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Regular Article

Mathematical modelling of unemployment as the effect of COVID-19 pandemic in middle-income countries

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Abstract WHO (World Health Organization) has stamped Coronavirus disease 2019 (COVID-19) as a pandemic. This disease has affected most of the population in every aspect. One of those main aspects is the economical burden on the middle class and poor people. To overcome these issues, there should be multi-stage precautions, and awareness was required apart from these, there should be a multi-level posterior effort on facing the challenges that were much needed. In that way, to join the dealing of facing the posterior challenges of this disease, we try to help the society in our way of modelling approach, so that we developed a mathematical model to identify the complexities of the virus and its actual phenomena along with its economical oriented effects and perspectives. During the time of the COVID-19 pandemic situation, millions of employees lost their jobs irrespective of age consideration. More youngsters in many countries have been looking for a new jobs even after facing so much of struggles as the effect of this pandemic. In these facts, a system of ordinary differential equations (ODEs) model is constructed and analysed the outbreak of COVID-19 and its effects on employment. The basic reproduction number R_0 has determined using the next-generation matrix technique. If $R_0 \leq 1$ disease-free equilibrium is globally asymptotically stable. If $R_0 \geq 1$, the endemic equilibrium is global asymptotic stability is achieved. To show the analytic findings, numerical simulations were performed. We hope that this work is suitable to control unemployment for all kinds of income populations.

Keywords Unemployment · COVID-19 · Stability analysis · Mathematical modelling

1 Introduction

In pandemic situations from Dec 2019 to Dec 2021 worldwide, many developed and developing countries face many ways of struggle came to control it. However, not clear till now, so many difficulties economically and socially. We took the issue of unemployment in middle-income countries. Many of the country execute the given categories of lockdown for the response to COVID-19 (e.g., school, business, and public park closures, as well as mobility limitations). Following the lockdown phase, additional focused efforts like contact tracing and isolation of possibly affected persons would be implemented, aided by testing.

Unemployment rates for young employees, which were already high, suddenly increased during the pandemic situation of COVID-19. The total unemployment rate among young working-age groups 16–24 moved from 8.4 to 24.4% the year from 2019 to 2020. Unemployment among those aged 25 and up increased from 2.8 to 11.3%. In the year 2020, the rate of unemployment had even higher for young workers (Black at 29.6%, Hispanic at 27.5%, and Asian American/Pacific Islander (AAPI) at 29.7%). The financial results of the COVID-19 economic system on younger employees may also persist for years [1].

This unique aspect of the COVID-19 crisis makes it easy to distinguish layoffs from job loss by looking at the impact of activity status on mental health. The main difference is that it reveals whether future employment aspirations give psychological protection against the loss of current wages and activities. Mental health and nervous system problems are the third leading cause of illness burden in South Africa, behind HIV and other infectious diseases [2].

The effect of COVID-19 cases and fatalities are backbone of deflation on economic and social changes in European countries and gives much impact on European economies also on the pandemic situation. The descriptive data suggest that COVID-19 casing and casualties are more common in Italy and the United Kingdom than in other European countries [4]. Furthermore, the progress of creating new vacancies takes time that is outstanding to the continual growth of the

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population, which is a slim chance that the number of jobless people will alter when the current vacancies are. In this instance, too, freshly created vacancies will be insufficient, resulting in an unmanageable situation. As a result, new opportunities must have produced as quickly as possible, as our findings suggest that delaying the creation of new vacancies destabilises the system [5].

Personal protection, therapy when individuals are diagnosed by treatment when the individuals are late-diagnosed, effective spraying of the environment, and cleaning probably contaminated surfaces are all part of the control set that can assist lower the virus's number [6]. The SARS-CoV-2 virus is changed with time, which is the standard characteristic of any virus. Generally, when a virus transforms into a mutant, its traits remain the same or have a minimum influence [7]. The comparative analysis of the COVID-19 for pandemic situations has concentrated on the first and second waves. It describes the growth of the virus, death rates, and rate of recovery of the people. To suggest control of the virus takes prevention actions for COVID-19. The statistical tool ANOVA is used to compare the fractal interpolation function [8].

