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Efficiencies of power plants, quasi-static models and the geometric-mean temperature

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Abstract. Observed efficiencies of industrial power plants are often approximated by the square-root formula: $1 - \sqrt{T_-/T_+}$, where $T_+ (T_-)$ is the highest (lowest) temperature achieved in the plant. This expression can be derived within finite-time thermodynamics, or, by entropy generation minimization, based on finite rates for the processes. In these analyses, a closely related quantity is the optimal value of the intermediate temperature for the hot stream, given by the geometric-mean value: $\sqrt{T_+ T_-}$. In this paper, instead of finite-time models, we propose to model the operation of plants by quasi-static work extraction models, with one reservoir (source/sink) as finite, while the other as practically infinite. No simplifying assumption is made on the nature of the finite system. This description is consistent with two model hypotheses, each yielding a specific value of the intermediate temperature, say T_1 and T_2. The lack of additional information on validity of the hypothesis that may be actually realized, motivates to approach the problem as an exercise in inductive inference. Thus we define an expected value of the intermediate temperature as the equally weighted mean: $(T_1 + T_2)/2$. It is shown that the expected value is very closely given by the geometric-mean value for almost all of the observed power plants.

1 Introduction

In recent years, there has been a great interest in extending thermodynamic models to justify the observed performance of industrial power plants [1–4]. The observed efficiencies are usually much less than the Carnot limit given by

$$\eta_C = 1 - \frac{T_-}{T_+}. \tag{1}$$

The above value involves only the ratio of the highest (T_+) and the lowest (T_-) temperatures for the particular plant. Naturally, real machines operate under irreversibilities caused by various factors, like finite rates of heat transfer and fluid flow, internal friction, heat leakage and so on, unlike the idealized quasi-static processes of a reversible cycle. Thus the analysis of irreversible models with finite-rate processes

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seems a reasonable goal to pursue. One often-studied measure is the efficiency at maximum power of an irreversible model which is then compared with the observed efficiency of these plants.

The earliest known such model is ascribed to Reitlinger [5], which involved a heat exchanger receiving heat from a finite hot stream fed by a combustion process. An analogous model was applied to a steam turbine by Chambadal [6]. The considered heat exchanger in these models is effectively infinite. Novikov [7] considered the heat transfer process between a hot stream and a finite heat exchanger with a given heat conductance. Two, simple but significant, assumptions enter these models: i) constant specific heat of the inlet hot stream and ii) validity of Newton's law for heat transfer. Further, there appears a floating, temperature variable in between the highest and the lowest values, such as the temperature of the exhaust warm stream, over which the output power can be optimized. This yields an optimal value of the intermediate temperature which is usually found to be $\sqrt{T_+ T_-}$, the geometric mean of T_+ and T_-. Related to this fact, is the conclusion that the efficiency at maximum power is given by an elegant expression:

$$\eta_{CA} = 1 - \sqrt{\frac{T_-}{T_+}}. \tag{2}$$

Due to historical imperative [8], the above expression may be called Reitlinger-Chambadal-Novikov efficiency. However, more recently it was rediscovered by Curzon and Ahlborn (CA) [1]. Thus in the physics literature, it is more popularly addressed as CA-value. This latter model considered finite rates of heat transfer at both the hot and the cold contacts, but also explicitly considered the times allocated to these contacts. The average power per cycle may be optimized over these times [1], or alternately, over the intermediate temperature variables [9]. This model spawned much activity and the new area borne thereforth was termed Finite-time Thermodynamics [10]. In the engineering literature, the corresponding approach is called Entropy Generation Minimization [2,11].

