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Multifractal formalism combined with multiresolution wavelet analysis of physiological signals

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Abstract An approach to the processing of physiological signals is considered combining multifractal formalism with multiresolution wavelet analysis, which involves the transition from the original signals to sets of detail wavelet-coefficients related to different levels of resolution. This transition could expand the possibilities of multifractal analysis from the viewpoint of physiological interpretation of the results. In particular, changes in the singularity spectra due to variations in system behavior are associated with specific frequency regions, what simplifies their description and can provide a link between observed phenomena and changes in rhythms of electroencephalograms (EEG) or other physiological processes when the method is applied to datasets of different origins. We illustrate this approach using EEG signals during mental tasks solving.

1 Introduction

Wavelet-based methods are widely applied in many fields of science and technology [1–4]. They can use both continuous and discrete wavelet transforms, which have noticeable distinctions in algorithms and bases for signal decomposition. Thus, the basic functions used in the continuous wavelet transform have an analytic expression and redundancy properties, what enables to perform a thorough analysis and visual representation of the results [3, 4]. The latter is important for studies where visual control of the estimated quantities is required for a deeper understanding of the dynamics and features of the system. In particular, such an analysis provides information about the temporal behavior of the frequencies and amplitudes of rhythmic components, changes in the distribution of local energy in distinct frequency ranges, etc. The bases used within the discrete wavelet transform (DWT) can be orthogonal and non-orthogonal depending on the purpose of the study [1, 2]. However, they do not have an analytic expression (except for the simplest Haar wavelet) and are given by a set of filter coefficients estimated numerically. A useful DWT-based approach is multiresolution wavelet analysis (MWA), which decomposes the signal by means of filter banks and a fast (pyramidal) decomposition scheme [5]. This approach was widely used for diagnostic purposes in various fields, including engineering [6], physics [7, 8], physiology [9, 10], etc.

Usually, sets of decomposition coefficients are obtained at different resolution levels, and their standard deviations are considered as informative measures of signal features. Such consideration, however, has some limitations since the complex shape of the probability density function for the decomposition coefficients is not described by its width alone, and other measures quantifying the distribution may be useful. In particular, accounting of its symmetric properties, the behavior of the tails can give additional information for diagnostic purposes. To provide a more thorough analysis of signals in wavelet space, we proposed to consider distribution cumulants and showed how this analysis can improve the characterization of changes in the system dynamics, especially for signals with specific features (extreme events) [11, 12]. In our opinion, a promising approach consists in the consideration of the decomposition coefficients as new data sets for application of signal processing methods. In [13, 14], we proposed the idea of enhanced MWA (EMWA), which is a combined MWA approach with the detrended fluctuation analysis of decomposition coefficients at different resolution levels.

Another wavelet-based approach to the statistical analysis of inhomogeneous processes is the multifractal formalism revised with wavelets [15, 16]. It performs characterization of complex processes in terms of the singularity spectrum or its important quantities such

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as the mean Hölder (Hurst) exponent or the spectrum width, which is a useful complexity measure of nonstationary datasets. One of the main problems with this tool is the rather complicated relation of the results to specific rhythmic components and regulatory control systems that limits the interpretation of changes in acquired signals in physiological studies and does not allow a deeper understanding of the mechanisms underlying the observed phenomena. To improve this interpretation, the method can be applied to band-pass filtered data sets, assuming that the filter is selected appropriately. In an effort to expand the possibilities of multifractal characterization of singular processes, here we propose another idea, namely to apply the wavelet-transform modulus maxima (WTMM) method to the decomposition coefficients at different resolution levels instead of the original datasets. The latter provides a way to characterize the multifractal structure of complex signals associated with distinct ranges of scales within the multiresolution WTMM approach and introduces a number of measures associated with different levels of resolution. The proposed approach is illustrated using electroencephalograms (EEG) related to different conditions.

The paper is organized as follows. Section 2 briefly describes the proposed idea of combining the wavelet-transform modulus maxima method with the multiresolution analysis and the considered experimental data. Section 3 includes the main results and discussion of the application of the multiresolution WTMM approach to EEG. Section 4 contains some concluding remarks.

