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Modelling mixing and transport of radioactive effluents in water reservoirs: an application to the operation of a fusion facility

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Abstract The aim of the present study is to model mixing and transport of radioactive effluents in the course of time between two water reservoirs. To test the model, a hypothetical case study is presented of liquid effluents potentially released during normal operation of a fusion neutron source devoted to radioisotopes production. A suitable example is an accelerator-driven intense D-T 14 MeV neutron source relying on T and D ion beams, with the potential to provide a neutron yield in the range $5-7 \cdot 10^{13}$ s^{-1}. It is expected that during normal operation a number of radionuclides will be produced and managed. The present report discusses the mechanisms and parameters which affect and control the fate of radionuclides potentially released into two connected water reservoirs during normal operation of the plant. A mathematical mixing model is developed that describes groundwater flow and radioactive transport between the two basins. The aim of this study is to estimate the amount of radioactivity concentration in both water reservoirs at any time, an information that can be used for radiation protection purposes.

1 Introduction

Intense D-T 14 MeV neutron sources provide a wide spectrum of applications spanning from fusion technology to medical applications. In particular, recent research has proved the feasibility of using a 14-MeV accelerator-driven neutron source to produce ^{99}Mo via the ^{100}Mo(n,2n)^{99}Mo reaction [1], [2]. This is a key feature, especially in the light of the experienced vulnerabilities of the ^{99}Mo/^{99m}Tc supply [3].

In this context, an area of major priority will be the management of the amounts of radioactive byproducts generated by activation reactions during the irradiation process [4]. As a starting hypothesis, we consider a source featuring a neutron emission rate of $5-7 \cdot 10^{13}$ s^{-1}, a preliminary operational plan for ^{99}Mo production that envisages a working load of 2 days per week and the production of about 5 Ci of ^{99}Mo obtained by irradiating about 10 kg of natural molybdenum every day. During each irradiation session, each sample is irradiated for 24 hours, and then processed. Molybdenum isotopes are recovered from the

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effluents. Therefore, about 1000 kg of natural molybdenum will be processed every year. With this operational programme, it is expected that a number of radionuclides will be produced during the normal operation of the experiment due to activation of the target. Table 1 shows the inventory of radionuclides obtained after 24 hours of irradiation of 10 kg of natural molybdenum and after 12 additional hours of processing of the irradiated sample in the radiochemical laboratory.

A hypothetical scenario is considered assuming that, during normal operation of the facility, every 3 days the produced radionuclides will be discharged as liquid effluents into a small water basin (lake X), without any filtering or preliminary treatment. As it often happens, the basin X is assumed to be connected by a reversible hydroelectric plant to a second water reservoir (lake Y).

In order to evaluate the impact on the environment of the potentially released substances, simple transport models in surface waters are available to describe the increase in environmental concentration of radioactive contaminants [5].

The international guidelines recommend to use these screening models with an iterative approach, that starts from a very simple assessment (the "no dilution model") to a generic environmental model accounting for the dispersion of radioactive materials in the environment, aimed to predict doses to the public. However, the more complex the situation, the more a site-specific model is needed.

In particular, when describing radionuclide transport in surface waters, generic methods for estimating radionuclide concentrations in water basins don't take into account the migration of water, and consequently radioactive contaminant, in the course of time among different connected water reservoirs, as often happens in actual situations.

To this aim, in the present study a mathematical mixing model is developed to estimate the amount of radioactivity concentration at any time in the two connected lakes of the scenario described.

The model can be adapted to describe any case of migration of contaminant between two different small water reservoirs, when in one of them some radioactive material is discharged.