A positive indication for the simple thermodynamic approach is that the actually observed efficiencies of industrial plants happen to be quite close to CA-value. Figure 1 shows this comparison as tabulated in Table 1. Although the agreement is close, the observed values can be higher, or lower, than CA-value. This apparent discrepency has stimulated further extensions of models, for instance using the low-dissipation assumption [3], which predict the efficiency at maximum power, to be bounded as:

$$\frac{\eta_C}{2} \leq \eta \leq \frac{\eta_C}{2 - \eta_C}. \tag{3}$$

It is then realized that most of the observed efficiencies fall within these bounds [3,4]. Naturally, the question of the actual working constraints and the real optimization targets for each plant, is also relevant. Still, the effectiveness of these simple models, in reproducing the gross features of diverse plants, cannot be denied.

Apart from finite-time models, the irreversibilities reducing the efficiency to a lower-than-Carnot value, may also be treated within a quasi-static work extraction models with finite source/sink [13–16]. Some of these studies also derive the optimal value of an intermediate temperature which is the geometric-mean value, and consequently the efficiency at maximum work equals the CA-value. However, here again, the simplifying assumption of a constant heat capacity, say, of the source or the working medium, heavily determines the conclusion.

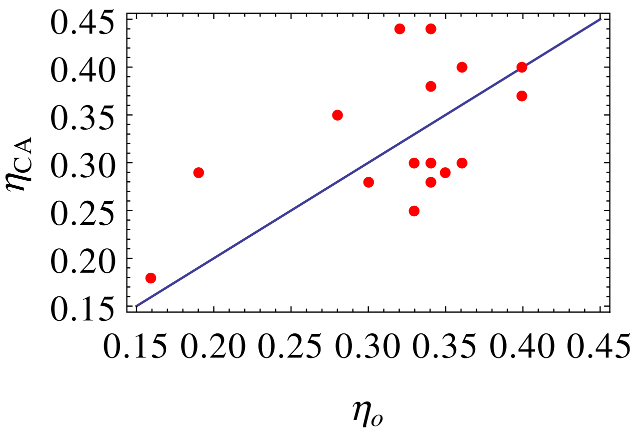

Fig. 1. Data on the observed efficiencies (η_o) of power plants, plotted against the CA-value, $\eta_{CA} = 1 - \sqrt{T_-/T_+}$ for respective plants as given in Table 1. A point lying on the straight line has the observed efficiency equal to CA-value. So for points above the line, η_o is below the CA-value, while the converse is true for the points below this line.

Recently, it was observed [17] that within linear response theory, the bounds such as equation (3), also follow within a quasi-static model of work extraction from finite-sized heat source and sink. Reference [17] makes no specific assumption about the nature of the heat source/sink. The finite size of source/sink is a consideration also from a practical point of view. Thus the hot source may be a finite amount of fuel, such as coal or natural gas in a power plant. Further, the sink may not be an infinite environment, as for instance in a congested environment such as cities, where heat dissipation is a concern in the overall design. The present study aims to carry forward this analogy [17] between quasi-static engine models with finite reservoirs, and finite-time models with infinite reservoirs, in the context of the performance of real industrial plants.

More precisely, we address an inverse question. Instead of finding the optimal intermediate temperature and then from it, the efficiency of the process, we use the information on the highest and the lowest temperatures, along with the value of observed efficiency, to infer the intermediate temperature at maximum work. The correspondence between this temperature and efficiency is as follows. If the former is exactly equal to the geometric-mean value, then the efficiency is equal to CA-value. The model of work extraction is based on one system as a finite source, or sink, and the other as an infinite reservoir, which allows for two alternate scenarios: i) when the source is finite and ii) when the sink is finite. The limited information on the actual situation being realized out of these two possibilities, motivates to do an inference analysis. Thus we estimate an expected value of the intermediate temperature. Interestingly, this value is found to be very close to the geometric mean of T_+ and T_-. We also present a quantitative argument for this proximity to the geometric-mean value. Thus our analysis indicates the role of geometric mean from a novel angle which has two distinctive features: first, it is based on maximum work approach, and second, we do not assume a specific nature of the finite source or the sink per se.

