

# Enhancing synchronization in chaotic oscillators by induced heterogeneity

Ranjib Banerjee<sup>1,a</sup>, Bidesh K. Bera<sup>2</sup>, Dibakar Ghosh<sup>2</sup>, and Syamal Kumar Dana<sup>3</sup>

<sup>1</sup> School of Engineering & Technology, BML Munjal University, Gurgaon, India

<sup>2</sup> Physics and Applied Mathematics Unit, Indian Statistical Institute, Kolkata, India

<sup>3</sup> Department of Mathematics, Jadavpur University, Kolkata 700032, India

Received 31 January 2017 / Received in final form 6 March 2017

Published online 21 June 2017

**Abstract.** We report enhancing of complete synchronization in identical chaotic oscillators when their interaction is mediated by a mismatched oscillator. The identical oscillators now interact indirectly through the intermediate relay oscillator. The induced heterogeneity in the intermediate oscillator plays a constructive role in reducing the critical coupling for a transition to complete synchronization. A common lag synchronization emerges between the mismatched relay oscillator and its neighboring identical oscillators that leads to this enhancing effect. We present examples of one-dimensional open array, a ring, a star network and a two-dimensional lattice of dynamical systems to demonstrate how this enhancing effect occurs. The paradigmatic Rössler oscillator is used as a dynamical unit, in our numerical experiment, for different networks to reveal the enhancing phenomenon.

## 1 Introduction

Various types of collective behaviors emerge when two or more oscillatory units interact with each other; synchronization is one of the most important collective behaviors due to a range of applications in different fields [1, 2], physics, chemistry, biology, network science and technology. The concept of chaos synchronization is useful in secure communications [3–7], for encoding and information processing in neuronal systems. Synchronization in two coupled chaotic oscillators and also in complex networks has been extensively studied in the last two decades [8, 9]. Some of the important forms of synchronization are complete synchronization (CS) or zero lag synchronization (ZLS) [10], phase synchronization (PS) [11], lag synchronization (LS) [12] and generalized synchronization (GS) [13] that might occur either in identical or nonidentical chaotic systems. In a large ensemble of oscillators, more varieties of collective behaviors such as clustering [14], partial synchronization [15], chimera states [16–18], relay synchronization (RLS) [10, 19–21] and remote synchronization [22, 23], were reported. In this paper, we emphasize on RLS [10] that defines a state of synchrony between two indirectly coupled oscillators interacting through an intermediate oscillator, called as a

<sup>a</sup> e-mail: [ranjib.b@gmail.com](mailto:ranjib.b@gmail.com)

relay unit in a network. We revisit our previous study [19] on RLS and the related enhancing effect in an open array of oscillators and further extend the results to different other networks. RLS was first reported [10] in diode lasers when two delay coupled laser sources were interacting via a relaying third laser source. The robustness of RLS against heterogeneity and noise were experimentally demonstrated in lasers [10, 24–26] and electronic circuits [27, 28]. The idea of RLS is useful [29] for transmitting and recovering encrypted messages. The dynamical relaying mechanism and the associated RLS is a possible recipe for isochronous synchronization between distantly located cortical areas of brain [30–32].

We established a type of RLS earlier [19] where the critical coupling for CS was found reduced in two identical chaotic oscillators when the parameter of the mediating third oscillator was detuned from the identical parametric condition. The induced parameter mismatch or heterogeneity in the mediating oscillator played a constructive role on CS of the indirectly interacting identical oscillators. Basically, LS emerged between both the identical units and the mismatched relay unit for a coupling strength lower than the coupling threshold for a CS state. This reduction in critical coupling of CS between the identical units via the relay unit was explained as enhancing of synchrony. We show here that the enhancing phenomenon is not limited to an open chain of oscillators rather it can emerge in other type of networks as well. RLS was also reported by others [21] during the onset of GS where the authors also found an enhancing effect. The role of heterogeneity in the enhancement of CS was also explored [33] in a complex network. Different processes such as induced heterogeneity [20, 33], coupling delay [34], noise [35–38] have been suggested for the enhancement of chaos synchronization in two or more oscillators.