2 The system of two connected water reservoirs

In the hypothetical scenario considered, during the day water flows from lake X (0.55 km^2; $6.6 \cdot 10^6$ m^3) to lake Y (1.5 km^2; $46 \cdot 10^6$ m^3) for the production of energy, while overnight water is pumped back to X from lake Y, thereby producing mixing of water contained in the two basins. The night flow is due to the hydroelectric plant, which during the day produces electricity through the penstocks that connect it to X and at night it inverts the turbines into pumps, filling back the X reservoir. The power of the pumps is assumed to be about 150 MW each, so that they pump about 72 m^3/s of water towards lake X, which is therefore able to fill up in 6 hours with the plant at full operation. The average flow rate of the penstocks over 12 hours is therefore 36 m^3/s. Lake Y is mainly fed by the penstocks connecting it to a third lake Z with a flow rate of 38 m^3/s, and itself feeds the hydroelectric power station through penstocks with a flow rate of 42 m^3/s.

The ENEA facility is supposed to discharge through a wastewater treatment system and collect negligible quantities for daily use compared to the hydroelectric flow.

Table 1 Inventory of radionuclides obtained after 24 hours of irradiation of 10 kg of natural molybdenum and after 12 additional hours of processing of the irradiated sample in the radiochemical laboratory

Radionuclide	Q(Bq/year)	λ (1/s)	Radionuclide	Q (Bq/year)	λ (1/s)
H-3	$2.36 \cdot 10^{6}$	$1.78 \cdot 10^{-9}$	Nb-91m	$1.38 \cdot 10^{11}$	$1.32 \cdot 10^{-7}$
Sr-89	$3.57 \cdot 10^{2}$	$1.59 \cdot 10^{-7}$	Nb-92	$3.09 \cdot 10^{2}$	$6.27 \cdot 10^{-16}$
Sr-90	$6.19 \cdot 10^{-1}$	$7.63 \cdot 10^{-10}$	Nb-92m	$4.21 \cdot 10^{11}$	$7.90 \cdot 10^{-7}$
Y-88	$4.44 \cdot 10^{5}$	$7.52 \cdot 10^{-8}$	Nb-93m	$2.74 \cdot 10^{7}$	$1.36 \cdot 10^{-9}$
Y-90	$2.22 \cdot 10^{5}$	$3.01 \cdot 10^{-6}$	Nb-94	$3.19 \cdot 10^{5}$	$1.10 \cdot 10^{-12}$
Y-90m	$7.86 \cdot 10^{3}$	$6.04 \cdot 10^{-5}$	Nb-95	$6.16 \cdot 10^{10}$	$2.29 \cdot 10^{-7}$
Y-91	$1.42 \cdot 10^{4}$	$1.37 \cdot 10^{-7}$	Nb-95m	$1.10 \cdot 10^{11}$	$2.22 \cdot 10^{-6}$
Zr-88	$4.24 \cdot 10^{7}$	$9.67 \cdot 10^{-8}$	Nb-96	$6.77 \cdot 10^{11}$	$8.25 \cdot 10^{-6}$
Zr-89	$3.30 \cdot 10^{11}$	$2.46 \cdot 10^{-6}$	Nb-97	$5.90 \cdot 10^{10}$	$1.60 \cdot 10^{-4}$
Zr-93	$1.11 \cdot 10^{3}$	$1.44 \cdot 10^{-14}$	Nb-98m	$1.74 \cdot 10^{7}$	$2.25 \cdot 10^{-4}$
Zr-95	$7.96 \cdot 10^{9}$	$1.25 \cdot 10^{-7}$	Tc-98	$1.48 \cdot 10^{-5}$	$5.11 \cdot 10^{-15}$
Zr-97	$5.40 \cdot 10^{10}$	$1.15 \cdot 10^{-5}$	Tc-99	$1.14 \cdot 10^{5}$	$1.03 \cdot 10^{-13}$
Nb-90	$4.39 \cdot 10^{4}$	$1.32 \cdot 10^{-5}$	Tc-99m	$1.37 \cdot 10^{13}$	$3.20 \cdot 10^{-5}$
Nb-91	$1.87 \cdot 10^{8}$	$3.23 \cdot 10^{-11}$	–	–	–

3 Reference scenario

The aim of the present section is to develop a system of differential equations describing the amount of activity $A(t)$ in lakes X and Y, at any time t. The migration of a radionuclide from a catchment is a very complex process that depends on a number of hydrological, climatic and geological characteristics of the constituent parts (subcatchments) of the drainage area. Unavoidably, a number of simplifications need to be introduced. In the present study, radionuclide sedimentation is not considered. Furthermore, two main assumptions are made:

- The activity concentration is uniform throughout the two lakes at any time.
- The volume of both lakes is constant during the entire cycle.

