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Quantum Zeno and anti-Zeno effect on a two-qubit gate by dynamical decoupling

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Abstract. Controlling the dynamics of entanglement and preventing its disappearance are central requisites for any implementation of quantum information processing. Many solid state qubits are affected by non-Markovian noise, often with $1/f$ spectrum. Leading order dephasing effects are captured by treating noise in the quasistatic approximation. In this article we investigate dynamical decoupling of a two-qubit gate studying the crossover from Zeno to anti-Zeno entanglement dynamics obtained by varying the sequence parameters.

1 Introduction

Protecting quantum coherence and entanglement are key requirements for any quantum information platform. Quantum error correcting codes set strict limits on the allowable error rates, which may be particularly demanding for solid-state implementations suffering from material inherent noise sources often characterized by strong coupling and memory effects [1]. Considerable progress in design and materials has led to significant advances. In superconducting and semiconducting qubits larger decoherence times are attained by operating qubits at "optimal points" where sensitivity to low-frequency fluctuations is cancelled to lowest order [2–5]. Further improvement can be achieved by dynamical decoupling (DD), i.e. by active control, driving the quantum system towards specific targets while effectively decoupling it from noise sources [6,7]. Originally conceived for single qubits, dynamical control has been extended to preserve entanglement in quantum registers [8,9] and during the operation of entangling gates [10,11].

DD in the "bang bang" limit is a manifestation of quantum Zeno effect [12,13], both being based on a strong and fast interaction with an external system or with a measurement apparatus. Quantum Zeno subspaces are generated where non trivial coherent evolution can take place [14]. This analogy has been pushed further since under specific circumstances pulsed control can enhance decoherence, a situation well known in the Zeno literature, as anti-Zeno effect [15–19]. The occurrence of anti-Zeno behavior in the dynamically controlled evolution of qubits subject to non-Markovian and non-Gaussian noise, under proper pulse rates and coupling regimes, has been

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predicted in references [20–22]. These concepts have been extended to bipartite open quantum systems. Entanglement protection via the quantum Zeno effect has been recently investigated [9,11,23–26].

In this article, we study DD *during entanglement generation* in the presence of local pure dephasing $1/f$ noise whose spectral characteristics have been inferred in experiments with superconducting qubits. We consider three procedures recently implemented [27–31]: periodic DD (PDD) [6,7], Carr Purcell DD (CP) [32] and Uhrig DD (UDD) [33] and evaluate the gate error at periodic times where the ideal unitary evolution leads to the target entangled state. We demonstrate that, depending on the specific gate time, either almost complete decoupling is achieved (Zeno regime) or an enhancement of the gate error occurs for small pulse rate (anti-Zeno effect) followed by a Zeno regime for sufficiently large rate.

2 Entangling gate and dynamical decoupling

We consider the $\sqrt{iSWAP}$ entagling gate implemented by coupling two resonant qubits as modeled by the Hamiltonian

$$\mathcal{H}_0 = -\frac{\Omega}{2}\,\sigma_{1z}\otimes\mathbb{I}_2 - \frac{\Omega}{2}\,\mathbb{I}_1\otimes\sigma_{2z} + \frac{\omega_c}{2}\,\sigma_{1x}\otimes\sigma_{2x}\,. \tag{1}$$

Here, $\sigma_{\alpha z}$ are Pauli matrices, $\mathbb{I}_\alpha$ is the identity in qubit-α Hilbert space ($\alpha = 1,2$), we put $\hbar = 1$. This model applies to the fixed, capacitive or inductive, coupling of superconducting qubits [1,34]. Individual-qubit control allows an effective switch on/off of the interaction. Unitary evolution starting from the factorized state $|+-\rangle$ periodically leads to the fully entangled state $|\psi_e\rangle = \pm(|+-\rangle - i|-+\rangle)/\sqrt{2}$ at times $t_e^{(n)} = (1+4n)\,\pi/(2\omega_c) \equiv (1+4n)\,t_e$, $n\in\mathbb{N}$. We consider the relevant situation [35–38] where each qubit is affected by pure dephasing local noise

$$\delta\mathcal{H}(t) = -\frac{1}{2}z_1(t)\,\sigma_{1z}\otimes\mathbb{I}_2 - \frac{1}{2}\mathbb{I}_1\otimes z_2(t)\,\sigma_{2z}\,, \tag{2}$$

where $z_\alpha(t)$ is a stochastic process whose power spectrum

$$S_\alpha(\omega) = \int_{-\infty}^{\infty} dt\,\langle z_\alpha(t)z_\alpha(0)\rangle\, e^{i\omega t}, \tag{3}$$