2 Methods and experimental data

2.1 Multifractal formalism revised with wavelets

The WTMM approach proposed by Muzy et al. [15, 16] and widely used in many fields of science [17–22] performs the continuous wavelet transform of the signal $x(u)$

$$W(a,t) = \frac{1}{a}\int_{-\infty}^{\infty} x(u)\psi\left(\frac{u-t}{a}\right)du, \qquad (1)$$

where a and t are the scale and translation parameters, and ψ is the wavelet. Theoretically, the singularity spectrum is independent of the choice of the wavelet-basis [23], although the processing of relatively short datasets contaminated by noise and artifacts may require an appropriate selection of ψ. Usually quite simple real-valued functions constructed as derivatives of the Gaussian are applied, such as the MHAT-wavelet that is used in the current study. To increase the speed of numerical estimates of the continuous wavelet transform, a method described in [24] can be applied. It can improve the performance of the wavelet-based multifractal formalism by providing faster computations.

A singularity of $x(u)$ for $t = t^*$ leads to the power-law behavior of the wavelet-coefficients for $a \to 0$

$$W(a,t^*) \sim a^{h(t^*)}, \qquad (2)$$

quantified by the Hölder exponent $h(t^*)$. Although it can be obtained based on the last equation, this is a very unstable procedure due to the presence of neighboring singularities that affect the estimates as a increases. The better way is based on the partition functions computed as follows:

$$Z(q,a) = \sum_{l\in L(a)} \mid W(a,t_l(a)) \mid^q, \qquad (3)$$

where $L(a)$ is the full set of skeleton lines (lines of local maxima and minima of $W(a,\, t)$ detected at each scale a), $t_l(a)$ is the time associated with the line l. To further increase the stability of the method, the values of $W(a,\, t)$ close to zero can be eliminated according to the equation

$$Z(q,a) = \sum_{l\in L(a)} \left(\sup_{a'\le a} \mid W(a',t_l(a')) \mid\right)^q. \qquad (4)$$

The parameter q determines the strength of the singularities under consideration. Negative q are selected to study small signal fluctuations (weak singularities). Positive q are used to quantify large fluctuations (strong singularities).

The power-law behavior of $Z(q,\, a)$

$$Z(q,a) \sim a^{\tau(q)} \qquad (5)$$

allows estimating the scaling exponents $\tau(q)$ and, at the next stage, the Hölder exponents with singularity spectrum $D(h)$

$$h(q) = \frac{d\tau(q)}{dq}, D(h) = qh - \tau(q), \qquad (6)$$

where $D(h^*)$ is the Hausdorff dimension for data subsets characterized by the Hölder exponent h^*. The mean Hölder exponent $H = h(0)$ and the singularity spectrum width $\Delta = h_{max} - h_{min}$ are important quantities of correlation properties and complexity of $x(u)$ [25, 26].

2.2 Multiresolution analysis and a combined approach

The multiresolution wavelet analysis decomposes a signal $y(t)$ using two sets of filters constructed from the scaling function $\varphi(t)$ and the wavelet $\psi(t)$, which are a low-pass filter and a high-pass filter, respectively. The sets are built by means of integer translations and dilations with the scaling factor 2^j of $\varphi(t)$ and $\psi(t)$. The decomposition can be done over all available resolutions or up to a fixed level j_m using a selected wavelet basis (e.g., the D^8 Daubechies wavelet, which will be used in the current study)

$$y(t) = \sum_k s_{j_m,k}\varphi_{j_m,k}(t) + \sum_{j\geq j_m}\sum_k d_{j,k}\psi_{j,k}(t). \quad (7)$$

This wavelet is a compromise between the regularity of the basic function and its support length. It is often used for signal processing and our analysis confirmed that D^8 provides a reliable diagnostic for the purpose of the study. The decomposition coefficients are called approximation ($s_{j_m,k}$) and detail coefficients ($d_{j,k}$). The latter are considered to describe the features of the signal at different scales (resolution levels). For this purpose, the standard deviations of $d_{j,k}$ as a function of the resolution level σ_j are typically used.

However, such consideration takes into account only limited information about the statistics of $d_{j,k}$. Their more thorough processing can expand the possibilities of characterizing signal features. Here, we propose to consider $d_{j,k}$ at each resolution level j as a data set whose scaling features can be characterized by the multifractal formalism to quantify singularities associated with different resolution levels.