With reference to Fig. 2, in the present analysis the assumption is made that irradiation of the sample begins on Monday at 8.00 a.m. and stops on Tuesday 8.00 a.m (24-hour irradiation). The discharge of radionuclides reported in Table 1 begins on Tuesday at 8.00 p.m and stops at 9.00 p.m., 12 hours after the end of irradiation. The considered scenario is conservative and assumes a 1-hour release of radioactive byproducts every 72 hours (about 2 discharges per week). The 72 hour time slot is divided into three phases, detailed below.

Phase A. With reference to Fig. 1, from 8.00 p.m. to 9.00 p.m a given amount of radionuclides and a negligible quantity of water enter lake X at a certain rate Q and (once fully operational) are mixed with radioactive water already in the lake. During phase A there is no water flowing from lake X to lake Y ($b = 0$). On the other hand in the same time slot a pipeline connected to lake Y pumps water back into basin X with flow rate e. Fresh water flows regularly into lake Y (with flow rate f, Fig. 1) from lake Z and water flows out of lake Y with flow rate d.

Phase B. During phase B (9.00 p.m.–8.00 a.m. of the next day) there is no release of radionuclides into lake X, i.e. $Q = 0$. As in phase A, $b = 0$ and $e \neq 0$, i.e. water flows from lake Y

Table 2 Parameters used to asses the potential discharge of radionuclides from the fusion facility, typical of systems of lakes of the Italian Apennines

Parameters		Phase
V_1	$6.6 \cdot 10^9$ (L)	
V_2	$4.6 \cdot 10^{10}$ (L)	
Q	$\neq 0$ (Bq/h)	
b	0 (L/h)	Phase A
e	$1.30 \cdot 10^8$ (L/h)	$1h$
d	$1.50 \cdot 10^8$ (L/h)	
Q	$= 0$ (Bq/h)	
b	0 (L/h)	Phase B
e	$1.30 \cdot 10^8$ (L/h)	$11h$
d	$1.50 \cdot 10^8$ (L/h)	
Q	$= 0$ (Bq/h)	
b	$1.30 \cdot 10^8$ (L/h)	Phase C
e	0 (L/h)	$12h$
d	$1.50 \cdot 10^8$ (L/h)	

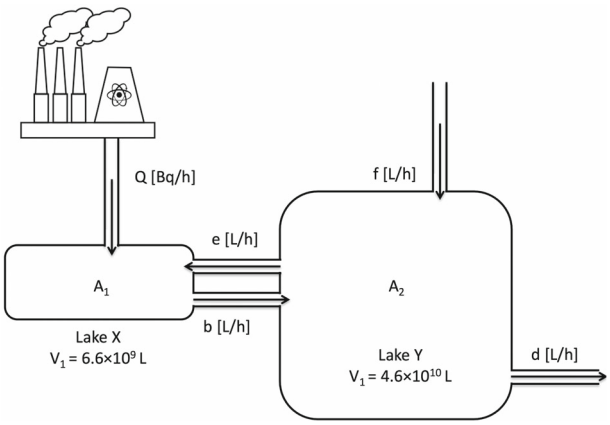

Fig. 1 Compartimental model used to assess the potential discharge of radionuclides from the fusion facility to lakes X and Y

to lake X, but no water flows out of lake X. At the same time, water enters and leaves lake Y with flow rate $f \neq 0$ and $d \neq 0$, respectively.