India has implemented public health and social measures based on local epidemiology and risk assessments; similarly, a full-day lockdown, limited mobility, and educational facility closures. In terms of the total infected population, it has a lower COVID-19 death rate [9]. Controlling individuals from travelling and rigidly implementing personal precautions at the same time can considerably minimise the COVID-19 outbreak. The overall infected population plot compares to the actual infected population of India's COVID-19 outbreak's second wave [10]. In addition, people throughout the country have been suffering from a shortage of medical resources. Mathematically, the unique coronavirus dynamics have been modelled to analyse its dynamics. However, the fractional-order COVID-19 model with control techniques is the subject of this paper. The COVID-19 model investigates using the Caputo fractional derivative, besides quarantine and hospitalised compartments [11].

This work is developing as follows: Section (2) describes the mathematical model and analysis that comprises the subsections as model assumption and determination, equilibria, stability analysis, and the global stability of our model. Section (3) describes the numerical results of the simulation of $R_0, E^0, \& E^*$ and the parameter effect. It simulates the design in Python to show the figures of the deterministic model and project the real-world data from WHO/UN WOMEN/ UNDP and Economic Policy Institute collections. At the end of the section (4), given our result discussion and conclusion.

2 Mathematical models: formulation and analysis

2.1 Model assumption with determination

The total population separates into four subclasses. Based on what we are considered the poor people. Let $P(t)$, $I(t)$, $U(t)$, and $R(t)$ be the fraction of the poor population, the COVID- 19 infected individuals, unemployment population, and the recovered population, respectively, at time t.

This model includes the following assumptions:

(i) Let μ be the inflow rate of individuals in a class, also show the natural death rate per capita in each division. The COVID-19-infected peoples increase through touch already affected by COVID-19 people, with poor populations are taking standard mass action incidence.

(ii) Let β be the rate of transmission of COVID-19 infection, so that $\frac{\beta PI}{N}$ denotes the infected people's contact with uninfected people and they are travel without any protection the individual.

(iii) We further assume that poor people are quitting individual quarantine at a rate αU.

(iv) μ_1 be the inflow of COVID-19 severely infected death values.

(v) γ is the rate of recovery from COVID-19 infection. COVID-19 infects people who have been taking individual quarantine or some medical treatment due to recovering from the COVID-19 infection.

(vi) δ rate of people the got jobless/unemployment. During the COVID-19 lockdown period, many people have stayed at home. Therefore, they have no job due to the social distancing and people communication stoping purpose closed all the Corporates/Industry/Hotels/Tourism. Daily labour is also affected in this way.

(vii) σ rate of people goes to recover. Some of the Corporate supported the employees working from home at the time few industries also like followed this way and created new vacancies to give opportunities for jobless people, those who got the job would move to R.

The flow of people from one class to the next has displayed in the diagram below:

Hence, we developed the following model:

$$\begin{aligned} \frac{dP}{dt} &= \Lambda - \frac{\beta PI}{N} - \alpha U - \mu P, \\ \frac{dI}{dt} &= \frac{\beta PI}{N} - (\gamma + \delta + \mu + \mu_1)I, \\ \frac{dU}{dt} &= \delta U + \alpha U - \sigma U - \mu U, \\ \frac{dR}{dt} &= \gamma I + \sigma U - \mu R, \end{aligned} \tag{1}$$

where $N = P + I + U + R$ with $P(0) \geq 0, I(0) \geq 0, U(0) \geq 0, R \geq 0$ as initial conditions.

