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Quantum fluctuations and density of states in low-dimensional superconductors

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Abstract. We investigate the influence of quantum phase fluctuations on electron density of states (DOS) in ultrathin superconducting films and nanowires. Using an effective action approach we derive a non-perturbative correction to DOS in such systems. The main effect of phase fluctuations in quasi-two-dimensional films is the appearance of electron states at subgap energies while in quasi-one-dimensional nanowires fluctuations also lead to smearing of the gap edge singularity in DOS.

1 Introduction

Fluctuations are known to play an important role in low-dimensional systems. In low-dimensional superconducting structures, they give rise to many intriguing phenomena which are not captured by the mean-field BCS theory and cannot be observed in bulk superconductors. For instance, in quasi-one-dimensional nanowires quantum fluctuations of the order parameter lead to substantial deviations from the BCS theory down to zero temperature T [1,2]. Perhaps one of the most striking features of such nanowires is the presence of the so-called quantum phase slips (QPS) – processes of simultaneous local suppression of the absolute value of the order parameter along with a jump of its phase by 2π [1]. As the result of such processes sufficiently thin nanowires acquire a non-vanishing resistance down to lowest T [3,4]. It is also predicted that QPS processes generate nonequilibrium voltage noise in such nanowires [5,6].

QPS in nanowires are controlled by QPS rate $\gamma_{\rm QPS}$ which decreases exponentially as the cross-section area of the wire becomes larger. This implies that these processes are of practical importance only in extremely thin wires. However, there exists a different type of fluctuations which cannot be completely neglected even in relatively thicker wires, namely smooth fluctuations of the phase of the order parameter φ. It was recently shown [7] that such fluctuations can significantly affect single-particle properties of superconducting nanowires, such as the single-particle (electron) density

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of states (DOS) – a quantity of prime importance in tunneling measurements. Note that previously the effect of superconducting fluctuations (both the magnitude and the phase of the order parameter) on DOS have been addressed at temperatures close to the superconducting critical temperature T_c (see, e.g., [2,8]). At temperatures much lower than T_c fluctuations of the absolute value of the order parameter become unimportant (at least in relatively thicker wires) and, hence, only phase fluctuations can be considered [7].

In this article, we extend our analysis [7] to the case of quasi-two-dimensional films and derive a non-perturbative expression for the electron DOS in ultrathin superconducting films. We analyze the effect in two opposite limits of unscreened and completely screened Coulomb interaction. The structure of the paper is as follows. In Section 2, we briefly outline our basic formalism and derive a general expression for the average DOS. In Section 3, we further specify our results in various limits. A brief discussion and concluding remarks are presented in Section 4.

2 The model and basic formalism

Below we will focus on the two systems, a long superconducting wire and a thin superconducting film. Transversal dimensions of both systems – the effective diameter of the wire $\sqrt{s}$ and the thickness of the film d – satisfy the condition $l \ll \sqrt{s}, d < \xi$, where l is the electron elastic mean free path and ξ is the superconducting coherence length. The first of these inequalities assures that the electron motion remains diffusive in the transverse direction, whereas the second one allows to neglect the dependence of the superconducting order parameter Δ on the transverse coordinates. In the case of films, we impose an additional condition $g \gg 1$ with $g = \nu_0 D d$ being the dimensionless conductance per square. Here, ν_0 stands for DOS in a normal metal at the Fermi level and $D = v_F l/3$ is the diffusion constant. The latter condition restricts our analysis to not very strongly disordered samples.

Taking into account that the electron mean-free path l is usually of order of several nanometers while the superconducting coherence length ξ is usually bigger by 1–2 orders of magnitude, we conclude that our results apply directly to films several tens of nanometers thick. Hence, we expect that our predictions can be observed, e.g., in conventional superconducting materials such as aluminum.

Finally, within our analysis we will disregard fluctuations of the absolute value of the order parameter and set it equal to a constant $|\Delta(x,t)| = \Delta$. This order parameter value (as well as the critical temperature T_c) can actually include the fluctuation correction [9,10] which remains small under the conditions adopted here. Our system is considered to be in a thermodynamic equilibrium at a temperature T which is assumed to remain sufficiently low ranging from $T \to 0$ up to $T \lesssim \Delta(T)$ throughout our calculation.

