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# Unification predictions with or without supersymmetry

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**Abstract.** Supersymmetric (SUSY) grand unified theories (GUTs) appear to be best motivated for understanding strong, weak and electromagnetic interactions of nature. We briefly review emergence of new formulas for running fermion masses valid in direct breaking of GUTs. High scale mixing unification of quark and neutrino mixings and existence of theorems on vanishing theoretical uncertainties in GUT predictions are discussed. D-Parity properties of $SO(10)$ representations leading to large number of intermediate breaking models are pointed out. Unification predictions of SUSY $SO(10)$ in the light of neutrino mass, lepton flavor violation, baryogenesis via leptogenesis within gravitino constraint, and proton decay are noted. We further discuss realisation of flavour unification and possibility of fitting all fermion masses through R-Parity and D-Parity conserving left-right symmetric intermediate breaking in SUSY $SO(10) \times S_4$. In the absence of SUSY, two interesting possibilities of minimal grand desert modifications by only one intermediate mass scalar in each case and their applications to dark matter decay through type-I seesaw are briefly noted. Heavy scalar triplet decay leptogenesis through new ansatz for type-II seesaw dominance in non-SUSY $SO(10)$, emergence of new CP asymmetry formulas and model capabilities to explain WIMP dark matter, vacuum stability of the scalar potential and experimentally observed limit on proton lifetime are briefly summarised.

## 1 Introduction

The standard model of electroweak and strong interaction gauge theory, $SU(2)_L \times U(1)_Y \times SU(3)_C$, enjoys a very special staus in the fundamental understanding of particle interaction and three forces of nature. It was a much sought after theoretical breakthough after Dirac theory of Quantum Electrodynamics (QED) [1,2] which achieved precision prediction in higher orders of electromagnetic gauge coupling successfully through its intrinsic capability of renormalizability. Dirac's idea manifested in the generalization Yang-Mills Lagrangian for non-Abelian gauge theories [3]. In sharp contrast with massless photon of $U(1)_Q$ invariant QED, major

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**Table 1.** Particle content of MSSM for three generations of fermions ($i = 1, 2, 3$). The SM particle content is devoid of superpartners and has only one Higgs scalar doublet $\phi(2, 1/2, 1)$ instead of two.

| Particle type | $G_{213}$ charges | Superpartners & charges |
|---|---|---|
| Gauge Bosons | $W_\mu(3, 0, 1), B_\mu(1, 0, 1), G_\mu(1, 0, 8)$ | $\tilde{W}_\mu(3, 0, 1), \tilde{B}_\mu(1, 0, 1), \tilde{G}_\mu(1, 0, 8)$ |
| Fermions of $i$th generation | $l_i(2, -1/2, 1), e_{R_i}(1, -1, 1)$ | $\tilde{l}_i(2, -1/2, 1), \tilde{e}_{R_i}(1, -1, 1)$ |
| | $Q_i(2, 1/6, 3), u_{R_i}(1, 2/3, 3),$ | $\tilde{Q}_i(2, 1/6, 3), \tilde{u}_{R_i}(1, 2/3, 3),$ |
| | $d_{R_i}(1, -1/3, 3)$ | $\tilde{d}_{R_i}(1, -1/3, 3)$ |
| Higgs scalars | $\phi_u(2, 1/2, 1), \phi_d(2, -1/2, 1)$ | $\tilde{\phi}_u(2, 1/2, 1), \tilde{\phi}_d(2, -1/2, 1)$ |

hurdles in achieving a renormalizable SM were the compelling experimental and phenomenological issues that demanded massive vector bosons to mediate weak interaction. The ingenuous idea of gauging the electroweak theory [4–6] combined with Higgs mechanism [7–11] finally resolved the long standing issue with the emergence of renormalisable electroweak theory even after its spontaneous symmetry breaking [12,13].

Even though the SM has been tested by numerous experiments , it fails to explain several issues, the most prominent being neutrino oscillation [14–18], baryon asymmetry of the universe (BAU) [19–23], nature of dark matter (DM) and its stability [24–31], and the origin of disparate values of gauge couplings. Besides these the SM faces the most fundamental issue of protecting the Higgs mass at the electroweak scale. This is due to the fact that the SM Higgs mass becomes quadratically divergent by radiative corrections against which there does not seem to be any natural solution except through supersymmetery [32–43]. Grand unified theories (GUTs) [44–52], originally aimed at unifying the three forces of nature, were subsequently supersymmetrised to confront the gauge hierarchy problem, exhibit explicit coupling unification through direct breaking to SM and address issues like neutrino masses and WIMP dark matter.

Including Fermi-Bose symmetry, the particle content of minimal supersymmetric standard model (MSSM) is shown in Table 1. The second column of Table 1 represents the SM particle content of one fermion generation except that instead of two Higgs dublets of MSSM, SM has only the standard doublet $\phi(2, 1/2, 1)$. The underlying Fermi-Bose symmetry of MSSM and SUSY GUTs naturally cancels out the quadratic divergence of the Higgs mass thus removing the gauge hirarchy problem. Another major theoretical achievement of MSSM descending from SUSY GUTs is the automatic natural explanation of three forces of SM as discussed below.

## 2 Renormalization group evolution of couplings and masses

### 2.1 SUSY grand desert unification

To understand failure of unification in SM and its success in MSSM and SUSY GUTs, the precision electroweak measurements are used to detemine the SM gauge couplings at the electroweak scale

$$\alpha_Y{}^{-1}(M_Z) = 59.8, \ \alpha_{2L}{}^{-1}(M_Z) = 29.6, \ \alpha_{3C}{}^{-1}(M_Z) = 8.54, \tag{1}$$

where $\alpha_i = g_i^2/(4\pi)$. The evolutions of gauge couplings are given by the renormalisation group equations (RGEs) [53–55]

$$\mu\frac{\partial g_i}{\partial \mu} = \frac{a_i}{16\pi^2} + \frac{g_i^3}{(16\pi^2)^2}\left(\sum_j b_{ij}g_j^2 - k_i y_{top}^2\right),\tag{2}$$

where $a_i(b_{ij})$ are one-loop (two-loop) coefficients, and $k_i = (17/10, 3/2, 2)$ for SM but $k_i = (26/5, 6, 4)$ for MSSM. Denoting the Dynkin indices due to gauge bosons, fermions, and Higgs scalars by $t_2(G_i), t_2(F_i)$ and $t_2(S_i)$, respectively, under gauge group $G_i$ the analytic formulas for one-loop beta function coefficients in non-supersymmetric (non-SUSY) and SUSY cases are.

