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## Time-dependent Ginzburg–Landau model for light-induced superconductivity in the cuprate LESCO

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Abstract. Cavalleri and coworkers have discovered evidence of lightinduced superconductivity and related phenomena in several different materials. Here, we suggest that some features may be naturally interpreted using a time-dependent Ginzburg–Landau model. In particular, we focus on the lifetime of the transient state in  $La<sub>1.675</sub>Eu<sub>0.2</sub>Sr<sub>0.125</sub>CuO<sub>4</sub> (LESCO<sub>1/8</sub>)$ , which is remarkably long below about 25 K, but exhibits different behavior at higher temperature.

## 1 Introduction

In this brief note, we suggest that time-dependent Ginzburg–Landau models may be useful in interpreting the experiments of Cavalleri and coworkers (and other groups) that have demonstrated ultrafast phase transitions in materials responding to femtosecond-scale laser pulses.

It is impossible to do justice here to the complete literature relevant to these experiments, which is vast because the interaction of spin-ordering, charge-ordering, and superconductivity has been one of the most central issues in condensed matter physics for more than 30 yr. There is reason to believe, in fact, that spin- and charge-ordering in stripes is closely related to the origin of high-temperature superconductivity. We will instead focus on just the papers that are most directly relevant to light-induced superconductivity in the specific material  $La_{1.675}Eu_{0.2}Sr_{0.125}CuO_4$  (LESCO<sub>1/8</sub>) [\[1](#page-4-0)[–5\]](#page-4-1), represented by the results of references [\[1\]](#page-4-0) and [\[3\]](#page-4-2) shown in Figure [1.](#page-1-0)

The work of references  $[1–5]$  $[1–5]$ , and that cited in these papers, indicate that coherent three-dimensional light-induced superconductivity emerges when the competing coherent three-dimensional phase of in-plane stripes is "melted" by an ultrafast laser pulse.

Here, we will consider a simple time-dependent Ginzburg–Landau model of these competing phases:

$$
-\tau_1 \frac{dn_1}{dt} = \frac{\partial F}{\partial n_1} n_1, \quad -\tau_2 \frac{dn_2}{dt} = \frac{\partial F}{\partial n_2} n_2 \tag{1}
$$

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<span id="page-1-0"></span>Fig. 1. Left panel, taken from reference  $[1]$  with the original caption: Transient c-axis reflectance of  $LESCO<sub>1/8</sub>$ , normalized to the static reflectance. Measurements are taken at 10 K, after excitation with IR pulses at  $16 \,\mu\text{m}$  wavelength. The appearance of a Josephson plasma edge at 60 cm<sup>−</sup><sup>1</sup> demonstrates that the photoinduced state is superconducting. Right panel, taken from reference [\[3\]](#page-4-2) with the original caption: Phase diagram of LESCO, based on Supplemental Material at [http://link.aps.org/supplemental/10.1103/PhysRevB.91.020505,](http://link.aps.org/supplemental/10.1103/PhysRevB.91.020505) indicating regions of bulk superconductivity (SC) and static spin (SO) and charge (CO) order. The static stripes suppress c-axis coupling of the  $CuO<sub>2</sub>$  planes (inset cartoon, left), with bulk superconductivity restored at dopings in which the stripe order is reduced (inset cartoon, right).

$$
F = -a_1 n_1 + \frac{1}{2} b_1 n_1^2 + q_1^2 A(t)^2 n_1 - a_2 n_2 + \frac{1}{2} b_2 n_2^2 + q_2^2 A(t)^2 n_2 + c n_1 n_2 \tag{2}
$$

where  $n_1$  and  $n_2$  respectively represent condensate densities for the three-dimensional superconducting and stripe phases.

All the coefficients  $a_i, b_i, q_i$ , and c are in principle temperature as well as materials dependent (with  $q_i$  also frequency dependent). The terms involving  $a_i$  and  $b_i$  are standard in a Ginzburg–Landau description of superconductors (and various other systems). The terms involving  $q_i^2 A(t)^2 n_i$  result from a Ginzburg–Landau description averaged over one wavelength of the laser radiation with

$$
\psi^* \frac{1}{2m} \left( -i \nabla - \frac{q_{\text{eff}}}{c} \mathbf{A}(t) \right)^2 \psi \longrightarrow q^2 A(t)^2 n, \quad n = \psi^* \psi \tag{3}
$$

if the wavelength of the radiation is large compared to the length scale for variations in the order parameter. (The bare kinetic energy from  $\nabla^2$  is contained in the other parameters, with any shift in kinetic energy approximately absorbed in the  $q_i^2 A(t)^2 n_i$ term. For a laser field oscillating with a single frequency  $\omega$ , the average intensity is proportional to  $A(t)^2$ .) We note that (i) charge- and spin-density waves are similar in some respects to superconductivity, so the symmetry in  $F$  is natural for a simplest model in the present context, and (ii) the essential point is just that both the stripe and superconducting phases couple to an oscillating electromagnetic field (with intensity proportional to  $A^2$ ). The term  $c n_1 n_2$  describes the fact that two competing phases – with very different length scales, textures, and even topologies – must both recruit the same electrons, so that one tends to frustrate the other, as has long been

known. The form for the time dependence is chosen because it gives an exponentially fast rise time for small  $n_i$ , and also an exponentially slow decay time, so that  $n_i$ remains positive. An extra feature of the model is that small random fluctuations are introduced in each  $n_i$  at each time step, to simulate the physical (thermal and quantum) fluctuations of an order parameter. Without these fluctuations,  $n_i$  could never recover after going to zero. Finally, we note that, with this form for the time dependence, there is an exponentially fast approach to equilibrium for both phases, from either below or above the equilibrium values of  $n_i$ ,

