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Approximate solution of impedance matching for nonmagnetic homogeneous absorbing materials

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Abstract Impedance matching is an essential concept in the design of absorbing materials. Its basic form $Z_{\text{in}}–Z_0 = 0$ derived from the expression of reflectivity transmission line is often used to explain the reason for the reflectivity peak. Still, the relationship between impedance matching and material parameters has not been accurately described so far. In this paper, for nonmagnetic homogeneous materials, a solution of impedance matching equation is proposed to illustrate the relationship between impedance matching and the complex permittivity of absorbing materials. The thickness required for impedance matching is also obtained in the proposed solution. An eigenvalue P containing the relationship between the real and imaginary parts of the permittivity is presented. When P is equal to a positive odd number squared, with a corresponding definite thickness, the material can achieve impedance matching. With the parameters data in the available references, the solution is verified to be effective.

1 Introduction

According to the equivalent transmission line theory, the reflectivity of absorbing materials can be expressed as [1–5]:

$$\mathrm{RL} = 20 \log \left| \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} \right|, \tag{1}$$

where Z_{in} and Z_0 are the input impedance of absorbing material and free space respectively(All the parameters in this paper are normalized values [6], herein, $Z_0 = 1$). In order for $\mathrm{RL} = -\infty$, the following conditions should be met:

$$Z_{\text{in}} - Z_0 = 0, \tag{2}$$

which is well-known as impedance matching.

Early researches tried to extract the relationship that material electromagnetic parameters should satisfy from Eq. (2). Musal et al. [7] proposed a graphical method to analyze the impedance matching conditions corresponding to different loss angle ranges. Furthermore, a universal design drawing was presented [8,9], giving an overall view of the interrelated numerical values of material properties required to implement an optimum electromagnetic absorber design using a single homogeneous uniform layer of material.

A derivative conclusion is proposed and appears as an inference in lots of researches, which is [10–14]:

$$|Z_{\text{in}}| = |Z_0|\,. \tag{3}$$

Equation (3) is frequently used to explain the principle of microwave absorption. For example, Wang et al. [15] took $Z = Z_{\text{in}}/Z_0$ as the characteristic parameter of impedance matching and made a Z curve to explain the reason for reflectivity peak. Chen et al. [16] calculated the Z values of the three absorbing materials, and believed that the reflectivity of the corresponding materials could achieve the best effect when Z was closest to 1.

In this paper, an approximate solution of impedance matching for nonmagnetic homogeneous absorbing materials is presented. The expression of the impedance matching condition was simplified by equivalent infinitesimal substitution, and then solved, and the relationship between material parameter and impedance matching was obtained. An eigenvalue P is proposed, to represent the condition that the real and imaginary parts of permittivity ε' and ε'' should satisfy when impedance is matching, and the corresponding thickness is determined. The proposed method was verified with the parameters data in the available references, and proved to be practical.

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2 Theory and method

The general expression for the input impedance is given by:

$$Z_{\text{in}} = j\sqrt{\frac{\mu_r}{\varepsilon_r}} \tan\left(\frac{2\pi f d\sqrt{\varepsilon_r \mu_r}}{c}\right) \tag{4}$$

For non-magnetic homogeneous isotropic dielectric materials, the real part of the relative permeability is 1 and the imaginary part is 0, so the normalized transmission impedance expression can be simplified as follows [17–21]:

$$Z_{\text{in}} = j\sqrt{\frac{1}{\varepsilon_r}} \tan\left(\frac{2\pi f d\sqrt{\varepsilon_r}}{c}\right) = j\sqrt{\frac{1}{\varepsilon_r}} \tan\left(\mathrm{g}\sqrt{\varepsilon_r} f\right) \tag{5}$$

where

$$g = \frac{2\pi d}{c} \tag{6}$$

In which d is the thickness of the dielectric material testing sample, c is the speed of light in the vacuum, ε_r is the relative permittivity of the dielectric material.

