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Review

Transitions without connections: quantum states, from Bohr and Heisenberg to quantum information theory

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Abstract. In his recent paper, L. Freidel noted that instead of representing the motion of electrons in terms of oscillators and predicting their future states on the basis on this representation, as in the previous, classical, electron theory of H. Lorentz, quantum theory was, beginning nearly with its inception, concerned with the probabilities of *transitions between states* of electrons, without necessarily representing how these transitions come about. Taking N. Bohr's and then W. Heisenberg's thinking along these lines in, respectively, Bohr's 1913 atomic theory and Heisenberg's quantum mechanics of 1925 as a point of departure, this article reconsiders, from a nonrealist perspective (which suspends or even precludes this representation of the mechanism behind these transitions), the concept of quantum state, as a *physical concept*, in contradistinction to *the mathematical concept of* quantum state, a vector in the Hilbert-space formalism of quantum mechanics. Transitions between quantum states appear, from this perspective, as "transitions without connections," because, while one can register the change from one quantum phenomena to another, observed in measuring instruments, we have no means of representing or possibly even conceiving of how this change comes about. The article will also discuss quantum field theory and, in closing, briefly quantum information theory as confirming, and giving additional dimensions to, these concepts of quantum state and transitions between them.

1 Introduction

Niels Bohr's atomic theory (hereafter Bohr's theory), introduced in 1913 and initially concerned with the hydrogen atom, was even more radical than those, already revolutionary, of his main predecessors, Max Planck and Albert Einstein, on whose ideas Bohr's theory was built.[1] Indeed, I would argue that Bohr's concept of "quantum jumps," defining his theory, was as radical and important as any innovation in the

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[1] I adopt the designation "Bohr's theory" to include its subsequent development, including in dealing with more complex atoms, by Bohr and others from 1913 until W. Heisenberg's introduction of quantum mechanics in 1925. Here I will only be concerned with fundamentals of Bohr's theory, rather offer a historical account of it. Two helpful treatments are Kragh (2012) and Folse (2014).

history of quantum theory, especially if one properly considers the architecture of this concept. Some aspects of this concept and of Bohr's theory or the old (semi-classical) quantum theory, which preceded quantum mechanics, are still often missed or, to take a more positive view, continue to be rediscovered. Thus, a new angle on the development of quantum theory from the old quantum theory to quantum mechanics was recently suggested by L. Freidel, with possible implications for quantum gravity (Freidel, 2016). Freidel noted that instead of representing the motion of electrons in terms of oscillators and predicting their future states on the basis on this representation, as in the previous, classical, electron theory of H. Lorentz and his followers, quantum theory was concerned with the probabilities of *transitions between states*. Freidel argues as follows:

> The classical dispersion formula of Drude-Lorentz contained the sum over s. The set of states s represented a collection of linear oscillators, classically thought of as a set of electron orbits, that is the set of classical objects. The main point and the deep shift behind [the development, leading to the discovery of] quantum mechanics, was a gradual realization that instead of describing the set s as a set of states of a classical system, one needs to replace it [with] a collection of processes $s \rightarrow (s \rightleftarrows m)$. That is, the transitions between the stationary states s under study, and all the other possible states of the system labeled by n. It might look at first sight like a philosophical consideration empty of physical consequences, a relabeling of our understanding. But this is a deep conceptual shift, because it implies that we should not assign a physical reality to [the] eternal state, as we do in classical physics, [but] only to processes described as relations between states. And this is the shift that opened up the way towards the discovery of quantum mechanics. (Freidel, 2016)

This article takes advantage of this important observation to define the physical concept of quantum state, along with that of transitions between quantum states, from a nonrealist perspective. From this perspective, "relations between states" cannot be seen as "processes," at least continuous and causal processes, but only as "transitions without connections." On the other hand, quantum states between which these transitions occur, and these transitions themselves can be assigned reality, but this assignment and the conception of this reality are very different from that of classical physics or relativity.

In Bohr's theory, these transitions were between states *associated* with the so-called "stationary states," conceived as classical orbits along which the electrons moved around the nucleus. I speak of states associated with stationary states, because "stationary" only means that the electrons remained in their orbits with the same energy, were in the same "energy-state," while continuously changing their position or their "position-state" along each orbit. On the other hand, the electrons would discontinuously, by "quantum jumps," change their energy states, or their other states, by moving from one orbit to another. Bohr's theory abandoned the aim of physically or mathematically representing such transitions. This makes even the term "jump" potentially misleading, as suggesting some representation. Electrons do not jump: quantum states discontinuously change, and there is no story to be told and no concept (such as "jump") to be formed about how they do this. Extended, with Heisenberg, to all quantum events, thus defined by transitions without connections between them, this is a decisive shift in our understanding of the nature of physical reality, which grounds my argument in this article.

Bohr's approach, by then used by others as well, was taken up by Heisenberg, leading him to his discovery of quantum mechanics in 1925, by not only readily accepting this lack of a representation of transitions between orbital quantum states but also by adopting an even more radical way of thinking about the atomic constitution of nature. He also abandoned a mechanical representation of stationary states (Heisenberg, 1925). In Heisenberg's scheme, there were only states and transitions between them, and probabilities of these transitions, which quantum mechanics could predict, without representing either these states or these transitions. If one were to use the language (again, ultimately inapplicable), of Bohr's theory, then in Heisenberg's theory there were only quantum states and quantum jumps. Heisenberg's scheme was developed into a proper matrix mechanics by M. Born, P. Jordan, and Heisenberg himself (Born and Jordan, 1925; Born et al., 1926), and was given its first interpretation by Bohr in 1927, defining in Heisenberg's phrase, "the Copenhagen spirit of quantum theory" [*Kopenhagener Geist der Quantenheorie*] (hereafter "the spirit of Copenhagen") (Heisenberg, 1930, p. iv).[2]

The concept of quantum state proposed here refers to a *physical state*, as against the mathematical concept of "quantum state," a vector in a Hilbert-space in the mathematical formalism of quantum mechanics, to which the term "quantum state" commonly refers. This use of the term "state" is misleading, especially in the present, nonrealist, view, according to which it is only part of the quantum-mechanical mathematical machinery for predicting, in probabilistic or statistical terms, the outcomes of quantum experiments, as effects of the interactions between quantum objects and measuring instruments. The transitions between quantum states are seen as "transitions without connections" because, while we can register a change observed in *quantum phenomena*, as a change in the (classical) state of the measuring instrument used impacted by a *quantum object*, we have no means of representing or even conceiving of how this change comes about.[3]

The next section outlines Bohr's atomic theory and the general conceptual framework offered in this article, based on this theory and Heisenberg's approach to quantum mechanics and Bohr's interpretation of it, considered in Section 3. Section 4 considers the implications of Heisenberg's thinking in his approach for the nature of fundamental physics and will comment on quantum field theory and quantum information theory.

2 Bohr's theory: from quantum postulates to quantum states

The starting assumptions of Bohr's theory, known as Bohr's "quantum postulates," were manifestly in conflict with both classical mechanics and classical electrodynamics. These postulates were as follows:

1. Bohr postulated the existence of stationary states of electrons in the atom, at which they could remain in orbital motion, and discontinuous shifts, "quantum jumps," between these states, resulting in the emission of Planck's quanta of radiation, with the energy change $h\nu = E_1 - E_2$ (h is Planck's constant and ν is the frequency of radiation), without electrons radiating continuously while remaining in orbit.

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[2] The designation "the spirit of Copenhagen" is preferable to a more common "Copenhagen interpretation," because there is no single such interpretation, even in the case of Bohr who changed his views several times. This is true about most prominent rubrics in a long list of interpretations of quantum mechanics – Copenhagen, many-worlds, consistent-histories, modal, relational, and so forth. Each contains multiple versions. There is a great deal of the literature dealing with each interpretation and the relationships among them. Standard reference sources, such as *Wikipedia* ("Interpretations of Quantum Mechanics"), would list and summarize most common rubrics.

[3] By quantum phenomena, I refer to those observed physical phenomena in considering which Planck's constant, h, must be taken into account. In the present view, reflected in my emphasis, quantum phenomena are essentially different from quantum objects, which can never be observed as such, and are beyond representation or even conception, physical or mathematical.

2. Bohr also postulated there would exist a lowest energy level at which electrons would not radiate, but would only absorb, energy.

Bohr abandoned, as hopeless, an attempt to offer a mechanical explanation for these transitions: "The dynamical equilibrium of the system in the stationary states can be discussed by help of the ordinary mechanics, while the passing of the system between different stationary states cannot be treated on that basis.... [T]here obviously can be no question of a mechanical foundation of the calculations given in this paper" (Bohr, 1913, pp. 7, 15). As noted above, the classical electron theory of Lorentz and his followers considered the probability of finding an electron in a given state, under the underlying realist assumptions, in particular that of (causally) representing the motion of electrons, say, in terms of oscillators. Bohr's theory was instead concerned with the probabilities of *transitions* between *states*, specifically those associated with stationary states, where the electrons have the same energy or are in the same energy state (while changing their position state along each orbit), without assuming the possibility of representing these transitions as continuous connections, or even assuming them to be. Hence, I speak of these transitions as "transitions without connections." As a result, Bohr's theory of these transitions abandoned causality or, more accurately, *classical* causality, roughly, a connection by means of which something, *the cause*, leads to something else, *the effect* (I shall discuss classical causality in more detail below). Neither the time of the emission of an energy quantum (when jumping to a lower energy state) nor the direction angle of a quantum jump was causally determined. This worried both Rutherford (upon his first reading of Bohr's paper, which he published nevertheless) and Einstein (upon reading it), but evidently not Bohr or, later, Heisenberg (Plotnitsky, 2016, pp. 60–61). However, from the classical viewpoint mysteriously, one could still estimate probabilities of transitions between states, which Bohr's theory was able to do and Heisenberg's quantum mechanics could do even better. Accordingly, one could focus not on discrete (or, as E. Schrödinger tried later, continuous, wave-like) quantum *objects* and their continuous behavior, as in classical mechanics, but on discrete *states* of these objects and probabilities of predicting the transitions between these states. Schrödinger's waves were, in this view, rethought as symbolic entities associated with probabilities of quantum predictions.

The situation, I argue here, invites a new concept of physical state, quantum state, in contradistinction to classical mechanics and relativity, where the concept of state is coextensive with the concept of motion, within the same mathematical representation and where it can, by the same token, be defined independent of our interaction with the object considered. By contrast, quantum states cannot be unambiguously considered apart from the interaction between quantum objects and measuring instruments, and are, in the present view, only definable in terms of effects of these interactions on these instruments (effects that are not assigned classical causes), but not in themselves. The deeper and more urgent reasons for this rethinking became apparent only with quantum mechanics.

Bohr's theory was a major step on this trajectory. In yet another audacious move, Bohr *dissociated* the frequency of the light emitted by the atom from the frequency at which the electron orbited the atom. In the Bohr formula, $h\nu = E_1 - E_2$, there are two electron frequencies for E_1 and E_2, that of the electron in its initial orbit and that of the electron in its final orbit; *neither* of these frequencies coincides with the frequency ν of the emitted radiation. Energies for orbiting electrons were whole number multiples of h multiplied by half of the final orbital frequency, $E = nh\nu$. It was thus half of the energy, $E = nh\nu$, that Planck in deriving his black body radiation law assumed for his oscillators. This was a very radical idea at the time. According to A. Stone, who noted that Einstein did not dare to take this step, even though he did consider something along these lines: "This was a pretty crazy notion to a classical physicist, for whom light was *created* by the acceleration of charges and must necessarily mirror the frequency of the charge motion" (Stone, 2015, p. 177). Bohr was well aware of the radical nature of this step. As he stated in his paper: "How much the above interpretation differs from an interpretation based on the ordinary electrodynamics is perhaps most clearly shown by the fact that we have been forced to assume that a system of electrons will absorb radiation of a frequency different from the frequency of vibration of electrons calculated in the ordinary way" (Bohr, 1913, p. 16). According to Heisenberg's later assessment on the situation, "the discrepancy between the calculated orbital frequency of the electrons and the frequency of the emitted radiation, had to be interpreted as a limitation to the concept of the electronic orbit" (Heisenberg, 1962, p. 41). In 1925, this view compelled Heisenberg to abandon the association of stationary states with orbits and thus any quantum behavior with a geometrical mechanical picture. Heisenberg's greatest contribution was to give a mathematics to this new physics, the mathematics of quantum mechanics, still our (nonrelativistic) theory of quantum phenomena. Before I consider Heisenberg's work on quantum mechanics, I would like to discuss the epistemological underpinnings and implications of this physics.