Phase C. The water flow is reversed, and water flows from lake X to lake Y with flow rate $b \neq 0$, while no water is pumped back from lake Y to X, i.e. $e = 0$. As in the previous phases, water regularly enters and leaves lake Y with flow rate $f \neq 0$ and $d \neq 0$, respectively. The complete workflow is detailed in Fig. 1.

Phases B and C are repeated every 12 hours for 72 hours, after which another discharge occurs (Phase A, Friday night, 8.00 p.m.–9.00 p.m.). The conservative assumption is made that the entire cycle is repeated over one year of operation.

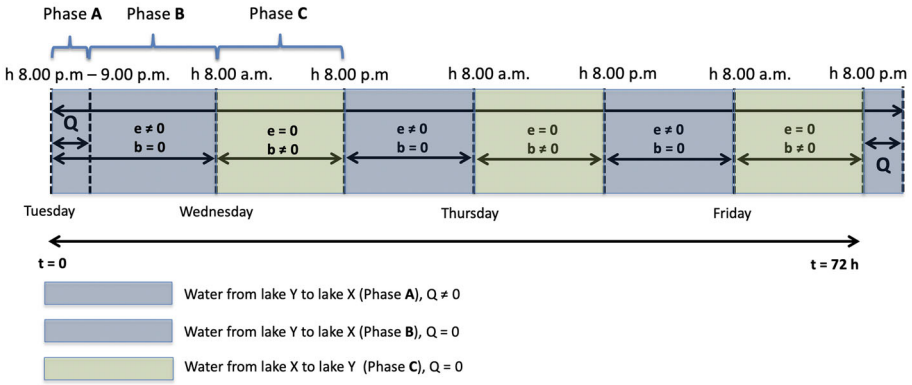

Fig. 2 Scenario of release of radionuclides considered in this study

4 Mathematical mixing model

As a general rule, the scenario described in Sect. 3 assumes that radionuclides mixed with water enter the two lakes at a certain rate, are mixed with what is already in the lake, and the mixture leaves at a certain rate. The aim of the present analysis is to derive a set of differential equations to model the reference scenario, as detailed in Fig. 2. The independent variable will be the time, t, while the dependent variable will be activity A in the water basins. The basic principle determining the differential equation is:

$$\frac{dA}{dt} = \text{rate of activity entering} - \text{rate of activity leaving} \tag{1}$$

With reference to Fig. 1 the solution to the following system of ordinary differential equations provides A_1 and A_2, i.e. the activity in lake X and Y, respectively, at any time t:

$$\frac{dA_1}{dt} = Q - (\frac{b}{V_1} + \lambda)A_1 + (\frac{e}{V_2})A_2 \tag{2}$$

$$\frac{dA_2}{dt} = (\frac{b}{V_1})A_1 - (\frac{e+d}{V_2} + \lambda)A_2 \tag{3}$$

with λ decay constant of the considered radionuclide, Q release rate of the radionuclide, b, e, d water flow rates in and out of the two basins, V_1 and V_2 volume of lake X and Y, respectively (Fig. 1). Equations (2) and (3) don't contain the flow rate f as this quantity represents a freshwater stream that carries no radioactivity into lake Y. Table 2 lists the parameters used in the model.

The Laplace transform method can be used to solve the system of ordinary differential Eqs. (2 and 3). We consider a first-order linear system with constant coefficients [7]:

$$y_1' = a_{11}y_1 + a_{12}y_2 + g_1(t) \tag{4}$$

$$y_2' = a_{21}y_1 + a_{22}y_2 + g_2(t) \tag{5}$$

writing $Y_1 = \mathcal{L}(y_1)$, $Y_2 = \mathcal{L}(y_2)$, $G_1 = \mathcal{L}(g_1)$, $G_2 = \mathcal{L}(g_2)$, we obtain the following subsidiary system:

$$sY_1 - y_1(0) = a_{11}Y_1 + a_{12}Y_2 + G_1(s) \tag{6}$$

$$sY_2 - y_2(0) = a_{21}Y_1 + a_{22}Y_2 + G_2(s) \tag{7}$$

By collecting the Y_1 and Y_2 terms we have:

$$(a_{11} - s)Y_1 + a_{12}Y_2 = -y_1(0) - G_1(s) \tag{8}$$

$$a_{21}Y_1 + (a_{22} - s)Y_2 = -y_2(0) - G_2(s) \tag{9}$$

By solving this system algebrically for $Y_1(S)$ and $Y_2(s)$ and taking the inverse transform we obtain the solutions $y_1 = \mathcal{L}^{-1}(Y_1)$ and $y_2 = \mathcal{L}^{-1}(Y_2)$ of the given system.

