcal{U}_{12} + \mathcal{U}_{13}/\sqrt{3}}{r_1} g\, v_u \tag{11}$$

where $\mathcal{V}, \mathcal{U}$ are the MSSM Higgs doublet mixing matrices at the GUT scale [46].

If we can guarantee that the mixings $\mathcal{U}, \mathcal{V}_{ij}$ and vevs $v_{u,d}$ and $\langle Z_\psi \rangle$ are real, the quark and lepton mass matrices will be hermitian, i.e. $M_q = M_q^\dagger$, which implies that at tree level, $\theta = \arg\det(M_u M_d) = 0$. At the same time, because of $i\tilde{g}$ term in the quark mass matrices, the CKM phase is arbitrary.

If this model is to be a solution to the strong CP problem, we have to show the following:

- the mixing parameters $\mathcal{U}, \mathcal{V}_{ij}$ in MSSM doublets which result from various GUT scales are real;
- the masses of all the GUT-scale colored fermions are real (e.g. $\arg\det M_C = 0$ for colored-Higgs mass matrices M_C);
- there are no higher order loop corrections to θ that are large; and
- the full superpotential is such that there are no dangerous sub-GUT-scale multiplets that affect coupling unification.

We show all these below. Clearly the first two require that all the couplings in the Higgs superpotential $\mathcal{W}$ are real. We show by an appropriate choice of the discrete symmetry G and choice of CP properties of the superfields that this condition is indeed satisfied in our model.

5.1 Superpotential

In addition to the above multiplets which play a role in generating fermion masses, we add the following multiplets: **45**, **54** and **210** (denoted by A, S and Φ respectively). The CP transformations of the various field in the model are given in Table 1. In column 3 of the Table, we give the transformation of the fields under a discrete group $G \equiv Z_{N_1} \times Z_{N_2} \times Z_{N_3}$. The purpose of the discrete group is to ensure that there are no complex parameters in the superpotential so that there will be at least one vacuum state which will have real vevs for the spurion fields forbidding any new contributions to the θ parameter. We have also used some of the Yukawa couplings and masses as spurion fields so that they have appropriate charges under G to make that desired field term G-invariant. We will show that the spurions will acquire GUT scale vevs to generate the Yukawa couplings and masses of the right order and discuss how those vevs for the gauge singlet spurion fields arise.

In Table 1, we have assigned the charges to the couplings and masses so that

- The Higgs doublets mixed by $H\Sigma A$, $\bar{\Delta}\Sigma A$ terms with a vev of A, as well as SAA, are allowed by the renormalizable coupling.
- The Yukawa coupling to Σ can be suppressed so the charge of Σ can be different from H (we certainly want to write $\psi\psi H$ Yukawa coupling, which generates the top mass, as a renormalizable term).
- The masses of Δ, A, and S, as well as some couplings such as SHH, are treated as spurion singlet fields.

Table 1. Charge assignment of the different superfields of the theory. It is understood that complex conjugate superfields have opposite discrete quantum numbers.

Field	CP transformation	$(Z_{N_1} \times Z_{N_2} \times Z_{N_3})$ charges
$\Psi(16)$	$\Psi^*(16)$	$(-1/2, 0, 0)$
$H(10)$	$H^*(10)$	$(1, 0, 0)$
$\bar{\Delta}(\overline{126})$	$\bar{\Delta}(\overline{126})^*$	$(1, 0, 0)$
$\Delta(126)$	$\Delta(126)^*$	$(1, 1, 0)$
$\Sigma(120)$	$-\Sigma(120)^*$	$(1, 0, 1)$
$A(45)$	$-A(45)^*$	$(-2, 0, -1)$
$S(54)$	$S(54)^*$	$(4, 0, 2)$
X_Δ,$\Phi(210)$	X_Δ^*,$\Phi(210)^*$	$(-2, -1, 0)$
X_Σ	X_Σ^*	$(-2, 0, -2)$
X_S	X_S^*	$(-8, 0, -4)$
X_A	X_A^*	$(4, 0, 2)$
X_Φ	X_Φ^*	$(4, 2, 0)$
Z_ψ	Z_ψ^*	$(0, 0, -1)$
Z_H	Z_H^*	$(-6, 0, -2)$
Z_Φ	Z_Φ^*	$(6, 3, 0)$
λ(Gaugino)	λ^*(Gaugino)	$(0, 0, 0)$