is $1/f$ for $f\in[\gamma_{m,\alpha},\gamma_{M,\alpha}]$. For the sake of simplicity, we assume identical local noise characteristics, $S_\alpha(\omega)\equiv S(\omega)$. Noise is simulated as the superposition of random telegraph processes with switching rates γ distributed as $1/\gamma$ in $[\gamma_m,\gamma_M]$ [1,39]. The spectrum reads $S(\omega)\approx\mathcal{A}/\omega$, with $\mathcal{A}=\pi\Sigma^2/\ln(\gamma_M/\gamma_m))$, for $\omega\le\gamma_M/2\pi$, with a roll-off to $1/f^2$ behavior at higher frequencies; Σ^2 is the noise variance. Under local dynamical control the system Hamiltonian takes the form

$$\tilde{\mathcal{H}}(t) \;=\; \mathcal{H}_0 + \delta\mathcal{H}(t) + \mathcal{V}_1(t)\otimes\mathbb{I}_2 \;+\; \mathbb{I}_1\otimes\mathcal{V}_2(t), \tag{4}$$

where $\mathcal{V}_\alpha(t)$ denotes the action of a sequence of local operations on qubit α applied at times $t = t_i$, $i\in\{1,m\}$. In order to reduce the effect of noise acting along $\sigma_{\alpha z}$ without altering the gate operation we apply an *even number* of simultaneous π-pulses around the y-axis of the Bloch sphere of each qubit, denoted as π_y [10]. The pulses are applied at times $t_i = \delta_i t_e$, where $0\le\delta_i\le 1$ with $i = 1,\ldots,m$. For the PDD

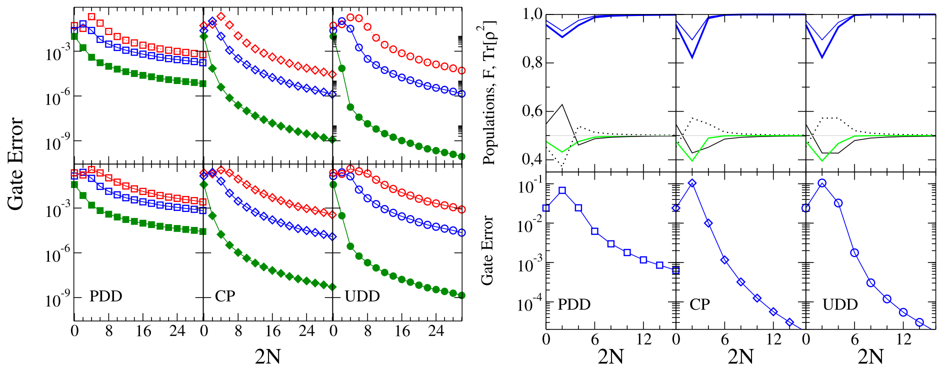

Fig. 1. Numerical results for quasistatic noise with $\Sigma = 10^9$ s^{-1}, $\gamma_m = \gamma_M = 1\,\mathrm{s}^{-1}$ under PDD, CP, UDD. Left panel: Gate error at times $t_e^{(n)}$ for $n = 0, 1, 2$ as a function of the number of pulses. Filled (green) symbols $t_e^{(0)} = t_e$, empty dark gray (blue) $t_e^{(1)} = 5t_e$, empty light gray (red) $t_e^{(2)} = 9t_e$. Top row $\omega_c = 5 \times 10^9$ rad/s, bottom row $\omega_c = 2.5 \times 10^9$ rad/s. Rigth panel: Top row $\mathrm{Tr}[\rho^2(t_e^{(1)})]$ (tick blue), fidelity (tin blue), populations (black continuous and dashed) and imaginary part of the coherence gray (green) in the factorized basis $\{|+-\rangle, |-+\rangle\}$ at $t_e^{(1)} = 5t_e$. Bottom row: Gate error at $t_e^{(1)}$.

sequence $\delta_i = i/m$, with the last pulse applied at time $t_e^{(n)}$, the pulse interval being $\Delta t = t_e^{(n)}/m$. For the CP sequences it is $\delta_i = (i - 1/2)/m$, for the UDD sequence $\delta_i = \sin^2[\pi i/(2m + 2)]$. In the limit of a two-pulse cycle, $m = 2$, UDD reduces to the CP sequence.