Our starting point is a reversible cycle, operating between two infinite heat reservoirs, for which the efficiency is the Carnot value η_C. As a first step that marks a deviation from reversibility, we consider one reservoir to be finite, while the other reservoir remains very large compared to the former, or practically infinite. Now, we

Table 1. Observed efficiencies (η_o) of power plants working between temperatures T_+ and T_-, compared with $\eta_{CA} = 1 - \sqrt{T_-/T_+}$. The effective temperatures are defined as: $T_m^{(+)} = T_+(1-\eta_0)$, $T_m^{(-)} = T_-/(1-\eta_0)$, $T_m^{(av)} = (T_m^{(+)} + T_m^{(-)})/2$, and $G(T_+, T_-) = \sqrt{T_+T_-}$.

Industrial Plant	T_+	T_-	η_o	η_{CA}	$T_m^{(-)}$	$T_m^{(+)}$	$T_m^{(av)}$	$G(T_+, T_-)$
Almaraz II (Nuclear, Spain) [2]	600	290	.34	.30	439.39	396.0	417.7	417.13
Calder Hall (Nuclear, UK) [2]	583	298	.19	.29	367.9	472.23	420.07	416.81
CANDU (Nuclear, Canada) [1]	573	298	.30	.28	425.71	401.1	413.41	413.22
Cofrentes (Nuclear, Spain) [2]	562	289	.34	.28	437.88	370.92	404.40	403.01
Doel 4 (Nuclear, Belgium) [2]	566	283	.35	.29	435.39	367.9	401.64	400.22
Heysham (Nuclear, UK) [2]	727	288	.40	.37	480.0	436.2	458.1	457.58
Larderello (Geothermal, Italy) [1]	523	353	.16	.18	420.24	439.32	429.78	429.67
Sizewell B (Nuclear, UK) [2]	581	288	.36	.30	450.0	371.84	410.92	409.06
West Thurrock (Coal, UK) [1]	838	298	.36	.40	465.63	536.32	500.97	499.72
Pressurized water nuclear reactor [12]	613	304	.33	.30	453.73	410.71	432.22	431.69
Boiling water nuclear reactor [12]	553	304	.33	.25	453.73	370.51	412.12	410.02
Fast neutron nuclear reactor [12]	823	296	.40	.40	493.33	493.8	493.57	493.57
(Steam/Mercury, US) [2]	783	298	.34	.38	451.52	516.78	484.15	483.05
(Steam, UK) [2]	698	298	.28	.35	413.89	502.56	458.22	456.08
(Gas Turbine, Switzerland) [2]	963	298	.32	.44	438.24	654.84	546.54	535.7
(Gas Turbine, France) [2]	953	298	.34	.44	451.52	628.98	540.25	532.91

first assume that system A acts as a finite heat sink at temperature T_-, relative to a very large heat source at temperature T_+. The two are coupled by an ideal engine which delivers work to a reversible work source, via infinitesimal heat cycles that successively increase the temperature of A, till A comes in equilibrium with the source, see Figure 2*(i)*. At this point, the extracted work is maximal under the given conditions. Suppose that in this process, the system A moves from an equilibrium state of energy U_- and entropy S_- to another equilibrium state with the corresponding values of U_+ and S_+. The heat extracted from the hot source is $Q_+ = T_+(S_+ - S_-)$. The heat discarded to the finite sink is $q_+ = U_+ - U_-$. The work extracted, $W_+ = Q_+ - q_+$, is given by

$$W_+ = T_+(S_+ - S_-) - (U_+ - U_-). \tag{4}$$

Then the efficiency at maximum work, $\eta_+ = W_+/Q_+$, is evaluated to be:

$$\eta_+ = 1 - \frac{T_m^{(+)}}{T_+}, \tag{5}$$

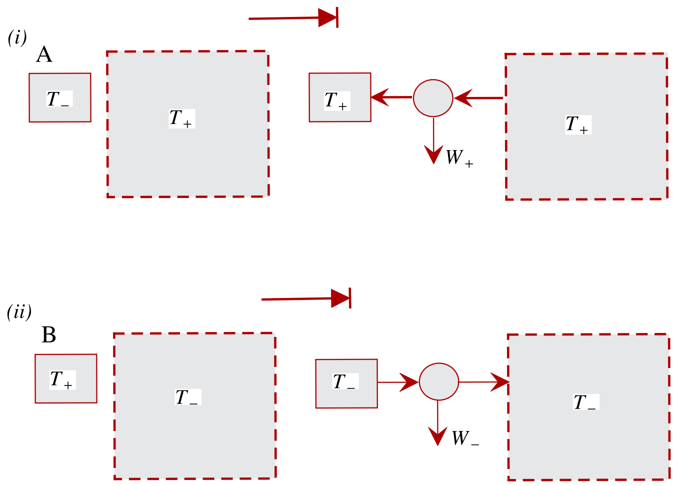

Fig. 2. Schematic of the engine between a finite system and a heat reservoir, for a given pair of initial temperatures (T_+, T_-): *(i)* System A as a finite sink at T_- and an infinite source at T_+, coupled with a reversible work source. Work extraction W_+, equation (4), is completed when the temperature of A becomes T_+. *(ii)* System B as a finite source at T_+ and an infinite sink at T_-. Maximum extracted work is W_-, equation (8), when the temperature of B becomes T_-.

where we define

$$T_m^{(+)} = \frac{U_+ - U_-}{S_+ - S_-}, \tag{6}$$

a quantity characteristic of system A. Further $T_m^{(+)}$ may be regarded as an effective temperature of an infinite reservoir [18], such that the present situation of an infinite source and a finite sink, at temperatures T_+ and T_- respectively, is equivalent to maximum work extraction from a reversible cycle between two infinite reservoirs at temperatures T_+ and $T_m^{(+)} (< T_+)$. In the latter case, the extracted work per cycle is: $W_+ = (T_+ - T_m^{(+)})(S_+ - S_-)$, with (Carnot) efficiency $1 - T_m^{(+)}/T_+$, which is the same as equations (4) and (5).

Now, we assume that a complete information on the states of system A is not available, or, in particular, $T_m^{(+)}$ is not known. But if the quasi-static model, with an infinite source and a finite sink, is a good model for the observed performance of an industrial plant, then we may infer the relevant value of the effective temperature $T_m^{(+)}$, by setting the theoretical efficiency (η_+) for the model to be equal to the observed value η_o. This implies that we can estimate

$$T_m^{(+)} = T_+(1 - \eta_o). \tag{7}$$

Such values of the intermediate temperatures based on the observed efficiencies of some of the industrial plants, are tabulated in Table 1, and also depicted in Figure 3 in comparison with the geometric mean value $G(T_+, T_-) = \sqrt{T_+ T_-}$. The latter value is chosen simply because when $T_m^{(+)} = G$, the observed efficiency is equal to CA-value. Thus the spread of observed values of the efficiency around CA-value in Figure 1, is translated here into a spread in the effective temperatures around the geometric-mean values. More precisely, $T_m^{(+)} \gtrless G$ indicates $\eta_o \lessgtr \eta_{CA}$, as may be seen also from Table 1. It is to be noted that inferring the effective temperature from the observed efficiency does not determine the nature of the system A or the form of fundamental

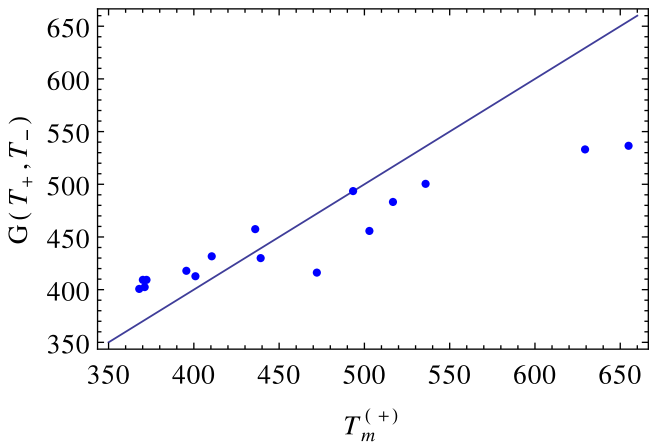

Fig. 3. The effective temperature $T_m^{(+)}$ plotted against the geometric mean $G(T_+, T_-) = \sqrt{T_+ T_-}$, the two quantities being equal along the straight line.

relation $U(S)$. The latter information is assumed not to be at our disposal within the premises of our method.