2.3 Experiments

The experiments were carried out at the Saratov State University (Saratov, Russia) on a group of 7 healthy volunteers (students) aged 18–21 years (men). Experimental procedures were carried out in accordance with the Declaration of Helsinki. EEGs were recorded at a sampling frequency of 200 Hz using the MP100 measuring complex (BIOPAC Systems, Inc.) and "AcqKnowledge" software. At the preprocessing stage, a Butterworth bandpass filter with cutoff frequencies of 1 Hz and 100 Hz and a notch filter 50 Hz were used. During the experiment, the volunteers sat on a chair. Baseline EEG (relaxation with open eyes) was recorded for 5 min, followed by recording during mental tasks (5 min). The latter involved solving arithmetic examples with four basic mathematical operations chosen at random. Artifacts were removed before signal processing.

3 Results and discussion

The multifractal structure of various natural signals, including physiological datasets, has been widely discussed in many studies [15–23]. Although the presence of complex scaling and multifractality is a typical phenomenon, changes in the singularity spectra can be used to describe transitions between states, responses to external stimuli, etc. The shape of the singularity spectrum can be quantified by different measures reflecting its features for weak and strong singularities (small and large fluctuations), including the degree of inhomogeneity (the range of Hölder exponents and associated Hausdorff dimensions) for all types of fluctuation and independently in the range of positive and negative q, spectrum asymmetry, its position on the h-axis and other quantities. Often the characterization using the measures H (mean Hölder exponent) and Δ (range of Hölder exponents, which can be treated as a measure of complexity) is rather informative and useful for diagnosing the state of the system.

In contrast to the traditionally used multifractal formalism applied to the acquired complex signals, we propose to make an intermediate stage consisting of multiresolution signal decomposition and obtaining sets of detail wavelet-coefficients. These sets also have a rather complex structure and can be used as inputs to the WTMM approach. In the latter case, each set is associated with a certain frequency range, which is convenient for further interpretation of the signal processing results. Figure 1a clearly shows the multifractal structures of the $d_{j,k}$ sets for the first 4 resolution levels. The transition from one level to another is accompanied here by the translation of the singularity spectrum along the h axis and by a change in its width. Therefore, the application of Δ and H measures seems to be quite informative for describing changes in the electrical activity of the brain related to different frequency ranges. Figure 1b illustrates the behavior of the spectrum of Hölder exponents and confirms that $h(q)$ values are useful for characterizing signal features related to distinct ranges of scales. Analysis for large negative values of q often provides unstable results for noisy data. For this reason, we limited ourselves to a reduced range of this parameter.

Using the measures Δ and H, we performed a comparison of two states: the baseline EEG (control state) and the solution of arithmetic examples. Figure 2a shows the statistical results for a group of volunteers (mean values $\pm$ SE). In the case under consideration, the level $j = 1$ corresponds to the frequency range [50, 100] Hz, $j = 2$ is related to [25, 50] Hz, $j = 3$ corresponds to [12.5, 25] Hz, and $j = 4$ refers to [6.25, 12.5] Hz. According to Fig. 2a, the strongest inter-state distinctions are related to the resolution level $j = 3$ ($p < 0.05$ according to the Mann–Whitney test). For other resolution levels considered, the distinctions are less pronounced, and the Mann–Whitney test does not recognize them as significant. The decomposition was performed until the number of detail wavelet-coefficients exceeds 128. For smaller numbers, the computing errors can be quite strong, and the method does not provide a reliable characterization of data sets. Note that the third level of resolution is mainly associated with the EEG β-rhythm, and the appearance of changes caused by mental tasks in this frequency range is in accordance with physiological assumptions. The results for measure H are shown in Fig. 2b. Here inter-state distinctions are less well expressed. Despite the fact that they are stronger at $j = 3$, in the given example the width of the singularity spectrum provided better diagnostics of