**Non-SUSY gauge theory:**

$$\begin{aligned}a_i &= -\frac{11}{3}t_2(G_i) + \frac{2}{3}t_2(F_i) + \frac{1}{3}t_2(S_i),\ (i \in SU(N)),\\ &= \frac{2}{3}t_2(F_i) + \frac{1}{3}t_2(S_i),\ (i \in U(1)).\end{aligned}\tag{3}$$

**SUSY gauge theory:**

$$\begin{aligned}a_i &= -\left[\frac{11}{3}t_2(G_i) + \frac{2}{3}t_2(G_i)\right] + \left[\frac{2}{3}t_2(F_i) + \frac{1}{3}t_2(F_i)\right]\\ &\quad + \left[\frac{1}{3}t_2(S_i) + \frac{2}{3}t_2(S_i)\right]\\ &= -3t_2(G_i) + t_2(F_i) + t_2(S_i),\ (i \in SU(N)),\\ &= t_2(F_i) + t_2(S_i),\ (i \in U(1)).\end{aligned}\tag{4}$$

The first two lines in equation (4) are derived from Non-SUSY equation (3) using Fermi-Bose symmetry. Similar analytic formulas exist for two-loop coefficients $b_{ij}$ of equation (5) given below. Then $a_i = (41/10, -19/6, -7)$ for SM but $a_i = (33/5, 1, -3)$ for MSSM for which particle contents are shown in Table 1. These coefficients are used in the integral form of evolution equations

$$\frac{1}{\alpha_i}(\mu) = \frac{1}{\alpha_i}(M_Z) - \frac{a_i}{2\pi}\ln\left(\frac{\mu}{M_Z}\right) - \frac{1}{4\pi}\sum_j B_{ij}\ln\left[\frac{\alpha_j(\mu)}{\alpha_j(M_Z)}\right],\tag{5}$$

where $\alpha_i(\mu) = \frac{g_i^2(\mu)}{4\pi}$ and $B_{ij} = b_{ij}/a_j$. The second (third) term in the RHS represents one (two-loop) effects [54]. For simplicity, the evolution of gauge couplings at one-loop level is shown in Figure 1 where the left (right)-panel is for SM (MSSM).

The presence of a triangle of finite area in the case of SM, instead of a single meeting point (or a much smaller triangle compatible with experimental errors), demonstrates inherent deficiency of the minimal SM to unify the three gauge couplings. On the other hand profound unification is exhibited in MSSM with the unification scale $2 \times 10^{16}$ GeV [40–43]. Thicker sizes of the three curves in SUSY case arise due to existing uncertainty at the electroweak scale.

About 8 years before CERN-LEP data inspired SUSY unification was noted [40–43], such unification was also observed in non-SUSY $SO(10)$ GUT with
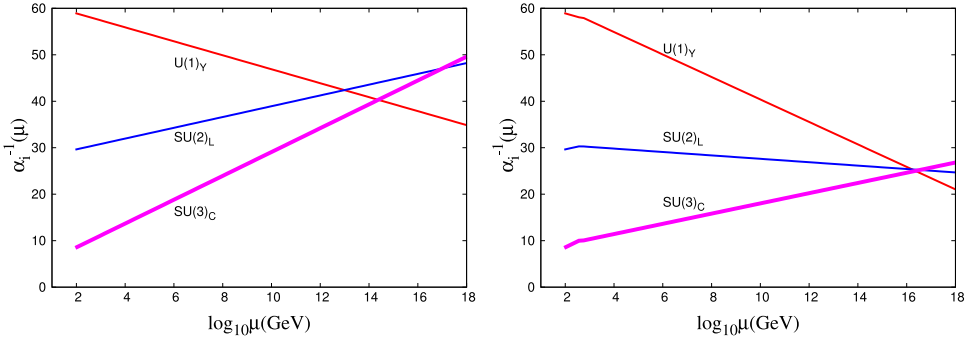
**Fig. 1.** Evolution of the three inverse fine strucure constants without unification in the SM (left panel), but with unification in the MSSM (right panel).

left-right intermediate gauge symmetries like $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (\equiv G_{2213})$, $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($\equiv G_{224}$), $G_{2213D}$ and $G_{224D}$ where symmetries with $D$ stand for D-Parity indicating $g_{2L} = g_{2R}$ [56]. It was further noted that each of the non-SUSY GUTs like $SO(10)$ or $E_6$ can accommodate one or more intermediate gauge symmetry breaking providing a variety of models for non-SUSY grand unification [57] that has resulted in a number of interesting applications [58] including prospects of low mass $W_R, Z_R$ bosons.

## 2.2 New formulas for running fermion masses in SM, 2HDM, and MSSM

Defining $\tan\beta = v_u/v_d =$ where $v_u(v_d) = \langle\phi_u(\phi_d)\rangle$, a very attractive aspect of MSSM and SUSY GUTs is $b - \tau$ Yukawa unification for smaller $\tan\beta \sim 1 - 5$ and approximate $t - b - \tau$ Yukawa unification for larger $\tan\beta \sim 40 - 55$. SUSY $SO(10)$ besides gauge and Yukawa unifications, also possesses ability for reasonable parameterisation of charged fermion masses at the GUT scale [60]. The new formulas for running fermion masses [61,62] were derived taking into account the scale dependence of VEVs $v_u, v_d$ in MSSM and 2HDM [63–71] which were also found to decrease with increasing mass scale instead of remaining constant. Using the correspoding running VEVs in MSSM, SM and 2HDM new formulas have been developed to extrapolate all charged fermion masses from their low energy values to the GUT scale values [61,62]. These extrapolated values have been found useful in testing $SO(10)$ model capabilities for representing fermion masses even without using flavour symmetries.

# 3 Unification of quark and neutrino mixings

## 3.1 Radiative magnification with quasi-degenerate neutrinos

In the presence of supersymmetry it was found that the mixing angle between two light neutrinos could be quite small near the SUSY GUT scale. But due to renrmalisation group evolution, the mixing angle gets magnified to be compatible with its large value in concordance with neutrino oscillation data. Initially this was realised only for atmospheric neutrino mixings. Radiative magnification was noted to be possile for two neutrinos with (i) quasi-degenerate neutrino masses, (ii) identical CP properties, and (iii) larger values of $\tan\beta = v_u/v_d$ [72].

## 3.2 High scale mixing unification

Despite the radiative magnification mechanism that applied for the atmospheric neutrino mixing, it was difficult to reconcile with neutrino data showing the general behaviour of three large values of neutrino mixings ($\theta_{ij}$) compared to correspondingly small quark mixings ($\theta_{ij}^q$) : $\theta_{23} \gg \theta_{23}^q, \theta_{12} \gg \theta_{12}^q, \theta_{13} \gg \theta_{13}^q$. In addition the initial input value of the mixing angle, dynamical origin of quasi-degenerate neutrinos and the necessity of large value of $\tau$-Yukawa coupling in the RGE were used as a matter of necessity [72–74] without deeper theoretical understanding. Through further development of neutrino RGEs [75,76] an interesting resolution of this puzzle has been suggested [76] using the underlying quark lepton symmetrty of supersymmetric Pati-Salam theory [44] (or SUSY $SO(10)$) along with $S_4$ flavour symmetry. The $G_{224D} \times S_4$ breaking through RH triplet VEV ($\langle \Delta_R \rangle \simeq V_R \simeq M_{GUT}$) generated small departure from degeneracy created through type-II seesaw induced VEV of LH triplet ($\Delta_L(3, 1, 10)$) at the highest scale. Because of Pati-Salam symmetric quark-lepton unification, identification of initial boundary values of quark mixings with lepton mixings was a natural pre-existing input for the RG evolutions. Large $\tau$-Yukawa coupling with large value of $\tan \beta \simeq 40-55$ was a necessary prediction of $b - \tau$ unification as shown earlier [61,62]. In this theory neutrino mixings are predicted to be unified with corresponding quark mixings at the SUSY GUT scale. The RGEs predict negligible changes for quark mixings because of their strong mass hirachy. On the other hand for large $\tan \beta$ and due to quasi-degenerate masses, $m_{\nu_1} \simeq m_{\nu_2} \simeq m_{\nu_3} \geq 0.2$ eV, the RGEs for neutrino mixings are magnified to their large low-energy values [76–81]. The RGEs for the mass eigen values can be written in a simpler form [76–80]

$$\frac{dm_i}{dt} = -2F_\tau m_i U_{\tau i}^2 - m_i F_u, \ (i = 1, 2, 3) \,.$$ 
(6)