Now, here enters the second important piece of the puzzle in our story. The scenario of work extraction which we have assumed in the above, involving a finite system coupled to an infinite reservoir via reversible process, is consistent with an alternate picture too. The latter picture suggests that instead of the finite system acting as a sink, it can well serve as a finite source, if we reverse the initial temperatures of the reservoir and the system. This implies that, for the same initial temperatures T_+ and T_-, we consider a finite source (B) at T_+, coupled to an infinite sink at T_-, see Figure 2*(ii)*. For the second configuration also, we can extract work by utilizing the temperature gradient between B and the reservoir, till B finally comes to be at temperature T_- [19]. Assuming that the system goes from some equilibrium state (U'_+, S'_+) to another one (U'_-, S'_-), the heat removed from the source is $Q_- = U'_+ - U'_-$ and the amount discarded to sink is $q_- = T_-(S'_+ - S'_-)$. So the extracted work is [20] $W_- = Q'_- - q'_-$, or

$$W_- = (U'_+ - U'_-) - T_-(S'_+ - S'_-). \tag{8}$$

The efficiency $\eta_- = W_-/Q_-$ is given by

$$\eta_- = 1 - \frac{T_-}{T_m^{(-)}}, \tag{9}$$

where

$$T_m^{(-)} = \frac{U'_+ - U'_-}{S'_+ - S'_-}. \tag{10}$$

It is clear from the expressions for W_- and η_-, that an equivalence exists between the above model and that of work extraction in a reversible cycle from two infinite reservoirs at T_- and $T_m^{(-)} (> T_-)$.

Again, to apply the above model to an industrial plant, we can equate the observed efficiency to the theoretical efficiency, as $\eta_o = \eta_-$ and infer the corresponding effective temperature $T_m^{(-)}$ from equation (9), as

$$T_m^{(-)} = \frac{T_-}{1 - \eta_o}. \tag{11}$$

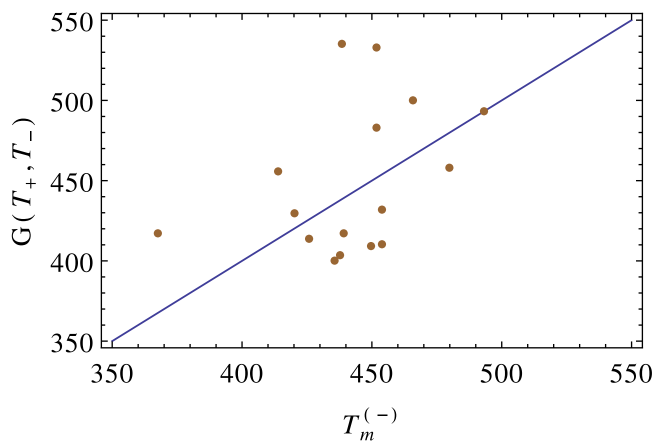

Fig. 4. The effective temperature $T_m^{(-)}$ plotted against the geometric mean $G(T_+, T_-) = \sqrt{T_+T_-}$, the two quantities being equal along the straight line.

It is clear that $T_+ > T_m^{(-)} > T_-$. The calculated values of $T_m^{(-)}$, based on the observed efficiencies of the plants, are also tabulated in Table I, and shown graphically in Figure 4 in comparison to $G(T_+, T_-)$.