3 Zeno and anti-Zeno regimes

In order to estimate the gate performance under DD we evaluate the fidelity with respect to the target state $|\psi_e\rangle$, $\mathcal{F}$, and the corresponding error ε defined as

$$\mathcal{F} = \langle\psi_e|\rho(t_e^{(n)})|\psi_e\rangle, \qquad \varepsilon = 1 - \mathcal{F}, \tag{5}$$

where $\rho(t)$ is the two-qubit density matrix and

$$\langle\psi_e|\rho(t_e^{(n)})|\psi_e\rangle = \int \mathcal{D}[\mathbf{z}(t)]\, P[\mathbf{z}(t)]\, \langle\psi_e|\rho\big(t|\mathbf{z}(t)\big)|\psi_e\rangle\,, \tag{6}$$

with $\mathbf{z}(t) = \{z_1(t), z_2(t)\}$. The path integral is evaluated by exact numerical solution of the stochastic Schrödinger equation of the coupled-qubits under the action of the considered DD sequences. The number of noise realizations over which the average is performed is $\geq 10^4$. Under this condition, the numeric simulation can be considered a reliable method for calculating the gate error. For realistic noise figures, the static path approximation, $\mathbf{z}(t) \approx \mathbf{z}$, with statistically distributed values of $\mathbf{z}$, captures the system's evolution for times of interest, $t \leq 1/\gamma_M$ [1].

Our results are reported in Figure 1 for quasistatic noise and in Figure 2 for different $1/f$ noise characteristics, either by changing the variance or the high-frequency cut-off (up to $\Omega \approx 10^{11}\,\mathrm{s}^{-1}$). We evaluate the gate error at times $t_e^{(n)}$ for $n = 0, 1, 2$ as a function of the number, N, of applied pulses for the considered decoupling procedures. In Figure 1 we observe that at the first entangling time, $t_e^{(0)} \equiv t_e$, the gate

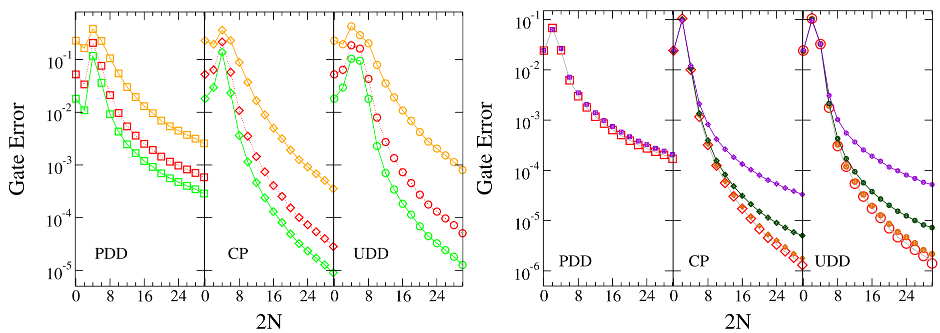

Fig. 2. Gate error for gate time $t_e^{(2)} = 9t_e$ under PDD, CP, UDD for different spectral characteristics, $\omega_c = 5 \times 10^9$ rad/s. Left panel: Results for quasistatic noise $\gamma_m = \gamma_M = 1\,\mathrm{s}^{-1}$ and different values of the noise variance: $\Sigma = 10^9\ \mathrm{s}^{-1}$, dark gray (red), $\Sigma = 2 \times 10^9\ \mathrm{s}^{-1}$ light gray (orange), $\Sigma = 1/\sqrt{2} \times 10^9\ \mathrm{s}^{-1}$ gray (green). Right panel: Results for fixed variance $\Sigma = 10^9\ \mathrm{s}^{-1}$, $\gamma_m = 1\,\mathrm{s}^{-1}$ and different high-frequency cut-off of $1/f$ noise. Empty (red) symbols are for quasistatic noise, gray line for $\gamma_M = 10^6\,\mathrm{s}^{-1}$, undistinguishable from the quasistatic noise effect. Filled gray (dark orange) symbols for $\gamma_M = 10^8\,\mathrm{s}^{-1}$, dark gray (green) symbols $\gamma_M = 10^9\,\mathrm{s}^{-1}$, gray (violet) symbols for $\gamma_M = 10^{10}\,\mathrm{s}^{-1}$.

error decreases with increasing N, indicating successfull decoupling. At the subsequent entangling times, $t_e^{(n)}$ $n \geq 1$, for small pulse numbers the gate error increases with N. After having reached a maximum for a certain $\bar{N}(n)$, the error decreases monotonically. Remarkably, $\bar{N}(n)$ does not depend on the decoupling sequence, nor on the qubit's coupling, ω_c, and the noise variance Σ (Figs. 1 and 2 left panels). These results can be interpreted as a crossover from an anti-Zeno regime $N \leq \bar{N}(n)$ to the Zeno regime for $N \gg \bar{N}(n)$.