4.1 Solutions for phase A

In this scenario, the assumption is made that despite being $e \neq 0$, there is no volume change during the discharge of radionuclides into lake X. Equations (2) and (3) become:

$$\frac{dA_1}{dt} = Q - \lambda A_1 + (\frac{e}{V_1})A_2 \tag{10}$$

$$\frac{dA_2}{dt} = -(\frac{e+d}{V_2} + \lambda)A_2 \tag{11}$$

Using the formalism described by Eq. (4), Eqs. (10) and (11) become:

$$y_1' = a_{11}y_1 +_{12} y_2 + g_1(t) \tag{12}$$

$$y_2' = a_{22}y_2 \tag{13}$$

with:

$$\begin{aligned} y_1' &= dA_1/dt \\ a_{11} &= -\lambda \\ a_{12} &= -(\frac{e}{V_1}) \\ g_1(t) &= Q \\ a_{21} &= 0 \\ a_{22} &= -(\frac{e+d}{V_2} + \lambda) \end{aligned}$$

4.1.1 ***Solution for phase A, t=0 (First day of operation, 8.00 p.m.–9.00 p.m)***

At $t = 0$ there is no build up of radioactivity into lake Y, i.e. $A_2 = 0$, and equation (12) becomes:

$$y_1' = a_{11}y_1 + g_1(t) \tag{14}$$

which can be solved according to the solution method proposed by equation (8), with $y_1(0) = 0$:

$$(a_{11} - s)Y_1 = -Q/s \tag{15}$$

which provides:

$$Y_1 = \frac{Q}{s(s - a_{11})} \tag{16}$$

The solutions to equation (16) can be obtained using the following property of the inverse Laplace transform:

$$\boxed{y_1(t) = \mathcal{L}^{-1}(\frac{K}{s(s+a)}) = \frac{K}{a}[1 - exp(-at)]} \tag{17}$$

4.1.2 Solution for phase A, $t \neq 0$

When $t \neq 0$, the initial conditions at any abritrary time t can be assumed to be $y_1(t) = \alpha$ and $y_2(t) = \beta$. Equations (8) and (9) then become:

$$(a_{11} - s)Y_1 + a_{12}Y_2 = -\alpha - Q/s \tag{18}$$

$$(a_{22} - s)Y_2 = -\beta \tag{19}$$

This leads us to the following result:

$$Y_1 = \frac{s^2\alpha + s(a_{12}\beta - \alpha a_{22} + Q) - Qa_{22}}{s(s - a_{11})(s - a_{22})} \tag{20}$$

$$Y_2 = \frac{\beta}{(s - a_{22})} \tag{21}$$

The solution to Eq. (20) can be obtained using the linearity properties of the inverse Laplace transform. Writing:

$$Y_1 = Y_a + Y_b + Y_c \tag{22}$$

with:

$$\begin{aligned} Y_a &= \frac{s\alpha}{(s - a_{11})(s - a_{22})} \\ Y_b &= \frac{(a_{12}\beta - \alpha a_{22} + Q)}{(s - a_{11})(s - a_{22})} \\ Y_c &= \frac{-Qa_{22}}{s(s - a_{11})(s - a_{22})} \end{aligned} \tag{23}$$

Then:

$$\mathcal{L}^{-1}(Y_1) = \mathcal{L}^{-1}(Y_a) + \mathcal{L}^{-1}(Y_b) + \mathcal{L}^{-1}(Y_c) \tag{24}$$

