



Chaos synchronization with coexisting global fields

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Received 12 August 2021 / Accepted 16 December 2021 / Published online 29 December 2021

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Abstract We investigate the phenomenon of chaos synchronization in systems subject to coexisting autonomous and external global fields by employing a simple model of coupled maps. Two states of chaos synchronization are found: (i) complete synchronization, where the maps synchronize among themselves and to the external field, and (ii) generalized or internal synchronization, where the maps synchronize among themselves but not to the external global field. We show that the stability conditions for both states can be achieved for a system of minimum size of two maps. We consider local maps possessing robust chaos and characterize the synchronization states on the space of parameters of the system. The state of generalized synchronization of chaos arises even when the drive and the local maps have the same functional form. This behavior is similar to the process of spontaneous ordering against an external field found in nonequilibrium systems.

1 Introduction

A global interaction in a system takes place when all its elements share a common influence or source of information. Global interactions occur in many physical, chemical, biological, social, and economic systems, such as parallel electric circuits, coupled oscillators [1, 2], charge density waves [3], Josephson junction arrays [4], multimode lasers [5], neural networks, evolution models, ecological systems [6], social networks [7], economic exchange [8], mass media influence [9, 10], and cross-cultural interactions [11]. A variety of phenomena can occur in systems subject to global interactions; for example, chaos synchronization, dynamical clustering, nontrivial collective behavior, chaotic itineracy [12, 13], chimera states [14, 15], quorum sensing [16]. These behaviors have been investigated in arrays of globally coupled oscillators in diverse experiments [17–21]. Global interactions can provide relevant descriptions in networks possessing highly interconnected elements or long-range interactions. Systems with global and local interactions have also been studied [22].

A global interaction field may consist of an external influence acting on all the elements of a system, as in a driven dynamical system [23]; or it may arise from the interactions between the elements, such as a mean field [24], in which case we have an autonomous dynamical system. In most situations, systems subject to either type of global field have been studied separately.

In this article, we investigate the dynamics of systems subject to the simultaneous influence of external and

autonomous global interaction fields. As a simple model for such systems, we study a network of coupled maps with coexisting external and autonomous global fields. Specifically, we focus on the important phenomenon of chaos synchronization. With this aim, we consider local map units possessing robust chaos dynamics. A chaotic attractor is robust if there exists a neighborhood in its parameter space where windows of periodic orbits are absent [25]. Robust chaos is an advantageous property in applications that require reliable functioning in a chaotic regime, since the chaotic behavior cannot be destroyed by arbitrarily small perturbations of the parameters.

In Sect. 2, we present the model for a system subject to coexisting autonomous and external global fields. We define two types of synchronization states for this system in relation to the external field, complete synchronization and generalized synchronization, and characterize them through statistical quantities. In Sect. 3, we carry out a stability analysis of the synchronization states and derive conditions for their stable behavior in terms of the system parameters. Section 4 contains applications of the model with coexisting global fields for maps exhibiting robust chaos. The states of chaos synchronization for the system and their stability boundaries are characterized on the space of parameters expressing the strength of the coupling to the global fields. In particular, we show that the state of generalized synchronization of chaos can appear even when the functional forms of the external field and the local maps are equal, a situation that does not occur for this family of maps if only one global field is present. This behavior represents a collective ordering of the system in a state

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alternative to that of the driving field. Conclusions are given in Sect. 5.

2 Coupled map network with autonomous and external global fields

As a model of a system of chaotic oscillators with coexisting autonomous and external global fields, we consider a network of coupled maps in the form

$$y_{t+1} = g(y_t), \tag{1}$$

$$x_{t+1}^i = (1 - \epsilon_1 - \epsilon_2) f(x_t^i) + \epsilon_1 h_t + \epsilon_2 g(y_t), \tag{2}$$

$$h_t = \frac{1}{N} \sum_{j=1}^N f(x_t^j), \tag{3}$$

where x_t^i represents the states variable of element i ($i = 1, 2, \dots, N$) at discrete time t ; N is the size of the system; $f(x_t^i)$ describes the local chaotic dynamics; $y_{t+1} = g(y_t)$ is an external global field that acts as a homogeneous drive with independent chaotic dynamics; $h_t(x_t^j)$ is an autonomous global field that corresponds to the mean field of the system; ϵ_1 is a parameter measuring the strength of the coupling of the elements to the mean field h_t ; and ϵ_2 expresses the intensity of the coupling to the external field. We assume a diffusive form of the coupling for both fields.