To simplify Eq. (5), write it as normalized admittance Y_{in}:

$$Y_{\text{in}} = \frac{1}{Z_{\text{in}}} = Y'_{\text{in}} - jY''_{\text{in}} = -j\sqrt{\varepsilon_r}\cot(g\sqrt{\varepsilon_r} f) \tag{7}$$

Using the first important limit:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \tag{8}$$

the trigonometric part of Eq. (7) can be simplified, as follows:

$$\lim_{g\sqrt{\varepsilon_r}f \to \frac{(2n+1)\pi}{2}} \cot\left(g\sqrt{\varepsilon_r} f\right) = \frac{(2n+1)\pi}{2} - g\sqrt{\varepsilon_r} f \tag{9}$$

According to quarter-wavelength theory [22–26], when d meets the following condition:

$$d = \frac{(2n+1)c}{4f\sqrt{\varepsilon_r}}, \quad n = 0, 1, 2, \ldots \tag{10}$$

the reflectivity curve has an absorption peak. Since when the impedance matching condition is satisfied, Eq. (10) must be true, which means $g\sqrt{\varepsilon_r} f \to (2n+1)\pi/2$ exists. Then, the equivalent infinitesimal substitution of Eq. (7) can then be made using Eq. (9):

$$Y_{\text{in}}' - jY_{\text{in}}'' = j\left(g\varepsilon_r f - \frac{(2n+1)\pi}{2}\sqrt{\varepsilon_r}\right) \tag{11}$$

Write ε_r in complex numbers:

$$\varepsilon_r = \varepsilon' - j\varepsilon'' \tag{12}$$

$\sqrt{\varepsilon_r}$ is equal to normalized wave impedance Y_o, expressed in the plural form:

$$\sqrt{\varepsilon_r} = \sqrt{\frac{\varepsilon_r}{\mu_r}} = Y_o = Y_o' - jY_o'' \tag{13}$$

Obviously, there is:

$$Y_o'^2 - Y_o''^2 = \varepsilon' \tag{14}$$

$$2Y_o' Y_o'' = \varepsilon'' \tag{15}$$

Plug Eq. (12) and Eq. (13) into Eq. (11):

$$Y_{\text{in}} = g\varepsilon'' f - \frac{(2n+1)\pi}{2} Y_o'' + j\left(g\varepsilon' f - \frac{(2n+1)\pi}{2} Y_o'\right) \tag{16}$$

If Z_{in} satisfies the impedance matching condition $Z_{\text{in}} = 1$, there should be $Y_{\text{in}} = 1$, then the real and imaginary parts of Y_{in} are:

$$\begin{cases} Y'_{\text{in}} = g\varepsilon'' f - \frac{(2n+1)\pi}{2} Y''_o = 1 \\ Y''_{\text{in}} = g\varepsilon' f - \frac{(2n+1)\pi}{2} Y'_o = 0 \end{cases} \tag{17}$$

Solve Y'_o and Y''_o, substitute into Eq. (14) and Eq. (15), there are:

$$\left(g\varepsilon' f\right)^2 - \left(g\varepsilon'' f - 1\right)^2 = \frac{(2n+1)^2\pi^2\varepsilon'}{4} \tag{18}$$

$$g\varepsilon' f\left(g\varepsilon'' f - 1\right) = \frac{(2n+1)^2\pi^2\varepsilon''}{8} \tag{19}$$

Considering establishment condition of Eq. (9) $g\sqrt{\varepsilon_r} f \to (2n+1)\pi/2$, when ε'' is sufficiently small, there could be $\varepsilon_r \approx \varepsilon'$, according to Eq. (10), ε' can be expressed as:

$$\varepsilon' \approx \frac{(2n+1)^2\pi^2}{4g^2 f^2} = \frac{(2n+1)^2 c^2}{16 f^2 d^2} \tag{20}$$

Substitute Eq. (20) into Eq. (18) and Eq.(19), two solutions can be obtained:

$$g\varepsilon'' f = 1 \tag{21}$$

$$g\varepsilon'' f = 2 \tag{22}$$

When Eq. (21) is true, according to Eq. (17), there is $Y_o'' = 0$, and therefore $\varepsilon'' = 0$, which is the property of lossless dielectric and can be skipped. Then, when Eq. (22) is true, ε'' can be solved as:

$$\varepsilon'' = \frac{2}{gf} = \frac{c}{\pi df} \tag{23}$$

Combine Eqs. (20) and (23), there are:

$$\frac{\varepsilon'}{\varepsilon''^2} = \frac{(2n+1)^2\pi^2}{16} \tag{24}$$

which is the condition that ε' and ε'' should satisfy in impedance matching. Meanwhile, two thicknesses are obtained by ε' and ε'' respectively:

$$d_1 = \frac{(2n+1)c}{4f\sqrt{\varepsilon'}} \tag{25}$$

$$d_2 = \frac{c}{\pi f \varepsilon''} \tag{26}$$

When Eq. (24) is satisfied, there should be $d_1 = d_2$.