I begin with the concept of reality, which is very general and applies to the concepts of reality found in most, even if not all (which would be impossible), currently available versions of realism and nonrealism. By *reality* I refer to that which exists or is assumed to exist, *without any claim concerning the character of this existence*, the type of claim that defines realism. I understand existence as a capacity to have effects on the world with which we interact and which has such effects upon itself. In physics, the primary reality considered is that of nature or matter, including that of fields or that to which the concept of field, classical or quantum, would relate. This or any other idea of nature or matter (such as that of fields) is still a product of thought, generally assumed, however, to be a product of material processes in the brain, and thus of matter. Matter is commonly, but not always (although exceptions are rare), assumed to exist independent of our interaction with it, and to have existed when we did not exist and to continue to exist when we will no longer exist, which may be seen as defining the independent existence of matter.

Physical theories prior to quantum theory have been realist, commonly *representational* realist theories. Such theories aim to represent, usually (classically) causally, the corresponding objects and their behavior by mathematical models, assumed to idealize how nature works, an assumption sometimes referred to as "scientific realism." Thus, classical mechanics (used in dealing with elemental individual objects and small classical systems), classical statistical mechanics (used in dealing, statistically, with large classical systems, whose individual constituents are assumed to be described by classical mechanics), chaos theory (used in dealing with classical systems that exhibit a highly nonlinear behavior), or classical electromagnetism (used in dealing with continuous fields) are realist theories. I shall be primarily concerned with these types of realist theories and the corresponding models.[4] Realism, representational or nonrepresentational (defined below), need not be mathematical. It could also, for example, in certain biological theories, be scientific without being mathematical, but not without being conceptual.

In his *Physics and Philosophy*, Heisenberg used the role of concepts in theoretical physics to argue that "the Copenhagen interpretation of quantum theory" (as he

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[4] Their status as realist could be questioned, for example, on Kantian lines, even in the case of classical mechanics, where the representational idealizations used are more in accord with our phenomenal experience, which is only partially the case in relativity theory. However, these theories still allow for viable idealized realist and causal models. This appears to be much more difficult in quantum theory, which, at least thus far, has to be irreducibly probabilistic on experimental grounds even in considering elementary individual quantum objects and processes, or in the present view, transitions. This excludes a deterministic model, although not a causal or realist model.

defined it, although this part of his argument applies to the spirit of Copenhagen in general and specifically to Bohr's view) was not positivistic. In this emphasis, Heisenberg followed Einstein, with whom he had important exchanges concerning the subject and the relationships between observation and theory in physics, following Heisenberg's invention of quantum mechanics (Heisenberg, 1962, pp. 45–46; Heisenberg, 1989, p. 30; Plotnitsky, 2016, pp. 42–44). The phrase "the elements of reality" used by Heisenberg in the passage I am about to cite is, too, borrowed from Einstein, who often used it, most famously in the EPR paper (Einstein et al., 1935, p. 138). Heisenberg says: "The Copenhagen interpretation of quantum theory is in no way positivistic. For, whereas positivism is based on the sensual perceptions of the observer as the elements of reality, the Copenhagen interpretation regards things and processes which are describable in terms of classical concepts, i.e., the actual, as the foundations of any physical interpretation" (Heisenberg, 1962, p. 145). "The actual" here refers, following Bohr, to what is observed in measuring instruments, but, in this or other interpretations in the spirit of Copenhagen used as part of classical physical concepts. I shall return to the role (often misunderstood) of classical concepts in Bohr's argumentation later. My main point at the moment is the fundamental role of concepts in theoretical physics. Quantum mechanics uses plenty of concepts that are not classical, both physical, such as complementarity or the uncertainty relations, and mathematical.

While in the spirit of Copenhagen, "the Copenhagen interpretation" invoked here and discussed in Heisenberg's book is shaped by a mixture of Heisenberg's own and Bohr's views, from which Heisenberg departs here and elsewhere in his later writings, in particular, in his understanding of complementarity and in introducing the concept of "potentiality" or *potentia*, not found in Bohr. Heisenberg, at this stage (or even already by 1930s), also appears to have believed in the capacity of mathematics to represent the ultimate structure of matter, a belief not shared by Bohr either. Bohr would, however, have agreed that his understanding of the quantum-mechanical situation was "in no way positivistic," in part for the reasons stated by Heisenberg. Moreover, Bohr's interpretation was in fact concerned, essentially concerned, with quantum objects, even though they could not be observed as such, but only inferred from their effects on measuring instruments. It is not us but nature that, in its interaction with us, via our experimental technology (which includes our bodies that observe these effects but which consists of so much more) is responsible for these effects, even though we cannot know or possibly even conceive how these effects come about. It is true that we, our bodies, and our technology are nature, too. But neither are sufficient in themselves to produce these effects, which require the quantum constitution of nature apart from us and our technology, or what compels us to speak of this constitution, assuming that the term "constitution" applies.

One could also define another type of realism. This realism encompasses theories that would presuppose an independent structure of reality governing the behavior of the ultimate objects these theories consider, while allowing that this architecture cannot be represented, even ideally, either at a given moment in history or perhaps ever, but if so, only due to practical limitations. In the first eventuality, a theory that is merely predictive may be accepted for lack of a realist alternative, but with the hope that a future theory will do better, in particular as a realist theory of the representational type.

Realism of either type is abandoned in nonrealist interpretations of quantum phenomena and quantum mechanics, beginning with that of Bohr, which was introduced around 1927 and developed along nonrealist lines by 1930s. Such interpretations assume quantum mechanics to be a strictly probabilistically or statistically predictive theory, while suspending or even precluding a representation or conception of the quantum-level reality. The probabilistic or statistical character of quantum predictions must, however, be equally maintained by realist interpretations of these theories or alternative theories (such as Bohmian mechanics) to accord with what is observed in quantum experiments, where only probabilistic or statistical predictions are possible. This is because the repetition of identically prepared experiments in general leads to different outcomes, and, unlike in classical physics, this difference cannot be diminished beyond the limit defined by Planck's constant, h, by improving the capacity of our instruments, as manifested in the uncertainty relations, which would remain valid even if we had perfect instruments.

Nonrealist interpretations of quantum mechanics, at least those, in the spirit of Copenhagen, considered in this article, do assume the concept of *reality*, without, in contrast to realist theories, making any claims concerning the *character* of this existence, which makes this concept of *reality* that of "reality *without* realism" (RWR), a concept introduced in (Plotnitsky, 2016; Plotnitsky and Khrennikov, 2015). The concept of "transitions without connections" between quantum states implies this concept of reality, applied to that which makes such transitions possible. The existence of quantum objects or of something that leads to this idealization (it is still an idealization, different as it is from those of classical physics) is inferred from the totality of effects they have on our world, specifically on experimental technology.[5] Nothing, however, could be said or, in the strongest form of the nonrealist, RWR-type, view even be thought concerning *what* happens between quantum experiments, thus placing the ultimate nature of reality, that of quantum objects, states, and processes, beyond conception – philosophical, physical, or mathematical. A mathematical conception of reality may be divorced from any physical or philosophical conception of it. The RWR-type view makes such words as "ultimate nature," or "objects," "process," and "quantum," or even "reality" or "existence," provisional and ultimately inapplicable. It is reasonable (although not in principle imperative) to assume that something "happens" or "changes," for example, that an electron changes its quantum state in an atom, say, from one energy level to another, between observations that then registers this change. But, in the RWR-type view, one can do so only if one keeps in mind the provisional nature of such words as "happen," "change," or "atom," which are ultimately inapplicable in this case, more than any other words or concepts. A change in the quantum state of an electron only manifests itself, as an effect, in what is observed in measuring instruments from one measurement to the other. Accordingly, quantum states are defined here only as manifested in such effects, but not as anything considered in terms of their independent properties. As Heisenberg argued:

> There is no description of what happens to the system between the initial observation and the next measurement. . . . The demand to "describe what happens" in the quantum-theoretical process between two successive observations is a contradiction in adjecto, since the word "describe" refers to the use of classical concepts, while these concepts cannot be applied in the space between the observations; they can only be applied at the points of observation. (Heisenberg, 1962, pp. 47, 145)

The same, it follows, applies to the word "happen" or any word we use. But we must use words, even when we try to restrict ourselves to mathematics as much as possible. As Heisenberg also noted, "the problems of language here are really serious. We wish to speak in some way about the structure of the atoms and not only about 'facts' – the latter being, for instance, the black spots on a photographic plate or the water droplets in a cloud chamber. But we cannot speak about the atoms in ordinary language" (Heisenberg, 1962, pp. 178–179). As indicated, however, in Heisenberg's

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[5] For an analysis of the concept of quantum object from a realist perspective, see Jaeger (2013).

later view, in place by the time this statement was made, mathematics could, in principle, represent the structure of the atomic or quantum constitution of matter.[6]

I now turn to the question of causality, which, along with and correlative to the question of reality and realism, has been central to the discussions of and debates concerning quantum theory. I shall first comment, briefly, on the concepts of indeterminacy, randomness, and chance, as these concepts are defined and used here, because these concepts, too, can be defined otherwise. In the present definition, indeterminacy or chance is a more general category, while randomness refers to a most radical form of indeterminacy, in which case, the events considered cannot be assigned a probability. Indeterminacy, including randomness, and chance are sometimes defined differently, too, giving chance a more ontological flavor, for example, but, I shall, for the sake of economy, consider them synonymous here and refer only to indeterminacy. An indeterminate, including random, event may or may not result from some underlying causal process, whether this process is accessible to us or not. The first eventuality defines classical indeterminacy or randomness, conceived as ultimately underlain by a hidden classically causal process; the second defines the irreducible indeterminacy and randomness, in the absence of such an underlying causal process, which absence is, as discussed below, nearly automatic in the nonrealist, RWR-type, views.[7] The ontological validity of the second concept cannot be guaranteed, because one cannot definitively ascertain that this indeterminacy is not underlain by a classically causal process.[8] Indeterminacy or randomness is an assumption that can only be practically justified, as it is in nonrealist interpretations of quantum mechanics.

The difference between probability and statistics is important in the present context. "Probabilistic" commonly refers to our estimates of the probabilities of either individual or collective events, such as that of a coin toss or of finding a quantum object in a given region of space. "Statistical" refers to our estimates concerning the outcomes of identical or similar experiments, such as that of multiple coin tosses or repeated identically prepared experiments with quantum objects, or to the average behavior of certain objects or systems.[9] The Bayesian understanding defines probability as a degree of belief concerning a possible occurrence of an individual event on the basis of the relevant information we possess. This makes the probabilistic estimates involved, generally, subjective, although there may be agreement (possibly among a large group of individuals) concerning such estimates. The frequentist understanding, also referred to as "frequentist *statistics*," defines probability in terms of sample data by emphasis on the frequency or proportion of these data, which is considered more objective, although this claim requires some caution. As explained earlier, in quantum physics, exact predictions are in general impossible even in dealing with elemental individual processes or, in the present terms, elemental individual transitions (without connections) between states, and events. This situation could,

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[6] Heisenberg's philosophical thinking is rarely given the attention and rigor it merits and, I would add, requires. Two treatments should, however, be credited here: Camilleri (2011), which also offers an extensive discussion of Heisenberg's view language, and, more technical, Jaeger (2018).

[7] It is possible to assume that the ultimate nature of reality is random or mixed, and, while causal conceptions of reality have been dominant, random ones have been around since the pre-Socratics, as in Democritus's and then Epicurus's and Lucretius's atomism. Such conceptions have a problem insofar as the dynamics leading to random or chance events is not given an explanation. On the other hand, such random events have not been seen, at least not prior to quantum theory in the RWR-type interpretation, as effects of the ultimate reality beyond representation or conception, which resolves this problem, albeit not to everyone's satisfaction. Besides, because of quantum correlations, quantum effects are not always random, although these correlations are statistical and defy causality.