For every $\sin \theta_{ij} = s_{ij}$, the corresponding RGEs are,

$$\frac{ds_{23}}{dt} = -F_\tau c_{23}{}^2 \left(-s_{12} U_{\tau 1} D_{31} + c_{12} U_{\tau 2} D_{32}\right),$$
(7)

$$\frac{ds_{13}}{dt} = -F_\tau c_{23} c_{13}{}^2 \left(c_{12} U_{\tau 1} D_{31} + s_{12} U_{\tau 2} D_{32}\right),$$
(8)

$$\frac{ds_{12}}{dt} = -F_\tau c_{12} \left(c_{23} s_{13} s_{12} U_{\tau 1} D_{31} - c_{23} s_{13} c_{12} U_{\tau 2} D_{32} \right.$$
$$\left. + U_{\tau 1} U_{\tau 2} D_{21}\right),$$
(9)

where $D_{ij} = (m_i + m_j)) / (m_i - m_j)$ and, for MSSM,

$$F_\tau = -h_\tau^2 / \left(16\pi^2 \cos^2 \beta\right),$$
$$F_u = \left(\frac{1}{16\pi^2}\right) \left(\frac{6}{5} g_1^2 + 6g_2^2 - 6\frac{h_t^2}{\sin^2 \beta}\right),$$
(10)

but, for SM,

$$F_\tau = 3h_\tau^2 / \left(32\pi^2\right),$$
$$F_u = \left(3g_2^2 - 2\lambda - 6h_t^2 - 2h_\tau^2\right) / \left(16\pi^2\right).$$
(11)

Natural occurence of a SUSY scale near $300-1000$ GeV does not permit radiative magnification below this scale where the mixing angles remain constant in the presence of the SM.

The resulting RG evolutions of quark and neutrino mixings in different cases have been shown in [75–81].

For such high scale mixing unification (HUM), the desired QD neutrino mass scale is $m_i \simeq m_0 > 0.15$ ($i = 1, 2, 3$) eV. WMAP data [82–84] suggest the bound $\Sigma_C \equiv \sum m_{\nu_i} < 0.69-1.3$ eV but, consistent with priors, it has been also noted that $\Sigma_C \leq 1$ eV [85,86]. Although recent Planck satellite data has determined a cosmological bound $\Sigma_C \leq 0.23$ eV [22,23] (or even lower [87]), the same data have been noted to admit $\Sigma_C \leq 0.71$ eV in the absence of $\Lambda$CDM based theory of the Universe [22,23]. However all neutrino mass values needed for HUM [76–81] are in concordance with the most recent laboratory bound from KATRIN collaboration [88] that has reached the limit $m_\nu < 1$ eV. QD neutrino masses needed elsewhere [68–74,89–94] are also allowed by KATRIN results. It has been further noted that the presence of SUSY substantially below the GUT-Planck scale is not a necessary criteria for understanding such RG origin of neutrino physics as the mechanism works profoundly even with very high scale split-SUSY [81].

# 4 Advantages of $SO(10)$

The discussions stated below apply to both SUSY or non-SUSY $SO(10)$ or $E_6$. The $SU(5)$ GUT predicts **15** SM fermions of one generation in two different representations $\bar{5}_F + 10_F$, but they are all unified with right-handed (RH) neutrino (N) into a single spinorial representation $16_F$ in $SO(10)$. Dirac neutrino mass generation in SM or $SU(5)$ needs introduction of N externally, but in $SO(10)$ this follows automatically from Yukawa interaction $Y 16_F 16_F 10_H$ where $10_H \supset \phi(2, 1/2, 1)$ which is the standard Higgs scalar doublet.

## 4.1 D-parity breaking and emergence of new $SO(10)$ models

Before 1984 the breaking of left-right discrete symmetry ($\equiv$ Parity(P) = space-inversion symmetry) was synonymous with $SU(2)_R$ breaking [44,95–97]. This did not permit low mass $W_R$ bosons or $SU(4)_C$ [44] breaking scales accessible to accelerators, although a two-step breaking of left-right symmetric gauge theory was shown to predict a low-mass $Z'$ boson [98] in concordance with $K_L - K_S$ mass difference. The discovery and identification of D-Parity properties of $SO(10)$ [56,57] representations paved the ways for lowering such mass scales substantially leading to new classes of $SO(10)$ accessible to experimental tests [58]. It was at first noted [56] that if the LRS theory $G_{2213D}$ has a scalar singlet $\sigma$ that is odd under the L(left) $\rightarrow$ R (right) transformation , then its VEV $\langle\sigma\rangle = V_\sigma$ would break the LR discrete symmetry without breaking the gauge symmetry leading to $G_{2213D} \rightarrow G_{2213}$ (or $G_{224D} \rightarrow G_{224}$). More important is the identification of such scalar singlets in $SO(10)$. Defining D-Parity as an element of $SO(10)$ gauge transformation that takes a fermion $\psi_L \subset 16_F$ to its conjugate $\psi_L^C (\propto \psi_R^*)$ which is also in the same $16_F$, invariance under $D$ guarantees left-right (LR) discrete symmetry with $g_{2L} = g_{2R}$, but spontaneous breaking of $D$ implies breaking of LRS with $g_{2L} \neq g_{2R}$ but without breaking the gauge symmetry $G_{224}$ or $G_{2213}$. The D-Parity properties of $SO(10)$ scalar components were identified for the first time [56,57]. Considering branching rules [59] of $SO(10)$ scalar representations under Pati-Salam symmetery ($G_{224D}$)

$\boldsymbol{SO(10)} \supset SU(2)_L \times SU(2)_R \times SU(4)_C (g_{2L} = g_{2R})(\equiv G_{224D})$:

$$
\begin{aligned}
10 &= (2, 2, 1) + (1, 1, 6), \\
16 &= (2, 1, 4) + (1, 2, \bar{4}), \\
45 &= S_o(1, 1, 15) + (3, 1, 1) + (1, 3, 1) + (2, 2, 6), \\
54 &= S_e(1, 1, 1) + (3, 3, 1) + (2, 2, 6) + (1, 1, 20), \\
126 &= (2, 2, 15) + \Delta_L(3, 1, 10) + \Delta_R(1, 3, \bar{10}) + (1, 1, 6), \\
210 &= S_o(1, 1, 1) + S_e(1, 1, 15) + (2, 2, 10) + (2, 2, \bar{10}) \\
&\quad + (3, 1, 15) + (1, 3, 15) + (2, 2, 6).
\end{aligned}
\tag{12}
$$