Define a parameter P as the eigenvalue of impedance matching, when:

$$P = \frac{16\varepsilon'}{\pi^2\varepsilon''^2} = (2n+1)^2 \tag{27}$$

impedance matching exists at the frequency where ε' and $\varepsilon''$are located, and the thickness of impedance matching can be calculated by Eq. (25) or Eq. (26).

To be specific, for a nonmagnetic homogeneous material, if there is a frequency f_x, at which point $P = (2n+1)^2$ can be obtained from ε' and ε'', then there is a thickness $d = d_1 = d_2$ that make impedance matching achieved at f_x.

In addition, for the low-loss magnetic materials [27, 28] of which the real part of the permeability μ' is greater than 1 and the imaginary part μ'' is almost 0, Eq. (15) should be amended as:

$$2Y_o^{'}Y_o^{''} = \frac{\varepsilon''}{\mu'} \tag{28}$$

And Eq. (20) should be amended as follows:

$$\varepsilon' \approx \frac{(2n+1)^2\pi^2}{4\mu' g^2 f^2} = \frac{(2n+1)^2 c^2}{16\mu' f^2 d^2} \tag{29}$$

Then, an eigenvalue of impedance matching for the low-loss magnetic materials material can be defined:

$$P_m = \frac{16\varepsilon'\mu'}{\pi^2\varepsilon''^2} = (2n+1)^2 \tag{30}$$

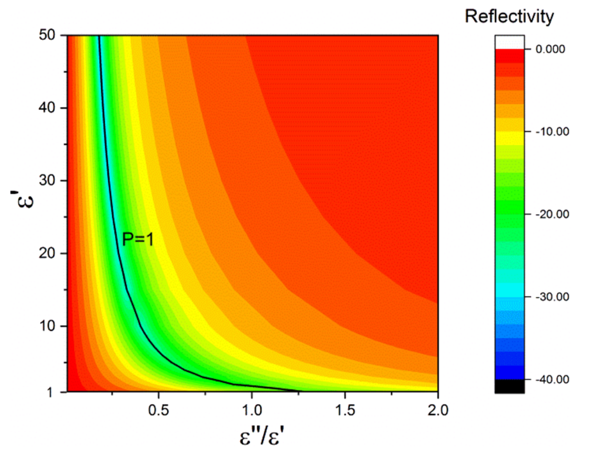


Fig. 1 Normal incidence reflectivity at 10 GHz with different permittivities

Meanwhile, d_1 should contain μ', and d_2 should remain the same:

$$d_{1m} = \frac{(2n+1)c}{4f\sqrt{\varepsilon'\mu'}} \tag{31}$$

Figure 1 shows the relationship between permittivity and reflectivity, the frequency is set at 10 GHz, which is the midpoint of the 2–18 GHz frequency band, and the material thickness is calculated by Eq. (25) with $n = 0$. The value range of ε' is from 1 to 50, $\varepsilon''/\varepsilon'$ is set as 0.1–2, thus setting a representative range of discussion. The curve of $P = 1$ is plotted, and it can be seen that the region closest to impedance matching is almost coincident with $P = 1$.

Imagine an ideal material whose ε' and ε'' satisfy Eq. (20) and (23) in a continuous frequency band when the thickness is given, then the ideal material has impedance matching everywhere in that frequency band. As shown in Fig. 2, when the thickness of the material is 2, 3, 4 mm, the ideal permittivity in the range of 2–18 GHz is calculated by Eq. (20) and (23), where n of Eq. (20) is 0. The corresponding reflectivity curves are shown in Fig. 3, which are very low in the entire frequency band 2–18 GHz. When the frequency is low, the value of ε' and ε'' differ significantly, the substitution condition of Eq. (9) is met, so it is closest to impedance matching, and the impedance matching performance is approaching the expectation. When the frequency becomes higher, the value of ε' and ε'' are close to each other, and the error caused by the approximation of Eq. (9) and Eq. (20) is gradually obvious.