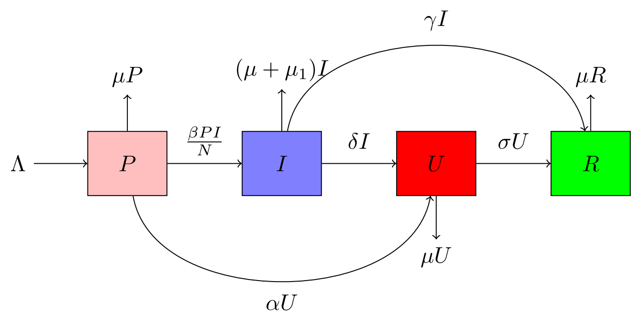


Fig. 1 Diagram of the model

Table 1 Description of parameters

Parameter	Description
Λ	Recruitment rate
μ	Natural death rate
μ_1	Death rate due to COVID-19 infection and with out immunity
β	Interaction rate of infected and poor people compartments
γ	Rate of progression from COVID-19 infection to recovery
α	Rate of people moves to unemployment
δ	Flow rate of infected to unemployment
σ	Rate at which the population moves to recovery from unemployment

As N is constant, we use following transformation $p = P/N, i = I/N, u = U/N, r = R/N$ to get:

$$\begin{aligned} p' &= \Lambda - \beta pi - \alpha u - \mu p, \\ i' &= \beta pi - ki, \\ u' &= \delta i + \alpha u - (\mu + \sigma)u, \\ r' &= \gamma i + \sigma u - \mu r, \end{aligned} \tag{2}$$

where $k = \gamma + \delta + \mu + \mu_1$

with $p(0) \geq 0, i(0) \geq 0, u(0) \geq 0, r(0) \geq 0$ as initial conditions. Here, note that $p + i + u + r = 1$.

Equation (2) has following two equilibriums:

(i) Trivial equilibrium $E^0(p^0, 0, 0, 0) = (\frac{\Lambda}{\mu}, 0, 0, 0)$

(ii) Endemic equilibrium $E^*(p^*, i^*, u^*, r^*) = \left(\frac{\Lambda - \alpha u}{\beta i + \mu}, \frac{\beta(\Lambda - \alpha u) - \mu k}{\beta k}, \frac{\delta i}{(-\alpha + \mu + \sigma)}, \frac{\gamma i + \sigma u}{\mu}\right)$ and $R_0 = \frac{\beta \Lambda}{\mu k}$ is the basic reproduction number of system (2).

Disease-free steady state E^0 exists always but endemic equilibrium E^* exists only if $R_0 > 1$. For finding the Basic reproduction number, we use the next-generation matrix method. Using this method, we obtain two non-negative matrices **F** and **V** evaluated at E^0, such that

$$\mathbf{F} = \begin{pmatrix} \beta P & 0 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{V} = \begin{pmatrix} k & 0 \\ -\delta & \mu + \sigma - \alpha \end{pmatrix}.$$

For definition and evaluation of matrices, **F** and **V** refer to [15]. Now, the spectral radius of $(\mathbf{FV}^{-1})$, which corresponds to basic reproduction number, is

$$R_0 = \frac{\beta \Lambda}{\mu k}.$$

2.2 Stability analysis

2.2.1 Stability analysis of disease-free equilibrium

Theorem 1 Disease-free steady state E^0 is locally asymptotically stable for $R_0 < 1$.

The $\mathbf{J^0}$ matrix of model Eq. (2)

$$\mathbf{J^0} = \begin{pmatrix} -\mu & -\beta p & -\alpha & 0 \\ 0 & -(k - \beta p) & 0 & 0 \\ 0 & \delta & -(\mu + \sigma - \alpha) & 0 \\ 0 & \gamma & \sigma & -\mu \end{pmatrix}.$$

The eigenvalues of this $\mathbf{J^0}$ matrix are given by the following roots:

$$\begin{aligned} \lambda_1 &= -\mu, \\ \lambda_2 &= -(k - \beta p), \\ \lambda_3 &= -(\mu + \sigma - \alpha) \\ \& \ \lambda_4 &= -\mu. \end{aligned} \tag{3}$$

We can show that all eigenvalues of $\mathbf{J^0}$ are negative. The disease-free equilibrium is locally asymptotically stable if $R_0 < 1$ [14].