The $G_{224D}-$ singlet $(1, 1, 1)$ in $54_H (210_H)$ was identified to be D-even (D-odd) leading to $SO(10) \to G_{224D}(G_{224})$ through its GUT scale VEV. Similarly the neutral component of $(1, 1, 15)_H$ in $45_H (210_H)$ was identified to be D-odd(D-even) leading to $G_{2213}(G_{2213D})$. In this manner a large number of new symmetry breaking chains were predicted [56–58] with interesting phenomenological consequences [56–58,99–103] including gauge coupling unification. Before the D-Parity properties of $210_H, 45_H, 54_H$ were known, LR discrte symmetry breaking at the GUT scale was employed through fine tuning assumption to predict left-right asymmetric gauge theory $G_{224}(g_{2L} \neq g_{2R})$ at $M_C \simeq 10^6$ GeV leading to bservable $n - \bar{n}$ oscillation and TeV scale $Z_R$ boson [104,105].

## 5 Theorems on vanishing uncertainties in GUTs

In a GUT, besides the RGE corrections due to running gauge couplings, there are other uncertainties due to GUT threshold corrections [106–110], Planck scale induced higher dimensional operators [111–113], and string threshold effects [114] on the model predictions of $\sin^2 \theta_W$ or $G_{224D}$ breaking intermediate scale $M_I = M_P$. This was also projected as a major source of uncertainty in $\sin^2 \theta_W$ prediction with Pati-Salam intermediate breaking [110]. In sharp contrast, the following three theorems were discovered to predict complete absence of such uncertainties establishing profoundly predictive nature of SUSY and non-SUSY GUTs with $G_{224D}$ intermediate symmetry.

**(1) Theorem-1** [111]: Whenever a grand unified theory possesses the gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C \times D (= G_{224D}, g_{2l} = g_{2R}))$ at the highest intermediate scale $(M_I)$, the one-loop GUT threshold contribution to $\sin^2 \theta_W(\mu), (\mu \geq M_I)$ by every class of superheavy particles (gauge bosons, Higgs scalars and additional fermions) vanishes. The result also applies with supersymmetry, infinite towers, or higher dimensional operators, and is independent of other intermediate symmetries at lower scales [111].

**(2) Theorem-2** [112]: In all symmetry breaking chains where the symmetry $G_{224D}$ occurs at the highest intermediate scale $M_I$, all higher order multi-loop corrections on $\sin^2 \theta_W(\mu)$ are absent in the mass range $\mu = M_I - M_U$. This theorem also holds with supersymmetry or string inspired models [114].

**(3) Theorem-3** [113]: In all symmetry breaking chains where $G_{224D}$ occurs at the highest intermediate scale $M_I$, the scale $M_I$ has vanishing contributions from all sources of corrections arising at mass scales $\mu > M_I$ [113].

These corrections also include those due to gravitational or Planck scale effects originating from higher dimensional operators and/or string threshold effects. Consequently, $M_I$ prediction of a SUSY $SO(10)$ has been shown to be unaffected by the

number of $16_H \oplus \overline{16}_H$ pairs above $\mu = M_I$ although it has the capability to change the value of $M_I$ only through lighter components in $16_H \oplus \overline{16}_H$ and/or $45_H$. One example of solutions is $M_I = 10^{12.5}$ GeV and $M_U = 10^{17.6}$ GeV indicating a very stable proton and string scale unification [113] with GUT fine-structure constant well below the perturbative limit. All the fields used in this SUSY $SO(10)$ belong to the string compacification model [114].

As a result of these theorems, it is interesting to examine SUSY and non-SUSY $SO(10)$ model predictions:

$$E_6 \text{ or } SO(10) \rightarrow G_{224D} \rightarrow SM. \tag{13}$$

In non-SUSY case, the intermediate scale remains fixed at $M_I \simeq 10^{13.6}$ GeV and $M_{GUT} = 10^{14.8}$ GeV. At first sight the GUT scale appears to be volnurable to Super Kamiokande limit on proton lifetime [115] for $p \rightarrow e^+\pi^0$. But the theorems also come to rescue. By fine-tuning when the scalar multiplet $\xi(2, 2, 15) \subset 126_H$ is placed at $M_I$, the theorem predicts no change in the values of $\sin^2 \theta_W$ or $M_I$ from the minimal case. But the RG effects do increases the unification scale leading to $M_{GUT} > 10^{15.5}$ GeV which easily evades the Super Kamiokande limit. In SUSY case $M_I \simeq 10^{14}$ GeV and $M_U \simeq 10^{16.5}$ GeV [113].

A theorem has been also proposed which is valid for threshold corrections due to Higgs representation that does not acquire VEV and has all degenerate components.

**(4) Theorem-4** [116]: Threshold corrections to unification scales, $\sin^2 \theta_W$ and $\alpha_S$ vanish for a Higgs multiplet of grand unification group which does not acquire VEV if we make the plausible assumption that all its submultiplets are degenerate in mass after symmetry breakings.

**(5) Vanishing Planck scale effect on SUSY $M_U$** [117]: Planck scale effects due to $5 - dim.$ operators are known to affect the GUT scale predictions substantially in a grand unified theory. But it has been shown that in SUSY $SO(10)$ breaking to $G_{2213}$ the D-Parity even and odd combinations arising from $210_H$ cancel out the effects of the two non-renormalisable corrections on $M_U$.

## 6 Leptogenesis within Gravitino constraint

In the RHN extended SM the reheating temperataure after inflation is to be as high as $T_{rh} \simeq 10^8 – 10^9$ GeV [118] leading to overproduction of gravitinos [119–121] and depletion of deuterium relic abundance below acceptable limits. Another draw back of high type-I seesaw scales is that the proposed mechanism can neither be directly verified in near future, nor can it be disproved. They also predict negligible LFV decay branching ratio (Br.) for $l_\alpha \rightarrow l_\beta \gamma (\alpha \neq \beta = e, \mu, \tau)$ and $\mu \rightarrow e\bar{e}e$ which have expermental limits $Br. \simeq 10^{-9} \rightarrow 10^{-13}$.