[8] For a comprehensive discussion of this subject, see Aronson (2013).

[9] The standard use of the term "quantum statistics" refers to the behavior of large multiplicities of identical quantum objects, such as electrons and photons, which behave differently, in accordance with, respectively, the Fermi–Dirac and the Bose–Einstein statistics.

however, be interpreted either on Bayesian lines, under the assumption that a probability could be assigned to individual quantum events, or on frequentist lines, under the assumption that each individual effect is strictly random. I here adopt the latter view (Plotnitsky, 2016, pp. 173–186; Plotnitsky and Khrennikov, 2015).[10]

This brief summary cannot do justice to the idea of probability and probability theory even in quantum theory, let alone in general, but it is sufficient for my purposes.[11] I might add that probability introduces an element of order into situations defined by the role of randomness in them and enables us to handle such situations better. In other words, probability or statistics is about the interplay of indeterminacy or randomness and order. This interplay takes on a unique significance in quantum physics, because of the presence of statistical correlations, such as the EPR or EPR-Bell correlations, found in the experiments of Einstein–Podolsky–Rosen (EPR) type (Einstein et al., 1935) and considered in Bell's and the Kochen–Specker theorems.

I am now ready to return to causality. I shall first discuss the concept of classical causality, as an ontological category, part of reality. It relates to the behavior of physical systems whose evolution is defined by the fact that the state of a given system (as idealized by a given theory or model) is determined at all moments of time by their state at a particular moment of time, indeed at any given moment of time. This concept is in accord with the *principle* of causality, introduced by Kant, although the history of the principle, and of the concept classical causality, is much longer, reaching all the way to Plato and even earlier (e.g., Phaedo, 96, a 6–10; Plato, 2005). According to Kant, "If, therefore, we experience that something happens, then we always presuppose that something else precedes it, which it *follows* in accordance with a rule" (Kant, 1997, pp. 305, 308). This presupposition also defines the *concept* of classical causality, proceeding from causes to effects. Quantum phenomena, in nonrealist interpretations, violate this principle, because no event could be determinately established as the cause of a given event, and only statistical correlations between certain events could be ascertained.

I also use the concept of determinism as distinct from classical causality, a distinction useful for historical and conceptual reasons. If causality is an ontological category and as such is part of *the architecture of reality*, "determinism," defined as an epistemological category, is part of our *knowledge concerning reality* or of observable effects of reality. It denotes our ability to predict the state of a system, at least, again, as defined by an idealized model, exactly, rather than probabilistically, at any moment of time once we know its state at a given moment of time. Determinism is sometimes used in the same sense as classical causality, and in the case of classical mechanics, which deals with single objects or a small number of objects, both coincide.[12] Factually, quantum phenomena only preclude determinism, because, as noted, identically prepared quantum experiments in general lead to different outcomes. Only

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[10] For a nonrealist Bayesian approach to QM, known as QBism, see Fuchs et al. (2014). There have of course been statistical interpretations of QM, along a spectrum different philosophical assumption underlying them, although, to my knowledge, not in the present reality-without-realism (RWR) sense. For a compelling example, see Allahverdyan et al. (2016), which starts with the quantum-mechanical formalism, assumed to be valid, and then establishes (minimal) postulates that should be added for an adequate theory of quantum measurement, which enables the authors to connect more rigorously the quantum-mechanical formalism and the measurement process, differently from approaches based on decoherence. Their position is close to Bohr or the present view, because they argue that one should interpret outcomes of pointer indications and leave the richer quantum structure, which has many ways of expressing the same identities, without interpretation. In the present view, this structure is seen as a technology of statistical predictions, without representing the quantum behavior leading to the outcome of quantum experiments.

[11] See Khrennikov (2009) and Hayek (2014) and references therein. On the Bayesian philosophy of probability, in two different versions of it, see Jaynes (2003) and De Finetti (2008).

[12] The term deterministic is sometimes used in quantum mechanics, for example, in referring to Schrödinger's equation, which I find misleading, for the reasons explained below.

the statistics of multiple identically prepared experiments are repeatable. It would be difficult to do science otherwise: something needs to be repeatable. The lack of classical causality or realism in the corresponding interpretations of quantum phenomena are *interpretive inferences* from this situation and additional quantum features such as correlations and the uncertainty relations, or complementarity.[13]

Nonrealist interpretations make the absence of classical causality nearly automatic. It is strictly automatic if one takes the strongest form of the RWR view and assumes the ultimate nature of reality to be beyond conception altogether, because the assumption that the ultimate nature of reality is classically causal would imply at least a partial conception of this reality. However, even if one adopts a weaker assumption, which only precludes a representation, rather than conception, of this reality, classical causality is difficult to maintain in considering quantum phenomena. This is because to do so requires a sufficient degree of representation, analogous to that found in classical physics, which appears to be prevented in the case of quantum phenomena, in particular, by the uncertainty relations. Schrödinger expressed this difficulty, while bemoaning quantum mechanics as "the doctrine born of distress," in his cat-paradox paper: "If a classical state [defined by the ideally definite position and the definite momentum of an object] does not exist at any moment, it can hardly change causally" (Schrödinger, 1935, p. 154). According to Bohr,

> [T]he recourse to probability laws [in quantum physics] is essentially different in aim from the familiar application of statistical considerations as practical means of accounting for the properties of mechanical systems of great structural complexity. In fact, in quantum physics we are presented not with intricacies of this kind, but with the inability of the classical frame of concepts to comprise the peculiar feature[s] of the elementary processes [in other words, with the impossibility of realism]. (Bohr, 1987, v. 2, p. 34)

I shall now discuss certain alternative conceptions of causality pertinent to this article. Thus, the term "causality" is often used in accordance with the requirements of special relativity, which restricts (classical) causes to those occurring in the backward (past) light cone of the event that is seen as an effect of this cause, while no event can be a cause of any event outside the forward (future) light cone of that event. In other words, no physical causes can propagate faster that the speed of light in a vacuum, c, which requirement also implies temporal locality. Technically, this requirement only restricts classical causality, by a relativistic antecedence postulate, rather than precludes it, and relativity theory itself, special or general, is a classically causal and indeed deterministic theory. By contrast, while, as a probabilistic or statistical theory of quantum phenomena, it lacks classical causality, quantum mechanics or quantum field theory (which, in its currently standard version, mathematically conforms to special relativity), the compatibility with relativity or, more generally, locality requirements would be maintained insofar as an already performed quantum experiment determines, probabilistically, a possible outcome of a future experiment, or statistically, possible outcomes of numerous future experiments.

Relativistic causality is, thus, a manifestation of a more general concept or principle, that of locality; and one can generalize relativistic causality accordingly, without assuming special relativity first. This principle states that no instantaneous transmission of physical influences between spatially separated physical systems ("action at a distance") is allowed or, which is a more current formulation, that physical systems

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[13] Such interpretations, again, do not exclude the possibility of causal or realist interpretations of QM, or alternative causal or realist quantum theories, such as Bohmian mechanics (which is nonlocal), or theories defined by deeper underlying causal dynamics, which makes QM an "emergent" theory. Among recent proposals is Khrennikov's "pre-quantum classical statistical field theory" (Khrennikov, 2012; Plotnitsky and Khrennikov, 2015).

can only be physically influenced by their immediate environment. Nonlocality in this sense is usually seen as undesirable. The standard quantum mechanics appears to avoid it. While in certain circumstances, as in the EPR-type situations, quantum mechanics can make *predictions* concerning the state of a spatially separated quantum, the physical circumstances of making these predictions and verifying them remains *local*, which allows one to avoid nonlocality, at least if one adopts a nonrealist interpretation (Bohr, 1935, pp. 700–701). However, the question of the locality of quantum mechanics is a matter of much controversy, especially in the wake of the Bell and Kochen–Specker theorems and related findings.[14]

Finally, I shall define the concept of quantum causality, which respects locality. I shall do this via Bohr's concept of complementarity, which Bohr saw as a generalization of classical causality. Complementarity is defined by

(a) mutual exclusivity of certain phenomena, entities, or conceptions; and yet

(b) the possibility of considering each one of them separately at any given point, and

(c) the necessity of considering all of them at different moments for a comprehensive account of the totality of phenomena that one must consider in quantum physics.

Complementarity may be seen as a reflection of the fact that, in a radical departure from classical physics or relativity, the behavior of quantum objects of the same type, say, electrons, is not governed, individually or collectively, by the same "physical law," in all possible contexts, specifically in complementary contexts (Plotnitsky, 2017). Speaking of "*physical* law" requires caution, because, in Bohr's and related nonrealist interpretations, there is no physical law representing this behavior, not even a probabilistic law if one adopts a statistical, rather than a Bayesian, view. On the other hand, quantum mechanics offers correct probabilistic or statistical predictions (no other predictions are, again, possible) *in all contexts*, in nonrealist interpretations under the assumption that quantum objects and behavior are beyond representation or even conception.

If one adopts this type of interpretation, the nature of both experimental and theoretical physics changes, which change is part of a broader transformation, discussed in Section 4, brought about by Heisenberg's discovery of quantum mechanics. Experimentally we no longer track, as we do in classical physics or relativity, the independent behavior of the systems considered, track what happens in any event. Instead we define what *will* happen in the experiments we perform, by *how* we experiment with nature by means of our experimental technology, even though and because we can only predict what will happen probabilistically or statistically. Thus, in the double-slit experiment, the two alternative setups of the experiment, whether we, respectively, can or cannot know, even in principle, through which slit each particle, say, an electron, passes, we obtain two different outcomes of the statistical distributions of the traces on the screen (with which each particle collides). Or, in effect equivalently, we can set up our apparatus so as to measure and correspondingly predict, again, probabilistically or statistically, either the position or the momentum of a given quantum object, but never both together. Either case requires a separate experiment, incompatible with the other, rather than representing an arbitrary selection of either type of measurement within the same physical situation, by tracking either one of its aspects or the other, as in classical mechanics. Quantum physics

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[14] The literature dealing with these subjects is nearly as immense as that on interpretations of quantum mechanics. Among the standard treatments are Bell (2004), Cushing and McMullin (1989), and Ellis and Amati (2000). See also Brunner et al. (2014) for a more current assessment of Bell's theorem. I have considered locality in the Bohr–Einstein debate in Plotnitsky (2016, pp. 136–154).

changes what experiments do: they define what will or will not happen in terms of probabilistic or statistical predictions, rather than follow what is bound to happen in accordance with classical causality.

It is this probabilistic or statistical determination (which precludes classical causality but respects locality) of what can happen as a result of our conscious decision concerning which experiment to perform at a given moment in time that defines what I call "quantum causality." Whatever is registered as a quantum event (providing the initial data) defines a possible set of, probabilistically or statistically, predictable future events, outcomes of possible future experiments. This definition is in accord with recent approaches to causality in considering quantum phenomena in quantum information theory (e.g., Branciard et al., 2016; D'Ariano et al., 2017; Fuchs et al., 2014; Hardy, 2010), except that it brings into consideration our conscious decision concerning experiments we perform, which is rarely expressly considered, although in effect implied (whether this implication is recognized or not).[15] It is, however, this aspect of the situation that brings complementarity into play, because, in certain situation, such a decision irrevocably rules out the possibility of our probabilistic or statistical predictions concerning certain other, complementary, events.

With these considerations in mind, one can understand Bohr's view of complementarity as a generalization of causality (Bohr, 1987, v. 2, p. 41). On the one hand, our "free choice" concerning what kind of experiment we want to perform is essential to complementarity (Bohr, 1935, p. 699). On the other hand, as against classical physics or relativity, implementing our decision concerning what we want to do will allow us to make only certain types of predictions and will exclude the possibility of certain other, *complementary*, types of predictions. Complementarity generalizes causality in the absence of classical causality and, in the first place, realism, because it defines which reality can and cannot be brought about by our decision concerning what experiment to perform.[16] The corresponding predictions will still be probabilistic or statistical, forming in Schrödinger's apt language, expectation-catalogs (Schrödinger, 1935, p. 154).[17]

3 Quantum mechanics, quantum states, and quantum phenomena, from Heisenberg to Bohr

Heisenberg's thinking leading him to his discovery of quantum mechanics both followed Bohr's thinking in his atomic theory and moved beyond it, toward a still more radical approach to quantum theory. Heisenberg abandoned the aim of representing the behavior of electrons in atoms altogether, rather than only when dealing with quantum jumps. He had his reasons to do so, although he was the only one at the time who adopted this radical approach. By the early 1920s, Bohr's theory, while developed to apply, with notable successes, to complex atoms by Bohr and others, ran into major difficulties and had ultimately proven unsustainable. It is in order to remedy these failures that Heisenberg decided to depart from classical physics even

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[15] A notable exception, also compelling because it deals with quantum correlations and, hence, causality is Mermin (1998), although N.D. Mermin only preliminarily considers the subject.