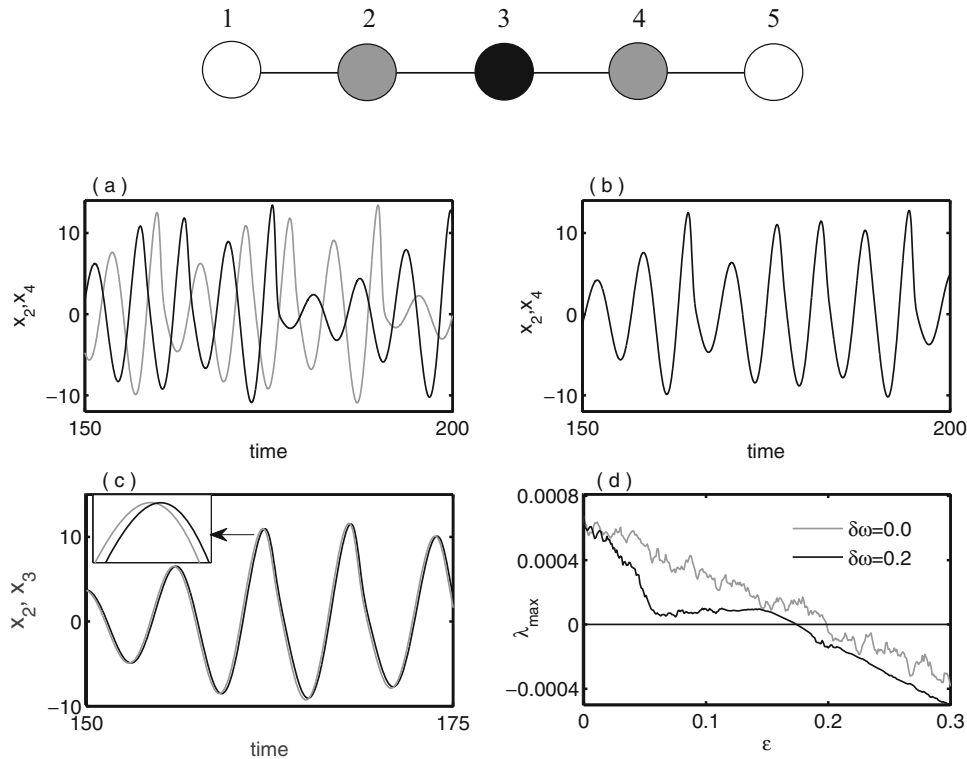
We emphasize here that a natural presence of heterogeneity in dynamical systems or an induced heterogeneity (positive and negative) can really play constructive role on synchrony. We explore this constructive role in chaotic oscillators using different coupling configurations, an one-dimensional open array, a star network, a ring of oscillators and a 2-dimensional (2D) lattice of oscillators. We consider the chaotic Rössler oscillator as a paradigmatic model, although the text, and always use mutually diffusive interactions between any two oscillators, although we know that the phenomenon can be seen in many other chaotic models. We mention here that the enhancing effect is observed in chaotic systems only [19] since LS emerges in such systems at a lower coupling than the critical coupling for CS. On the other hand, in limit cycle systems, CS emerges in identical systems for a coupling smaller than the coupling for LS in mismatched systems.

The rest of the paper is arranged as follows: the enhancement of CS under RLS in one dimensional linear array, ring network are presented in Section 2. The study has been extended to star networks as illustrated in Section 3. The case of two dimensional array is explained in Section 4. The manuscript ends with a conclusion in Section 5.

## 2 One dimensional array and a ring lattice

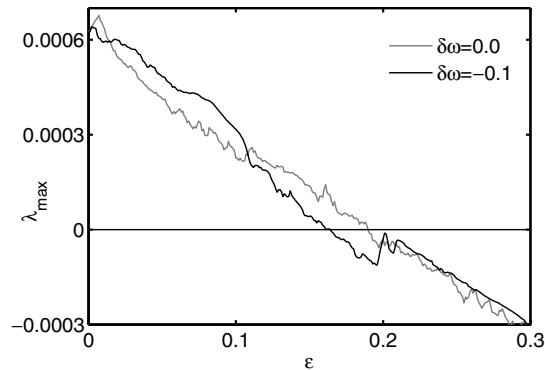
We consider an open array and a ring of  $N$ -coupled Rössler oscillators with nearest neighbor diffusive coupling. The dynamical equation of the networks is

$$\begin{aligned}\dot{x}_i &= -\omega_i y_i - z_i + \epsilon(x_{i-1} + x_{i+1} - 2x_i) \\ \dot{y}_i &= \omega_i x_i + a y_i \\ \dot{z}_i &= b + z_i(x_i - c), \quad i = 1, \dots, N.\end{aligned}\tag{1}$$