A synchronization state for the system Eqs. (1)–(3) occurs when the N elements share the same state; that is, $x_t^i = x_t^j, \forall i, j$. Then, the mean field becomes $h_t = f(x_t^i), \forall i$.

Two types of synchronization states can be defined in relation to the external global field $g(y_t)$: (i) complete synchronization, where the N elements in the system are synchronized among themselves and also to the external driving field; i. e., $x_t^i = x_t^j = y_t, \forall i, j$, or $h_t = f(x_t^i) = g(y_t)$; and (ii) internal or generalized synchronization, where the N elements get synchronized among themselves but not to the external global field; i. e., $x_t^i = x_t^j \neq y_t, \forall i, j$, or $h_t = f(x_t^i) \neq g(y_t)$.

To characterize the synchronization states of the system Eqs. (1)–(3), we calculate the asymptotic time average $\langle \sigma \rangle$ (after discarding transients) of the instantaneous standard deviation of the distribution of state variables σ_t , defined as

$$\sigma_t = \left[\frac{1}{N} \sum_{i=1}^N (x_t^i - \bar{x}_t)^2 \right]^{1/2}, \tag{4}$$

where

$$\bar{x}_t = \frac{1}{N} \sum_{i=1}^N x_t^i. \tag{5}$$

Additionally, we calculate the asymptotic time average $\langle \delta \rangle$ (after discarding transients) of the instantaneous difference

$$\delta_t = |\bar{x}_t - y_t|. \tag{6}$$

Then a complete synchronization state $x_t^i = x_t^j = y_t, \forall i, j$, where the maps are synchronized to the external global field, is characterized by the values $\langle \sigma \rangle = 0$ and $\langle \delta \rangle = 0$. An internal or generalized synchronization state $x_t^i = x_t^j \neq y_t, \forall i, j$, where the maps are synchronized among themselves but not the external field, corresponds to $\langle \sigma \rangle = 0$ and $\langle \delta \rangle \neq 0$.

In practice, we set the numerical condition $\langle \sigma \rangle < 10^{-7}$ and $\langle \delta \rangle < 10^{-7}$ for the zero values of these statistical quantities to characterize the above synchronization states.

3 Stability analysis of synchronized states

The system Eqs. (1)–(3) can be written in vector form as

$$\mathbf{x}_{t+1} = \mathbf{M}\mathbf{f}(\mathbf{x}_t), \tag{7}$$

where \mathbf{x}_t and $\mathbf{f}(\mathbf{x}_t)$ are $(N+1)$ -dimensional state vectors expressed as

$$\mathbf{x}_t = \begin{pmatrix} y_t \\ x_t^1 \\ x_t^2 \\ \vdots \\ x_t^N \end{pmatrix}, \quad \mathbf{f}(\mathbf{x}_t) = \begin{pmatrix} g(y_t) \\ f(x_t^1) \\ f(x_t^2) \\ \vdots \\ f(x_t^N) \end{pmatrix}, \tag{8}$$

and \mathbf{M} is the $(N + 1) \times (N + 1)$ matrix

$$\mathbf{M} = (1 - \epsilon_1 - \epsilon_2) \mathbf{I} + \frac{1}{N} \mathbf{C}, \tag{9}$$

where \mathbf{I} is the $(N + 1) \times (N + 1)$ identity matrix and \mathbf{C} is the $(N + 1) \times (N + 1)$ matrix that represents the coupling to the global fields, given by

$$\mathbf{C} = \begin{pmatrix} (\epsilon_1 + \epsilon_2)N & 0 & \dots & 0 \\ \epsilon_2 N & \epsilon_1 & \dots & \epsilon_1 \\ \vdots & \ddots & \vdots & \vdots \\ \epsilon_2 N & \epsilon_1 & \dots & \epsilon_1 \end{pmatrix}. \tag{10}$$

The linear stability condition for synchronization can be expressed in terms of the Lyapunov exponents for the system Eq. (7). This requires the knowledge of the $(N + 1)$ eigenvalues of matrix \mathbf{M} , given by

$$\mu_k = (1 - \epsilon_1 - \epsilon_2) + \frac{1}{N} c_k, \quad k = 1, 2, \dots, N + 1, \tag{11}$$

where c_k are the eigenvalues of matrix \mathbf{C} , corresponding to $c_1 = (\epsilon_1 + \epsilon_2)N$, $c_2 = \epsilon_1 N$, and $c_k = 0$ for $k > 2$, which is $(N - 1)$ -times degenerated.