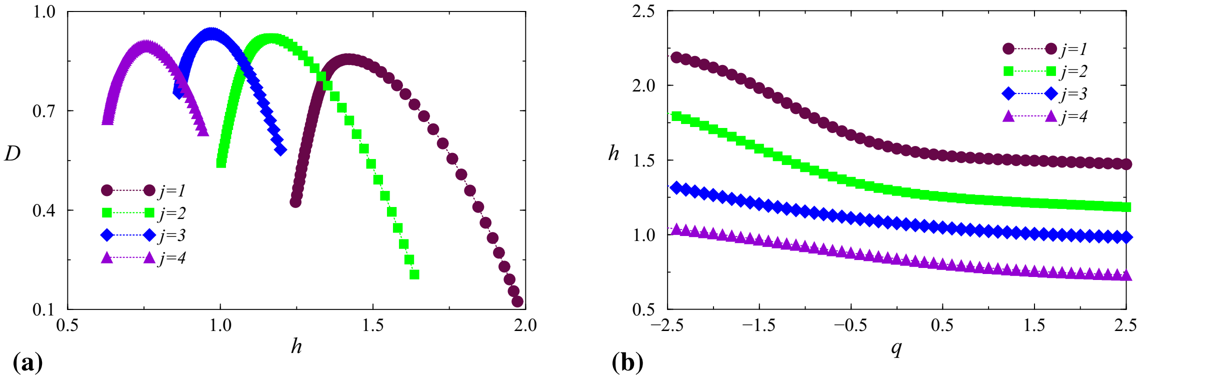

Fig. 1 Examples of singularity spectra (**a**) and the corresponding Hölder exponents (**b**) estimated from detail wavelet coefficients at the first four levels of resolution (computations were made for the background EEG). Here, H=1.42, Δ=0.72 ($j = 1$); H=1.17, Δ=0.64 ($j = 2$); H=0.97, Δ=0.33 ($j = 3$); H=0.76, Δ=0.31 ($j = 4$)

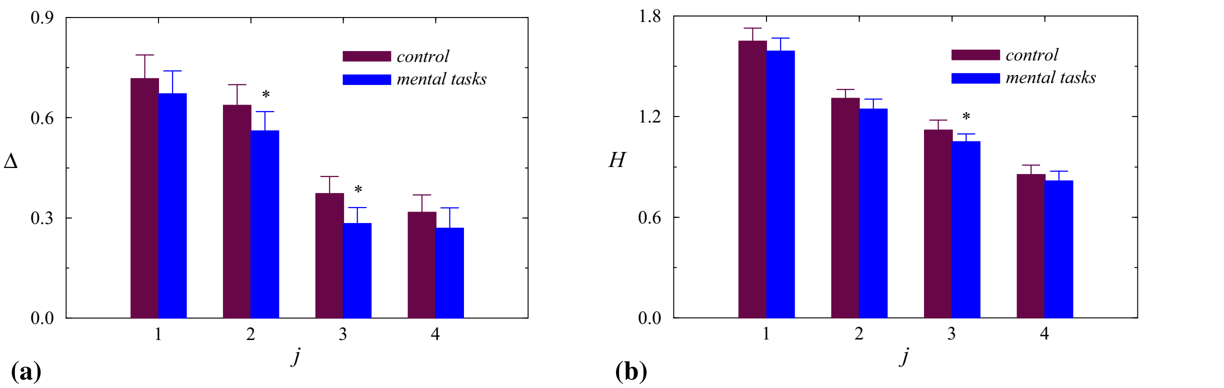

Fig. 2 Statistical results for a group of volunteers: differences in the width of the singularity spectrum (**a**) and the position of $D(h)$ (**b**) for two states. Asterisks mark significant differences between the states according to the Mann–Whitney test (p <0.05)

changes in the electrical activity of the brain compared to the position of the spectrum $D(h)$.

The main idea of this study was to propose an approach to multifractal analysis which could expand its capabilities in terms of the physiological interpretation of the results. Combining the WTMM-approach with multiresolution wavelet analysis and transitions from the original signals to sets of detail wavelet-coefficients related to distinct resolution levels is a way to improve the performance of the method, which was illustrated in this study using an example of two states of the electrical activity of the brain.

4 Conclusion

Multifractal analysis revised with wavelets is probably the most powerful approach to the statistical analysis of nonstationary and inhomogeneous processes. However, its performance is limited by the lack of a clear and simple association of the features of the singularity spectrum with the physiological control mechanisms responsible for the observed changes in the dynamics of physiological systems. In other words, the WTMM method is capable of diagnosing dynamical changes in the behavior of complex systems with time-varying characteristics, but does not provide a mechanism-based interpretation of these phenomena. Here, we considered an alternative approach that performs a multifractal description of individual sets of wavelet coefficients obtained within the multiresolution analysis with filter banks. Each set is associated with a specific region of frequencies, therefore, changes in the singularity spectrum, such as its translation and width variations, may be related to changes in EEG rhythms or other physiological processes when the method is applied to datasets of a different origin. An illustration of the proposed approach for the case of mental tasks is described. The proposed idea of the combined method of multiresolution WTMM can be applied in many areas of research, dealing with processing of nonstationary dynamics of complex systems, not limited to EEG signals.

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