**Fig. 1.** One-dimensional array of five Rössler oscillators (top). The oscillator in black color has parameter mismatch. Pair of  $(x_2, x_4)$  times series of oscillators 2 and 4 shown for  $\epsilon = 0.19$ , (a) in black and gray lines in all identical case when they are in a desynchronized state, (b) CS emerges when a mismatch  $\delta\omega = 0.2$  is introduced in the relay oscillator numbered 3 and, (c) pair of  $(x_2, x_3)$  time series for oscillators 2 and 3 shows a LS state; time lag of the adjacent oscillators with the mismatched oscillator. (d) Plots of master stability function  $\lambda_{max}$  against the coupling strength  $\epsilon$  shows (gray line for  $\delta\omega=0$ , black line for  $\delta\omega=0.2$ ) a drifting to a lower critical value for induced heterogeneity.

where  $\epsilon$  is the coupling strength. The uncoupled system ( $\epsilon = 0$ ) exhibits chaotic dynamics for a choice of parameters,  $\omega_i = \omega + \delta\omega_i$ ,  $a = 0.2$ ,  $b = 0.4$ , and  $c = 7.5$ . For all the oscillators,  $\omega = 1$  and  $\delta\omega = \delta\omega_i = 0$ , except for the mismatched oscillator, for which  $\delta\omega = 0.2$ . Depending on the boundary conditions, equation (1) represents either an open array or a ring topology. We consider five ( $N = 5$ ) Rössler oscillators, as an example, where a boundary condition  $x_0 = x_1$  and  $x_{N+1} = x_N$  represents an open array. For the ring configuration, a periodic boundary condition  $x_0 = x_N$  and  $x_{N+1} = x_1$  is considered. An open array of five oscillators is shown in the upper panel of Figure 1 where the black circle represents the relay oscillator with a mismatch  $\delta\omega$  and indirectly coupled identical oscillators in open and gray circles. In this open array, the critical coupling for CS between the identical oscillators in symmetric positions to the relay oscillator in the center, reduces. A LS state emerges between the central relay oscillator and its neighboring relayed systems, and thereby CS is enhanced between all the identical oscillators in symmetric positions on both sides. All identical oscillators ( $N = 5$ ) emerge into a CS state for a coupling strength  $\epsilon \geq 0.1985$ . Figure 1a shows a desynchronized state between the indirectly coupled oscillators (2, 4) for a coupling



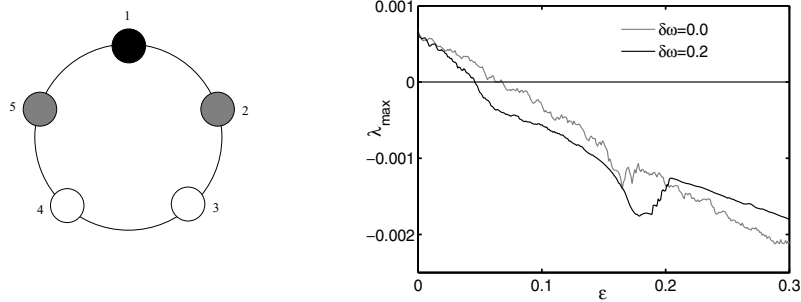
**Fig. 2.** Plots of master stability function  $\lambda_{max}$  against the coupling strength  $\epsilon$  show (gray line for  $\delta\omega=0$ , black line for  $\delta\omega = -0.1$ ) a drifting to a lower critical value for induced heterogeneity. For identical case, when  $\delta\omega=0$  for all the oscillators in the lattice,  $\omega_i=1.1$  is used and in the mismatched case,  $\omega_i = 1.1 + \delta\omega$  is used only for the oscillator in black color. Rest of the oscillators have the same set of parameter values.

strength  $\epsilon = 0.19$  that is lower than the critical coupling when all the oscillators are identical. When a mismatch  $\delta\omega = 0.2$  is introduced in the central oscillator (black circle oscillator 3), then CS is observed between  $(x_2, x_4)$ - and  $(x_1, x_5)$ -pair and time lag is observed between the time series of  $(x_2, x_3)$ - or  $(x_3, x_4)$ -pair in Figures 1b and 1c, respectively. The inset figure of Figure 1c shows the LS in an enlarged part of the time series. Interesting point to note that the amount of delay does not propagate as the distance of the relayed systems increases from the relay unit. Rather, it remains constant for all the oscillators in the network. Figure 1d shows the variation of master stability function (MSF)  $\lambda_{max}$  with coupling strength  $\epsilon$ . Pairs of (2, 4)-oscillators and (1, 5)-oscillators emerge into a CS state for  $\epsilon > 0.1985$  (gray line) when five oscillators are identical ( $\delta\omega = 0$ ).  $\lambda_{max}$  crosses the line to a negative value at a lower critical coupling strength  $\epsilon \geq 0.174$  (black line) when a mismatch is introduced in the central oscillator. For a negative mismatch ( $\delta\omega = -0.1$ ), the effect is found similar as shown in Figure 2.