The solution to (24) can be obtained using the following key properties of the inverse Laplace transform:

$$\boxed{\mathcal{L}^{-1}(Y_a) \rightarrow \mathcal{L}^{-1}\left(\frac{sK}{(s + a)(s + b)}\right) = \frac{K}{b - a}[(b)exp(-bt) - (a)exp(-at)]} \tag{25}$$

$$\boxed{\mathcal{L}^{-1}(Y_b) \rightarrow \mathcal{L}^{-1}\left(\frac{K}{(s + a)(s + b)}\right) = \frac{K}{b - a}[exp(-at) - exp(-bt)]} \tag{26}$$

$$\boxed{\mathcal{L}^{-1}(Y_c) \rightarrow \mathcal{L}^{-1}\left(\frac{K}{s(s+a)(s+b)}\right) = \frac{K}{ab}\left[1 - \frac{b}{b-a}exp(-at) + \frac{a}{b-a}exp(-bt)\right]} \tag{27}$$

On the other hand, the solution to equation (21) can be obtained using the following property of the inverse Laplace transform:

$$\boxed{y_2(t) = \mathcal{L}^{-1}\left(\frac{K}{(s + a)}\right) = Kexp(-at)} \tag{28}$$

being K, a and b constant terms.

4.2 Solutions for phase B

In this scenario, Eqs. (2) and (3) become:

$$\frac{dA_1}{dt} = -\lambda A_1 + (\frac{e}{V_2})A_2 \tag{29}$$

$$\frac{dA_2}{dt} = -(\frac{e+d}{V_2} + \lambda)A_2 \tag{30}$$

Since during phase B there is no release of radioactive liquids from X to Y, $Q = 0$. As a consequence, using the formalism described by Eqs. (4 and 5), Eqs. (29 and 30) become:

$$y_1' = a_{11}y_1 + a_{12}y_1 \tag{31}$$

$$y_2' = a_{22}y_2 \tag{32}$$

with:

$$\begin{aligned} y_1' &= dA_1/dt \\ y_2' &= dA_2/dt \\ a_{11} &= -\lambda \\ a_{12} &= (\frac{e}{V_2}) \\ a_{21} &= 0 \\ a_{22} &= -(\frac{e+d}{V_2} + \lambda) \end{aligned}$$

4.2.1 ***Solution for phase B, t=0 (First day of operation, 9.00 p.m. 8.00 p.m.)***

At $t = 0$, $A_2 = 0$, $Q = 0$ and a given amount of radioactivity has build up in lake X, so that $y_1(t) = \alpha$ and equation (31) becomes:

$$Y_1 = \frac{\alpha}{(s - a_{11})} \tag{33}$$

which provides the following solution:

$$\boxed{y_1(t) = \mathcal{L}^{-1}(\frac{K}{(s+a)}) = Kexp(-at)} \tag{34}$$

As expected, since there is no source term Q and there is no water flowing from lake X to Y, the activity build up in lake X decays exponentially with decay constant $a_{11} = \lambda$.

4.2.2 Solution for phase B, $t \neq 0$

After a given time t, some amount of radioactivity has build up both in X and Y ($A_2 \neq 0$). Assuming $y_1(t) = \alpha$ and $y_2(t) = \beta$ Equations (8) and (9) then become:

$$(a_{11} - s)Y_1 + a_{12}Y_2 = -\alpha \tag{35}$$

$$(a_{22} - s)Y_2 = -\beta \tag{36}$$

This leads us to the following result:

$$Y_1 = \frac{s\alpha - \alpha a_{22} + \beta a_{12}}{s^2 - s(a_{22} + a_{11}) + (a_{11}a_{22})} \tag{37}$$

$$Y_2 = \frac{\beta}{(s - a_{22})} \tag{38}$$

For each radionuclide, the solutions to equations (37) and (38) can be obtained using the following properties of the inverse Laplace transform:

$$\boxed{y_1(t) = \mathcal{L}^{-1}\left(\frac{K(s+c)}{(s+a)(s+b)}\right) = \frac{K}{b-a}[(c-a)exp(-at) - (c-b)exp(-bt)]} \tag{39}$$

$$\boxed{y_2(t) = \mathcal{L}^{-1}\left(\frac{K}{(s+a)}\right) = Kexp(-at)} \tag{40}$$

being K, a, b and c constant terms.