2.2.2 Stability analysis of endemic equilibrium

Theorem 2 Endemic equilibrium E^*, whenever it exists, is locally asymptotically stable when $R_0 > 1$ [15, 16].

Proof The $\mathbf{J^1}$ matrix of the system 2

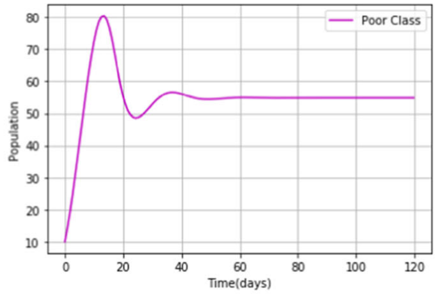
$$\mathbf{J^1} = \begin{pmatrix} -\beta i - \mu & -\beta p & -\alpha & 0 \\ \beta i & \beta p - k & 0 & 0 \\ 0 & \delta & \alpha - (\mu + \sigma) & 0 \\ 0 & \gamma & \sigma & -\mu \end{pmatrix}.$$

One eigenvalue of this $\mathbf{J^1}$ matrix is -μ and the other three eigenvalues are the roots of the following cubic equation:

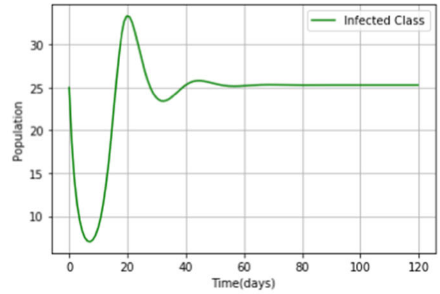
$$\lambda^3 + h_1 \lambda^2 + h_2 \lambda + h_3 = 0,$$

where

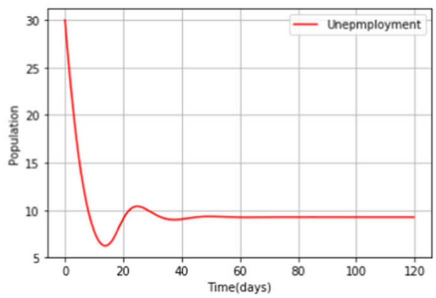
$$\begin{aligned} h_1 &= -[c_2 + a_1 + b_2], \\ h_2 &= a_1 c_2 + b_2 c_2 + a_1 b_2 - b_1 a_2, \\ h_3 &= -(-a_1 b_2 c_2 + b_1 a_2 c_2 - b_1 c_1 a_3). \end{aligned}$$



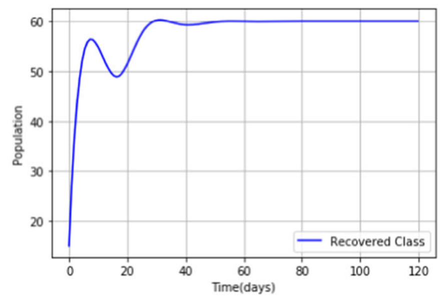
(a) Variation of the poor population(P) with time showing the stability of E^*



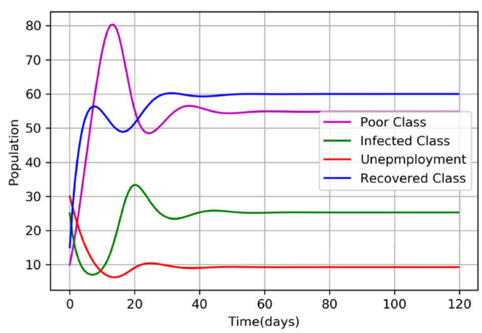
(b) Variation of the infected population(I) with time showing the stability of E^*