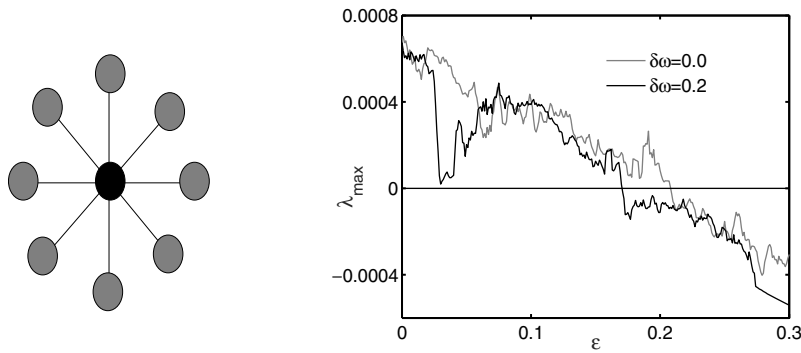
Next we consider a ring of oscillators shown in Figure 3. The dynamical equation for this network is represented by equation (1) with the boundary conditions,  $x_0 = x_N$  and  $x_{N+1} = x_1$ . For this ring network, five ( $N = 5$ ) mutually coupled Rössler oscillators are chosen, with any one of them having a parameter mismatch: here the black color circle is the mismatched oscillator with an amount of mismatch  $\delta\omega$ . The nodes at equal distances on both sides of the relay oscillator show an enhancing effect and the distance from the central node does not affect the time lag. Variation of  $\lambda_{max}$  with respect to coupling strength  $\epsilon$  is shown in the right panel of Figure 3 for identical (gray) and mismatched (black) cases. We observe again that the critical coupling strength for CS in all the identical oscillators is lowered when a mismatch is introduced in any one of the oscillator.

### 3 Star lattice

A similar enhancing of synchronization due to heterogeneity is also observed in a star network. In a star network shown in the left panel of Figure 4, one central node (black circle) communicates directly with all the remaining nodes (gray color circles), and the other nodes communicate between themselves via the central node considered as a hub. The central node relays the information to all the other oscillators. The



**Fig. 3.** Left panel shows five oscillators in a closed ring. The oscillator in black color has parameter mismatch. Right panel: Variation of MSF with respect to coupling strength  $\epsilon$ .



**Fig. 4.** Left panel: Coupling configuration of star network. All the gray color oscillators are identical and black oscillator has parameter mismatch. Right panel: Variation of MSF for synchronization between identical nodes (gray circle).

heterogeneity is induced in the hub as a mismatch  $\delta\omega$ . The dynamical equation of the star network is

$$\dot{x}_1 = -\omega_1 y_1 - z_1 + \frac{\epsilon}{N-1} \sum_{j=2}^N (x_j - x_1)$$

$$\dot{y}_1 = \omega_1 x_1 + a y_1 \tag{2a}$$

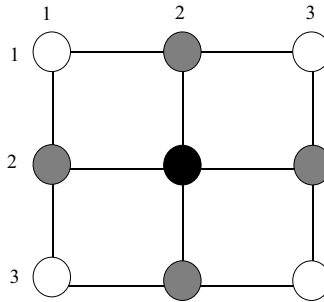
$$\dot{z}_1 = b + z_1(x_1 - c)$$

$$\dot{x}_i = -\omega_i y_i - z_i + \epsilon(x_1 - x_i)$$

$$\dot{y}_i = \omega_i x_i + a y_i \tag{2b}$$

$$\dot{z}_i = b + z_i(x_i - c), \quad i = 2, \dots, N.$$

Equation (2a) describing the dynamics of  $[x_1, y_1, z_1]^T$  represents the hub. A parameter mismatch  $\omega_1 = 1 + \delta\omega$  is introduced in the hub. All indirectly connected outer nodes are identical ( $\omega_i = 1$  for  $i = 2, \dots, N$ ), which are represented by their state variables  $[x_i, y_i, z_i]^T, i = 2, 3, \dots, N$  in equation (2b). To reveal enhancing of synchrony in the star network, we consider  $N = 15$  Rössler oscillators and their system parameters in a chaotic regime. Variation of  $\lambda_{max}$  with coupling strength is shown in the right panel of Figure 4 for identical (gray) and mismatched (black) cases. Figure 4b of  $\lambda_{max}$  clearly shows a lowering of critical coupling (black) of CS in the outer oscillators for



**Fig. 5.** Two dimensional grid of bidirectionally coupled Rössler oscillator where black color oscillator represents the mismatched oscillator and open circles and gray colors circles are identical oscillators.