Analogous behaviors have been reported in reference [21] where PDD was applied to a qubit subject to a structured environment formed by a bistable quantum impurity [40] or a collection of non-Gaussian fluctuators having $1/f$ spectrum. It was demonstrated that, at a general working point [41], contrary to the semiclassical picture that expects decoupling for pulse rates $\Delta t < 1/\gamma_M$, efficient noise compensation can only be obtained if also the condition $2\Delta t \ll 2\pi/\Omega$ is satisfied. Our results for the entangling gate can be explained similarly. In fact, if the system is initially prepared in the factorized state $|+-\rangle$, evolution under the total Hamiltonian (4) takes place within the bi-dimensional swap-subspace $\{|+-\rangle, |-+\rangle\}$. In the absence of noise, the restriction of (1) to the swap-subspace can be expressed in terms of a pseudo-spin as $-\omega_c\,\tau_x/2$, with eigenstates $|1\rangle = (-|+-\rangle + |-+\rangle)/\sqrt{2}$, $|2\rangle = (|+-\rangle + |-+\rangle)/\sqrt{2}$. Local pure dephasing noise is transverse in the entangled basis and π_y pulses maintain the system within the swap subspace. Therefore, we can rephrase the result of reference [21] for the entangling gate and expect efficient decoupling for pulse intervals $\Delta t \ll \min\{1/\gamma_M, \pi/\omega_c = 2t_e\}$. For an entangling gate of fixed time duration $t_e^{(n)} = t_e(1+4n) = 2N\Delta t$, the two conditions require $1 + 4n \ll 4N$ and $\Delta t \ll 1/\gamma_M$. Therefore, for $t_e^{(0)}$ the decoupling sequences induce error reduction for any N, instead for $n > 1$, a minimum number of pulses $\bar{N}(n) = (1+4n)/4$ is required. For $N < \bar{N}(n)$, pulses induce an acceleration of decay, analogous to the anti-Zeno effect. The maximum value of the gate error is larger the longer is the gate time and the larger is the noise strength, as measured by the noise variance. For $N > \bar{N}(n)$, towards the Zeno regime, the considered procedures have different performances. At t_e the most efficient procedure is UDD, while at longer gate times, $t_e^{(n)}$, for $N > \bar{N}(n)$ CP has a better performance than UDD (Fig. 1) Successful decoupling yields a pure state belonging to the dynamically generated Zeno subspace, in our case the swap-subspace. To better emphasize the approach to the Zeno regime we plot in Figure 1 (right panel) $\mathrm{Tr}\rho^2(t_e^{(n)})$, whose logarithm is the purity, and the relevant matrix elements of the two-qubit density matrix in the factorized basis, as a function of N for $t_e^{(1)}$. Purity and fidelity to the target state actually depend on the same matrix elements, being $\mathcal{F} = \frac{1}{2} + \mathrm{Im}[\rho_{+-,-+}(t_e^{(n)})]$, $S = \ln\{\mathrm{Tr}[\rho^2(t_e^{(n)})]\} = \ln\{\rho^2_{+-,+-}(t_e^{(n)}) + \rho^2_{-+,-+}(t_e^{(n)}) + 2|\rho_{+-,-+}(t_e^{(n)})|^2\}$. We observe that projection to the pure state, i.e. the asymptotic Zeno regime, is indeed already reached for relatively small numbers of pulses.

Finally we investigate the effect of higher frequency components in the $1/f$ spectrum, see Figure 2 (right panel) where we consider spectra having the same low-frequency behavior with progressively increasing γ_M. For small number of pulses the gate error is conditioned by low frequency components in the specrtum, therefore the anti Zeno regime is not influenced by the different high-frequency cut-off of the three procedures. The gate error under PDD remain quite insensitive to the spectrum at high frequencies also for large pulse number. This is to be expected provided it is $\Delta t < 1/\gamma_M$. CP and UDD turn out to be much more sensitive to the higher frequencies of the spectrum for $N > \bar{N}(n)$. Still these procedures guarantee about three-orders of magnitude decrease of the average error with respect to the unconditioned evolution until $\gamma_M \approx 10^8\,\mathrm{s}^{-1}$. This is a remarkable result, considering that at present $1/f$ noise has not been detected for frequencies higher than $\gamma_M \approx 20\,\mathrm{MHz}$ [1,27,28].

4 Conclusions

In conclusion, we investigated dynamical decoupling of an entangling gate subject to local pure dephasing $1/f$ noise with spectral characteristics inferred from experiments with superconducing qubits and DD procedures recently implemented. We demonstrated a crossover from a Zeno to an anti-Zeno regime for the entanglement depending on the periodic times, $t_e^{(n)}$, where the ideal gate leads to the target entangled state. The onset of the two regimes has a simple explanation within the Zeno subspace where the dynamics takes place and the number of pulses separating the two regimes does depend only on the chosen period, i.e. n. By evaluating the purity it was possible to verify that the Zeno regime is achieved for relatively small number of pulses with an efficiency depending on the noise characteristics at higher frequencies for the most sensitive decoupling procedures, CP and UDD.

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