Following [7] we will evaluate the nonequilibrium quasiclassical electron Green function [11,12]

$$\check{G}(t,t',x) = \begin{pmatrix} G^R(t,t',x) & G^K(t,t',x) \\ 0 & G^A(t,t',x) \end{pmatrix} \tag{1}$$

averaged over fluctuations of the phase of the order parameter φ and electromagnetic fields represented by the scalar and vector potentials V, A and then find the average (local) DOS $\nu(E,x)$ as

$$\nu(E,x) = \nu_0 \,\mathrm{tr} \frac{\sigma_3}{4} \left(G^R(E,x) - G^A(E,x) \right), \tag{2}$$

where

$$\check{G}(E,x) = \int d(t-t')\mathrm{e}^{iE(t-t')}\check{G}(t,t',x). \tag{3}$$

The Green function itself can, in principle, be found as a solution of the Usadel equations [11,12]. However, as it was demonstrated in [7], there is no actual need to solve this equation since the result may be derived making use of the gauge invariance of the theory combined with the Josephson relation between the phase and the voltage. Then for any configuration of φ, V, A we get

$$\check{G}(t,t',x) \simeq \mathrm{e}^{\frac{i}{2}\check{\varphi}(t,x)\sigma_3}\check{\Lambda}(t-t')\mathrm{e}^{-\frac{i}{2}\check{\varphi}(t',x)\sigma_3}. \tag{4}$$

Here, $\check{\Lambda}$ is the Green function of a uniform BCS superconductor and

$$\check{\varphi} = \begin{pmatrix} \varphi_+ & \varphi_- \\ \varphi_- & \varphi_+ \end{pmatrix}. \tag{5}$$

$\varphi_\pm = (\varphi_F \pm \varphi_B)/2$ is expressed via the phase values on the forward (F) and backward (B) branches of the Keldysh time contour. Averaging (4) over fluctuations yields the result for the DOS expressed in terms of the equilibrium Keldysh propagator

$$\mathcal{V}_{ab}(t,x) = -\frac{i}{4}\langle\varphi_a(t,x)\varphi_b(0,0)\rangle = \begin{pmatrix} \mathcal{V}^K(t,x) & \mathcal{V}^R(t,x) \\ \mathcal{V}^A(t,x) & 0 \end{pmatrix}_{ab} \tag{6}$$

of the phase variable φ [7],

$$\begin{aligned}\nu(E) &= \nu_0 \int d(t-t')\mathrm{e}^{iE(t-t')}\,\mathrm{tr}\left\langle \frac{\tau_3\sigma_3}{4}\mathrm{e}^{\frac{i}{2}\check{\varphi}(t,x)\sigma_3}\check{\Lambda}(t-t')\mathrm{e}^{-\frac{i}{2}\check{\varphi}(t',x)\sigma_3}\right\rangle_\varphi \\ &= \nu_0\int dt\,\mathrm{e}^{iEt}\,\mathrm{tr}\left(\frac{\tau_3\sigma_3}{4}\,\tau_a\check{\Lambda}(t)\tau_b\mathbb{B}^{ab}(t)\right) = \int \frac{d\epsilon}{2\pi}\nu_{\mathrm{BCS}}(\epsilon)\mathbb{B}^K_{E-\epsilon}\left(1+F_\epsilon F_{E-\epsilon}\right)\end{aligned} \tag{7}$$

where $a,b = \{0,1\}$,

$$\mathbb{B}(t) = \begin{pmatrix} \mathbb{B}^K(t) & \mathbb{B}^R(t) \\ \mathbb{B}^A(t) & 0 \end{pmatrix} = \mathrm{e}^{i(\mathcal{V}^K(t)-\mathcal{V}^K(0))}\begin{pmatrix} \cos\left(\mathcal{V}^R(t)-\mathcal{V}^A(t)\right) & i\sin\left(\mathcal{V}^R(t)\right) \\ i\sin\left(\mathcal{V}^A(t)\right) & 0 \end{pmatrix}, \tag{8}$$