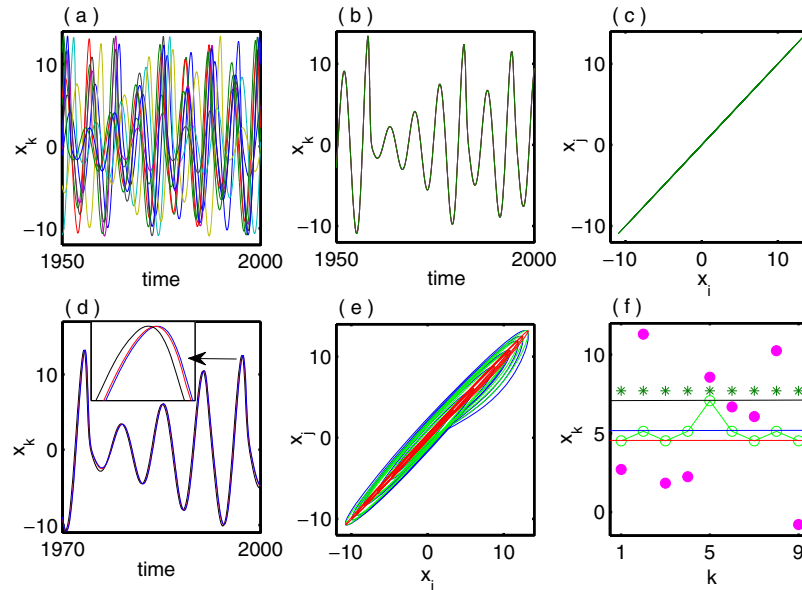
a mismatch in the hub. Plot of  $\lambda_{max}$  for all identical case shows a larger critical coupling (gray) for CS.

#### 4 Two dimensional lattice

Finally, we consider a 2D lattice of chaotic Rössler oscillators shown in Figure 5. The dynamical equation of the 2D lattice is

$$\begin{aligned}\dot{x}_{i,j} &= -\omega_{i,j}y_{i,j} - z_{i,j} + \epsilon(x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1} - 4x_{i,j}) \\ \dot{y}_{i,j} &= \omega_{i,j}x_{i,j} + ay_{i,j} \\ \dot{z}_{i,j} &= b + z_{i,j}(x_{i,j} - c)\end{aligned}\quad (3)$$