A remark seems to be in place here. An analogy may be drawn between the above model and an earlier irreversible model proposed by Chambadal [2,6], in which an intermediate temperature T_w (of the warm exit stream) enters into the analysis and the theoretical efficiency of the model is $1 - T_-/T_w$ (compare with Eq. (9)). For an optimal value $T_w = \sqrt{T_+T_-}$, the power output becomes optimal with the corresponding efficiency equal to CA-value. The crucial difference, from our model, is twofold: a) we model work extraction by quasi-static processes so no notion of time enters here; b) the nature of system B (finite source) is not specified, whereas in earlier approaches, the warm stream is often assumed to follow a temperature-independent heat capacity.

Now as observed in Figures 3 and 4, the values of effective temperatures seem to be distributed apparently in a random fashion about the geometric-mean value. However, it is remarkable to note that for a given plant, the calculated values of $T_m^{(+)}$ and $T_m^{(-)}$ are such that, to a high accuracy, they are equidistant from the geometric mean G. More precisely, if we define an average value of temperature $T_m^{(av)}$, as $T_m^{(+)} - T_m^{(av)} = T_m^{(av)} - T_m^{(-)}$, then this value $T_m^{(av)}$ is very close to the G value for that situation. In other words, we define an average scale of temperature as the arithmetic mean of the two inferred temperatures $T_m^{(\pm)}$, and so given by

$$T_m^{(av)} = \frac{1}{2}\left[T_+(1-\eta_o) + \frac{T_-}{1-\eta_o}\right]. \tag{12}$$

Then the above average value is found to be closely approximated by $G(T_+, T_-)$ for most observed cases of industrial plants, see Figure 5, as well as Table 1.

Now, we turn to a more quantitative characterization of the above observation. It is easy to see that $T_m^{(av)}$ takes a minimum value of $\sqrt{T_+T_-}$, w.r.t to η_o ($dT_m^{(av)}/d\eta_o = 0$, and $d^2T_m^{(av)}/d\eta_o^2 > 0$), at $\eta_o = 1 - \sqrt{T_-/T_+}$. This implies that any possible value of $T_m^{(av)}$ is equal to or greater than $\sqrt{T_+T_-}$, and so will lie on or below the straight line plotted in Figure 5.

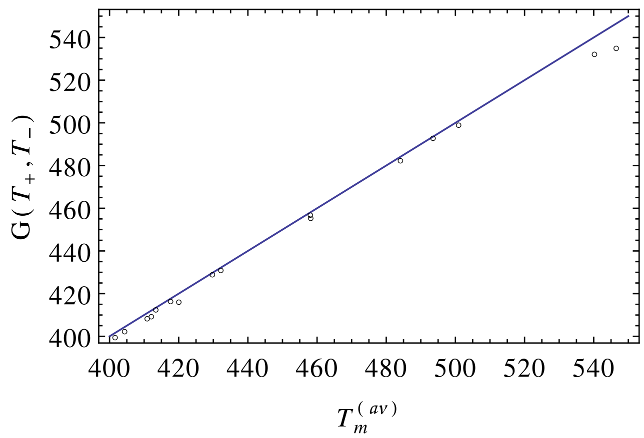

Fig. 5. The average effective temperature $T_m^{(av)}$ plotted against the geometric mean $G(T_+,T_-) = \sqrt{T_+T_-}$, the two quantities being equal along the straight line. $T_m^{(av)}$ values calculated from the observed data on most of the power plants, is remarkably close to $G(T_+,T_-)$.

The second aspect is related to the observation made earlier that the deviations in values of effective temperatures $T_m^{(\pm)}$ from the corresponding G-values, reflect the fact that the observed efficiency deviates from the CA value. However, deviations from the G-values, are suppressed considerably in case of $T_m^{(av)}$. This may be argued by considering small deviations (ϵ) in the observed efficiency from CA value, and expanding $T_m^{(av)}$ in powers of ϵ. Let $\eta_o = \eta_{CA} + \epsilon$. Then we see that up to second order:

$$T_m^{(av)} = \sqrt{T_+T_-} + \frac{1}{2}\sqrt{\frac{T_+^3}{T_-}}\epsilon^2 + \mathcal{O}(\epsilon^3). \tag{13}$$

Thus, the first non-zero correction from the geometric-mean value is of second order in ϵ, while it is of the first order for η_o, or $T_m^{(\pm)}$. Clearly, the magnitude of fluctuations is suppressed in the case of $T_m^{(av)}$.