Then the eigenvalues of matrix \mathbf{M} are

$$\mu_1 = 1, \tag{12}$$

$$\mu_2 = 1 - \epsilon_2, \tag{13}$$

$$\mu_k = 1 - \epsilon_1 - \epsilon_2, \quad k > 2, \text{ (} N - 1\text{)-times degenerated.} \tag{14}$$

The eigenvectors of the matrix \mathbf{M} satisfying $\mathbf{M}\mathbf{u}_k = \mu_k\mathbf{u}_k$ are also eigenvectors of the matrix \mathbf{C} , and they are given by

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{u}_k = \begin{pmatrix} 0 \\ a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}, \tag{15}$$

where the components a_k of the eigenvectors \mathbf{u}_k , $k > 2$, satisfy the condition $\sum_{k=1}^N a_k = 0$ since the eigenvectors \mathbf{u}_k are orthogonal to the eigenvectors \mathbf{u}_1 and \mathbf{u}_2 .

The eigenvectors of the matrix \mathbf{M} constitute a complete basis where the state \mathbf{x}_t of the system Eq. (7) can be expressed as a linear combination. In particular, the complete synchronization state of \mathbf{x}_t is associated to the eigenvector \mathbf{u}_1 , while the generalized synchronization state is represented by the eigenvector \mathbf{u}_2 .

The $(N + 1)$ Lyapunov exponents $(A_1, A_2, \dots, A_{N+1})$ of the system Eq. (7) are defined as

$$\begin{aligned} & (e^{A_1}, e^{A_2}, \dots, e^{A_{N+1}}) \\ &= \lim_{T \rightarrow \infty} (\text{magnitude of the eigenvalues of} \\ & \left| \prod_{t=0}^{T-1} \mathbf{J}(\mathbf{x}_t) \right|)^{1/T}, \end{aligned} \tag{16}$$

where \mathbf{J} is the Jacobian matrix of the system Eq. (7), whose components are

$$J_{ij} = \left[(1 - \epsilon_1 - \epsilon_2) \delta_{ij} + \frac{1}{N} c_{ij} \right] \frac{\partial [\mathbf{f}(\mathbf{x}_t)]_i}{\partial x_j}, \tag{17}$$

where c_{ij} are the ij -components of matrix \mathbf{C} and $[\mathbf{f}(\mathbf{x}_t)]_i$ is the i -component of vector $\mathbf{f}(\mathbf{x}_t)$. Then we obtain

$$e^{A_k} = \lim_{T \rightarrow \infty} \left| \mu_k^T \prod_{t=0}^{T-1} f'(x_t^k) \right|^{1/T}, \quad k = 1, \dots, N + 1, \tag{18}$$

where μ_k , $k = 1, 2, \dots, N + 1$, are the eigenvalues of matrix \mathbf{M} . Substitution of the eigenvalues μ_k , gives the Lyapunov exponents for the system Eq. (7),

$$A_1 = \lambda_g, \tag{19}$$

$$A_2 = \ln(1 - \epsilon_2) + \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \ln |f'(x_t^1)|, \tag{20}$$

$$A_k = \ln(1 - \epsilon_1 - \epsilon_2) + \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \ln |f'(x_t^k)|, \quad k > 2, \tag{21}$$

where λ_g is the Lyapunov exponent of the driven map $g(y_t)$, which is positive since $g(y_t)$ is assumed chaotic. Note that, in general, the limit terms in Eqs. (20) and (21) depend on ϵ_1 and ϵ_2 since the iterates x_t^i are obtained from the coupled system Eqs. (1)–(3). At synchronization, these terms are equal and we denote them by λ_f .

The stability of the synchronized state is given by the condition

$$e^{A_k} = |\mu_k e^{\lambda_k}| < 1, \tag{22}$$

where $\lambda_1 = \lambda_g$; $\lambda_k = \lambda_f$, for $k > 1$.