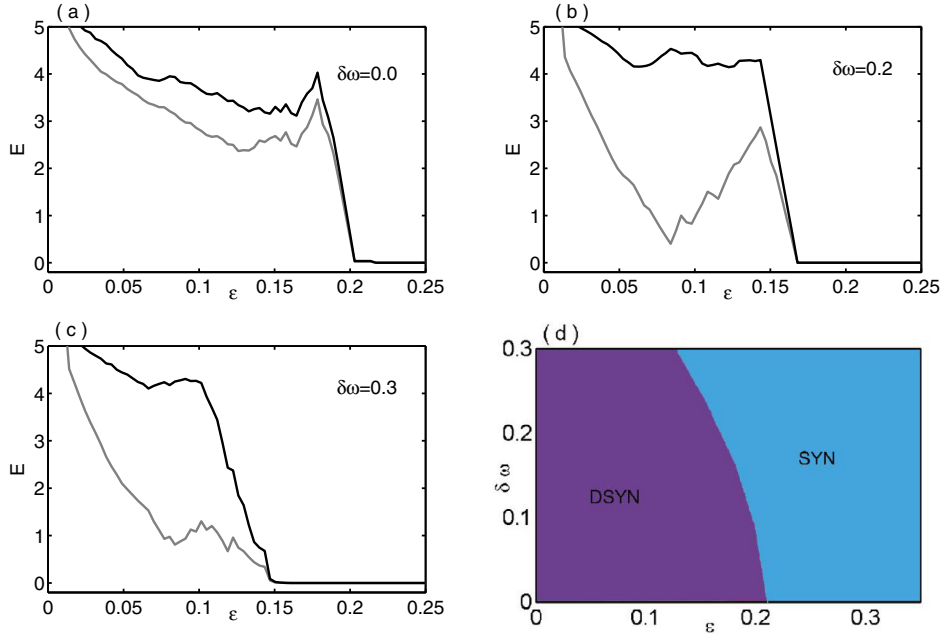
where  $i, j = 1, 2, 3$ . Individual oscillators are chaotic in absence of coupling ( $\epsilon = 0.0$ ) for  $\omega_{i,j} = 1$  for all  $i, j$  and parameters,  $a = 0.2, b = 0.4, c = 7.5$ . As an example, we consider a  $3 \times 3$  lattice in Figure 5: all open circles and gray circles represents identical oscillators,  $\omega_{i,j} = 1$  and the central black node represents the mismatched oscillator  $\omega_{2,2} + \delta\omega$ . All open circle nodes communicates with the black node via two links whereas the gray color nodes form direct links with the black node. For the identical case (i.e.  $\delta\omega = 0.0$ ), we consider a lower coupling strength  $\epsilon = 0.19$  than the critical coupling when they are all desynchronized: chaotic time series of all identical oscillators confirm this in Figure 6a. When the coupling strength is increased to  $\epsilon = 0.25$  (above the critical value), all the oscillators reaches a CS state in chaotic motion as confirmed by their corresponding time series in Figure 6b and the associated synchronization manifold  $x_i$  vs.  $x_j$  plot in Figure 6c. When we induce a mismatch in the hub (black circle) at a lower coupling  $\epsilon = 0.19$ , we observe three separate groups or clusters: open circle nodes form a cluster in a CS state, gray color nodes form another cluster in a CS state but maintain a LS with the open circle nodes. On the other hand, the central hub remain isolated and forms a separate group, however, it also maintains a lag with gray color nodes but different from that of open circle nodes. We refer to this overall state as cluster synchronization. For demonstration, we induce a small mismatch  $\delta\omega = 0.1$  in the central oscillator (black circle) when all 9-oscillators are divided into three clusters: one cluster contains degree 2 nodes (open circles) far-away from the central node (black circle), another cluster contains the degree 3 nodes (gray color circles) and at a relay distance 1 from the central node (black circle) and, the central mismatched oscillator remains isolated from other two clusters. The time series of all oscillators are plotted in Figure 6d for  $\delta\omega = 0.1$  and  $\epsilon = 0.19$  which are



**Fig. 6.** Clustering synchronization in a 2D lattice Rössler oscillators. Time series of 9-identical oscillators show a desynchronized state for  $\epsilon = 0.19$ . In (a). In absence of parameter mismatch ( $\delta\omega = 0$ ), time series in (b) of all the oscillators are in CS for a larger  $\epsilon = 0.25$ , corresponding synchronization manifolds in (c). Under induced heterogeneity  $\delta\omega = 0.1$  and the lower coupling strength,  $\epsilon = 0.19$ , three clusters form. Two clusters emerge into separate CS states but they develop LS (blue and red lines) as shown in their time series plot in (d) when the central oscillator (black line) is isolated but having a lower lag time with the nearest neighbor. The corresponding LS manifolds are shown in (e). Snapshots of all the oscillators at a particular instant are plotted in (f) where magenta (solid circles), dark green (star) and green (open circles) colors represent the desynchronized state ( $\delta\omega = 0.0, \epsilon = 0.19$ ), CS state ( $\delta\omega = 0.0, \epsilon = 0.25$ ) and cluster synchronization ( $\delta\omega = 0.1, \epsilon = 0.19$ ), respectively. In cluster state, two clusters green color (open circles) are seen along the blue and red lines, while the isolated central node is seen along the black line.

magnified in the inset where three distinct colors, namely, black, blue and red lines are shown. It indicates that all open circle nodes are in a CS state and, the gray color nodes are also in a separate CS state but they maintain a lag. The black line represents the hub node which also maintain LS with the other two clusters, however, its lag time is different from red and blue lines. The amount of lag time between the central node and the open circle nodes is more than the lag between the central node and the gray color nodes. The synchronization manifolds  $x_{1,1}$  vs.  $x_{2,2}$  (blue color),  $x_{1,2}$  vs.  $x_{2,2}$  (green color) and  $x_{1,1}$  vs.  $x_{1,2}$  (red color) are plotted in Figure 6e. Snapshots of the amplitude at a particular instant are plotted in Figure 6f where magenta (solid circles), green (open circles) and dark green (stars) colors represents the desynchronization ( $\epsilon = 0.19, \delta\omega = 0.0$ ), cluster synchronization ( $\epsilon = 0.19, \delta\omega = 0.1$ ) and CS ( $\epsilon = 0.25, \delta\omega = 0.0$ ) in the network.