3 Method proving

Consider a non-magnetic microwave absorbing material 2.5 vol% GNSs/MgO(Graphene nanosheets/magnesia composites) investigated in Ref. [29], its reflectivity curves, P value and matching thicknesses are shown

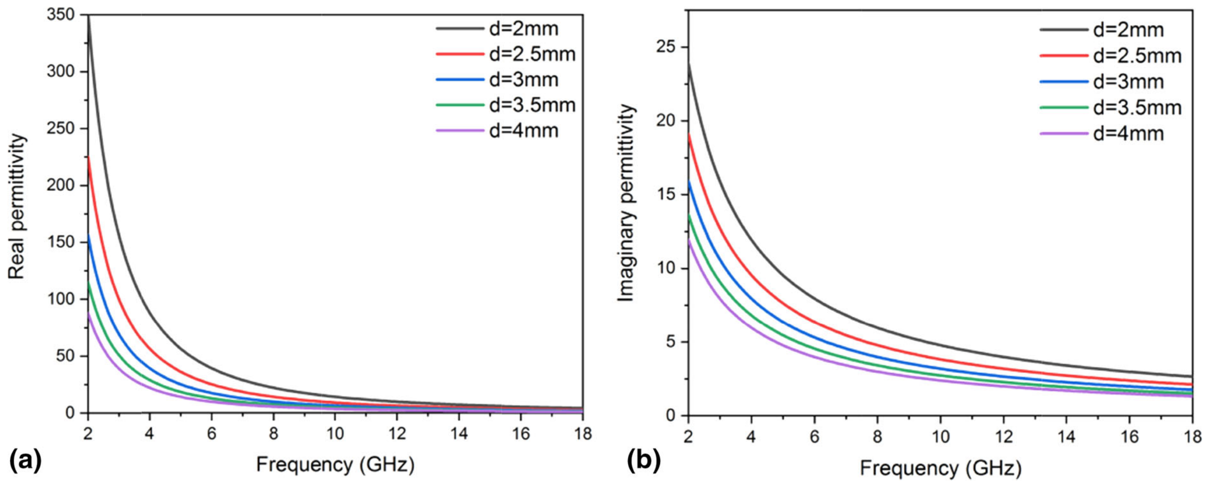
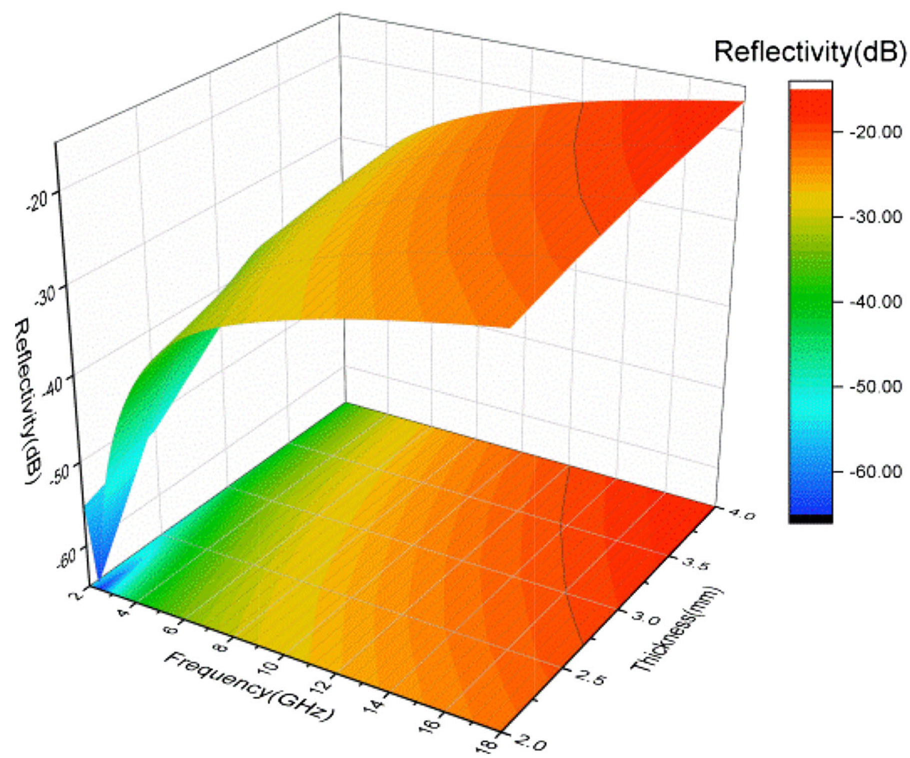


Fig. 2 Real part (**a**) and imaginary part (**b**) of ideal complex permittivity spectra for given thicknesses

Fig. 3 Normal incidence reflectivity curves of ideal materials



in Fig. 4. Two thicknesses were calculated, respectively $d = 1.53$ mm and $d = 1.48$ mm, and a better matching thickness was found near $d = 1.53$ mm, which is $d = 1.54$ mm, the difference may be caused by approximations of Eqs. (9) and (20). The reflectivity curves obtained from impedance matching thicknesses $d = 1.48$ mm and $d = 1.54$ mm are superior to the 1.5 mm curve in Ref. [29], yet neither of them was mentioned.