The superpotential invariant under $SO(10) \times CP \times G$ (in addition to the Yukawa-like terms in Eq. (1)) is given by:

$$\begin{aligned}\mathcal{W} = &\sum_\varphi X_\varphi\, \varphi^2 + X_\Delta \Delta\bar{\Delta} + \lambda_2\, \Sigma\, A\, H \\ &+\lambda_4\, \bar{\Delta}\, A\, \Sigma + \lambda_5\, S\, A\, A + \lambda_6\, H \Delta \Phi + Z_H\, S\, H\, H/\Lambda \\ &+\lambda_7\, \Phi \Delta \bar{\Delta} + \alpha_0 Z_\Phi \Phi^3/\Lambda, \end{aligned} \tag{12}$$

with $\varphi = \Sigma, S, A, \Phi$. Note that in this superpotential, all the coupling parameters are real due to CP invariance and the scale Λ, which is assumed to be the string scale, is much larger than the unification scale M_U. The reality of the couplings implies that all the GUT scale vevs of Higgs fields and the spurions will be real, as will be the mixing parameters $\mathcal{U}, \mathcal{V}_{ij}$. Moreover, all the colored Higgs fields will have real mass matrices so that they will not contribute any new phase to the tree level θ parameter. This establishes our condition (i) and (ii) above at the leading order in the superpotential. Higher order non-renormalizable terms can still disturb the conditions (i) and (ii); we will show that their contributions to theta are at or below the current bound.

Additionally, we choose the specific symmetry $G = Z_{24} \times Z_6 \times Z_4$ so that we can add the following superpotential terms:

$$\begin{aligned} W' = &\alpha_1 \frac{(X_\Delta \Delta \bar{\Delta})^2}{\Lambda^3} + \alpha_2 \frac{S^6}{\Lambda^3} + \alpha_3 X_S^3 + \alpha_4 \frac{Z_\Phi^4}{\Lambda} \\ &+\alpha_5 \frac{Z_\Phi X_\Delta^3}{\Lambda} + \alpha_6 \frac{X_A^6}{\Lambda^3} + \alpha_7 \frac{Z_\psi^4}{\Lambda} + \alpha_8 \frac{Z_\psi^2 X_A X_\Sigma^2}{\Lambda^2} \\ &+\alpha_9 \frac{Z_H^3 X_\Sigma^3}{\Lambda^3} + \alpha_{10} \frac{Z_H^4}{\Lambda} + \alpha_{11} \frac{X_\Sigma^6 Z_H^2}{\Lambda^5} \\ &+\alpha_{12} \frac{X_S^2 Z_H X_\Sigma}{\Lambda} + \alpha_{13} X_A^2 X_S + \alpha_{14} \frac{X_\Phi^3 Z_\Phi^2}{\Lambda^2} \\ &+\alpha_{15} X_\Phi X_\Delta^2 + \alpha_{16} \frac{X_\Phi^6}{\Lambda_3} \\ &+(\text{higher order terms in } 1/\Lambda). \end{aligned} \tag{13}$$

With very mild fine tuning of the α_i couplings, the F-term minimization can give real GUT scale vevs to all spurion fields as desired.

5.2 GUT symmetry breaking and particle spectra

In this section, we show by analyzing the superpotential that there are no undesirable light particles below the GUT scale that could destroy coupling unification. First note that the higher dimensional terms in equation (13), as well other terms, help to give vevs to all the spurion fields. If we choose the cutoff scale Λ to be the reduced Planck scale (2×10^{18} GeV), by appropriate choice of the coefficients of the higher dimensional terms, we can keep the spurion vevs near the GUT scale ($M_U \sim 2 \times 10^{16}$ GeV). This will lead to spurion masses below the GUT scale, but being gauge neutral, they do not affect the running of gauge couplings. We have checked that with a mild smallness of the coefficients of the higher dimensional operators, we can keep all the singlet vevs near the GUT scale.