[16] These considerations ground Bohr's argument in his reply to EPR (Bohr, 1935), as considered in detail in Plotnitsky (2016, pp. 136–154), and accordingly, have important connections to Bell's and Kochen–Specker theorems (and the question of counterfactual reasoning in quantum theory), as well as to the works, cited here, on causality and correlations in quantum information theory. These subjects are, however, beyond my scope.

[17] If Schrödinger's equation may be seen as "*deterministic*," as it is sometimes, it is only in the sense that it determinately provides such expectation-catalogs, which are, however, not deterministic.

beyond Bohr's theory, rather than attempting to restore more classical thinking to quantum theory, as was expected by most at the time. It was this decision that led him to his discovery of quantum mechanics. In assessing the new (matrix) quantum mechanics, in the wake of Heisenberg's discovery of the theory and Born and Jordan's work, that gave the theory a more rigorous form, but before Schrödinger introduced his wave version, Bohr said:

> In contrast to ordinary [classical] mechanics, the new quantum mechanics does not deal with a space–time description of the motion of atomic particles. It operates with manifolds of quantities [matrices] which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states in conformity with the correspondence principle [which requires that quantum and classical predictions coincide in the classical limit]. These quantities satisfy certain relations which take the place of the mechanical equations of motion and the quantization rules [of the old quantum theory]. (Bohr, 1987, v. 1, p. 48)

Like Bohr, Heisenberg was, thus, no longer thinking in terms of predictions, even if only probabilistic ones, concerning *a moving object*, say, an electron, free or orbiting the nucleus of an atom, but instead in terms of the probabilities of *transitions between the states of an electron*, transitions without connections. As Heisenberg said even before he completed his paper introducing quantum mechanics: "What I really like in this scheme is that one can really reduce *all interactions* between atoms and the external world ... *to transition probabilities*" (W. Heisenberg, Letter to R. Kronig, 5 June 1925; cited in Mehra and Rechenberg, 2001, v. 2, p. 242; emphasis added).[18]

Heisenberg's scheme, thus, moved beyond Bohr's 1913 theory in two key respects. First, stationary states were rethought in terms of energy levels of electrons, without a mechanical model or geometrical representation of their behavior, thus excluding from his theory the mechanical picture altogether, and replacing them with measurable quantities, numbers, and changes between these numbers. In the language of Bohr's atomic theory, in Heisenberg's scheme there were only quantum states and quantum jumps, or in a more rigorous language adopted here, only quantum states and transitions, transitions without connections, between them. Any physical state of an electron was just a quantum state: an energy-state, a position-state, a momentum-state, and so forth, depending on the measurement or, correspondingly, prediction one decided to make.

In addition, in the second departure from Bohr's theory, any quantum-mechanical situation was now defined in terms of events and probabilistic or statistical connections between events, as manifested only in the measuring instruments involved. Heisenberg's statement just cited clearly suggests that his scheme was about the *interactions* between atoms in the observed external world, specifically the measuring instruments involved. While not found in Bohr's theory, this view became the foundation of Bohr's concepts of phenomena, defined by these interactions, and complementarity, and his interpretation of quantum mechanics. All that one could say about quantum objects, states, and processes could only concern their effects on measuring instruments. One could speak of a physical state of a quantum object, such as an electron, only insofar as a certain change is registered in the measuring instruments associated with this electron, say, a registered change in two energy

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[18] More classical views of the situation have persisted as well, especially following Schrödinger's introduction of his wave mechanics, which appeared more amenable to such views. As I said, the debates concerning the subject have never subsided.

levels, which one might associate with the corresponding energy states, or a registered change in a position state of a free electron. While, as discussed earlier, it is reasonable to assume that something has "changed" or "happened" between these two registered events, showing a difference in some measured quantity, such as energy, one must, especially if one adopts a nonrealist, RWR-type, view, keep in mind, that, to return to Heisenberg's formulation, "these concepts cannot be applied in the space between the observations; they can only be applied at the points of observation" (Heisenberg, 1962, p. 145).[19] They can only be applied to what is physically observed in measuring instruments and thus can be described in classical terms, to which even such concepts as "position" (or "point in time"), "momentum," or "energy" belong as well. I shall return to this, potentially controversial, claim below. Importantly, measuring instruments also have their quantum strata, which enable their interactions with quantum objects, interactions that produce the effects in questions. The present concept of a quantum state is in accord with this view, because any such state can only be specified in terms of an effect a quantum object in this state had upon a measuring instrument and not in itself. It cannot be specified in advance before a measurement has taken place because we can only predict a probability of finding the quantum object concerned in a certain range of states, again, manifested only in a range of effects. Thus, one can, by using Schrödinger equation, predict the probability or statistics that an electron will be found in a certain area. In the RWR-type view, however, this only means that one can predict the probability or statistics that a black spot or, in repeated experiments, a set of such spots will be found in a certain area of the photographic plate used, spots that are assumed to be the effects of the interactions between the electrons and the plate. Some of these electrons will not be found in this area: the corresponding runs of the experiment will not result in a spot in this area. In sum, a quantum object can be assumed to exist and be in a certain state, or change this state, but neither this object nor this state, nor this change, could be given a representation or even conception, which makes the use of the terms "state," "exists," or "change" provisional and ultimately inapplicable.

The correspondence principle, which states that the predictions of the quantum theory must coincide with those of the classical theory in the classical limit, motivated Heisenberg's decision to retain the equations of classical mechanics, thus giving a more rigorous mathematical meaning to the principle itself, previously used in a more ad hoc manner by Bohr and others. One the other hand, Heisenberg had to introduce the mathematically different variables used in these equations; using classical variables would not give correct predictions, except in the classical limit. Because these variables were different, the correspondence with classical theory was established by the fact that new quantum variables could be substituted for by the conventional classical variables in the classical limit, when, for example, the electrons were far away from the nuclei and when, accordingly, classical concepts, such as orbits, could be retained. This treatment is an idealization because this behavior is quantum and could lead to quantum effects, not observable when dealing with strictly classical objects. The old quantum theory was defined by the strategy of retaining the variables of classical mechanics while adjusting the equations. Heisenberg's reversal of this strategy was

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[19] J. Barbour's concept of "Platonia," an underlying reality without change and motion (the idea originating with Parmenides and then adopted by Plato) appears to derive from this circumstance (Barbour, 1999). From the present, RWR-based, viewpoint, it does not follow that everything "stands still" at the ultimate level of reality, because the concept of reality without change and motion would not apply any more than that of change or motion. It only follows that no human concept of time or space (there is no other such concept than human) would be applicable either. Heisenberg would not have subscribed to Barbour's argument, nor would have Bohr.

unexpected, while clearly motivated by his own logic, as just explained, especially the correspondence principle.[20]

Heisenberg's discovery was a remarkable achievement.[21] A detailed discussion of his derivation of quantum mechanics is beyond my scope.[22] Several key features of his thinking are, however, worth commenting on. Heisenberg's new quantum variables were infinite unbounded matrices with complex elements. Their multiplication, which Heisenberg, who was famously unaware of the existence of matrix algebra and reinvented it, had to define, is in general not commutative. Essentially, these variables are operators in Hilbert spaces over complex numbers, although Heisenberg did not know this at the time either. Such mathematical objects had never been used in physics previously, and their noncommutative nature was, initially, questionable and even off-putting for some, including Heisenberg himself and Pauli. In fact, the *unbounded* infinite matrices were not previously studied. As became apparent later, matrices of this kind are necessary to derive the uncertainty relations for continuous variables.

Most crucial is that the concept was used physically in a fundamentally different way from that in which the representational concepts of classical physics or relativity were used. Heisenberg's variables were mathematical entities enabling probabilistic or statistical predictions concerning the relationships between *quantum phenomena*, observed in measuring instruments, without providing a mathematically idealized representation of the behavior of the *quantum objects* responsible for the appearance of these phenomena. Accordingly, although understandable historically, using the term "observables" for these variables is misleading. By the same token, the equations of quantum mechanics become no longer equations of motion, at least in the sense of classical physics or relativity. Instead they formed a part of the probabilistically predictive machinery of quantum mechanics, enabling one to compile, to return to Schrödinger's language, "expectation-catalogs" concerning possible quantum events, defined by the transitions between quantum states, transitions that were not represented by these equations.

Heisenberg begins his derivation with an observation that reflects a radical departure from the classical ideal of continuous mathematical representation of individual physical processes in dealing with discrete quantum events, while still using continuous mathematics, but now only in probabilistically predictive, rather than representational, way. He says: "in quantum theory it has not been possible to associate the electron with a point in space, *considered as a function of time*, by means of observable quantities. However, even in quantum theory it is possible to ascribe to an electron the emission of radiation" (the effect of which could be observed in a measuring instrument) (Heisenberg, 1925, p. 263; emphasis added). My emphasis reflects the fact that, in principle, a measurement could associate an electron with a point in

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[20] Intriguingly, S. Weinberg thinks otherwise: "I have tried several times to read the paper that Heisenberg wrote on returning from Helgoland [his first paper of quantum mechanics], and, although I think I understand quantum mechanics, I have never understood Heisenberg's motivations for the mathematical steps in his paper" (Weinberg, 1994, p. 67; cited in Freidel, 1916). It seems to me that, while it might be difficult to surmise how Heisenberg came up with his mathematics, as there is some mysteriousness to his remarkable intuitions (which is perhaps what Weinberg has in mind), his *motivations* for his steps are clear. It might be that, being part of the generation trained when it was no longer a tool of quantum theory, Weinberg underappreciated the role of the correspondence principle, which was central to Heisenberg and motivated many of his steps.

[21] This does not mean that Heisenberg's invention of quantum mechanics was independent of or was not helped by preceding contributions, even beyond the old quantum theory, in particular, H. Kramers's work on dispersion. See Mehra and Rechenberg (2001, v. 2) for this history.

[22] I have considered this derivation on previous occasions (e.g., Plotnitsky, 2016, pp. 68–83).

space, but not by linking this association to a function of time representing the continuous motion of this electron, as it is done in classical mechanics.[23] If, however, one adopts, as Bohr eventually did, an RWR-type interpretation, one cannot assign any properties to quantum objects themselves, not even a single such property, such as that of having a position, rather than only (simultaneously) certain joint ones, which is precluded by the uncertainty relations. One could only assign physical properties to the measuring instruments involved. For the moment, in his paper, Heisenberg described his next task as follows: "In order to characterize this radiation we first need the frequencies which appear as functions of two variables. In quantum theory these functions are in the form:

$$v(n, n-\alpha) = 1/h\{W(n) - W(n-\alpha)\}$$

and in classical theory in the form

$$v(n, \alpha) = \alpha v(n) = \alpha/h(dW/dn)'' \text{(Heisenberg, 1925, p. 263).}$$

This difference leads to a difference between classical and quantum theories as regards the combination relations for frequencies, which, in the quantum case, correspond to the Rydberg–Ritz combination rules. However, "in order to complete the description of radiation [in correspondence, by the mathematical correspondence principle, with the classical Fourier representation of motion] it is necessary to have not only frequencies but also the amplitudes" (Heisenberg, 1925, p. 263). On the one hand, then, by the correspondence principle, the new, quantum-mechanical equations must formally contain amplitudes, as well as frequencies. On the other hand, these amplitudes could no longer serve their classical physical function (as part of a continuous representation of motion) and are instead related to the discrete transitions between stationary states. In Heisenberg's theory and in quantum mechanics since then, these "amplitudes" are no longer amplitudes of physical motions, which makes the name "amplitude" itself an artificial, *symbolic* term. They are instead linked to the probabilities of transitions between stationary states: they are essentially what we now call probability amplitudes. The corresponding probabilities are derived by a form of Born's rule for this limited case. (Technically, one needs the probability density functions, but this does not affect the essential point in question.) The standard rule for adding the probabilities of alternative outcomes is changed to adding the amplitudes and deriving the final probability by squaring the modulus of the sum.