Perturbations of the state \mathbf{x}_t along the homogeneous eigenvector $\mathbf{u}_1 = (1, 1, \dots, 1)$ do not affect the coherence of the system; thus, the stability condition corresponding to the eigenvalue μ_1 is irrelevant for the complete synchronized state. Then condition Eq. (22) with the next eigenvalue μ_2 provides the range of parameter values where the complete synchronized state $x_t^i = y_t$, $\forall i$, is stable, i.e.,

$$|(1 - \epsilon_2)e^{\lambda_f}| < 1 \Rightarrow 1 - \frac{1}{e^{\lambda_f}} < \epsilon_2 < 1 + \frac{1}{e^{\lambda_f}}. \tag{23}$$

Equivalently, complete synchronization takes place when $A_2 < 0$.

On the other hand, the internal or generalized synchronization state \mathbf{x}_t of the system is proportional to the eigenvector $\mathbf{u}_2 = (1, 1, \dots, 0)$. The stability condition of this state is given by the next degenerate eigenvalue μ_k , $k > 2$, that is,

$$\begin{aligned} |(1 - \epsilon_1 - \epsilon_2) e^{\lambda_f}| < 1 \Rightarrow 1 - \epsilon_2 - \frac{1}{e^{\lambda_f}} \\ < \epsilon_1 < 1 - \epsilon_2 + \frac{1}{e^{\lambda_f}}. \end{aligned} \tag{24}$$

The condition for stable generalized synchronization can be also be expressed as $A_k < 0$.

Because of the eigenvalues Eqs. (12)–(14), the stability conditions Eqs. (23) and (24) can be achieved for any system size $N \geq 2$. Equations (23) and (24) describe the regions on the space of the coupling parameters (ϵ_1, ϵ_2) , where complete and internal synchronization can, respectively, occur in the system with coexisting global fields, Eqs. (1)–(3).

4 Applications

We consider the system Eqs. (1)–(3) with local dynamics described by the tent map

$$f(x_t^i) = \frac{r}{2} |1 - 2x_t^i|, \tag{25}$$

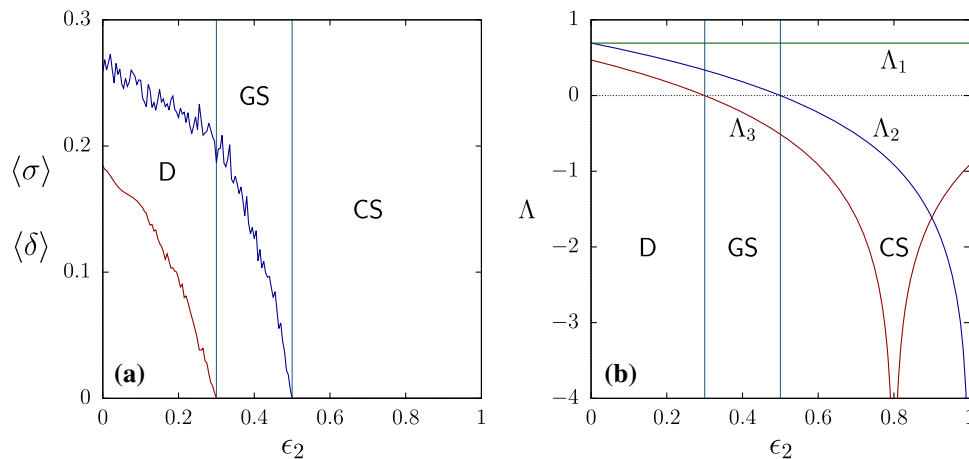


Fig. 1 **a** Statistical quantities $\langle\sigma\rangle$ (red line) and $\langle\delta\rangle$ (blue line) as functions of ϵ_2 for the system Eqs. (1)–(3) with local tent map Eq. (25) with $r = 2$, and external field $g = f$. Fixed $\epsilon_1 = 0.2$, size $N = 5000$. For each value of ϵ_2 both quantities are averaged over 20000 iterates after discarding 5000 transients. **b** Lyapunov exponents $\Lambda_1 = \lambda_g$ (black line), Λ_2 (blue line) and Λ_3 (red line) as functions of ϵ_2 for the sys-