$\mathcal{V}(t) = \mathcal{V}(t,0)$, $F_\epsilon = \tanh\left(\frac{\epsilon}{2T}\right)$ is the fermionic equilibrium distribution function and $\nu_{\mathrm{BCS}}(E) = \nu_0\theta(\epsilon^2-\Delta^2)\frac{\epsilon}{\sqrt{\epsilon^2-\Delta^2}}$ is the usual DOS of a BCS superconductor. The phase propagator $\mathcal{V}$ itself can be found with aid of the effective action [1] by integrating over A, V,

$$\mathrm{e}^{iS_{\mathrm{eff}}[\varphi]} = \mathrm{e}^{\frac{i}{8}\,\mathrm{tr}\left(\vec{\varphi}^T\mathcal{V}^{-1}\vec{\varphi}\right)} = \int DVDA\,\mathrm{e}^{iS[\varphi,V,A]}. \tag{9}$$

As usual, magnetic effects can be neglected. Then the integration yields the result [1]

$$\left(\mathcal{V}^R\right)^{-1}(\omega,q) \simeq 2s\frac{\left(\frac{\chi_J(\omega+i0)^2}{e^2}-\frac{\chi_L q^2}{m^2}\right)\left(U_c^{-1}/s+\chi_D q^2\right)-\frac{\chi_L\chi_J q^2}{m^2}}{U_c^{-1}/s+\chi_J+\chi_D q^2}. \tag{10}$$

This formula holds for nanowires and can be directly extended to superconducting films by means of the substitution $s \to d$. In the limit $\omega, Dq^2 \ll \Delta$ the kernels $\chi_{J,L,D}$ read [4,13],

$$\chi_J \simeq e^2\nu_0, \quad \chi_L \simeq \pi m^2 \nu_0 D\Delta, \quad \chi_D \simeq \frac{e^2\nu_0 D}{8\Delta}. \tag{11}$$

For the effective inverse Coulomb interaction one needs to use $U_c^{-1} = C$ in the case of strong screening in the substrate with C being the capacitance per unit length/area, or $U_c^{-1} = |q|/2\pi$ in the unscreened 2D case. The Josephson relation holds as long as $\chi_J \gg U_c^{-1}/s, \chi_D q^2$. At low energies this condition is satisfied in generic experimental setups which allows for yet one more simplification:

$$(\mathcal{V}^R)^{-1}(\omega, q) = \begin{cases} \frac{2C(\omega+i0)^2}{e^2} - 2s\pi\nu_0 D\Delta q^2, & \text{1D,} \\ \frac{|q|(\omega+i0)^2}{\pi e^2} - 2d\pi\nu_0 D\Delta q^2, & \text{2D, unscreened,} \\ \frac{2C(\omega+i0)^2}{e^2} - 2d\pi\nu_0 D\Delta q^2, & \text{2D, screened.} \end{cases} \tag{12}$$

It is now straightforward to evaluate both the effective propagator $\mathbb{B}$ and the DOS.

3 Density of states

In order to proceed let us first establish the plasmon spectral density $J(\omega)$ defined as

$$J(\omega) = -\frac{1}{\pi}\int \frac{d^D q}{(2\pi)^D}\,\mathrm{Im}(\mathcal{V}^R(\omega, q)) = \begin{cases} 1/2g\omega, & \text{1D,} \\ \mathrm{sign}(\omega)/4\pi^2 g\Delta, & \text{2D, unscreened,} \\ \mathrm{sign}(\omega)/8\pi^2 g\Delta, & \text{2D, screened.} \end{cases} \tag{13}$$

Here, the dimensionless parameter g equals to $g = 2\pi\sqrt{\pi\nu_0 D\Delta s C/e^2}$ in the 1D case and to $g = \nu_0 D d$ in the 2D one corresponding respectively to the wire inverse impedance (normalized by the quantum resistance) and to the dimensionless conductance per square. In the 2D case, we will restrict our analysis to not too strongly disordered films with $g \gg 1$. In the two limiting 2D regimes, the function $J(\omega)$ (13) differs only by the factor 2 implying that the effect of fluctuations in 2D is largely independent from the substrate, unlike in the 1D case. Hence, in what follows it suffices to restrict our analysis, e.g., to the unscreened 2D case and then to account for the opposite fully screened limit by a trivial substitution $g \to 2g$.