The mathematical structure thus emerging is in effect that of vectors and (in general, noncommuting) Hermitian operators in complex Hilbert spaces, which are infinite-dimensional, given that one deals, as Heisenberg did, with continuous variables. Heisenberg explains the situation in these terms in Heisenberg (1930, pp. 111–122). In his original paper, which reflects his initial thinking more directly, he argues as follows:

The amplitudes may be treated as complex vectors, each determined by six independent components, and they determine both the polarization and the phase. As the amplitudes are also functions of the two variables n and α, the corresponding part of the radiation is given by the following expressions:

Quantum-theoretical:

$$\text{Re}\{A(n, n-\alpha)e^{i\omega(n,n-\alpha)t}\}$$

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[23] Matrix mechanics did not offer a treatment of electrons in stationary states, in which case and only then one could speak of the position or the position-state of an electron in an atom.

Classical:

$$Re\{A_{\alpha}(n)e^{i\omega(n)\alpha t}\}(\text{Heisenberg, 1925, p. 263}).$$

The problem – a difficult and, "at first sight," even insurmountable problem – is now apparent: "the phase contained in A would seem to be devoid of physical significance in quantum theory, since in this theory frequencies are in general not commensurable with their harmonics" (Heisenberg, 1925, pp. 263–264). This incommensurability, which is in an irreconcilable conflict with classical electrodynamics, was, again, one of the most radical features of Bohr's 1913 atomic theory. Just as Bohr before him, Heisenberg converts this, to the classical way of thinking, insurmountable problem into a possible solution, in effect saying: "This is not a problem; the classical way of thinking is."

In his paper, Heisenberg says next: "However, we shall see presently that also in quantum theory the phase has a definitive significance which is *analogous* to its significance in classical theory" (Heisenberg, 1925, p. 264; emphasis added). "Analogous" could only mean here that, rather than being analogous physically, the way the phase enters mathematically is analogous to the way the classical phase enters mathematically in classical theory, in accordance with the *mathematical* form of the correspondence principle, insofar as quantum-mechanical equations are formally the same as those of classical physics. Heisenberg only considered a toy model of an aharmonic quantum oscillator, and thus needed only a Newtonian equation for it, rather than the Hamiltonian equations required for a full-fledged theory, developed by Born and Jordan, based on thinking in terms of transitions without connections between quantum states (Born and Jordan, 1925; Born et al., 1926; Freidel, 2016). As Heisenberg explains, if one considers "a given quantity $x(t)$ [a coordinate as a function of time] in classical theory, this can be regarded as represented by a set of quantities of the form

$$A_{\alpha}(n)\mathrm{e}^{i\omega(n)\alpha t},$$

which, depending on whether the motion is periodic or not, can be combined into a sum or integral which represents $x(t)$:

$$x(n,t) = \sum_{-\infty}^{+\infty} \alpha A_{\alpha}(n)\mathrm{e}^{i\omega(n)\alpha t}$$

or

$$x(n,t) = \int_{-\alpha}^{+\infty} A_{\alpha}(n)\mathrm{e}^{i\omega(n)\alpha t}\mathrm{d}\alpha\text{''} \quad (\text{Heisenberg, 1925, p. 264}).$$

Heisenberg next makes his most decisive and most extraordinary move. He notes that "a similar combination of the corresponding quantum-theoretical quantities seems to be impossible in a unique manner and therefore not meaningful, in view of the equal weight of the variables n and $n - \alpha$" (Heisenberg, 1925, p. 264). "However," he says, "one might readily regard the ensemble of quantities $A(n,\ n - \alpha)\mathrm{e}^{i\omega(n,n-\alpha)t}$ [an infinite square matrix] as a representation of the quantity $x(t)$" (Heisenberg, 1925, p. 264). The arrangement of the data in these ensembles (square) tables is a brilliant and, in retrospect, but only in retrospect, natural way to connect the transitions between energy levels, up or down. However, it does not by itself establish an *algebra* of these arrangements, for which one needs to find the rigorous rules for adding and multiplying these elements, beginning with $x(t)^2$. Otherwise, Heisenberg cannot use these variables in the equations of his new mechanics. "In quantum theory," Heisenberg proposes, "it seems that the simplest and most natural assumption would be to replace classical [Fourier] equations ... by

$$B(n, n-\beta)\mathrm{e}^{i\omega(n,n-\beta))t} = \sum_{-\infty}^{+\infty} \alpha A(n, n-\alpha)A(n-\alpha, n-\beta)\mathrm{e}^{i\omega(n,n-\beta))t}$$

$$= \int_{-\infty}^{+\infty} A(n, n-\alpha)A(n-\alpha, n-\beta)\mathrm{e}^{i\omega(n,n-\beta)t}\mathrm{d}\alpha\text{''} \quad \text{(Heisenberg, 1925, p. 265).}$$

This is the main mathematical postulate, the matrix multiplication postulate, of Heisenberg's new theory, "and in fact this type of combination is an almost necessary consequence of the frequency combination rules" (Heisenberg, 1925, p. 265). This bringing together of the particular arrangement of the data and the construction of an algebra of multiplying his new variables was Heisenberg's great invention.

Although it is commutative in the case of squaring a given variable, x^2, this multiplication is in general noncommutative, expressly for position and momentum variables, and Heisenberg, without quite realizing it, used this noncommutativity in solving his equation, as Dirac was the first to notice. Taking his inspiration from Einstein's "kinematics" of special relativity, Heisenberg spoke of his new algebra of matrices as the "new kinematics." This was not the best choice of term because his new variables no longer described or were even related to motion as the term kinematic would suggest, one of many, historically understandable, but potentially confusing terms. Technically, the theory, as Einstein often complained, was not even a mechanics, insofar as it did not offer a representation of individual quantum processes, or for that matter of anything else. As noted earlier, "observables," for the corresponding operators, and "states," for Hilbert-space vectors are other such terms: we never observe these "observables" or "states," or physically assign them to quantum objects (or to anything else), but only use them to predict, probabilistically, what will be observed in measuring instruments. While, Planck's constant, h, a dimensional, dynamic entity, has played no role thus far in Heisenberg's derivation, in order to make these predictions one does need h, which thus enters this new (nonrealist) relation, established by Heisenberg, between the data in question and the mathematics of the theory.

The quantum-mechanical situation that emerged in Heisenberg's paper was eventually, in the late 1930s, recast by Bohr in terms of his concept of "phenomenon." This recasting took a while and was shaped by a number of intervening developments in quantum theory and several changes in Bohr's views (Plotnitsky, 2016, pp. 107–172). According to Bohr:

> I advocated the application of the word phenomenon exclusively to refer to the *observations* obtained under specified circumstances, including an account of the whole experimental arrangement. In such terminology, the observational problem is free of any special intricacy since, in actual experiments, all observations are expressed by unambiguous statements referring, for instance, to the registration of the point at which an electron arrives at a photographic plate. Moreover, speaking in such a way is just suited to emphasize that the appropriate physical interpretation of the symbolic quantum-mechanical formalism amounts only to predictions, of determinate or statistical character, pertaining to individual phenomena appearing under conditions defined by classical physical concepts [describing the observable parts of measuring instruments]. (Bohr, 1987, v. 2, p. 64)

Part of Bohr's concept of phenomenon and the main reason for its introduction, especially following his exchange with Einstein concerning the EPR experiment (Bohr, 1935) was that this concept "*in principle* exclude[s]" any representation or analysis, or even conception, of quantum objects and their behavior by means of quantum mechanics or otherwise (Bohr, 1987, v. 2, p. 62). Bohr's statement occurs in response to Einstein's discontent with quantum mechanics as a fundamental physical theory, essentially because it was irreducibly probabilistic or statistical even in the case of elemental individual quantum processes, especially if understood, as it was in the spirit of Copenhagen, on nonrealist lines. Einstein admitted that such a theory is "logically possible without contradiction," but found it "so very contrary to his scientific instinct that [he could not forgo] the search for a more complete conception," by which he meant a realist and causal theory, preferably a field theory of the type general relativity was, on the model of Faraday's and Maxwell's electromagnetic field theory (Einstein, 1936, p. 375; cited in Bohr, 1987, v. 2, p. 62). While, however, responding to this objection by Einstein, Bohr's statement, just cited, expressing his epistemological position was clearly general and reflected his nonrealist position under discussion at the moment. I cite it more fully:

> Even if such an attitude [of Einstein] might seem well balanced in itself, it nevertheless implies a rejection of the whole argumentation exposed in the preceding [essentially explaining Bohr's interpretation], aiming to show that in quantum mechanics, we are not dealing with an arbitrary renunciation of a more detailed analysis of atomic phenomena but with a recognition that such an analysis is *in principle* excluded. (Bohr, 1987, v. 2, pp. 61–62)

The concept of phenomenon, which responds to this situation, becomes thus correlative to the RWR-type view, reached by Bohr at this stage. Physical quantities obtained in quantum measurements and defining the physical behavior of certain (classically described) parts of measuring instruments are *effects* of the interactions between quantum objects and these instruments, and do not pertain to quantum objects themselves. The measuring instruments used in quantum experiments also have quantum part through which they interact with quantum objects, for otherwise quantum measurements and the effects in question would not be possible. These effects are, thus, defined by classical states of these parts of measuring instruments, to which, in Bohr's language, these quantum interactions are "irreversible amplified" or, in the language of decoherence, into which these interactions decohere (Bohr, 1987, v. 2., p. 51, v. 3, p. 3). The language of effects (in the absence of classical causes), found throughout Bohr's writings on quantum mechanics, becomes especially prominent at this stage as well, including in the same article (e.g., Bohr, 1987, v. 2, pp. 46-47, 56, 62).[24]

Quantum states, as *physical* states, are defined here in accordance with this view: as states of quantum objects that, in their interaction with measuring instruments, produce such effects, which are (classical) physical states of measuring instruments, and are manifested only in these effects. They are so manifested, however, only as revealing the existence or reality of quantum objects by the nature these (quantum)

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[24] Intriguingly, P. Bush, in his excellent entry on "effect" in "Compendium of Quantum Physics," does not mention Bohr, focusing on G. Ludwig's formal definition of the concept (Bush, 2009; Ludwig, 1964). This definition, however, may be seen as formalizing Bohr's concept of "effect," as considered here. As Bush himself says, "Intuitively, this term refers to the 'effect' of a physical object on a measuring device" (Bush, 2009), which is fully in accord with Bohr's view, keeping in mind that in Bohr and Ludwig alike these effects are probabilistic, on lines of quantum causality, rather than classically causal. This formalization is both a testimony to Bohr's intuition and an important contribution in its own right, also as an example of the crucial role of mathematics, as transcending the limitations of intuition and language, in QM.

physical states of measuring instruments, without being but instead "*in principle* excluding" a representation by means of the formalism of quantum mechanics or otherwise, or even conception of these objects or these states. These effects, as classical physical states, only tell us that quantum objects are real and, as such, are in some physical states, without telling us anything about either quantum objects or their quantum states, even as concerns the space and time of these states. Whatever happens to them always happens in Lucretius' great phrase "at quite uncertain times, and uncertain places," (Lucretius, 2009, Book Two, ll. 218–219, p. 42).[25] Even this statement is provisional because it uses our language, including words like "happens" or even "space" and "time," that, as Heisenberg said in the passage cited above, does not apply to quantum objects and their behavior (Heisenberg, 1962, pp. 178–179). In Bohr's ultimate view (or in the view adopted by this article), even the mathematics of quantum theory, free from these limitations of language as it might be, only predicts, probabilistically or statistically, the outcomes of quantum experiments, expectation-catalogs, defined the effects of the interactions between quantum objects and measuring instruments, and not the behavior of quantum objects. "Quantum states" (Hilbert-space vectors) in the formalism of quantum mechanics are, again, merely part of this probabilistically predictive machinery, and do not represents any physical states.