tem Eqs. (1)–(3) with minimum size $N = 2$ and $g = f$ with fixed $\epsilon_1 = 0.2$. For each value of ϵ_2 the Lyapunov exponents were calculated with 25000 iterations after discarding 5000 transients. In this case, $\lambda_f = \lambda_g = \ln 2$. Labels on both figures indicate *D* desynchronized or incoherent state, *GS* generalized or internal synchronization, *CS* complete synchronization

which exhibits robust chaos for $r \in (1, 2]$ with $x_t^i \in [0, 1]$. We fix the local parameter at the value $r = 2$ and assume the external driving field equal to the local dynamics, i.e., $g = f$.

Figure 1a shows the statistical quantities $\langle\sigma\rangle$ and $\langle\delta\rangle$ that characterize the collective synchronization states for this system as functions of the coupling parameter ϵ_2 , with fixed $\epsilon_1 = 0.2$. System size is $N = 5000$. Labels indicate the regions of the parameter ϵ_2 where different synchronization states take place: *D* (desynchronized state) where $\langle\sigma\rangle \neq 0$ and $\langle\delta\rangle \neq 0$; *SG* (generalized or internal synchronization) corresponding to $\langle\sigma\rangle = 0$ and $\langle\delta\rangle \neq 0$; and *SC* (complete synchronization) characterized by $\langle\sigma\rangle = 0$ and $\langle\delta\rangle = 0$. Figure 1b shows the Lyapunov exponents Λ_1 , Λ_2 , and Λ_3 as functions of ϵ_2 , with fixed $\epsilon_1 = 0.2$, for a system of minimum size $N = 2$, since the stability conditions for the synchronized states are satisfied for $N \geq 2$. The Lyapunov exponent $\Lambda_1 = \lambda_g = \ln 2$ is positive. The transition of the exponent Λ_3 from positive to negative values signals de onset of stable generalized synchronization (GS), while $\Lambda_2 = 0$ indicates the boundary of the complete synchronization state (CS) that is stable for $\Lambda_2 < 0$. The corresponding boundaries of the regions GS and CS coincide exactly in both figures.

Figure 2 shows the collective synchronization states of the system Eqs. (1)–(3) with the local tent map and external drive $g = f$ with $r = 2$ on the space of parameters (ϵ_1, ϵ_2) . The regions on this space where the different states occur are indicated by labels. The boundaries of the synchronized states are calculated analytically from conditions Eqs. (23) and (24). These boundaries coincide with the criteria for the quantities $\langle\sigma\rangle$ and $\langle\delta\rangle$ characterizing each state, as explained above.

To understand the nature of the collective behaviors, Fig. 3 shows the attractors corresponding to the dif-

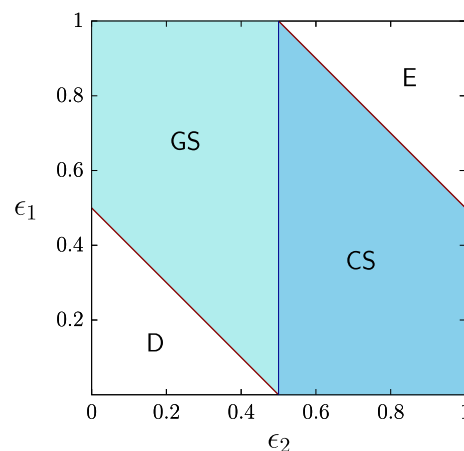


Fig. 2 Synchronization states for the system Eqs. (1)–(3) with local tent map and external drive $g = f$ on the space of parameters (ϵ_1, ϵ_2) . Fixed parameters: $r = 2, N = 5000$. Labels indicate the regions where these states can be found: *D* desynchronization, *GS* generalized synchronization, *CS* complete synchronization, *E* escape. The boundaries determined analytically with Eqs. (23) and (24) and by the quantities $\langle\sigma\rangle$ and $\langle\delta\rangle$ coincide exactly. The region labeled *E* corresponds to coupling parameter values (ϵ_1, ϵ_2) for which the state variables of the system escape to infinite. The boundary for region *E* is given by the upper stability boundary of the generalized synchronization state given by Eq. (24)

ferent synchronization states for the reduced size system Eqs. (1)–(3), with the local tent map and external drive $g = f$. The desynchronized state (*D*) in Fig. 3a has all positive Lyapunov exponents and shows no definite structure. In the internal synchronization state (*GS*) with $\Lambda_1 > 0$, $\Lambda_2 > 0$, and $\Lambda_3 < 0$, displayed in Fig. 3b, the dynamics collapses onto an attractor lying