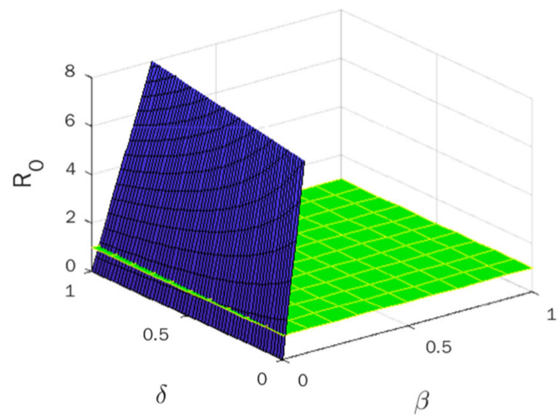
(c) Variation of the unemployment population(U) with time showing the stability of E^*



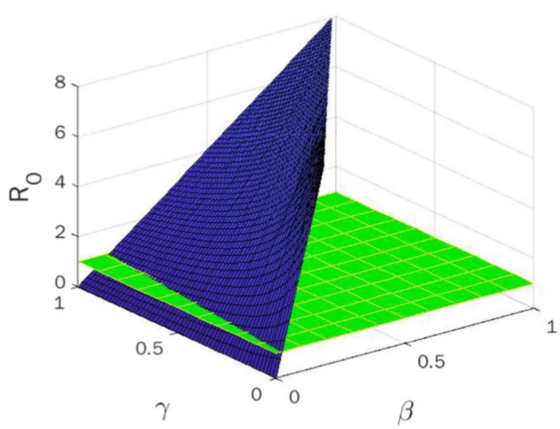
(d) Variation of the recovery population(R) with time showing the stability of E^*



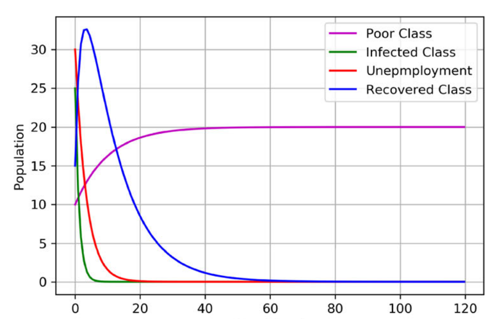
(e) Stability of the endemic equilibrium point at $R_0 > 1$



(f) The relationship between R_0, δ and β



(g) The relationship between R_0, γ and β



(h) Stability of the disease free equilibrium point at$R_0 < 1$

Fig. 2 Stability analysis of Eq. (2) and 3D Plot of R_0

By Routh–Hurwitz criteria, roots of the cubic equation will have negative real parts if $h_1 > 0, h_3 > 0, h_1 h_2 > h_3$. Hence, the equilibrium point E^* is locally asymptotically stable provided that $R_0 > 1$. □

2.3 Global stability

2.3.1 Global stability of disease-free equilibrium

Theorem 3 The disease-free equilibrium E^0 is globally asymptotically stable when $\mathcal{R}_0 \leq 1$.

Proof Consider Lyapunov function as

$$V(p,i,u,r) = \frac{1}{2}((p-p^0)+i+u)^2 + i.$$

The derivative of V along solutions of system (2) is given by

$$\begin{aligned}\dot{V}(p,i,u,r) &= ((p-p^0)+i+u)(\dot{p}+\dot{i}+\dot{u})+\dot{i}\\ &= (\beta p-(\mu+\alpha))i,\\ &\leq -(\mu+\alpha)(1-R_0)i.\end{aligned}$$

If $R_0 \leq 1$, then $\dot{V} \leq 0$. Using Lyapunov LaSalle theorem [3], we conclude that E^0 is globally asymptotically stable for $R_0 \leq 1$. □

2.3.2 Global stability of endemic equilibrium

Theorem 4 The endemic equilibrium E^1 is globally asymptotically stable when $\mathcal{R}_0 > 1$.