Next, due to the absence of $A\Delta\bar{\Delta}$ term, the F-flatness conditions of $\Delta, \bar{\Delta}, \Phi$ and S,A are separated. The Δ vev, which is at GUT scale along the SU(5)-singlet direction i.e. $\langle \Delta_{13579} \rangle = \langle \bar{\Delta}_{13579} \rangle = v_R \neq 0$ (to get the D-terms to be zero), can be generated by $(\Phi + X_\Delta)\Delta\bar{\Delta} + \Phi^3 + X_\Phi \Phi^2$ term. The vevs of A can be generated by $X_A A^2 + X_S S^2 + SAA$. For group theory of such models see [51–53].

Note that in the absence of the $\Phi\Delta\bar{\Delta}$ term, the superpotential is only a function of the singlet contraction of $\Delta\bar{\Delta}$, which implies that the superpotential has a large global symmetry, and thus the decomposed multiplets under SU(5) are massless. However, the presence of $\Phi\Delta\bar{\Delta}$ term cures this problem and makes all submultiplets of $\Delta \oplus \bar{\Delta}$ massive.

5.3 Proton decay from higher dimensional operators

As is well known grand unified theories predict proton decay with life times at the level of 10^{32-34} years. Current limits for the SUSY mode i.e. $p \to K^+\bar{\nu}$ is: $\tau_p \geq 8 \times 10^{33}$ yrs [54]. In discussing proton decay, there are two classes of contributions that need to be considered: (i) contribution from the Yukawa sector and (ii) and that from Planck suppressed higher dimensional operators. As far as the first class of operators go, they have been analyzed for both classes of renormalizable SO(10) models i.e. **10**+**126** Higgs [49] as well as **10**+**126**+**120** Higgs cases [50]. In the first class of models, current proton decat lifetime limits imply that the SUSY breaking scale must be larger than 100 TeV [49]. In the second class of models, the current limits can be satisfied with SUSY scale of a few TeV [50].

Turning to the second class of operators, in view of the fact that we have chosen $\Lambda \sim 2 \times 10^{18}$ GeV in our discussion above, one might wonder whether the model leads to rapid proton decay via higher dimensional operators scaled by Λ as in usual SUSY GUT models. However, our model has a discrete symmetry $Z_{24} \times Z_6 \times Z_4$, which all higher dimensional terms must also respect. As a result, the leading dangerous terms (for proton decay) such as $[\psi]^4$ are forbidden. The leading order term that contributes to proton decay after symmetry breaking is $\lambda[\psi\psi\bar{\Delta}]^2$ and this is suppressed by Λ^3 and as a result it quite is compatible with current limits on proton lifetime for $\lambda \leq 1$.

6 Strong CP phase in SUSY SO(10) with parity

We just showed that $\theta_{tree} = 0$ in these models. We now study possible generation of the strong CP phase from (i) higher dimensional terms, (ii) loop correction to the gluino mass, (iii) renormalization contribution to the quark Yukawa coupling as we extrapolate from GUT scale to the weak scale.

6.1 Higher dimensional terms

In general one could envision two types of non-renormalizable contribution to θ: (i) one set which are invariant under the discrete symmetry $CP \otimes G$ and (ii) terms induced by global symmetry-breaking, non-perturbative gravitational effects. We ignore the latter since there seem to be different opinions on whether black holes really break global symmetries.