Finally, we address the meaning of the average effective temperature. If we again consider the two extreme situations envisaged in Figure 2, then they are mutually exclusive, or one may say, they are counterfactual. The average temperature is not necessarily seen in an actual realization, except for the special case $T_m^{(+)} = T_m^{(-)} = \sqrt{T_+T_-}$. In this sense, the meaning one can attach to the definition of $T_m^{(av)}$, can be given best in the language of inductive inference [21,22]. In latter terms, the average temperature represents an expected scale of the effective temperature, in view of our inability to choose between two alternatives *(i)* and *(ii)*, where each represents our hypothesis for the model of work extraction applicable to the plant. In inference, when any of the mutually exclusive hypotheses cannot be given a preference over the others, then we must assign equal weights to the inferences derived from each hypothesis [22,23]. Any deviation from equal weights would imply that we have some extra piece of information about the process, and thus would be inconsistent [24]. The effective temperatures $T_m^{(\pm)}$ are our inferred values from the given data on the observed efficiency, and, so $T_m^{(av)}$, defined with equal weights assigned to both the inferences, represents the expected value of the effective temperature, commensurate with the information at our disposal.

Concluding, we have proposed to model the observed efficiencies of power plants, using the quasi-static models of work extraction where one reservoir (hot or cold) is finite while the other is practically infinite. This brings an extra scale of intermediate temperature into the analysis. We note two models consistent with the hypothesis as to which reservoir is taken as finite, and consequently, we get two possible values of the intermediate temperature. The limited information on the working conditions does not allow to prefer one model hypothesis over the other and so an equally-weighted average represents the expected scale of intermediate temperature. For most of the power plants, this inferred value is found to be quite close to the geometric mean of the highest and the lowest temperatures. Thus we rediscover the significance of the geometric-mean temperature, which was emphasized in earlier irreversible models of plants [5–7, 25], but there it was often based on simplifying assumptions such as constant heat capacity for the hot stream, and Newton's law for heat transfer. Such assumptions are not relevant in our analysis and we do not specify the particular nature of the finite reservoir. In the present approach, if the deviations in the observed efficiency from the CA-value are small, then the geometric-mean temperature appears as a rational estimate for the intermediate temperature.

An inference based approach has been used earlier to study models of thermodynamic machines with limited information. The emergence of CA efficiency from inference has been noted in quantum heat engines [26] and mesoscopic models like Feynman's ratchet [27]. For classical models of work extraction from two finite reservoirs, the results for efficiency at maximum work are reproduced through inference, beyond linear response [28]. Further, reversible models with limited information have been related to irreversible models through inference based reasoning [29]. In the present context, it is remarkable how simple, but general considerations can lead to estimates close to the geometric-mean value for the intermediate temperature. More research is needed for a deeper understanding of the connection between the use of limited information and thermodynamic modelling.

The utility of inferential approach is that it may give insight into the actual state of affairs, while incorporating the prior information normally not considered in thermodynamic models. In this context, we note that although most of the data on plants yield an expected value of the intermediate temperature close to geometric-mean value, still there are a couple of significant deviations in Figure 5, near the top right corner. A valid question is, why do these examples differ from the rest of the cases? Does it indicate other measures of optimization being used in the actual operation of these cases? As far as the available data is concerned, we note these plants operate under higher temperature gradients than most other plants. It may be the case that our model and the assumptions may not serve as good approximations for large temperature differences. In any case, further studies on the actual working conditions may yield information in order to extend the inference analysis, or may help to improve the performance, at par with the other plants.

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