Enhancing of synchronization between the indirectly coupled oscillators of the 2D lattice is described in Figure 7. In absence of any mismatch ( $\delta\omega = 0.0$ ), the two groups (open and gray color circles) are completely synchronized for  $\epsilon \geq 0.22$ . Figure 7a shows the variation of synchronization errors  $E$  of the open and gray color circles nodes by changing the coupling strength  $\epsilon$ . From this figure it is shown that the oscillators in the two groups are synchronized at the same critical coupling strength. If we



**Fig. 7.** Synchronization error  $E$  against coupling strength  $\epsilon$  of the bidirectionally coupled 2D lattice, (a)  $\delta\omega = 0.0$ , (b)  $\delta\omega = 0.2$  and (c)  $\delta\omega = 0.3$ . The black and gray colors represent the corresponding synchronization error of two cluster synchronization (open circle and gray color oscillators in 2D coupling topology). (d) The enhancement of relay synchronization is shown by varying the coupling strength  $\epsilon$  and mismatched parameter  $\delta\omega$  in two dimensional phase plane  $\epsilon - \delta\omega$  where violet (black) and cyan (gray) colors represent the corresponding desynchronized (DSYN) and synchronized (SYN) regions respectively.

introduce heterogeneity in the central oscillator ( $\delta\omega = 0.2$ ), the whole network forms two clusters (except the central node) and the oscillators (open and gray color circles) in each cluster are fully synchronized for lower value of  $\epsilon \geq 0.17$ . The corresponding synchronization error with respect to coupling strength is shown in Figure 7b. By further increasing the heterogeneity at the relay unit by  $\delta\omega = 0.3$ , synchronization in the two clusters is more enhanced (Fig. 7c). To explore the whole scenario by changing the heterogeneity  $\delta\omega$  and the coupling strength  $\epsilon$ , we plot a phase diagram in  $\epsilon - \delta\omega$  plane in Figure 7d where violet (black) and blue (gray) colors represent the desynchronized and synchronized region. The borderline of the synchronous and asynchronous regions indicates the critical coupling strength. It is clear that the critical coupling  $\epsilon = \epsilon_c$  decreases with increase of mismatch  $\delta\omega$ .

## 5 Conclusion

The effect of heterogeneity was studied on CS of chaotic oscillators using different configurations ranging from 1D array, ring, star and 2D lattice. It was shown that an induced heterogeneity or parameter mismatch in a suitably located central oscillator works as a relaying device to enhance CS in other identical nodes which interacted via the central node. The effect was found general in all the given example networks. The heterogeneity in the form of a parameter mismatch induced a LS between the mismatched oscillator and all the other identical oscillators, which, in turn, helped the identical oscillators at symmetric positions with respect to the relay



unit, synchronize at a lower coupling strength than the critical coupling of CS in all identical oscillators. The time-lag remained unaffected by the distance of the relayed units from the relay unit in 1D open array. In case of 2D lattice the time-lag changed with the degree of the relayed units and thereby they evolved into cluster synchronization. Each cluster remained in CS states but with separate lag between different clusters.

D.G. was supported by SERB-DST (Department of Science and Technology), Government of India (Project no. EMR/2016/001039).