Given the charge assignments in Table 1, we find that the leading operators which can generate phases in the doublet Higgs mixings and the masses of colored Higgsinos are of dimension-9:

$$\begin{aligned}
&A\Delta\bar{\Delta}X_\Delta Z_H Z_\psi S^2/\Lambda^5,\\
&H\Sigma\Phi\Delta\bar{\Delta}Z_H Z_\psi X_A/\Lambda^5,\\
&A\Phi\Phi X_\Phi Z_H Z_\psi X_A^2/\Lambda^5,
\end{aligned} \tag{14}$$

and the suppression factor will be $(M_U/\Lambda)^5$, where M_U is a GUT unification scale $\sim 2\times 10^{16}$ GeV. For Λ to be the reduced Planck mass, 2.4×10^{18} GeV, the bound for $|\theta| < 10^{-10}$ can be satisfied by these contributions to θ.

It is also the case that a $\psi\psi AH$ term can break the hermitian nature of the Yukawa coupling matrices, but such a term is generated only by a dimension-10 operator under the above charge assignment, which leads to a θ below the current bound.

Note that the hierarchy between the cutoff (string scale) and the GUT unification is essential to suppress the θ parameter in the current model.

6.2 Loop corrections and gluino phase

Since phases in all colored fermion fields contribute to the θ parameter, we have to track the phases in the gluino mass term in addition to the quark and GUT-scale colored Higgsino mass matrices. At the tree level, gluino mass term is real due to CP symmetry. However, since CP symmetry is broken, quantum loops induce non-zero gluino phases. To estimate this, we assume a framework where supersymmetry is broken by gravity mediation so that (i) $A_q \propto M_q$ and (ii) the squark masses are flavor diagonal at the susy breaking scale. This assumption is crucial in what follows. We have not examined what happens for other ways of supersymmetry breaking.

Under these assumptions, the loop contribution to gluino phase can be estimated. At the one-loop level, the gluino mass is real due to hermiticity of the quark mass matrices, since the contribution from quarks is of the form $\mathrm{tr}\, M_q^\dagger A_q$ where $A_q \propto M_q$ if as just noted, we assume gravity mediated supersymmetry breaking, and all other colored fermion fields have no phases.

The loop correction from the pure-imaginary Yukawa coupling g can induce a phase for the gluino mass. At the two-loop level, however the contribution is always proportional to $\mathrm{Tr}\, YY^\dagger$, where Y is a Yukawa coupling to colored Higgs, and the contribution is real. From the three-loop level diagram in which the doublet Higgs also propagates (Fig. 2), the imaginary part of the gluino mass is given by

$$\sum_{a,b} \frac{g_s^2}{(16\pi^2)^3} \mathrm{Tr}\,(Y_{T_a} Y^*_{D_b} Y_{\bar{T}_a} Y^*_{\bar{D}_b}) F\left(\frac{M_{H_{D_b}}}{M_{H_{T_b}}}\right) A_{\mathrm{tri}}, \tag{15}$$

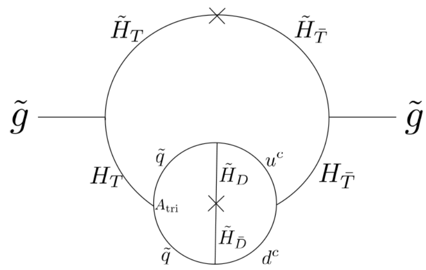

Fig. 2. The 3-loop diagram which induces the phase of the gluino mass. There are also diagrams in which $u^c e^c$-$q\ell$ and $\nu^c d^c$-ℓq propagate. In **126** and **120**, there are other colored components, e.g. $(\mathbf{8}, \mathbf{2}, 1/2)$, which have bi-fermion couplings, and there are similar loop-diagrams in which the other colored components propagate for the gluino mass correction.