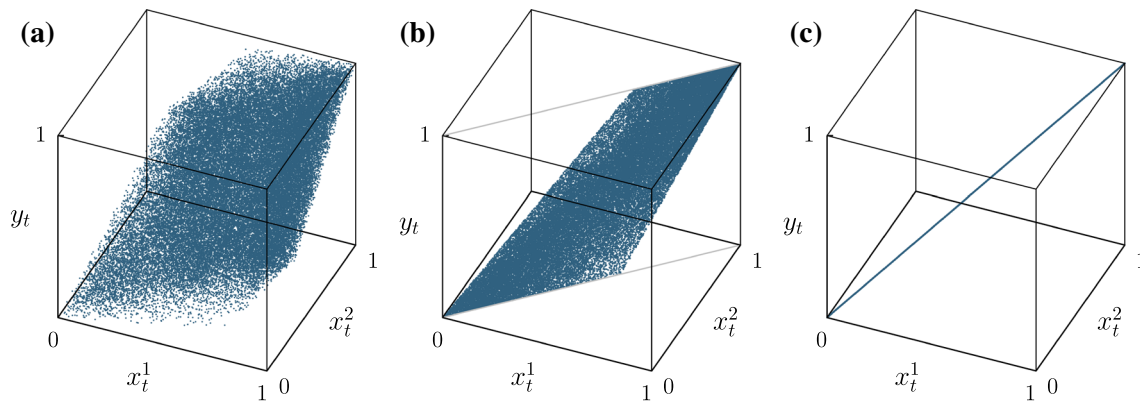


Fig. 3 Attractors on the three-dimensional phase space of the reduced size system Eqs. (1)–(3) with the local tent map and external drive $g = f$. Fixed parameters: $r = 2, \epsilon_1 = 0.2$.

a Desynchronized state (D), $\epsilon_2 = 0.2$. **b** Generalized synchronization (GS), $\epsilon_2 = 0.4$. **c** Complete synchronization (CS), $\epsilon_2 = 0.6$

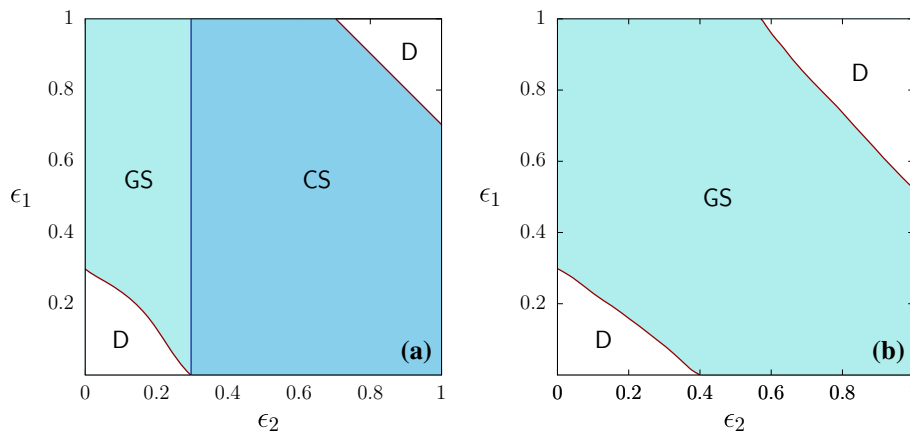


Fig. 4 Synchronization states for the system Eqs. (1)–(3) with local logarithmic map Eq. (26) and $N = 5000$ on the space of parameters (ϵ_1, ϵ_2) . **a** External drive function equal to the local dynamics, $g = f = -0.7 + \ln|x|$. **b** External drive different from the local map, with $g = 0.5 + \ln|x|$ and