It follows that physical properties or "elements of reality" defining these effects (realism, again, applies at the level of the observed parts of measuring instruments) are no longer assumed to correspond to any properties of quantum objects themselves, even single such properties, rather than only certain joint properties involved in the uncertainty relations. Bohr's earlier views allowed for this type of attribution of single properties *at the time of measurement* and only then. However, even this less radical view implied that the physical state of an object cannot be defined on the model of classical physics. This is because this definition requires an unambiguous determination of both conjugate quantities for a given object at any moment of time and independent of measurement, which is not possible in quantum physics because of the uncertainty relations. In any event, in Bohr's ultimate view, an attribution *even of a single property* or even of the corresponding conception (such as that of "position," "moment in time," "momentum," or "energy") to any quantum object as such is *never possible – before, during, or after measurement.* As just explained, one could only rigorously specify measurable quantities that could physically pertain to measuring instruments. Even when we do not aim to determine the momentum or energy of a given quantum object and thus need not worry about the uncertainty relations, neither the exact *position* of this object itself nor the actual time at which this "position" is established is ever available and, hence, in any way verifiable, again, strictly referring by all these terms to the corresponding effects observed in measuring instruments. Any possible information concerning quantum objects as independent entities is lost in "the finite [quantum] and uncontrollable interaction" between them and measuring instruments (Bohr, 1935, pp. 697, 700).

However, this interaction leaves a mark in measuring instruments, a mark, a bit of information, that can be treated as a part of a permanent, objective record, which can be discussed, communicated, and so forth (e.g., Bohr, 1987, v. 2, p. 74, v. 3).[26] The uncertainty relations remain valid, of course. But they now apply to the corresponding (classical) variables of suitably prepared measuring instruments, impacted

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[25] See Rovelli (1998) for a different viewpoint on this situation.

[26] Heisenberg appears to maintain this type of view in his later works as well (e.g., Heisenberg, 1955), even though his overall interpretation, while in the spirit of Copenhagen, is different from that of Bohr, arguably in any of its versions, but especially the ultimate one. Just as those of Bohr, Heisenberg's views had changed, but they were never quite identical to those of Bohr, although, as noted, Heisenberg's earlier views were closer to Bohr.

by quantum objects. We can prepare our instruments so as to either measure or predict a change of momentum of certain parts of those instruments or so as to locate the spot that registers an impact by a quantum object, but never do both in the same experiment. The uncertainty relations are correlative to the complementary nature of these arrangements.[27] This view implies and in fact arises from the irreducible difference between quantum phenomena and quantum objects. According to Bohr:

> This necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation may indeed be said to form a *principal distinction between classical and quantum-mechanical description of physical phenomena.* It is true that the place within each measuring procedure where this discrimination is made is in both cases largely a matter of convenience. While, however, in classical physics the distinction between object and measuring agencies does not entail any difference in the character of the description of the phenomena concerned, its fundamental importance in quantum theory, as we have seen, has its root in the indispensable use of classical concepts in the interpretation of all proper measurements, even though the classical theories do not suffice in accounting for the new types of regularities with which we are concerned in atomic physics. (Bohr, 1935, p. 701, 697–697n)

This statement might suggest, and has suggested to some, that, while observable parts of measuring instruments are described by means of classical physics, the independent behavior of quantum objects is described by means of the quantum-mechanical formalism, which, as explained, need not imply a phenomenal representation or visualization of these processes. This, however, is not the case. Bohr does say here that observable parts of measuring instruments are described by classical physics, keeping in mind that measuring instruments also have quantum strata, through which they interact with quantum objects. But he does not say and does not mean (there is no evidence to conclude otherwise) that the independent behavior of quantum objects is represented by means of the quantum-mechanical formalism. This formalism is assumed by Bohr to have a strictly probabilistically or statistically predictive role, while, what "happens" between experiments cannot be represented, even mathematically, let alone conceptually.

While "it is true that the place within each measuring procedure where this discrimination [between the object and the measuring instrument] is made is … *largely* a matter of convenience" (emphasis added), it is true only largely but not completely. As Bohr says: "In fact, it is an obvious consequence of [Bohr's] argumentation that in each experimental arrangement and measuring procedure we have only a free choice of this place within a region where the quantum-mechanical description of the process concerned is effectively equivalent with the classical description," in accordance with the correspondence principle (Bohr, 1935, p. 701). Quantum objects and quantum states are always on the other side of the "cut," as it became known. They can

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[27] An argument for the feasibility of the simultaneous measurement of the noncommuting (discrete) variables was offered in Perarnau-Llobet and Nieuwenhuizen (2017). Their proposal, however, is not in conflict with my claim here, because it only allows for statistical estimates of the values of these variables. As they say: "The measurement is found to be nonideal, as (i) the joint statistics do not coincide with the one obtained by separately measuring each spin component, and (ii) the density matrix of the spin does not collapse in either of the measured observables. However, we give an operational interpretation of the process as a generalized quantum measurement, and show that it is fully informative: The expected value of the measured spin components can be found with arbitrary precision for sufficiently many runs of the experiment." The uncertainty relations for the corresponding spin-components remain valid. The proposal also confirms the irreducibly statistical nature of quantum predictions.

never be isolated, materially or phenomenally. This impossibility reflects the indivisible wholeness of phenomena or their "atomicity," in the original Greek sense, but now used epistemologically rather than physically. In introducing this concept, Bohr said that "Planck's discovery of the quantum of action . . . disclosed a novel feature of atomicity in the laws of nature supplementing in such unsuspected manner the old doctrine of the limited divisibility of matter" (Bohr, 1938, p. 94).

The basis for Bohr's interpretation (in any of its versions) was, I contend, Heisenberg's derivation of quantum mechanics and then the uncertainty relations, aided by Bohr's concept of complementarity, a major contribution. Bohr, who, in his subsequent thinking remained closer to that of Heisenberg's thinking in his derivation of quantum mechanics than Heisenberg himself, was also the first to realize that Heisenberg radically changed the nature of the relationships between mathematics and physics, and, as I shall now argue, the nature of physics itself, at least in the case quantum physics (Bohr, 1987, v. 1, p. 51).

4 Physics after Heisenberg: quantum objects, quantum states, and quantum fields

Heisenberg's thinking leading him to his discovery of quantum mechanics revolutionized the very practice of theoretical physics, at least in quantum theory, and it also redefined the practice of experimental physics, when dealing with quantum phenomena. While made in the spirit of Copenhagen, this is a broader claim concerning the nature and practice of physics, rather than any particular interpretation of quantum mechanics, such as that of Bohr (in any of its version) or that of Heisenberg, again, different (in any of its versions) from that of Bohr, although all these interpretations are in accord with my claim.

In view of this change, the practice of experimental physics no longer consists, as in classical or relativistic experiments, in tracking the independent behavior of the systems considered, but in *unavoidably* creating configurations of experimental technology that reflect the fact that what happens is *unavoidably* defined by what kinds of experiments we perform, by how we affect quantum objects. I emphasize "unavoidably" because, while the behavior of classical or relativistic objects may be affected by experimental technology, in general we can observe them, such as planets moving around the Sun, without appreciably affecting their behavior. This does not appear to be possible in quantum experiments.

The practice of theoretical physics no longer consists, as in classical physics or relativity, in offering an idealized mathematical representation of quantum objects and behavior, but in inventing mathematical machinery enabling us to predict, probabilistically or statistically, the outcomes of quantum events and correlations between these events.

This situation became equally manifested, beginning with Dirac's work, inspired by Heisenberg and representing the shift in theoretical physics in question equally and even more markedly, in quantum electrodynamics (hereafter QED) and then quantum field theory (hereafter QFT), and experimental physics in the corresponding high-energy regimes. Indeed, this situation acquired a more complex and more radical form in these cases than in the low-energy quantum phenomena and quantum mechanics (hereafter QM). QED and QFT are characterized by, correlatively:

(1) more complex configurations of phenomena observed and hence experimental technology involved, and more complex configurations of effects of the interactions between quantum objects and measuring instruments;

(2) a more complex nature of the mathematical formalism of the theory, its mathematical technology, in part reflected in the necessity of renormalization;

(3) a more complex character of the quantum-field-theoretical predictions and, hence, of the relationships between the mathematical formalism and the measuring instruments involved.

I cannot address QFT in proper detail without extending this article too much beyond its scope.[28] I would like, however, to comment on those aspects of QFT that impact the concepts of quantum state and transitions between them, as transitions without connections, advanced in this article.

The mathematical architecture of QFT responds to, and, with Dirac's work, *led to* a discovery of, the following physical situation, not found in QM. The primary motivation of Dirac and other founding figures of QED and QFT was creating an adequate relativistic extension of QM. However, Dirac's equation contained mathematical features that led to a different picture of physics itself in high-energy quantum regimes, even as against QM. Suppose that one arranges for an emission of an electron, at a given high energy, from a source and then performs a measurement at a certain distance from that source, say, by placing a photographic plate at this point. The probability of the outcome would be properly predicted by QED. But what will be the outcome? The answer is not what our classical or even our quantum-mechanical intuition would expect, and this unexpected answer was a revolutionary discovery of QED, beginning with Dirac's equation. To appreciate the revolutionary nature of this discovery, let us consider, first, what happens if we deal with a classical object analogous to an electron and then a low-energy quantum electron in the same type of arrangement. I speak of a classical object analogous to an electron because the "game of small marbles" for electrons was finished even before QM. An electron, say, a Lorentz electron, of a small finite radius, would be torn apart by the force of its negative electricity. This required theoretical physics to treat the electron mathematically as a dimensionless point, without giving it a physical architecture, even in conjunction with spin (e.g., Dirac, 1928, p. 610).

We can consider as an example of the classical situation a small ball that hits a metal plate, which situation could be used for either a position or a momentum measurement, or indeed a simultaneous measurement of both, and time t. The place of the collision could, at least in an idealized representation of the situation, be predicted exactly by classical mechanics, and we can repeat the experiment with the same outcome on an identical or even the same object. Most importantly, regardless of where we place the plate, we always find the same object, at least when the experiment is shielded from outside interferences.

By contrast, if one considers an electron in the quantum-mechanical regime, beyond the fact that it is impossible, because of the uncertainty relations, to predict the place of collision exactly or with the degree (in principle, unlimited) of approximation possible in classical physics, there is a nonzero probability that we will not observe such a collision at all. It is also impossible to distinguish two observed traces

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[28] I have considered QFT in detail from a similar perspective in Plotnitsky (2016, pp. 207–264), on which my discussion here builds, while, however, recasting this earlier argument in terms of the concept of quantum state and modifying it, sometimes significantly, in several other respects, and thus, hopefully making it more precise. I am grateful to both anonymous reviewers on the article for their suggestions for revising this part of my argument. For an introduction to the current state of QFT, see Kuhlman (2015) and references there, which include most standard physical and philosophical treatments of the subject, such as, to give a representative physical and a representative philosophical example (Weinberg, 2005; Teller, 1995). For a technical textbook, see Peskin and Schroeder (1995). An exceptionally lucid nontechnical account of QED is given by Feynman (1985). For an elegant nontechnical account of more advanced developments and some future prospects, see Wilczek (2009).

as belonging to two different objects of the same type, or to distinguish such objects in the first place, a circumstance that becomes even more crucial in high-energy regimes. QM, however, gives us correct probabilities or statistics for such events, including correlations, such as those of the EPR type, without, in nonrealist, RWR-type, interpretations, representing the quantum objects and processes responsible for them. In a single experiment, an emitted electron could be found anywhere or not found at all. Nor can an emission of an electron be guaranteed.