A low-loss magnetic material microwave absorbing material PB@MoS_2 (Prussian blue and MoS_2) is proposed in Ref. [30], samples with 40 wt% filler loading performed excellent absorption properties. Its reflectivity curves, P value and matching thicknesses are plotted in Fig. 5. Two points of $P = 1$ are found and the calculated values of matching thickness are 2.06 mm and 2.41 mm, respectively. The 2.06 mm curve is closer to impedance matching than the 2.1 mm curve in the original text, while the 2.41 mm curve is worse than the 2.5 mm curve in the original text, respectively reflecting the reliability and error of the approximate solution.

It is noticed that a non-magnetic material is mentioned in Ref [31] marked as GO (Graphene oxide), its reflectivity curves, P value and matching thicknesses are presented in Fig. 6. The curve of P intersects at one point with $P = 9$, three points with $P = 25$, and one point with $P = 49$, corresponding matching thicknesses were calculated. Thicknesses that closest to impedance matching were selected around the calculated thicknesses d_1 and d_2, with varying degrees of deviation. Accordingly, the frequencies where reflectivity peaks appears with the chosen thicknesses deviate from that determined by the P curve. Both the calcu-

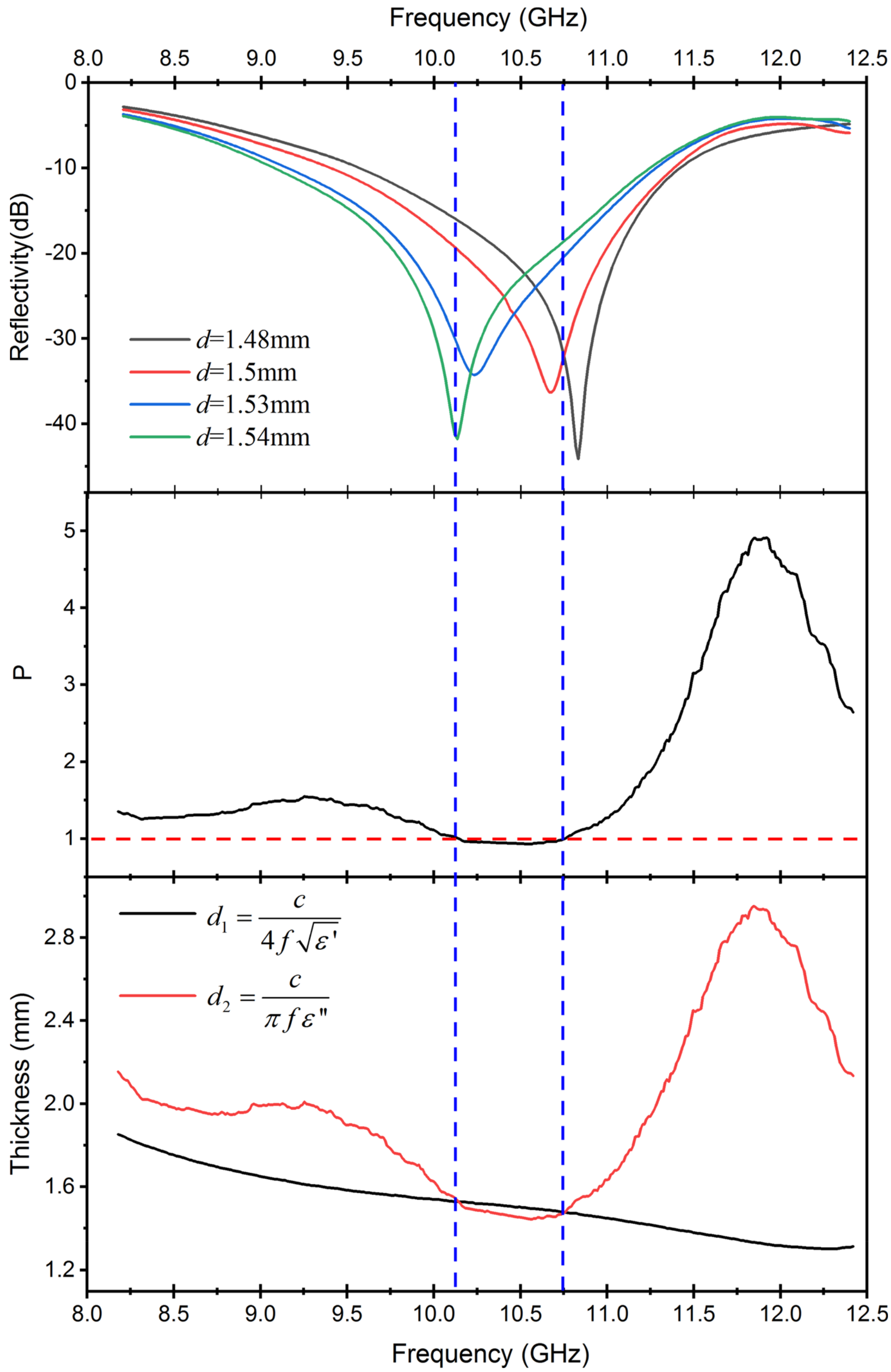


Fig. 4 Normal incidence reflectivity curves, P value and matching thicknesses of 2.5 vol% GNSs/MgO in Ref. [29]

lated and chosen thicknesses are far more than the value range discussed in Ref. [31]. Significant strong absorption peaks are obtained with the chosen thicknesses, while GO was believed to have negligible absorption performance in Ref. [31].

4 Conclusion

In this paper, for nonmagnetic homogeneous absorbing materials, a method to determine the frequency and thickness of impedance matching of materials by the mathematical relationship between ε' and ε'' is proposed. When the effect of permeability is negligible, the basic formalization of impedance matching can be simplified by equivalent infinitesimal substitution. Thus, an approximate solution of impedance matching is obtained, including only the simple mathematical relation of ε' and ε'', which is given by Eq. (24). An impedance matching eigenvalue P is defined to find the complex permittivity conforming to Eq. (24), shown in Eq. (27). Then, impedance matching would achieve at the frequency point where ε' and ε'' make $P = (2n{+}1)^2$