where Y_T and $Y_{\bar{T}}$ are Yukawa couplings to (diagonalized) colored Higgs fields ($H_{T,\bar{T}}$), and Y_D and $Y_{\bar{D}}$ are the Yukawa couplings to doublet Higgs fields ($H_{D,\bar{D}}$), and $A_{\rm tri}$ is the SUSY breaking scalar trilinear coupling, and F is a loop function. (We note that the heavy Higgs doublets and the Higgs triplets with GUT scale mass propagate in the loop diagram.) The Yukawa couplings are given by the linear combination of h, f and g. Noting that $\mathrm{Tr}\,(gX) = 0$ for symmetric matrix X, one finds that $\mathrm{Tr}\,(gh^3)$, $\mathrm{Tr}\,(ghfh)$, etc vanish, and $\mathrm{Tr}\,(ghhf)$ and $\mathrm{Tr}\,(gffh)$ etc can contribute.As a result, the leading contribution can be estimated to be

$$\mathrm{Im}\, m_{\tilde{g}} \sim \frac{g_s^2 g_{23} f_{23} h_{33}^2}{(16\pi^2)^3} A_{\rm tri}. \tag{16}$$

For $A_{\rm tri} \sim m_{\tilde{g}}$, we roughly estimate the contribution to θ as

$$\Delta\theta \sim 10^{-9} \left| \frac{f_{23} g_{23}}{10^{-3}} \right|. \tag{17}$$

This is on the borderline of satisfying the neutron EDM bounds, taking into account that f and g are the original couplings and not multiplied by the Higgs mixing. Note that $\tilde{f}_{23}$ and $\tilde{g}_{23}$ (which are f and g multiplied by Higgs mixings) can be estimated to be as large as V_{cb}. If $A_{\rm tri} \ll m_{\tilde{g}}$, the neutron EDM bounds can be safely satisfied; in this sense, gauge-mediated SUSY breaking, rather than the gravity mediation, is preferable to suppress the θ parameter. In any case, loop correction to the gluino mass can generate a borderline value for the θ parameter, and therefore, the model would predict an observable neutron EDM in current experiments. As for the squarks, their mass matrices are chosen diagonal at the GUT scale and while RGEs will induce some off-diagonal phases, they will not contribute to θ and any contribution to neutron EDM will be suppressed.

6.3 Extrapolation of θ from GUT to weak scale

The hermiticity of the quark mass matrices holds at the GUT scale. This means that the value of tree level θ is zero at that scale and a finite, non-zero θ will be induced at the weak scale due to renormalization extrapolation of the various Yukawa. This issue for a parity solution to strong CP has been considered in reference [48], and it is shown that if the weak scale theory is MSSM with a soft breaking scalar masses

M_{SUSY} above 5 TeV, the corrections to θ are given by

$$\delta\theta \simeq \left(\frac{1}{16\pi^2}\ln\frac{M_U}{M_{SUSY}}\right)^4 \left[c_1 \mathrm{ImTr}[Y_u^2 Y_d^4 Y_u^4 Y_d^2]\right.$$
$$\left. + c_2 (u \leftrightarrow d)\right]. \tag{18}$$

This can be estimated to be $\delta\theta \sim 10^{-26}\,(\tan\beta)^6(c_1 - c_2)$, which is below the experimental upper limit even for $\tan\beta = 50$. We note that one obtains $c_1 = c_2$ and $\delta\theta$ vanishes without an electroweak gauge loop at the MSSM scale and any extrapolation from there to the weak scale produces negligible change.

Finally, we note that since the discrete symmetries of our model break at the GUT scale, domain walls associated with them will get "inflated away" as long as the reheat temperature is below the GUT scale and will not lead to any anisotropy in cosmic microwave background.

7 Conclusion

In conclusion, we have shown that it is possible to have a solution to the strong CP problem without the need for the axion. The alternative solution uses the parity symmetry which bypasses the need for the axion. We show that in the minimal matter content version of the model, supersymmetry plays an essential role to keep θ suppressed. We demonstrate this using the minimal supersymmetric left-right model and its embedding in SUSY SO(10) grand unified theory for fermion masses. Unlike some other no axion solutions to the strong CP problem, in parity symmetric models, large CKM CP phase is easy to understand. These models have the additional virtue that they provide a realization of the seesaw mechanism for neutrino masses while giving a fit to the fermion masses and are therefore realistic models of particle interactions. In particular, the SO(10) GUT model is predictive in the neutrino sector. These models predict neutron electric dipole moment not far from the current upper limits and neutron edm can therefore be used as a test of these models.

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