Once one moves to processes that occur at a high energy, governed by QED, the situation is still different, radically different. One might find, in the corresponding region, not only an electron, as in classical physics, or an electron or nothing, as in the quantum-mechanical regime, but also other particles: a positron, a photon, an electron–positron pair, that is, the events, manifested in measuring instruments, that we associate with such entities. QED, beginning, again, with Dirac's equation, predicts which among such events can occur, and with what probability or statistics, in accordance with the observations just described, without, in nonrealist interpretations, describing the corresponding quantum processes themselves. In order to do so, however, the corresponding Hilbert-space machinery becomes much more complex, making the wave function ψ a four-component Hilbert-space vector, as opposed to a one-component or, if one considers spin, two-component Hilbert-space vector, as in quantum mechanics, keeping in mind that each component is infinite-dimensional. The reason that one needs four components is that, although Dirac did not realize it when he wrote down his equation, Dirac's equation is an equation for both the free electron and the free positron (including their spins), and they can transform into each other or photons in the high-energy processes covered by Dirac's equation or the more advanced QED formalism developed subsequently. Ultimately, one needs a state vector with an infinite number of such Hilbert-space components. While wave functions or density matrices are still used in QED or QFT (one can, for example, have Schrödinger's equation for an electron in an atom, although it is no longer soluble exactly), operators become more dominant, in which case to every spacetime point x a Hilbert-space operator is associated, rather than a state-vector as in QM. The mathematical concept of quantum field (different from the physical concept of quantum field defined below) is commonly defined in terms of operators, too, a linear combination of operators that work either on other operators or on the initial density matrix.

Once one moves to still higher energies or different domains governed by QFT, the panoply of possible outcomes becomes much greater. The Hilbert spaces and operator algebras involved have still more complex structures, linked to the appropriate Lie groups and their representations, defining (when these representations are irreducible) different elementary particles (Wigner, 1939). In the case of QED, we only have electrons, positrons, and photons, single or paired; in QFT, depending how high the energy is, one can literary find any known and possibly yet unknown elementary particle or combination (within the constraints of all pertinent conservation laws in each given case). An investigation of a particular type of quantum object irreducibly involves not only other particles of *the same type* but also *other types* of particles. The underlined qualification is important because the identity of particles within each type is strictly maintained in QFT, just as it is in QM. One cannot distinguish different particles of the same type in terms of their specificity, although it is of course possible to speak, say, of two or more different electrons existing in an atom. High-energy regimes compel us to think in terms of an incessant emergence and disappearance, creation and annihilation, of particles, theoretically governed by a new set of concepts, such as, in the first place, that of quantum field, virtual particle formation, and so forth, and mathematical features corresponding to them in QFT.

While QM, at least in the RWR-type interpretation, precludes the applicability of classical concepts, such as particles (or waves) and motion, at the quantum level, it still preserves the identity of quantum objects and of the types of quantum objects within the same experiment. It is possible to speak of this identity, even though, in RWR-type interpretations, these objects themselves are unrepresentable or even unthinkable and only manifest themselves and the type of their identity, say, electrons vs. photons, in their effects on measuring instruments. As explained earlier, these effects, while physically classical and thus corresponding to classical physical states of the classical part of measuring instruments, are defined by the quantum states of quantum objects and the quantum states of the quantum parts of measuring instruments, with which parts quantum objects interact. This interaction is, to return to Bohr's language, "irreversible amplified" to the effects in question. These effects are sufficient to adequately define each type of elementary particles. However, it is no longer possible to preserve this identity within the same experiment in high-energy quantum regimes: one may register effects corresponding to different types of particles in the course of the same experiment. This leads to what may be called the particle transformation, PT, principle. This principle was at work, in conjunction with or as correlative to various and invariance symmetry principles, in the QFT of nuclear forces, for example, and governed the practice of theoretical physics, leading to many discoveries of new particles. Thus, QED is an abelian gauge theory with the symmetry group U(1) (which is commutative), and it has one gauge field, with the photon being the gauge boson. The standard model is a non-abelian gauge theory with the symmetry group U(1) × SU(2) × SU(3) and broken symmetries, and it has a total of 12 gauge bosons: the photon, 3 weak bosons, and 8 gluons. While not, to my knowledge, stated under this heading, the PT principle is one of the most important principles of high-energy theoretical physics, and is implicit in virtually all accounts of QFT, usually presented in terms of creation and annihilation operators (e.g., Peskin and Schroeder, 1995). My appeal to the PT principle follows Heisenberg's reflections on QFT, from Dirac's work on, in his later writings (e.g., Heisenberg, 1989, pp. 31–35, 71–88). These writings also emphasize the role of symmetry groups, indeed to the point of identifying particles with them and giving them the status of reality, thus, in contrast to Bohr, in effect adopting a realist view, close to what is known as structural realism (e.g., Ladyman, 2016). While the present, RWR-type view, which follows Bohr (whose thinking has remained nonrealist throughout his life), would not allow for this type of realism any more than any other, it gives equal significance to the role of symmetry groups. This role does not depend on realism, physical or mathematical, because symmetry groups can be viewed as part of the probabilistically or statistically predictive machinery of QFT (or already QM), although there are also ontological symmetries, for example, those embedded in conservations laws by Noether's theorems.

In general, the QFT situation just outlined is fully in accord with the RWR view of quantum phenomena as considered here in the case of QM, specifically the concept of quantum state and the transitions between quantum states, as transitions without connections, while adding new dimensions to this view by virtue of the QFT-type multiplicity of quantum effects, multiplicity not found in QM regimes. First, as just indicated, low-energy (QM) regimes already permit a concept of elementary particle applicable in high-energy (QFT) regimes, beginning with the fact that elementary particles within the same type cannot be distinguished from each other in terms of their essential specificity, while these types themselves are rigorously distinguishable. Both features are consistently defined by the corresponding sets of effects manifested in measuring instruments, and thus are in accord with the RWR-based view that the *character* of elementary particles and their states and behavior is beyond representation or possibly even conception, a view that precludes one from attributing any physical properties to elementary particles themselves or their quantum states. An elementary particle of a given type, say, an electron, is specified by a set of possible phenomena or events (the same for all electrons), defined by their quantum states and observable in measuring instruments in the experiments associated with particles of this type, such as all electrons. The *elementary* character of an elementary particle is defined by the fact that there is no experiment that allows one to associate the corresponding effects on their interactions with measuring instruments with more elementary individual quantum objects. Once such an experiment becomes conceivable or performed the status of a given object as an elementary particle could be experimentally disproved or challenged (this challenge may also result from a theoretical argument), as it happened when hadrons and mesons were discovered to be composed of quarks and gluons.

With this possibility in mind, one could speak of "elementary particles" as the ultimate elemental constituents of matter. In the present view, these constituents are not "particles" in any specifiable sense that we can give this term. They cannot be comprehended by any concept of particle, any more than by any other concept, such as wave or field, although, as will be seen presently, "quantum field" could be defined as a physical concept otherwise. Instead, elementary particles and their states and behavior could be understood in terms of particular types of effects of their interactions with measuring instruments upon those instruments, effects that we can predict, probabilistically or statistically, by using quantum theory, QM or QFT. The permanent or invariant characteristics associated with elementary particles, such as mass, charge, or spin, could also be understood in terms of such effects, effects produced by such objects in their interactions with measuring instruments and reflecting something in the nature of these objects, without the properties (classical in nature) defining these effects being rigorously attributable to these objects themselves.

While, however, this conception of elementary particles is retained in high-energy quantum regimes and QFT in nonrealist, RWR-type, interpretations, it is not sufficiently adequate in view of the PT principle and needs to be supplemented by additional concepts, such as, commonly, that of quantum field. The nature and even the very possibility of such a concept, as a physical concept, is a subject of fluctuations in its definition and seemingly interminable debates, as reflected in most works on QFT cited here. While there is a strong general sense concerning the mathematics involved (although the range of specific mathematical tools offers one quite a few choices) and while there is a large consensus (although not a uniform one) that a physical concept of quantum field is necessary, most of the proposals concerning such a concept proceed along realist lines, rather than nonrealist ones, such as the one, RWR-based, pursued here.[29] I would like now to suggest a *physical* concept of quantum field, defined by the present quantum phenomena and quantum theory (in all regimes), which is, I would argue, consistent with the mathematics of QFT associated with the mathematical concepts of quantum field, say, in terms of second quantization, based on the Hamiltonian formulation of QM, or those of a Lagrangian formulation, as used in the Feynman path integral approach.

As understood here, a quantum field is a quantum object of a specific type, defined by the nature of its states and their effects on measuring instruments or other objects (in effect, equivalent to measuring instruments) in the world we observe, states and effects more multiple than those of quantum objects in QM regimes and their effects. Although, in the view adopted here, these states are still beyond representation or even conception, one could still speak of them as different from each other and, in these cases, about their greater multiplicities because of the kind of effects they produce in their interactions with measuring instruments. A quantum field is this type of

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[29] See Kuhlman (2015), which confirms this point, keeping in mind that the term nonrealist is sometimes used for interpretations that would qualify as realist in the present definition.

a transforming quantum object, which, while, along with its multiple states, beyond representation or even conception, produces the multiplicity of effects observed in high-energy quantum phenomena, as described above, specifically insofar as, unlike in low-energy (QM) regimes, some of these effects are associated with different *types* of quantum objects, such as elementary particles, even within the same single experiments. The multiplicities of elementary particles and states of the corresponding quantum field become progressively greater and more complex once we move to higher energy regimes. I stress that as a quantum object, a quantum field, including that associated with a given elementary particle, cannot, in the present, RWR-type, view be assigned any conceptual or mathematical architecture; it can only be associated with a particular architecture of specifiable effects, corresponding to a given type of particles, and predictable by means of the mathematics, the mathematical architecture, of QFT, defined by one or another the mathematical concept of quantum field. This concept brings together the irreducibly unthinkable, discovered by QM, and the irreducibly multiple, discovered by QFT. It may be useful to comment on the classical (realist) concept of field to have a better sense of the present concept of quantum field.

A classical field is, or is represented by, a differential manifold with a set of scalar (a scalar field), vector (a vector field), or tensor (a tensor field) variables associated with each point and the rules for transforming these variables, usually by means of differential functions, from point to point of this manifold. One can also think of it as a fiber bundle over a manifold with a connection. The concept of fiber bundle is used in QFT, where it is associated with local gauge symmetry, in the present, RWR-type, view, without representing, any more than any part of the mathematical formalism of QFT, any quantum physical process but only being part of the probabilistically or statistically predictive machinery of QFT. In classical physics or relativity, the variables in question map measurable quantities associated with the field, thus providing a field ontology associated with a given phenomenon, which also allows for (ideally) exact predictions concerning future events associated with this field via certain measurable field quantities. In nonrealist, RWR-type, interpretations of quantum phenomena, this type of ontology is impossible. One only deals with a discrete manifold of phenomena and sets of quantities associated with each phenomenon, and hence a discrete manifold of such quantities. As does QM, QFT relates, in terms of probabilistic or statistical predictions, the continuous (technically, differential) mathematics to the discontinuous configurations of the observed data, without representing the ultimate physical objects, states, and processes that it considers.

In QFT, however, while a given field could still be associated with a given type of particles, these quantities could no longer be limited to those associated with particle of this type and their quantum states. As understood here, then, a quantum field is a quantum object that generates this transforming multiplicity from the initial states, a quantum object, again, only manifested in its effects on measuring instruments. In other words, a quantum field is a quantum object responsible for a multiple and transformational architecture of these effects. By contrast, a quantum field itself, responsible for the effects defining this architecture in a given case, cannot be assigned this type of architecture, any more than any other. As in the case of quantum objects in QM regimes, as discussed earlier, for example, for energy levels of the electrons in the atoms, one can say that something "happened" or that there was a "change" of a quantum state. However, there is nothing we can say or even think about the character of this change (ultimately even as a "happening," "change," or a "change of state") apart from its effects, the architecture of which are more multiple and richer in QFT regimes than in QM regimes. One could speak of such field-like quantum objects and thus of quantum fields there as well, but in a reduced form that preserves the particle identities – photons always remain photons (or disappear), electrons remain electrons (or disappear), and so forth. In QFT regimes, particles or fields themselves transform into one another, producing the corresponding effects in measuring instruments.

Although, in this view, quantum objects of a given type could be understood or distinguished from quantum objects of other types only in terms of these effects, this suffices for rigorously distinguishing quantum fields, keeping in mind that elementary particles of the same type and hence the fields associated with them are not generally distinguishable. They are only distinguishable in terms of their variables quantities, such as position, momentum, or energy, of a given particle, manifesting its corresponding field in a given experiment. A particle or a set of particles (for example, consisting of a particle and its antiparticle) of a given type can give a specificity to a quantum field, even though it will still transform into other particles and, in effect, quantum fields of other types, a transformation that a measurement can register. In sum, while a quantum field is *real*, its existence is only manifested in its effects on the measuring instruments, and not in itself. Both quantum fields and their effects are real, but only their effects are physically manifested and, by virtue of their classical nature, are representable. In this regard, this situation is the same as that of quantum objects in QM.