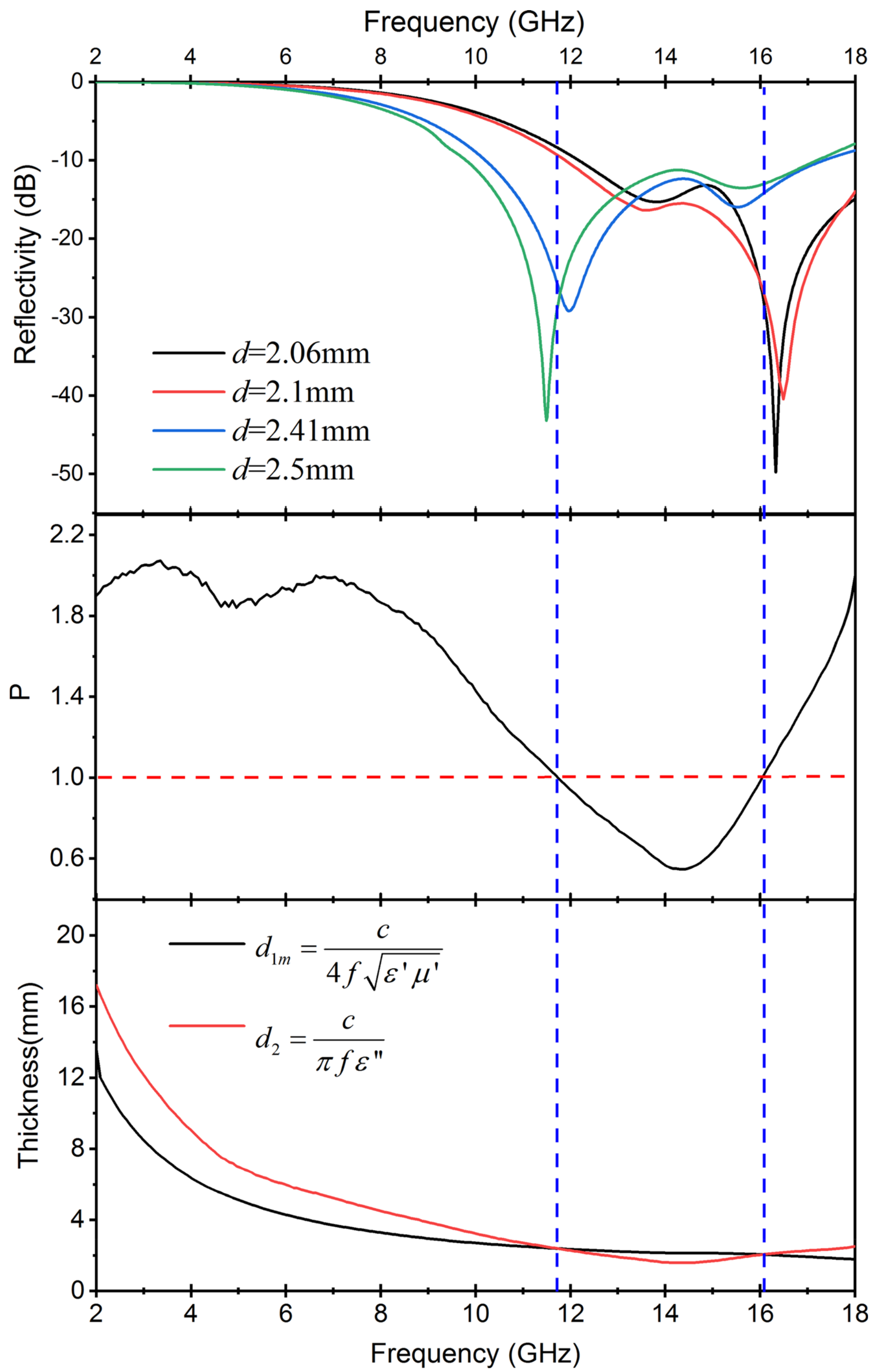