These issues have been addressed in the the SUSY $SO(10)$ breaking models [122–124]

$$SO(10) \xrightarrow{(M_U)} [\mathcal{G}_{2213D}] \xrightarrow{(M_P)} [\mathcal{G}_{2213}]$$
$$\xrightarrow{(M_R)} [\mathcal{G}_{213}] \xrightarrow{(M_Z)} SU(3)_C \times U(1)_Q. \tag{14}$$

The first stage of spontaneous symmetry breaking (SSB) is carried out by assigning GUT scale vacuum expectation values to the $\Phi_{54}$ of $SO(10)$ along the direction of a singlet under $G_{224}$ and $G_{2213D}$. The second step os SSB occurs when $(1, 1, 15)$

under $G_{224}$ contained in $210_H$ gets $VEV \simeq M_P$. At this stage D-parity remains intact with equal LR gauge couplings of $SU(2)_L$ and $SU(2)_R$, $g_{2L} = g_{2R}$ [56]. The second stage of SSB takes place by assigning vacuum expectation value to the D-Parity odd singlet also contained in $\Phi_{210}^{(2)}$ of $SO(10)$. By suitable fine tunings of the trilinear couplings beteen $\mathbf{210_H}$ and the $\mathbf{126_H} \oplus \mathbf{\overline{126}_H}$ or $\mathbf{16_H} \oplus \mathbf{\overline{16}_H}$ the right handed triplets $\mathbf{\Delta_R} \oplus \mathbf{\overline{\Delta}_R} \subset \mathbf{126_H} \oplus \mathbf{\overline{126}_H}$ and the RH doublets $\chi_{\mathbf{R}} \oplus \overline{\chi}_{\mathbf{R}} \subset \mathbf{16_H} \oplus \mathbf{\overline{16}_H}$ are made much lighter compared to their left-handed counterparts. By adopting higher degree of fine tuning for the RH triplet compared to the RH doublet, the components of the RH triplet pairs can be assigned masses between 100 GeV to a few TeV while the RH doublet pairs are kept heavier, but sufficiently lighter than the GUT scale. Although we do not assign any VEV directly to the neutral components of the RH-triplets in $\mathbf{126_H} \oplus \mathbf{\overline{126}_H}$, we find that the assigned VEV of the RH-doublet in $\mathbf{16_H}$, automatically induces the RH-triplet VEV. Smaller is the RH-triplet mass fixed by the D-parity breaking mechanism, larger is the induced triplet VEV.

This symmetry breaking gives the Yukawa Lagrangian near the intermediate scale

$$\mathcal{L}_Y = Y\overline{\psi}_L \psi_R \Phi + f\psi_R^T \tau_2 \psi_R \bar{\Delta}_R + F\overline{\psi}_R S \chi_R + \mu S^T S + H.c, \tag{15}$$

where $\psi_{L,R}$ are left- (right-) handed lepton doublets. In the $(\nu, N, S)$ basis this leads to the mass matrix

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & M_N & M_X \\ 0 & M_X^T & \mu \end{pmatrix}. \tag{16}$$

Here the $N - S$ mixing matrix arises through the VEV of the RH-doublet field with $M_X = Fv_\chi$, where $v_\chi = \langle \chi_R^0 \rangle$, and the RH-Majorana neutrino mass is generated by the induced VEV of the RH-triplet with $M_N = fv_R$, with $v_R = \langle \overline{\Delta}_R^0 \rangle$. The VEV of the weak bi-doublet $\Phi(2, 2, 0, 1) \subset 10_H$ of $SO(10)$ yields the Dirac mass matrix for neutrinos, $m_D = Y\langle \Phi^0 \rangle$.

The block diagonalization of this mass matrix results in a cancellation among the Type-I see-saw contributions and the light neutrino mass $m_\nu$ is dominated by the inverse see-saw,

$$m_\nu = -m_D \left[ M_X^{-1} \mu (M_X^T)^{-1} \right] m_D^T, \tag{17}$$
$$M_T = \mu - M_X \ M_N^{-1} \ M_X^T, \tag{18}$$
$$M = M_N + M_X M_N^{-1} M_X^T. \tag{19}$$

In this model the type-II seesaw contribution is negligible [124].

Assuming diagonal basis for RHN, $M_N = \text{diag}(M_{N_1}, M_{N_2}, M_{N_3})$, the model generates $N_i - S_j$ mixing angles,

$$\sin \xi_{ij} \simeq \frac{M_{X_{ij}}}{M_{N_i}}. \tag{20}$$

Purely from SUSY $SO(10)$ considerations, the method of keeping the relevant Higgs scalars substantially lighter than the GUT scale needed for pecision coupling unification has been discussed in [125]. As a typical example, the evolution of the gauge couplings and unification at the GUT scale are shown in Figure 2 that led to the solution

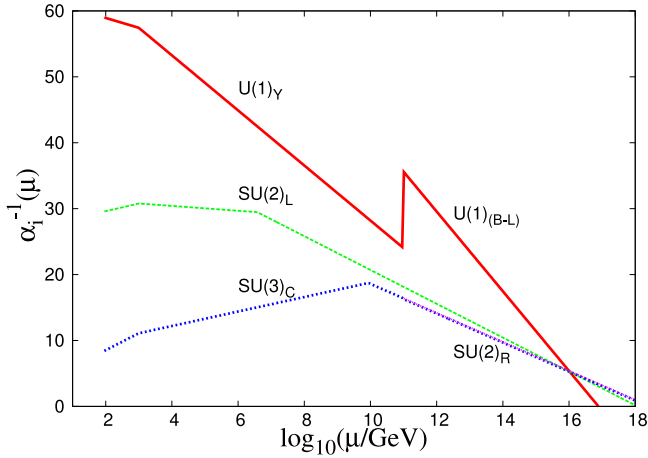$$M_R = 10^{11} \text{ GeV}, \quad M_U = 10^{16} \text{ GeV}, \tag{21}$$

**Fig. 2.** Evolution of gauge couplings leading to unification at the SUSY $SO(10)$ GUT scale $M_U = 10^{16}$ GeV.
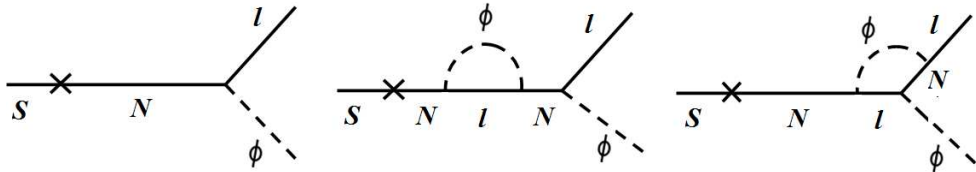


**Fig. 3.** The siglet fermion decay leptogenesis in SUSY $SO(10)$.

with $\alpha_G^{-1} = 5.3$ which is well within the perturbative limit. In Figure 2 the couplings for $SU(2)_R$ and $SU(3)_C$ are found to be almost ovelapping above the scale $M_R$ because of a fortuitous identity of their respective beta function coefficients and near equality of the boundary values at $M_R$ in this example. The changes in slopes at $M_\sigma$ and $M_C$ are clearly noticeable.

Proton lifetime in this model is cosistent with Super Kamiokande limit [115] As pointed out neutrino masses and mixings are fitted by inverse seesaw mediated by a singlet which also generates lepton asymmetry through its decay as shown in Figure 3.