## References

1. S. Boccaletti, *The Synchronized Dynamics of Complex Systems* (Elsevier, Amsterdam, 2008), Vol. 6
2. L.M. Pecora, T.L. Carroll, G.A. Johnson, D.J. Mar, J.F. Heagy, *Chaos* **7**, 520 (1997)
3. K. Murali, H. Yu, V. Varadan H. Leung, *IEEE Trans. Consum. Electron.* **47**, 709 (2001)
4. A.N. Miliou, I.P. Antoniadis, S.G. Stavrinos, A.N. Anagnostopoulos, *Nonlinear Analysis: Real World Applications* **8**, 1003 (2007)
5. D. Ghosh, S. Banerjee, *Phys. Rev. E* **78**, 056211 (2008)
6. D. Ghosh, S. Banerjee, A.R. Chowdhury, *Europhys. Lett.* **80**, 30006 (2007)
7. S. Banerjee, D. Ghosh, A. Ray, A.R. Chowdhury, *Europhys. Lett.* **81**, 20006 (2008)
8. A. Pikovsky, M. Rosenblum, J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge, 2003), Vol. 12
9. S. Boccaletti, J. Kurths, G. Osipov, D.L. Valladares, C.S. Zhou, *Phys. Rep.* **366**, 1 (2002)
10. I. Fischer, R. Vicente, J.M. Buldú, M. Peil, C.R. Mirasso, M.C. Torrent, J. García-Ojalvo, *Phys. Rev. Lett.* **97**, 123902 (2006)
11. M.G. Rosenblum, A.S. Pikovsky, J. Kurths, *Phys. Rev. Lett.* **76**, 1804 (1996)
12. M.G. Rosenblum, A.S. Pikovsky, J. Kurths, *Phys. Rev. Lett.* **78**, 4193 (1997)
13. N.F. Rulkov, M.M. Sushchik, L.S. Tsimring, H.D.I. Abarbanel, *Phys. Rev. E* **51**, 980 (1995)
14. V.N. Belykh, I.V. Belykh, E. Mosekilde, *Phys. Rev. E* **63**, 036216 (2001)
15. C.V. Vreeswijk, *Phys. Rev. E* **54**, 5522 (1996)
16. D.M. Abrams, S.H. Strogatz, *Phys. Rev. Lett.* **93**, 174102 (2004)
17. B.K. Bera, D. Ghosh, M. Lakshmanan, *Phys. Rev. E* **93**, 012205 (2016)
18. B.K. Bera, D. Ghosh, *Phys. Rev. E* **93**, 052223 (2016)
19. R. Banerjee, D. Ghosh, E. Padmanaban, R. Ramaswamy, L.M. Pecora, S.K. Dana, *Phys. Rev. E* **85**, 027201 (2012)
20. A. Bergner, M. Frasca, G. Sciuto, A. Buscarino, E.J. Ngamga, L. Fortuna, J. Kurths, *Phys. Rev. E* **85**, 026208 (2012)
21. R. Gutiérrez, R. Sevilla-Escoboza, P. Piedrahita, C. Finke, U. Feudel, J.M. Buldú, G. Huerta-Cuellar, R. Jaimes-Reategui, Y. Moreno, S. Boccaletti, *Phys. Rev. E* **88**, 052908 (2013)
22. A. Bergner, M. Frasca, G. Sciuto, A. Buscarino, E.J. Ngamga, L. Fortuna, J. Kurths, *Phys. Rev. E* **85**, 026208 (2012)
23. L.V. Gambuzza, A. Cardillo, A. Fiasconaro, L. Fortuna, J. Gómez-Gardenes, M. Frasca, *Chaos* **23**, 043103 (2013)
24. A. Englert, W. Kinzel, Y. Aviad, M. Butkovski, I. Reidler, M. Zigzag, I. Kanter, M. Rosenblum, *Phys. Rev. Lett.* **104**, 114102 (2010)
25. J. Tiana-Alsina, K. Hicke, X. Porte, M.C. Soriano, M.C. Torrent, J. Garcia-Ojalvo, I. Fischer, *Phys. Rev. E* **85**, 026209 (2012)
26. J.G. Wu, Z.M. Wu, G.Q. Xia, T. Deng, X.D. Lin, X. Tang, G.Y. Feng, *IEEE Phot. Tech. Lett.* **23**, 1854 (2011)

27. I.G. Da Silva, J.M. Buldú, C.R. Mirasso, J. García-Ojalvo, *Chaos* **16**, 043113 (2006)
28. A. Wagemakers, J.M. Buldú, M.A.F. Sanjuán, *Chaos* **17**, 023128 (2007)
29. A. Wagemakers, J.M. Buldú, M.A.F. Sanjuán, *Europhys. Lett.* **81**, 40005 (2008)
30. A.K. Engel, P.König, A.K. Kreiter, W. Singer, *Science* **252**, 1177 (1991)
31. P.R. Roelfsema, A.K. Engel, P. König, W. Singer, *Nature* **385**, 157 (1997)
32. R. Vicente, L.L. Gollo, C.R. Mirasso, I. Fischer, G. Pipa, *Proc. Natl. Acad. Sci. USA* **105**, 17157 (2008)
33. L.V. Gambuzza, M. Frasca, L. Fortuna S. Boccaletti, *Phys. Rev. E* **93**, 042203 (2016)
34. M. Dhamala, V.K. Jirsa, M. Ding, *Phys. Rev. Lett.* **92**, 074104 (2004)
35. A. Ray, A. Roy Chowdhury, D. Ghosh, *Physica A* **392**, 4837 (2013)
36. S. Majhi, B.K. Bera, S. Banerjee, D. Ghosh, *Eur. Phys. J. Special Topics* **225**, 65 (2016)
37. S.K. Bhowmick, B.K. Bera, D. Ghosh, *Commun. Nonlinear Sci. Numer. Simulat.* **22**, 692 (2015)
38. D.V. Senthilkumar, J. Kurths, *Eur. Phys. J. Special Topics* **187**, 87 (2010)