For the effective propagator we obtain

$$\mathbb{B}^K(t) = \mathrm{e}^{-\int_{-\omega_c}^{\omega_c} d\omega J(\omega)\coth\left(\frac{\omega}{2T}\right)(1-\cos(\omega t))} \cos\left(\int_{-\omega_c}^{\omega_c} d\omega J(\omega)\sin(\omega t)\right), \tag{14}$$

which yields

$$\mathbb{B}^K_\omega \approx \begin{cases} \cosh\left(\frac{\beta\omega}{2}\right)\left(\frac{2\pi T}{\omega_c}\right)^{1/g} \frac{\left|\Gamma\left(\frac{1}{2g} + \frac{i\omega}{2\pi T}\right)\right|^2}{2\pi T\Gamma(1/g)}, & \text{1D} \\ \frac{\pi}{T\Gamma(\alpha)}\left(\frac{\omega}{T}\right)^{\alpha-1} \mathrm{e}^{-|\omega|/T}, & \text{2D} \end{cases} \tag{15}$$

with $\alpha = T/\pi^2 g\Delta$ and $\omega_c \sim \Delta$ being the cutoff frequency. Combining this result with equation (7) we arrive at the expression for DOS.

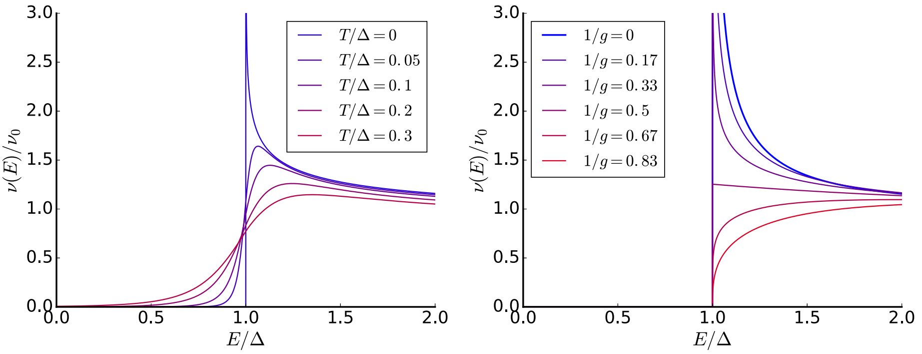

Fig. 1. The energy dependent DOS in the 1D case for $g = 3$ at different temperatures (left panel) and at $T = 0$ for different g (right panel). The depletion of DOS near the gap edge due to fluctuations is compensated at higher energies (outside the plot frame) in a way to assure the conservation of the total number of electron states.

We observe that for $\epsilon \gtrsim E + 2T$ the combination $1 + F_\epsilon F_{E-\epsilon}$ decays as $\propto \exp((E - \epsilon)/T)$. Hence, for $|\epsilon| < \Delta$ the electron DOS is suppressed exponentially by the factor $\sim \exp((|E| - \Delta)/T)$, i.e. at $T \to 0$ no subgap states emerge in both 1D and 2D superconductors. Furthermore, since $\mathbb{B}^K(t = 0) = 1$ one has $\int(\nu(E) - \nu_{\mathrm{BCS}}(E))dE = 0$, implying that the electron states can only be redistributed over energies; however, the total (energy integrated) DOS remains unaffected by fluctuations.

The behavior of the DOS in the immediate vicinity of the gap edge is qualitatively different in different dimensions. In 1D at any non-zero temperature the BCS singularity at $E = \Delta$ gets smeared, whereas at $T = 0$ the DOS exhibits a power law behavior

$$\nu(E) \simeq \frac{\nu_0\sqrt{\pi}\theta(E-\Delta)}{\sqrt{2}\Gamma(\frac{1}{2}+\frac{1}{g})}\left(\frac{E-\Delta}{\Delta}\right)^{\frac{1}{g}-\frac{1}{2}}, \quad |E-\Delta| \ll \Delta. \tag{16}$$

In the 2D case, the effect of fluctuations turns out to be much less pronounced. In this case, the gap edge singularity survives even at $T \neq 0$ (no suppression of the coherence peaks occurs), however its power changes to $(E - \Delta)^{\alpha-1/2}$. Perhaps the most important effect of phase fluctuations in 2D is the appearance of non-zero DOS in the subgap region. At $\alpha \ll 1$ one obtains

$$\nu(\Delta-\omega) \approx \frac{\alpha\nu_0}{2}\sqrt{\frac{\Delta}{2\omega}} \times \begin{cases} \pi, & \omega \ll T \\ \sqrt{\frac{2\pi T}{\omega}}\mathrm{e}^{-2\omega/T}, & T \ll \omega \ll \Delta. \end{cases} \tag{17}$$

The effect is proportional to α down to extremely low temperatures and vanishes as $T \to 0$. At $T \sim \Delta$ the supgap DOS exhibits a power-law behavior diverging at $E = \Delta$. At such temperatures this correction to DOS can be significant even in samples which are relatively far from both the superconducting phase transition and the superconductor–insulator transition (SIT).