The formula for the singlet fermion decay rate assumes the form,

$$\Gamma_{S_1} = \frac{1}{8\pi} M_{S_1} \frac{K_1}{K_2} \left[ (|\tilde{U}_{11}|)^2 \sin^2 \xi_{11} (Y_D^\dagger Y_D)_{11} + (|\tilde{U}_{12}|)^2 \sin^2 \xi_{32} (Y_D^\dagger Y_D)_{33} \right.$$
$$\left. + (|\tilde{U}_{13}|)^2 \sin^2 \xi_{23} (Y_D^\dagger Y_D)_{22} \right], \tag{22}$$

where $K_1$, $K_2$ are modified Bessel functions. Even though $Y_D$ is of the same order as the up-quark Yukawa matrix, the smallness of $\Gamma_{S_1}$, compared to the Type-I see-saw case, originates from two sources: (i) Allowed values of $M_{S_1} \ll M_{N_i} (i = 1, 2, 3)$, (ii) $\sin^2 \xi_{jk} \ll 1$ $(j, k = 1, 2, 3)$. These two features achieve the out-of-equilibrium condition at temperature $\sim M_{S_1}$ satisfying the gravitino constraint. A compact formula for CP-asymmetry has been also derived as a function of model parameters [124].

**Table 2.** Particle content of the model and their transformation properties under $S_4 \times SO(10)$.

| Fermions | | Higgs bosons | | | | | |
|---|---|---|---|---|---|---|---|
| $\Psi_i, (i = 1, 2, 3)$ | $S$ | $\Phi$ | $A_{1,2,3}$ | $\overline{\Sigma}_0 \oplus \Sigma_0$ | $H_0$ | $H_{1,2}$ | $H_{3,4,5}$ |
| $3' \times 16$ | $1 \times 54$ | $1 \times 210$ | $3 \times 45$ | $1 \times \overline{126} \oplus 126$ | $1 \times 10$ | $2 \times 10$ | $3 \times 10$ |

Defining $n_B$ as the net baryon number density over anti baryons and $n_\gamma$ as photon number density, the estimated baryon asymmetry turns out to be

$$\eta_B \equiv \frac{n_B}{n_\gamma} \simeq 10^{-2} \kappa \epsilon_1 \tag{23}$$

in good agreement with [22,23]:

$$(\eta_B)_{\text{expt}} = (6.15 \pm 0.25) \times 10^{-10}. \tag{24}$$

## 7 Flavour unification and fermion masses through SUSY $SO(10) \times S_4$

A number of attempts exist to explain neutrino masses as well as chargrd fermion masses using flavour symmetries [126]. In this section we discuss briefly how conservation of both the symmetries, D-Parity and R-Parity, guarantees an intermediate scale and all the fermion mass fittings through SUSY $SO(10) \times S_4$ flavour symmetry [125]. SUSY $SO(10)$ predicts R-Parity ($R_p = (-1)^{3(B-L)+2S}$) as its intrinsic gauged discrete symmetry for the stability of dark matter (wino, bino,neutralino etc.) whenever spontaneously broken through $126_H \oplus \overline{126}_H$. It has also the ability to predict space-time left-right discrete symmetry as a remnant of continuous gauge symmetry (=D-Parity) to survive down to lower intermediate scale. However minimal SUSY $SO(10)$ models with interesting prediction of type-I + type-II seesaw is known to forbid intermediate gauge symmetry breaking although, as we have seen in the previous section, lighter degrees of freedom resulting from fine tuning do permit intermediate left-right gauge symmetry with $M_R \ll M_{GUT}$. We consider $S_4$ flavor symmetry for three fermion generations through $SO(10) \times S_4$ [127–129] in the following symmetry breaking model $SO(10) \times S_4 \to_{M_U}^{210} G_{2213} \times S_4 \to_{M_R}^{126+\overline{126}} G_{213} \to_{M_W}^{10} U(1)_{em} \times SU(3)_C$.

The representation content of the $SO(10) \times S_4$ theory is shown in Table 2. In this model the $G_{2213}$ representations which have masses at the intermediate scale are

$$\Delta_L(3, 1, -2, 1) \oplus \Delta_R(1, 3, -2, 1) \oplus \overline{\Delta_L}(3, 1, 2, 1) \oplus \overline{\Delta_R}(3, 1, 2, 1),$$
$$6(2, 2, 0, 1), 3(1, 1, 0, 8), \tag{25}$$

where $\mathbf{6} = \mathbf{3} + \mathbf{2} + \mathbf{1}$, and $\mathbf{3}, \mathbf{2}$ and $\mathbf{1}$ are triplet, doublet, and singlet, respectively, under $S_4$. These result in the respective beta-function coefficients in the mass range

$\mu = M_R - M_U$:

$$a'_{BL} = 24, \ a'_{2L} = a'_{2R} = 10, \ a'_{3C} = 6. \tag{26}$$
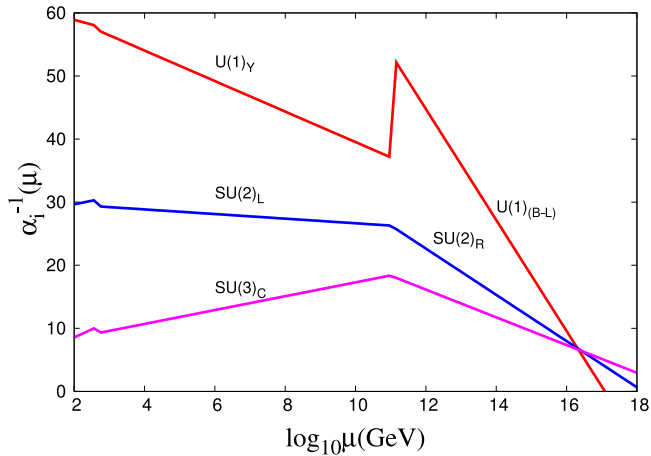
**Fig. 4.** Evolution of gauge couplings leading to unification at the SUSY $SO(10) \times S_4$ GUT scale $M_U = 2 \times 10^{16}$ GeV.

We find that values of the left-right symmetry breaking scale $M_R$ are permitted over a wide range,

$$5 \times 10^9 \text{ GeV} \leq \ M_\mathrm{R} \ \leq \ 10^{16} \text{ GeV} \tag{27}$$

but having almost the same value of unification scale $M_U = 2 \times 10^{16}$ GeV for all solutions. One example with $10^{13}$ GeV is shown in Figure 4. This model has capability to fit all fermion masses and mixings including neutrino data [125] by Type-I seesaw. The predicted proton lifetime is about $1-2$ orders longer than the Super Kamiokande limit [115].

## 8 Other applications in SUSY $SO(10)$

SUSY $SO(10)$ model building with $45_H$, or $54_H$, or $210_H$ combined with $126_H \oplus \overline{126}_H$ has a number of attractive predictions in neutrino physics, cosmology and all charged fermion mass fitting. It predicts type-I$\oplus$ type-II ansatz that fits oscillation data. The heavy RHN's mediating type-I or LH scalar triplet mediating type-II are capable of explaining baryon asymmetry of the universe via leptogenesis. Such $SO(10)$ breaking predicts R-Parity as gauged discrete symmetry that guarantees stability of dark matter. Despite these attractions, the SUSY $SO(10)$ theory starts becoming non-perturbative [130,131] at mass scales few times larger than the GUT scale $\mu \geq$ (few) $\times 2 \times 10^{16}$ GeV. A resolution of this difficulty has been suggested via GUT threshold effects [132] ensuring perturbativity till the Planck mass.