Proof we consider the following Lyapunov function:

$$\begin{aligned}V(p,i,u,r) &= \frac{1}{2}((p-p^*)+(i-i^*)+(u-u^*))^2\\ &+\frac{1}{2}(p-p^*)^2+(i-i^*-i^*log(\frac{i}{i^*})).\end{aligned}$$

This is a positive definite function in 1 and its derivative along solutions of system (2) is given by

$$\begin{aligned}\dot{V}(p,i,u,r) &= ((p-p^*)+(i-i^*)+(u-u^*))(\dot{p}+\dot{i}\\ &\quad+\dot{u})+(p-p)\dot{p}+(i-i^*)\frac{\dot{i}}{i}\\ &= (p-p^*)(\mu-\beta pi+\alpha u-\mu p)+\mu+\alpha u\\ &\quad+(i-i^*)(\beta p-\mu-\alpha)\\ &= -(\beta i+\mu)(p-p^*)^2.\end{aligned}$$

Clearly, if $R_0 > 1$, $\dot{V} \leq 0$. Using Lyapunov LaSalle theorem, we conclude that E^* is globally asymptotically stable whenever $R_0 > 1$ [17]. □

3 Numerical results and discussion

We used numerical simulation to verify our analytical conclusions under this section. Outcomes that validate are presented here with rationale.

The system (2) is simulated for various set of parameters satisfying the condition of local asymptotic stability of equilibria E^*. The fourth-order Runge–Kutta technique is used to numerically integrate the dynamic behaviour of the model for reaching equilibrium of the system. We took the parameter set $S_1 = \{\Lambda = 20, \mu = 0.01, \mu_1 = 0.2, \alpha = 0.423, \beta = 0.650, \gamma = 0.0541, \delta = 0.0651, \sigma = 0.501, \}$, and the parameters have been considered to have a reasonable value.

We evaluated the equilibrium stability with respect to the initial conditions to make sure it was stable (Fig. 2a–e). Figure 2a, b draws that the stability of endemic equilibrium point to different initial values on poor population and COVID-19-infective populations. And how they come to move other can stables at a point. Figure 2c, d depicts the stability of endemic equilibrium point for different initial values on both poor people and unemployment population. Figure 2e depicts the stability of endemic equilibrium point for different initial values on all compartment dynamics behaviour at time t.

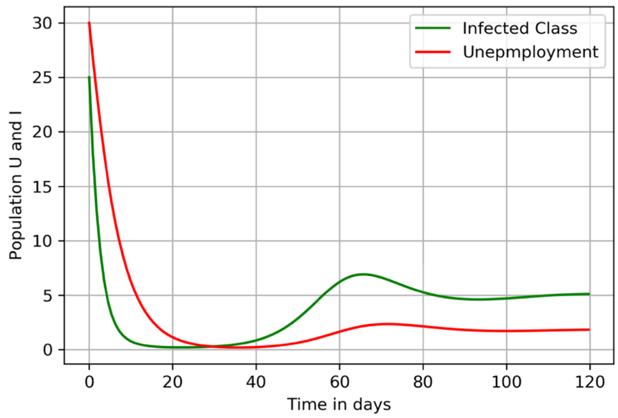
Figure 2f, g explores the relation between R_0 with other parameters. In Fig. 2f, we to look about the relations between R_0, β and β. In Fig. 2g, we look about the combined effect of R_0, γ, and β.

The pandemic situation has low-, middle-, and high-income country; poor populations have unemployment during the lockdown periods. In this aspect embrace, a mathematical model finds in the data. We are trying to our results along with data. In this view, we have absorbed the data on the COVID-19 situation.