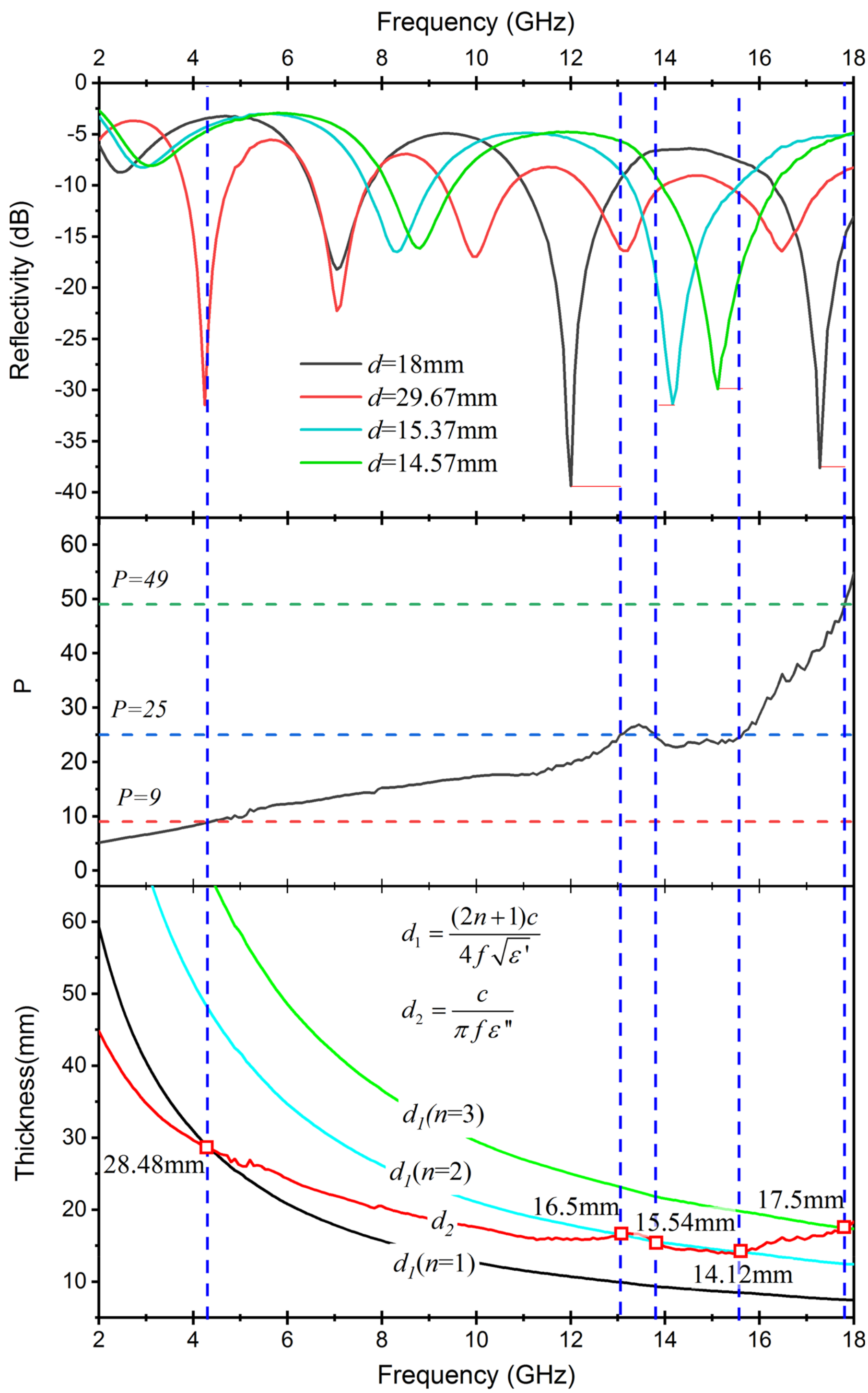