Currently Starobinsky [133,134] type inflation appears to describe the big-bang comology most effectively. Using the identified D-Parity properties of $SO(10)$ such an inflationary picture has been realised in SUSY $SO(10)$ [135] with double seesaw ansatz for neutrino masses and verifiable proton lifetime predictions in near future. With D-Parity broken at the GUT scale, SUSY $SO(10)$ predictions of low-mass $W_R$ bosons, proton decay, inverse seesw, and leptogenesis have been also investigated [136–138].
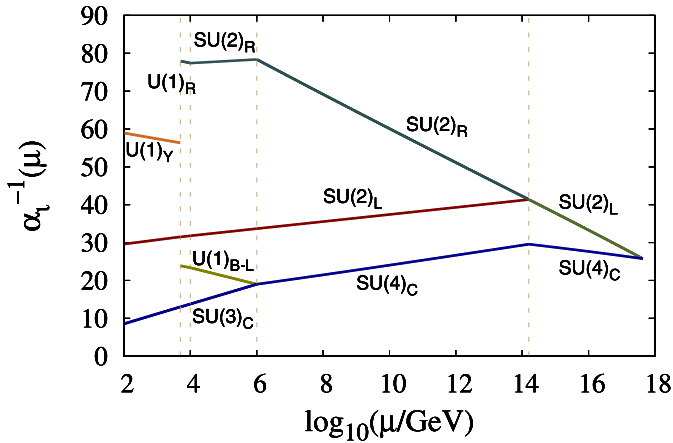
**Fig. 5.** High scale unification in Non-SUSY $SO(10)$ with prospects for verifiable $W_R, Z_R$ bosons, LFV and LNV decay.

## 9 Non-SUSY GUTs confronting particle physics issues

### 9.1 Intermediate breaking models

Even before emergence of MSSM unification, especially from CERN-LEP data [40–43], it was noted that SUSY may not be a compelling requirement for unification. This was worked out in detail in non-SUSY $SO(10)$ with one, two, or more intermediate gauge symmetries [56–58]. Two minimal examples with $G_{2213}$ or $G_{2213D}$ and $G_{224}$ or $G_{224D}$ intermediate gauge symmtries and others have been discussed including threshold effects [99,100,103].

Generally high scale seesaw models are not directly verifiable; they also predict negligible LFV branching ratios. However, it has been found [139,140] that if $G_{224D}$ occurs at higher scale, $G_{224}$ symmetry can survive down to $10^5$ GeV. In such a model the $G_{2213}$ breaking may occur at TeV scales leading to $W_R, Z_R$ bosons accessible to LHC. The model also has capability to predict LFV decay branching ratios only about $2-3$ orders lower than the current experimental limits. Neutrinoless double beta decay is predicted to be accessible by ongoing experimental searches even for normally ordered (NO) or invertedly ordered (IO) neutrino mass hierarchies as the LNV decay process is predicted by a low mass sterile neutrino of mass $\sim$10 GeV which is found to be a generic feature with Higgs representations $126_H \oplus 16_H$ as noted below. Gauge coupling unification in this model [139] is shown in Figure 5.

### 9.2 Verifiable $W_R, Z_R$, inverse seesaw, LFV, and $(\beta\beta)_{0\nu}$ with $G_{2213}$ intermediate breaking

Interestingly, if Planck scale effects are utilised [103], non-SUSY $SO(10)$ with D-Parity broken at the GUT scale having only the lone $G_{2213}$ intermediate symmetry gurantees verifiable $W_R, Z_R$ bosons with masses $1-10$ TeV, neutrino masses by inverse seesaw, experimentally accessible lepton flavour violating branching ratios, and neutrinoless double beta decay close to the current experimental limits even with hirarchical neutrino masses in concordance with cosmological bounds [141–144]. Other recent applications with D-Parity broken intermediate gauge symmetry have been discussed in [145–147].
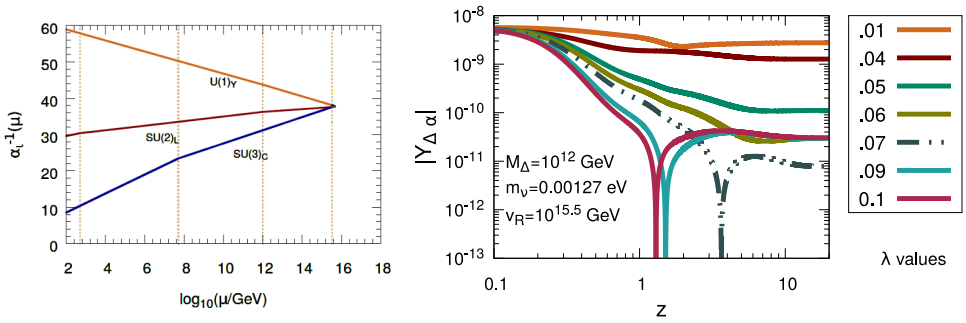
**Fig. 6.** Evolution of gauge couplings in the hybrid seesaw model of $SO(10)$ [148] as shown in the left panel. Prediction of baryon asymmetry as shown in the right panel.
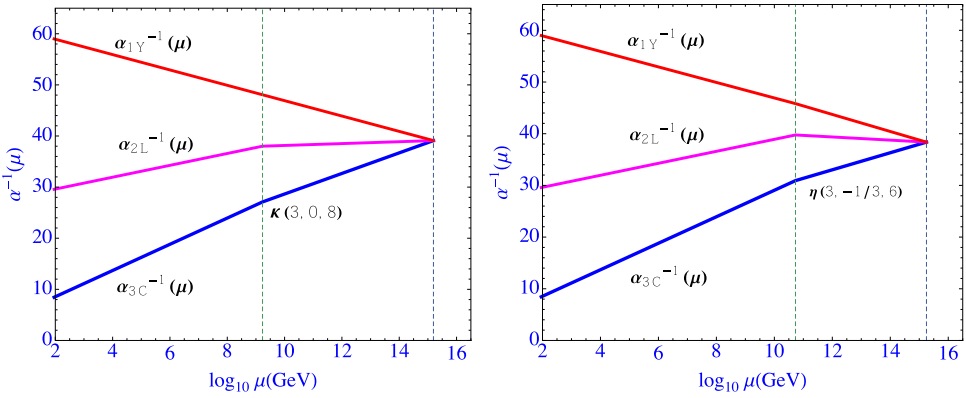


**Fig. 7.** Evolution of gauge couplings in Model-I with the real scalar submultiplet $\kappa(3,0,8) \subset 210_H$ of mass $M_\kappa = 10^{9.2}$ GeV as depicted by the first vertical line in the left panel. Unification of gauge couplings in Model-II due to the presence of the complex scalar component $\eta(3,-1/3,6) \subset 126_H$ at $M_\eta = 10^{10.7}$ GeV as shown in the right panel.

### 9.3 Hybrid seesaw, dark matter and leptogenesis

Without using any intermediate gauge symmetry but using only few lighter fields precision gauge coupling in one example [148] is shown in the left panel of Figure 6. Baryon asymmetry prediction of this model has been shown in the right panel of the same Figure 6.