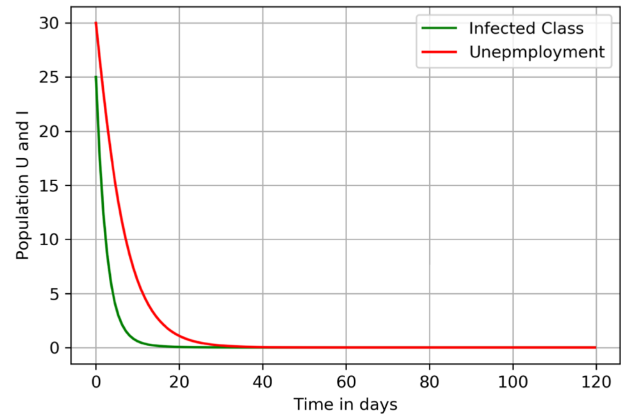
In Fig. 4a, b, the rate of unemployment has been assimilated by a pooled average of April, May, and June data in the years 2019 and 2020. The category of racial and ethnic is entirely incompatible. Hispanic refers to Hispanic/Latinx of any race through the White, Black, and AAPI refers to non-Hispanic white, non-Hispanic Blacks, and non-Hispanic Asian American/Pacific Islanders, in orderly [18] [*Source:* Economic Policy Institute Current Population Survey. http://microdata.org].

In 2020, 8.8% of worldwide working hours will be lost, equating to 255 million full-time employment, compared to the fourth quarter of 2019. Even though overall working-hour losses are largish reduction. It indicates the fact that adjustments occurred through reduced per week workdays in the lodging and dining services sector fell by more than 20% [12].

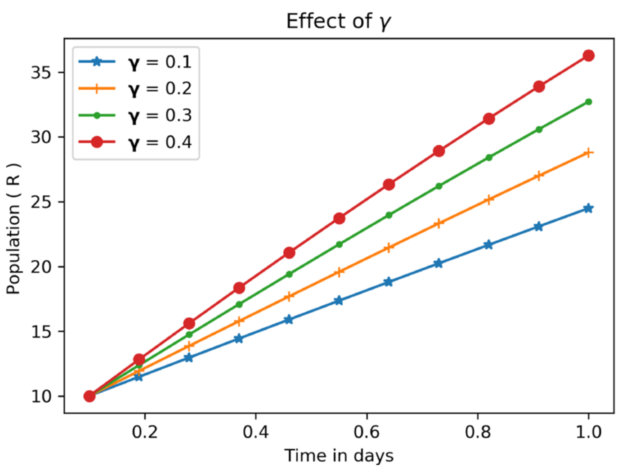
India's unemployment rate was over 8% in December 2021, up from 7% the previous month. While the unemployment rate had decreased dramatically since peaking in April 2020, the outbreak of new coronavirus strains, combined with periodic lockdowns, resulted in a shifting pattern of unemployment gripping the country in 2021 [13].



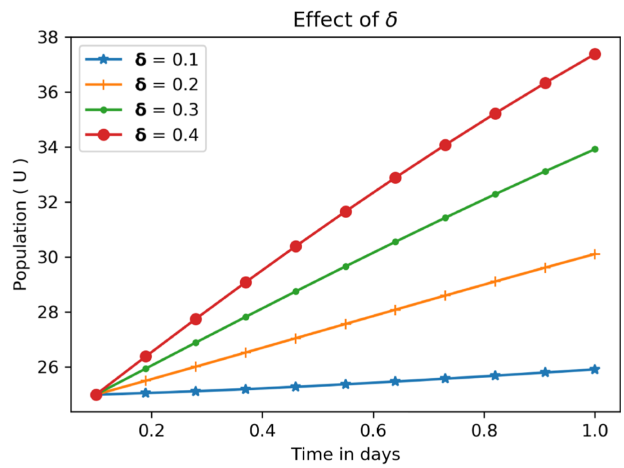
(a) Unemployment rate is decreased when the infected parameter $\beta = 0.7$ to 0.5 rate decreasing time of effect



(b) Unemployment rate is decreased when the infected parameter $\beta = 0.5$ to 0.3 rate decreasing time of effect