4.3 Solutions for phase C

In this scenario, Eqs. (2 and 3) become:

$$\frac{dA_1}{dt} = -\left(\frac{b}{V_1} + \lambda\right)A_1 \tag{41}$$

$$\frac{dA_2}{dt} = \left(\frac{b}{V_1}\right)A_1 - \left(\frac{d}{V_2} + \lambda\right)A_2 \tag{42}$$

Using the formalism described by Eqs. (4 and 5, 41 and 42) become:

$$y_1' = a_{11}y_1 \tag{43}$$

$$y_2' = a_{21}y_1 + a_{22}y_2 \tag{44}$$

with:

$$\begin{aligned} y_1' &= dA_1/dt \\ y_2' &= dA_2/dt \\ a_{11} &= -\left(\frac{b}{V_1} + \lambda\right) \\ a_{12} &= 0 \\ a_{21} &= \left(\frac{b}{V_1}\right) \\ a_{22} &= -\left(\frac{d}{V_2} + \lambda\right) \end{aligned}$$

4.3.1 ***Solution for phase C, t=0 (First day of operation, 8.00 p.m. 8.00 a.m.)***

At $t = 0$, $y_2 = 0$, $Q = 0$ and a given amount of radioactivity has build up in lake X $y_1(t) = \alpha$, which begins flowing into lake Y. Equations (41 and 42) become:

$$(a_{11} - s)Y_1 = -\alpha \tag{45}$$

$$a_{21}Y_1 + (a_{22} - s)Y_2 = 0 \tag{46}$$

This leads us to the following result:

$$Y_1 = \frac{\alpha}{(s - a_{11})} \tag{47}$$

$$Y_2 = \frac{a_{21}\alpha}{(s - a_{22})(s - a_{11})} \tag{48}$$

The solutions to Eqs. (47 and 48) can be obtained using the following properties of the inverse Laplace transform:

$$\boxed{y_1 = \mathcal{L}^{-1}(\frac{K}{(s+a)}) = Kexp(-at)} \tag{49}$$

$$\boxed{y_2 = \mathcal{L}^{-1}(\frac{K}{(s - a_{22})(s - a_{11}))} = \frac{K}{b-a}[exp(-at) - exp(-bt)]} \tag{50}$$

4.3.2 Solution for phase C, $t \neq 0$

Assuming that after a given time t $y_1(t) = \alpha$ and $y_2(t) = \beta$, equations (8) and (9) become:

$$(a_{11} - s)Y_1 = -\alpha \tag{51}$$

$$a_{21}Y_1 + (a_{22} - s)Y_2 = -\beta \tag{52}$$

This leads us to the following result:

$$Y_1 = \frac{\alpha}{(s - a_{11})} \tag{53}$$

$$Y_2 = \frac{s\beta - \beta a_{11} + \alpha a_{21}}{s^2 - s(a_{22} + a_{11}) + (a_{11}a_{22})} \tag{54}$$

For each radionuclide, the solutions to Eqs. (53 and 54) can be obtained using the following properties of the inverse Laplace transform:

$$\boxed{Y_1 = \mathcal{L}^{-1}(\frac{K}{(s+a)}) = Kexp(-at)} \tag{55}$$

$$\boxed{Y_2 = \mathcal{L}^{-1}(\frac{K(s+c)}{(s+a)(s+b)}) = \frac{K}{b-a}[(c-a)exp(-at) - (c-b)exp(-bt)]} \tag{56}$$

being K, a and b constant terms.