The results for the electron DOS affected by phase fluctuations are displayed in Figures 1 and 2.

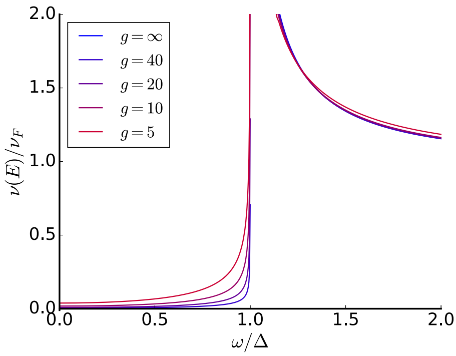

Fig. 2. The energy dependent DOS in the 2D case for $T \sim \Delta$ and different g.

4 Discussion

In this paper, we investigated the effect of phase fluctuations on the electron DOS in quasi-1D and quasi-2D superconductors. This effect turns out to be equivalent to that of a quantum dissipative environment with spectral density defined in equation (13). Interaction with such an environment depends on the system dimensionality, temperature and the dimensionless parameter g (defined differently in 1D and 2D cases) and causes a redistribution of the electron states over energies. Overall, the fluctuation effect is more pronounced in nanowires than in ultrathin films.

In 1D the coupling between electrons and the wire plasma modes is controlled by the parameter g representing the ratio between the quantum resistance unit R_q and the wire impedance $Z_{\rm w}$. At $g \gg 1$ and low enough temperatures fluctuations weakly affect the DOS except in the vicinity of the gap edge. At any non-zero T fluctuations suppress the gap edge singularity and induce nonvanishing DOS at energies below Δ. It is interesting that even at $T = 0$ phase fluctuations may significantly change the DOS above the gap softening the singularity and even leading to its total disappearance for $g < 2$. We also note that some of our predictions were verified in recent experiments with superconducting nanowires [14,15].

In 2D we investigated the effect of fluctuations in both limits of screened and unscreened Coulomb interaction. It turns out that the only difference between these two regimes is the factor of 2 in the plasmon spectral density (see Eq. (13)) implying that in 2D the influence of phase fluctuations on DOS is weakly affected by the substrate, unlike in 1D. The magnitude of the effect is determined by both the dimensionless conductance per square g and the temperature. At low enough $T \lesssim \Delta$ the deviation of the DOS from its BCS form is controlled by the parameter $\alpha = T/\pi^2 g\Delta$ which remains small as long as $g \gg 1$. Perhaps the most prominent effect in this case is the appearance of an exponentially decaying "tail" of states below the gap, cf. equation (17). At not too low $T \sim \Delta$ the supgap DOS exhibits a power-law dependence on $\Delta - E$ and survives at energies well below the gap, as it is indicated in Figure 2. As the temperature becomes closer to T_c the effect becomes even more pronounced. Using the corresponding asymptotics of the kernels $\chi_{J,L,D}$ [4,13] one comes to the conclusion that fluctuations in this case yield qualitatively similar corrections which are now controlled by parameter $\tilde{\alpha} = \frac{4T^2}{\pi g \Delta^2}$ which differs from α by an additional large factor of $\frac{4\pi T}{\Delta}$. At temperatures close to T_c there are additional effects associated with fluctuations of the absolute value of the order parameter and those due to the formation of vortices which can also contribute to the broadening of the coherence peaks and the appearance of states below the gap. However, for temperatures not too close to the transition, $(T_c - T)/T_c > Gi_{2\mathrm{D}} \sim 1/g$, we expect the largest contribution to be caused by smooth fluctuations of phase.

Recently, an analogous effect in disordered superconducting films was investigated in reference [16] where authors considered the regime below the SIT. They obtained fluctuation corrections to the electron DOS by means of a diagrammatic resummation using a phenomenological order parameter correlation function. Here, evaluate determine the corresponding correlation function entirely from the microscopic theory which, we believe, should also be possible in the situation considered in [16].

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Author contribution statement

We declare that each of the three authors equally contributed to both the scientific contents and writing of this manuscript.

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