### 9.4 Minimally modified grand deserts

In contrast to non-SUSY GUTs with one or more intermediate scales it has been recently shown that unification is possible with only one non-standard Higgs scalar $\kappa(3,0,8) \subset 210_H$ [149–151] or $\eta(3,-1/3,6) \subset 126_H$ with the respective masses $M_\kappa = 10^{9.2}$ GeV or $M_\eta = 10^{10.7}$ GeV. Coupling unification is shown in Figure 7. As outlined below these minimal $SO(10)$ models have interesting applications in dark matter decay [150] manifesting as monochromatic PeV enegy neutrinos detected at Ice-Cube
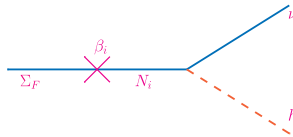
**Fig. 8.** Feynman diagram for dark matter decay $\Sigma_F \to \nu h$ manifesting as monochromatic PeV energy neutrinos at IceCube [152–155].

[152–155], Type-II seesaw prediction of heavy scalar triplet leptogenesis in $SO(10)$, verifiable LNV and LFV decays with type-II seesaw neutrino mass [156,157] in $SU(5)$ along with WIMP DM prediction, vacuum stability and observable proton decay [150].

### 9.5 Dark matter decay for PeV energy IceCube neutrinos

Each of the two models, Model-I and Model-II shown in Figure 7, predict fermionic DM decay [150] manifesting as monochromatic PeV energy neutrinos detected recently at IceCube [152–155]. Both the models account for neutrino mass via heavy RHN mediated canonical seesaw mechanism in concordance with neutrino data. These models predict three hierarchical RHNs which mix by different amounts with the fermionic singlet DM $\Sigma_F(1,0,1) \subset 45_F$ as a result of which the latter decays to produce the standard Higgs and the PeV energy neutrino: $\Sigma_F \to \nu h$. The decay mode is shown in Figure 8.

### 9.6 Triplet leptogenesis with new CP-asymmetry formulas

Ealier type-II seesaw dominance in SUSY or non-SUSY $SO(10)$ was achieved with an extended particle spectrum near the TeV scale [158–160]. But as noted above, even without having such extended spectrum near TeV scale, two minimal models [149,151] have been found to exhibit type-II seesaw dominance as they are also predicted by $SO(10)$ breaking through $SU(5)$ route

$$SO(10) \to SU(5) \to SM. \tag{28}$$

The RG evolution of gauge couplings in the two corresponding models are depicted through Figure 9 where in the left- panel (right-panel) unification is achieved by $\kappa(3,0,8)$ ( $\eta(3,-1/3,6)$).

Unlike such minimal grand desert modifications by only one non-standard lighter field below the GUT scale [149–151], unification models also exist with more than one non-standard lighter fields [161–164]. Whereas triplet fermionic DM $\sigma_F(3,0,1)$ of mass 2.7 TeV has been predicted in [161], the three unification models discussed in [151] predict a real scalar siglet DM or a real scalar singlet plus a fermionic triplet as DM with masses near $\simeq 1.0$ TeV. In addition they complete vacuum stability of the scalar potential and predict baryon asymmetry through new CP-asymmetry formulas in triplet leptogenesis. Proton lifetimes predicted by all the three models are accessible to ongoing experimental searches.

## 10 Summary and outlook

Besides the natural resolutions of gauge hierarchy problem and origin of three forces of nature, SUSY GUTs also accomplish the desired expectations for dark matter
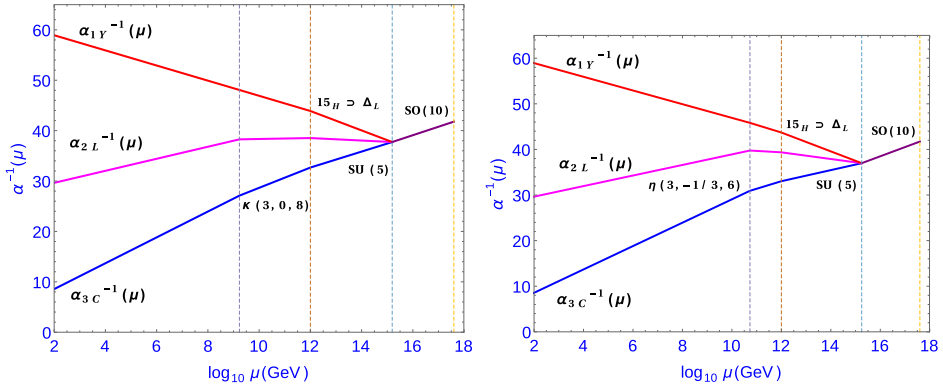
**Fig. 9.** Evolution of gauge couplings in Model-I with the real scalar submultiplet $\kappa(3,0,8) \subset 210_H$ of mass $M_\kappa = 10^{9.2}$ GeV as depicted by the first vertical line in the left-panel. Unification of gauge couplings in Model-II due to the presence of the complex scalar component $\eta(3, -1/3, 6) \subset 126_H$ at $M_\eta = 10^{10.7}$ GeV as shown in the right-panel.

and their stability, and baryogenesis via leptogenesis while matching the neutrino oscillation data through attractive seesaw mechanisms. They can also predict LFV decays closer to current experimental limits and verifiable proton decays. As pointed out SUSY $SO(10)$ is capable of representing all charged fermion masses with or without $S_4$ flavour symmetry [125]. If neutrinos are quasi-degenerate, SUSY GUTs with $S_4$ or $G_{224} \times S_4$, or even $G_{2213} \times S_4$ unify quark and lepton mixings at high scale and are capable of answering the puzzle as to why neutrino mixings are so different. It is high-time that evidence of SUSY shows up at LHC energies [165–167]. Once the hitherto non-appearance of SUSY is reconciled with anthropic principles [168,169], a large number of different GUT solutions with or without intermediate symmetries are capable of resolving puzzles confronting the standard model including gauge coupling unification, origins of neutrino and charged fermion masses, baryon asymmetry of the universe, dark matter with matter parity [170,171] as stabilising gauged discrete symmetry, vacuum stability of Higgs potential and proton decay prediction accessible to ongoing experiments. A novel ansatz for type-II seesaw dominance in a class of non-SUSY $SO(10)$ not only predicts new CP-asymmetry formulas for leptogenesis leading to baryon asymmetry of the universe, but it resolves the issues on dark matter, vacuum stability, and verifiable proton decay showing wide range of capabilities of these models [151]. Two interesting unification possibilities through minimal grand desert modifications [149,151] by only one intermediate mass scalar in each case and their various applications in solving puzzles confronting the SM are emphasized.

## Author contribution statement

M. K. Parida (Corresponding author) has made original contributions in different areas of High Energy Physics as cited and summarised in this paper which has been planned and outlined by him. Under the supervision of M. K. P. the co-author Riyanka Samantaray has rederived all equations involved and understood the ideas pertaining to this work. She has also carried out computations leading to different figures presented here.

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