Fig. 5 Normal incidence reflectivity curves, P value and matching thicknesses of PB@MoS_2 in Ref. [30]

exists, and the required thicknesses is determined by Eq. (25) or (26). The impedance matching eigenvalue P_m and corresponding thickness d_{1m} of low-loss magnetic materials are obtained by the same way, as in Eqs. (30) and (31). The above methods are verified by data from available references and proved to be effective. It can be observed that there is some error between the calculated value and the actual value of the matching thickness, which may be caused by approximations in Eqs. (9) and (20). The closer ε' and ε'' are, the greater the error of the calculated thickness is.

Fig. 6 Normal incidence reflectivity curves, P values and matching thicknesses of GO in Ref. [31]



References

1. X. Guan et al., J. Alloys Compd. **893**, 162317 (2022)
2. M. Zhang et al., Nanotechnology **31**, 275707 (2020)
3. B. Fu et al., J. Mater. Sci. Mater. Electron. **32**, 9611 (2021)
4. Z. Peng, J.-Y. Hwang, M. Andriese, Appl. Phys. Express **5**, 077301 (2012)
5. C. Liao et al., RSC Adv. **8**, 15315 (2018)
6. Y. Wang et al., Ceram. Int. **44**, 3606 (2018)
7. H.M. Musal, H.T. Hahn, IEEE Trans. Magn. **25**, 3851 (1989)
8. H.M. Musal, D.C. Smith, IEEE Trans. Magn. **26**, 1462 (1990)
9. H.M. Musal, H.T. Hahn, G.G. Bush, J. Appl. Phys. **63**, 3768 (1988)
10. Y. Huang et al., Compos. Part B Eng. **163**, 598 (2019)
11. Z. Ye et al., J. Alloys Compd. **893**, 162396 (2022)
12. X. Zhao et al., J. Colloid Interface Sci. **607**, 192 (2022)
13. R. Shu et al., J. Colloid Interface Sci. **606**, 1918 (2022)

14. Y. Sun et al., Carbon **186**, 333 (2022)
15. K. Wang et al., ACS Appl. Nano Mater. **2**, 8063 (2019)
16. J.-P. Chen et al., Carbon **164**, 59 (2020)
17. B. Quan et al., Adv. Funct. Mater. **29**, 1901236 (2019)
18. G. Wan et al., J. Phys. D Appl. Phys. **49**, 235104 (2016)
19. X. Liu, Z. Zhang, Y. Wu, Compos. Part B Eng. **42**, 326 (2011)
20. H. Cai et al., J. Alloys Compd. **893**, 162227 (2022)
21. T. Wang et al., J. Colloid Interface Sci. **595**, 1 (2021)
22. S. Zhang et al., J. Phys. D Appl. Phys. **53**, 265004 (2020)
23. L.T. Hall et al., Microw. Opt. Technol. Lett. **45**, 491 (2005)
24. Y. Wang et al., J. Mater. Chem. A **9**, 24503 (2021)
25. B. Quan et al., Nano Res. **14**, 1495 (2020)
26. X. Huang et al., Nano Res. **14**, 4006 (2021)
27. Y. Li et al., Nano **13**, 1850125 (2018)
28. L. Wang et al., Carbon **155**, 298 (2019)
29. C. Chen et al., J. Eur. Ceram. Soc. **38**, 1639 (2018)
30. Z. Zhao et al., ACS Sustain. Chem. Eng. **7**, 7183 (2019)
31. Q. Zhang et al., Appl. Surf. Sci. **473**, 706 (2019)