The time-dependent equations are

$$
\tau_1 \frac{dn_1}{dt} = \left[ a_1 - \left( b_1 n_1 + q_1^2 A(t)^2 + c n_2 \right) \right] n_1 \tag{4}
$$

$$
\tau_2 \frac{dn_2}{dt} = \left[ a_2 - \left( b_2 n_2 + q_2^2 A(t)^2 + c n_1 \right) \right] n_2 \tag{5}
$$

with the following time-independent solutions: either  $n_1 = 0$  or

$$
n_1 = \frac{a_1 - q_1^2 A(t)^2 - c n_2}{b_1} \tag{6}
$$

and either  $n_2 = 0$  or else

$$
n_2 = \frac{a_2 - q_2^2 A(t)^2 - c n_1}{b_2} \tag{7}
$$

(with unphysical negative solutions excluded and never reached in a numerical solution).

One can solve for  $n_1$  and  $n_2$ , but what is most interesting here is the qualitative behavior:

- 1. If  $q_2^2 A^2 > a_2$ , there will be a nonthermal "melting" of an initial stripe phase. Then if  $a_1 > q_1^2 A^2$ , the superconducting phase will emerge, as observed.
- 2. Depending on the specific parameters for a given material and set of conditions, there may be no ordered phase, or either, or both coexisting, as is consistent with a large body of experimental and theoretical work.
- 3. There is a reciprocity inherent in the free energy: the superconducting phase is just as effective in blocking the stripe phase as vice-versa, in the sense that the same coefficient  $c$  is involved. This can explain why the superconducting phase persists for an extremely long time in the low-temperature results of references [\[1\]](#page-4-0) and [\[3\]](#page-4-2) – at least 100 picoseconds and perhaps up to nanoseconds and longer, for temperatures below about 25 K.
- 4. However, the coefficient c depends on the character of both phases. This appears to be reflected in the experimental results above about  $25 K [3]$  $25 K [3]$ , where the spinand charge-ordering undergoes a change of character to a different phase, as can be seen in the right-hand panel of Figure [1,](#page-1-0) taken from reference [\[3\]](#page-4-2). According to reference  $[3]$ , "Below  $T_{\text{SO}}$ , the lifetimes remain temperature independent. Above  $T_{\rm SO}$ , where only static charge order remains, the lifetime drops exponentially with base temperature.... The exponential dependence of the relaxation between  $T_{\text{SO}} < T < T_{\text{CO}}$  can be reconciled with the expected kinetic behavior for a transition between two distinct thermodynamic phases separated by a free energy barrier." If  $c$  is smaller for the higher-temperature phase, then



<span id="page-3-0"></span>Fig. 2. Simulation for two competing phases with order parameters  $\psi_1$  and  $\psi_2$ . The order parameter  $\psi_i$ , which for the present simple model is real, is related to the condensate density by  $n_i = \psi_i^2$ . When the dominant phase 2 ("stripes") is suppressed by the laser pulse between  $t = 10$  and  $t = 30$ , the other phase 1 ("superconductivity") emerges and persists indefinitely after the laser pulse has finished.

the metastability of the superconducting phase will be weakened, permitting a relatively rapid activated transition back to the more stable phase.

A typical numerical solution of the above equations for a qualitative model is shown in Figure [2,](#page-3-0) with a model laser pulse having the form

$$
A(t) = A_0 \sin \left( \pi (t - t_0)/2\tau \right) \sin \left( \omega (t - t_0) \right), \qquad t_0 < t < t_0 + \tau \tag{8}
$$

where  $t_0 = 10$ ,  $\tau = 20$ ,  $A_0 = 10$ , and  $\omega = 2$ . (This form closely resembles a Gaussian envelope modulated by oscillations with frequency  $\omega$ .) The dominant phase ("stripes") has parameters  $\tau_2 = 5$ ,  $a_2 = 2$ ,  $b_2 = 1$ ,  $q_2^2 = 2$ , and the other phase ("superconductivity") has  $\tau_1 = 5$ ,  $a_1 = 1.8$ ,  $b_1 = 0.9$ ,  $q_1^2 = 0$ , with  $c = 2$ .

Both of the main qualitative features of Figure [2](#page-3-0) are similar to what is observed in the experiments: First, when the dominant phase is suppressed by the laser pulse, the other phase quickly emerges. Second, the other phase persists for an indefinite period of time after the pulse is finished, in a robust metastable state.

The present model can clearly be extended in many ways, with realistic models constructed for specific materials, but the present note is meant only to demonstrate its qualitative potential.

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## Author contribution statement

Roland E. Allen originated the project in consultation with M. Ross Tagaras and Jian Weng, who performed the calculations.

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