$f = -0.7 + \ln|x|$. Labels indicate the states *D* desynchronization, *GS* generalized synchronization, *CS* complete synchronization. The boundaries determined from conditions Eqs. (23) (blue line) and (24) (red lines) coincide with those obtained with the quantities $\langle \sigma \rangle$ and $\langle \delta \rangle$

on the plane $x_t^1 = x_t^2$. This plane constitutes the synchronization manifold where $x_t^1 = x_t^2 = \bar{x}_t$. Thus, the chaotic attractor on this plane represents a nontrivial functional relation, different from the identity, between \bar{x}_t and the drive y_t . In general, for the state of generalized synchronization with $N > 2$, a chaotic attractor arises between the times series of the mean field \bar{x}_t and that of the drive signal y_t . The complete synchronized state (CS), possessing $\Lambda_1 > 0, \Lambda_2 < 0, \Lambda_3 < 0$, is characterized by the attractor lying along the diagonal line $x_t^1 = x_t^2 = y_t$, as shown in Fig. 3c. In this situation, $\bar{x}_t = y_t$.

The emergence of a chaotic attractor is a characteristic feature of generalized synchronization in a drive-response system when the drive function g is different from the response system function f . The generalized or internal synchronization state does not arise if only the external drive $g = f$ acts on the system of tent maps Eqs. (1)–(3), i.e., if $\epsilon_1 = 0$; nor can it appear with mean field coupling alone. The emergence of the GS

state in this system when $g = f$ requires the coexistence of both, the autonomous global field and the external drive. Thus, we have a situation where the presence of an autonomous global interaction allows the synchronization of the maps in a state alternative to that of the forcing external field.

As another example, we consider a local chaotic dynamics given the logarithmic map

$$f(x_t^i) = b + \ln|x_t^i|. \tag{26}$$

This map is unbound and possesses robust chaos, with no windows of periodicity, for the parameter interval $b \in [-1, 1]$ [26]. Figure 4a shows the synchronization states of the system Eqs. (1)–(3) with the local chaotic map Eq. (26) and external drive $g = f$ on the plane (ϵ_1, ϵ_2) . Labels indicate the regions where the different synchronization states take place. The boundaries of the synchronized states are calculated numerically from Eqs. (23) and (24). The lower boundary of the

GS state is calculated from Eq. (24) where $\lambda_f \neq \lambda_g$. For the upper boundary of the CS state, the local maps are already synchronized to the drive y_t and therefore $\lambda_f = \lambda_g$; thus Eq. (24) gives a straight line on the plane (ϵ_1, ϵ_2) . Figure 4b shows the synchronization states corresponding to an external drive $g \neq f$; only generalized synchronization (GS) can occur in this case. There is no escape in either situation, since the map dynamics is unbounded.

5 Conclusions

We have studied a coupled map model for a system subject to coexisting autonomous and external global fields. We have investigated the states of chaos synchronization in this system, consisting of (i) complete synchronization, where the maps synchronize among themselves and to the external global field, and (ii) generalized or internal synchronization, where the maps synchronize among themselves but not to the external field. The generalized synchronization state can be described by the appearance of a chaotic attractor between the time series of the mean field of the system and the external driving field.

We have performed the stability analysis for both synchronization states and found that the stability conditions can be achieved for a system of minimum size of two maps subject to a common drive. The equivalence of the dynamics for a minimum size system is a characteristic feature of systems with global interactions [27].

By considering local tent maps and logarithmic maps that possess robust chaos dynamics, we have focused on the chaotic synchronization behavior of the system. We have characterized the synchronization states on the space of the coupling parameters by using the stability conditions of these states as well as statistical quantities, with complete agreement in all cases. The emergence of the state of generalized synchronization of chaos, when the drive and the local maps have the same functional form, requires the presence of both global fields. This behavior is similar to the phenomenon of spontaneous ordering against an external field found in some nonequilibrium systems [9].

Our results suggest that, in addition to chaos synchronization, other collective behaviors either observed or absent in a system with only one type of global interaction can be modified when both autonomous and external global fields are present.

Acknowledgements This work was supported by Vice-Cancillería de Investigación e Innovación, Universidad Yachay Tech, Ecuador, through Proyectos de Fondos Internos.

Data Availability Statement This manuscript has associated data in a data repository. [Authors' comment: Please note that this is a theoretical and computational work; there is no experimental or statistical data repository associated with this manuscript. The authors have elaborated the computer programs employed in the numerical calcula-

tions, and they can be shared with interested researchers upon request.]

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