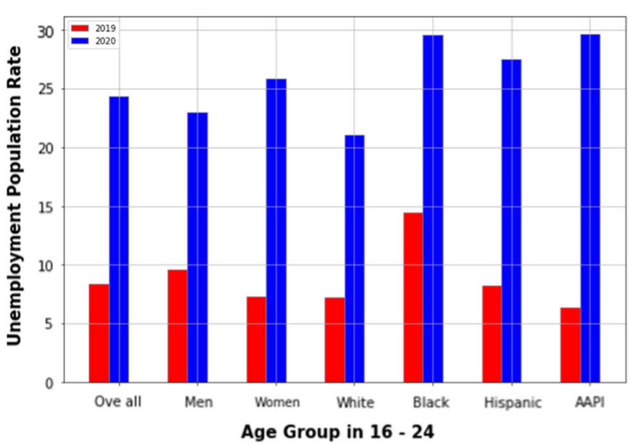


(c) Parameter effect of γ explore the infected peoples to recovered different level of time

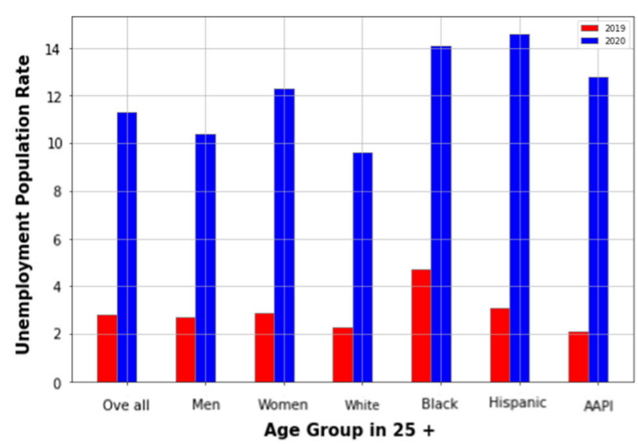


(d) Parameter effect of δ explore the infected peoples to unemployment in different level of time

Fig. 3 Plot in how unemployment rate decreasing and parameter effects



(a) Unemployment rate in 2019 and 2020 by age(16 - 24), gender, and race/ethinicity



(b) Unemployment rate in 2019 and 2020 by age 24+, gender, and race/ethinicity

Fig. 4 Unemployment rate in the spring of 2019 and 2020 young workers in the COVID-19 labour market

Table 2 The unemployment rate during the COVID-19 pandemic time from Jan 2020 to Dec 2021

Month →												
Year ↓	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2020 (%)	7.22	7.76	8.75	23.52	21.73	10.18	7.4	8.35	6.68	7.02	6.5	9.06
2021 (%)	6.53	6.89	6.5	7.97	11.84	9.17	6.96	8.32	6.86	7.74	6.97	7.91

4 Conclusion

Globally, the impact of COVID-19 is well noticed the numbers of the terms of death or terms of people who got unemployed. Particularity, this affected much in middle-income countries. We may consider the middle-income countries from Asia and Europe. We tried to convey that unemployment reduces if we reduce the effectiveness of COVID-19 spread.

Our numerical simulation indicates controlling the number of new confirmed cases and infections by lowering the transmissive. We also suggest that the SOPs for controlling infections results were helping the jobless people expected much reduced/decreased in the future.

The regain of the parameter γ shows that people who get treatment due to the COVID-19 infection will have been recovering. We have shown this fact in numerical simulation also.

The parameter δ (infected to unemployed) is showed some increase due to the effect of lockdown and its consequences. Figure 3c, d shows the β (poor people infected rate) which is compared for $\beta = 0.5$, 0.3 decreasing the unemployment rate, and also decreasing at the same time the peoples are moving from one place to another place certainly.

Therefore, they will have new job recruitment are available at that time to improve the lifestyle of jobless to employment the way new vacancies originated. During the COVID-19 pandemic, governments and officials attempted to alleviate unemployment, and the labour market improved by expanding job openings. Over the COVID-19 recession, governments also revised their job reallocation strategies. There is one proposal from general employees to the government that the government has to support an "unemployment insurance policy' which may introduce which might result in a dramatic drop in labour demand [4].

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