As a concept of quantum object, the concept of quantum field thus defined is a *physical* rather than *mathematical concept*. It can be associated with a mathematical concept, commonly also called "quantum field," defined in terms of a predictive Hilbert space formalism with a particular vector and operator structure (a linear combination of operators), enabling the proper probabilistic predictions of the QFT phenomena concerned. The operators enabling one to predict the probabilities for the "annihilation" of some particles and "creation" of other particles, that is, for the corresponding measurable quantities observed in measuring instruments, are called annihilation and creation operators. In RWR-type interpretations, these operators do not describe anything either; they only enable one to calculate the probabilities or statistics of the outcomes of quantum experiments, just as the wave functions do in quantum mechanics. Both, to return to Schrödinger's language, provide expectation-catalogs for the outcomes of possible experiments. These catalogs are, however, different from those of QM, because they give probabilities or statistics of the appearance of quantities associated with other types of particles even in experiments initially defined by a particle of a given type.

The wave-function formalism of QM in low-energy regimes, say, for an electron in an atom, can be recast in terms of annihilation and creation operators as well, by means of the second quantization. One sometimes speaks, appealingly but loosely, of the first quantization as making particles into waves, and the second quantization as making waves particles again, but in a new sense. While the procedure was developed to deal with quantum many-body systems, and reflects the indistinguishability of elementary particles of the same type, it is applicable even to a single nonrelativistic electron in an atom, normally described by a wave function, now replaced by annihilation and creation operators, which is indeed even more in accord with the concept of transitions without connections between quantum states. In high-energy regimes, governed by QFT, it is meaningless to ever speak of a single electron even in the hydrogen atom. In A. Pais's words, "the hydrogen atom can no longer be considered to consist of just one proton and one electron. Rather it contains infinitely many particles" (Pais, 1986, p. 325). As any claim concerning the physically infinite, the last claim needs to be qualified and in fact modified. I shall do so below, merely noting here that the multiplicity of particles in question cannot be contained (which is not the same as being infinite), in the present view in the sense of the multiplicity of possible effects, which, as already explains, could no longer be associated with a single particle or a single pair of one proton and one electron. Besides, we still have quarks and gluons inside this proton in the same transformational existence, although we cannot register their effects apart from those of the proton because of the "confinement" of quarks.

I shall now briefly comment on the so-called "virtual particle formation," central to QFT, from the perspective just outlined. The subject, still little explored philosophically, would require a more sustained analysis from the present perspective as well. My aim here is only to suggest that this concept could be adequately considered from this perspective. I shall briefly revisit, first, the defining characteristic of the observable phenomena, associated with real or actual particles, in QFT regimes. For simplicity, I shall, again, consider an experiment in the QED regime in which we begin with a single electron, registered at time t_0, the registered outcome of which is, say, a positron at time t_n, although it could also be an electron–positron pair, a photon, etc. This positron or whatever else is observed at this moment in time is a real particle or set of particles, again, using the term particle provisionally, while keeping that the corresponding quantum object, such an electron, is defined by the particular set of effects any electron (indistinguishable from any other) can have on measuring instruments or their equivalents. The theory predicts the probabilities or statistics of these outcomes, extensively confirmed by experiments, which makes QED the best-confirmed physical theory ever. Now, consider a discrete sequence of times: t_1, t_2, t_{n-1}, between t_0 and t_n, and a discrete sequence of possible measurements, at which, *if they were performed*, any of the outcomes just mentioned could take place, again, with the probability properly predicted by a theory. Once such a measurement is performed, a new expectation-catalog is defined for future measurements.

One cannot speak of such possible observations in terms of undetected particles, especially, from the present RWR-type perspective, for one thing, because, as noted from the outset, nothing could be said about what actually happened between measurements, but only, probabilistically or statistically, of possible outcomes of future measurements. Nor, however, are such particles, if observed (via their effects), are virtual particles. Particles that *could be in principle observed* (insofar as the corresponding quantities associated with them would have been measured), even if they are not actually observed, are considered real particles, although "actual" may be a better term, because virtual particles are equally real. In other words, virtual particles cannot be associated with any actually observed quantum phenomena, which make their description or even definition difficult and a matter of debate, even though there is more agreement concerning their existence. In what sense, then, can they be said to exist from the present, RWR-type, view? The answer is not so difficult. By the present definition of reality and existence, possible without realism by RWR principle (to which definition and possibility the present view of real particles conforms as well), virtual particles exist, exists as fluctuations of quantum fields, if they have specifiable effects, which distinguish them from real particles. And they do.

In conventional, rather than RWR, terms, virtual particles (again, using the term particle provisionally) are entities that are born and disappear very quickly, but that exist long enough to have measurable effects. The concept emerges in the perturbation QFT, which considers the interactions between real particles in terms of exchange of virtual particles. These exchanges are represented by Feynman diagrams, with virtual particles represented by internal lines. While they conserve energy and momentum, virtual particles do not necessarily have the same mass as their real counterparts. They are considered "off-shell" in the standard terminology because they do not satisfy the standard equations on motion (as "on-shell" objects do), for example, insofar as they do not strictly open the energy momentum relation $m^2c^4 = E^2 - p^2c^2$ and their kinetic energy may not have the standard relationships to their velocity. The probability amplitude for the existence of a virtual particle interferes with that for its nonexistence, while for a real or actual particle the cases of existence and nonexistence are not coherent and do not interfere with each other. What is crucial here, in particular if one adopts an RWR-type view, is that even though they are not observed, virtual particles are real, part of the reality of quantum fields, because they have statistically ascertainable effects on actual particles and our measurements concerning the latter. These effects are crucial to high-energy quantum physics: the Lamb shift, the Casimir effect, and the interaction of virtual gluons, are among them (e.g., Wilczek, 2009, pp. 45–50). In other words, in the present, RWR-type view, the assumption of the existence of virtual particles is justified by its effects, as is an assumption of any quantum objects, special only because of their near identity to real or actual particles.

There are situations when virtual particles can become real or actual. Thus, while virtual particles are generally seen as appearing in pairs of a particle and an antiparticle, which exist for a very short time and then mutually annihilate, there are situations (such as the Unruh effect and vacuum decay) when it is possible to separate the pair by external energy so that they avoid annihilation and become actual particles. Nevertheless, it would be incorrect to see what I call here quantum field in terms of a kind of virtual particle "foam," which may also give rise to actual particles, or even a "foam" that combines virtual and real particles, akin what J.A. Wheeler called "quantum foam" (Wheeler and Ford, 2000, pp. 245–263). First of all, such a picture would have a potential to reinstate realism (as it might in Wheeler's scheme), while, as defined here, a quantum field is as beyond this type of conception as it is beyond any other, or birth and disappearance of anything are only manifest at the level of effects of quantum fields manifested in measuring instruments, as manifolds of always discrete phenomena and sets of quantities associated with each phenomenon. Secondly, even though (for the moment, again, speaking conventionally) virtual particles can sometimes become real or actual, in terms of the corresponding effects, generally, we do deal with two different types of effects. The way the present concept of quantum field connects the unrepresentable, or the unthinkable, and the multiple – the unrepresentable or the unthinkable nature of quantum objects and the multiple of their effects on the world with observe – allows one to maintain this difference, as is required in the regimes governed by QFT.

There is something, some "it," inferred from "bit" (as information found in measuring instruments), in Wheeler's language (Wheeler, 1990, p. 309), something that may be assumed to be changing, but in a way that we cannot know or even conceive of, ultimately, even as change. This, again, need not mean that there is only permanence at the ultimate level, as the concept of permanence or standing still is no more applicable than that of change. There are effects of change, different in the case of actual and virtual particles (each type, again, being defined only by these effects), and QFT enables us to probabilistically or statistically predict them in the multiplicity of these effects and of their types, types associated with elementary particles, actual and virtual.

QFT thus becomes a far-reaching extension of Bohr' and Heisenberg's thinking concerning transitions without connections between quantum states and of the revolution of fundamental physics ushered in by Heisenberg, an extension clearly realized by Bohr in the wake of Dirac's discovery of his equation and antimatter. He spoke of "Dirac's ingenious quantum theory of the electron," as "a most striking illustration of the power and fertility of the general quantum-mechanical way of description" (Bohr, 1987, v. 2, p. 64). Heisenberg, who made major contributions to QFT (his main interest from the late 1920s on), was even more emphatic. He saw Dirac's theory as "perhaps the biggest change of all the big changes in physics of our century. It was a discovery of utmost importance because it changed our whole picture of matter. . . . It was one of the most spectacular consequences of Dirac's discovery that the old concept of the elementary particle collapsed completely" (Heisenberg, 1989, pp. 31–33).

Finally, quantum information theory, the most recent development in the history of quantum theory, may also be seen as an outcome of Heisenberg's revolution. It would not be possible to address the subject in detail.[30] It is, however, fitting to note that in retrospect, Heisenberg's approach in his creation of QM may be considered in quantum-informational terms (Plotnitsky, 2002; Plotnitsky, 2016, pp. 72–73). The quantum-mechanical situation, as he conceived of it (initially dealing with hydrogen spectra), was defined by

(a) certain *already obtained* information, concerning the energy of an electron, derived from spectral lines (due to the emission of radiation by the electron), *observed* in measuring instruments; and

(b) certain possible future information, concerning the energy of this electron, *to be obtainable* from spectral lines *to be observed* in measuring instruments and predictable, by means of one or another quantum theory.

As we have seen, Heisenberg's strategy was to develop a mathematical formalism that would connect these two sets of data, in strictly probabilistically or statistically predictive terms (on experimental grounds), without assuming that this formalism needed to represent how these two sets of data or information are connected by a spatiotemporal process or how each set comes about, in the first place. This type of representation or the physical conception of the processes that would be thus represented mathematically did not appear possible at the time and has not been possible since, at least not in a way generally agreed upon. Heisenberg's mathematical scheme did not represent anything at the time of measurement either: it only predicted transition probabilities between situations defined by measurements, already performed, which provide the numerical data that serve as the basis for these predictions, and possible future ones.

Quantum objects, in their quantum states, create in their interactions with measuring instruments, specifically organized collections of information (composed of classical bits) and make possible certain calculations, by using mathematical structures or models, but we cannot know and possibly cannot conceive how quantum processes do this. The ultimate (quantum) constitution of matter is, to return to Wheeler's language, "it from bit," "it" inferred from "bit" (Wheeler, 1990, p. 309). Heisenberg's approach was, thus, quantum-informational in spirit, and what he called "the spirit of Copenhagen" was also the spirit of quantum information theory, defined by the concept of transitions without connections. Wheeler's visionary manifesto of quantum information theory, from which I cite, was inspired by Bohr, whom Wheeler invoked, when he announced his "it from bit": "The overarching principle of 20th-century physics, the quantum – and the principle of complementarity that is the central idea of the quantum – leaves us no escape, Niels Bohr tells us, from 'a radical revision of our attitude [towards the problem of] of physical reality"' (Wheeler, 1990, p. 309; Bohr, 1935, p. 697). (I correct Wheeler's slight misquotation of Bohr.)

Although prepared by earlier momentous contributions of Planck and Einstein, this revision began with Bohr's 1913 concept of discrete quantum states and transitions between them, transitions without connections, as neither Planck nor even Einstein were ready to go that far. This concept has reached present-day physics through Heisenberg's discovery of quantum mechanics and Bohr's interpretation of it in terms of complementarity, Dirac's work and quantum field theory, and finally quantum information theory, all of which depended on this concept. It is difficult for a concept to have a more impressive record, and we might be far from the end of its trajectory and new complexities that the next stage of this concept might bring, complexities also transforming this concept and making it a new concept, just as it happened with each previous development just mentioned. Thus, while it is difficult to predict that a theory that will resolve the present conflict between quantum theory and general relativity, would require this type of concept, but if this theory is

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[30] I have considered it in detail in Plotnitsky (2016, pp. 247–264).

quantum, quantum gravity, as it is generally (even if not universally) expected to be, it is likely to do so.

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