5 Results and discussions

The model described by the set of ordinary differential Eqs. (2 and 3) was applied to the reference scenario illustrated in Sect. 3 and detailed in Fig. 2. The solutions derived in Sect. 4 were implemented to assess A_1 and A_2, i.e. the activity in lake X and Y, respectively, at any time t, using the parameters reported in Table 2. The solutions to Eqs. (2 and 3) derived in Sect. 4 were implemented to each of the radionuclides listed in Table 1. The results are presented in Figs. 3 and 4 up to $t = 350$ h, i.e. 15 days of full operation, beginning at time $t = 0$.

Figure 3 shows the activity concentration in lake X. The peak discharge of radionuclides (phase A) results in the highest activty concentration in the basin (about 25 Bq/L). As expected, peak discharges occur every 72 hours. Due to the largest Q value ($1.37 \cdot 10^{13}$

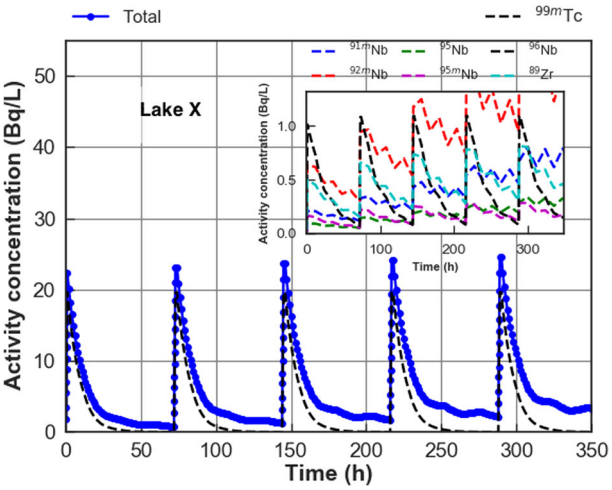

Fig. 3 Total activity concentration (Bq/L) and activity concentration of the most relevant radionuclides for lake X

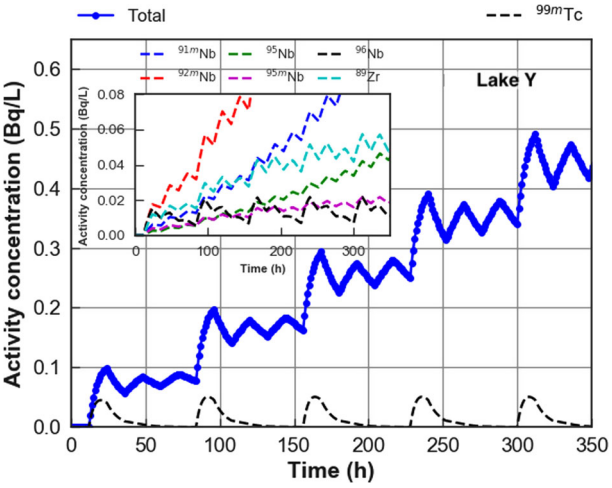

Fig. 4 Total activity concentration (Bq/L) and activity concentration of the most relevant radionuclides for lake Y

Bq/year) ^{99m}Tc is the radionuclide contributing the most to the early activity concentration in lake X. However, its short half-life (about 6 hours) produces a quick drop of the activity concentration after the first 12 hours (phase B). As expected, a further decrease in the activity concentration is observed during phase C, when water flows from lake X to lake Y. Due to the high Q values and to their relatively long half-lives, ^{91m}Nb, ^{92m}Nb, ^{95}Nb, ^{95m}Nb, ^{96}Nb, ^{89}Zr also contribute considerably to the total activity concentration. Of note, ^{92m}Nb (half life 10.15 days), ^{91m}Nb (half life 60.86 days) and ^{89}Zr (half life 3.27 days) tend to build up in lake X, contributing to an increase of the minimum activity concentration over time.

Similar considerations apply to the activity concentration in lake Y. Figure 4 is interesting in several ways. First, due the larger volume the activity concentration in lake Y is significantly

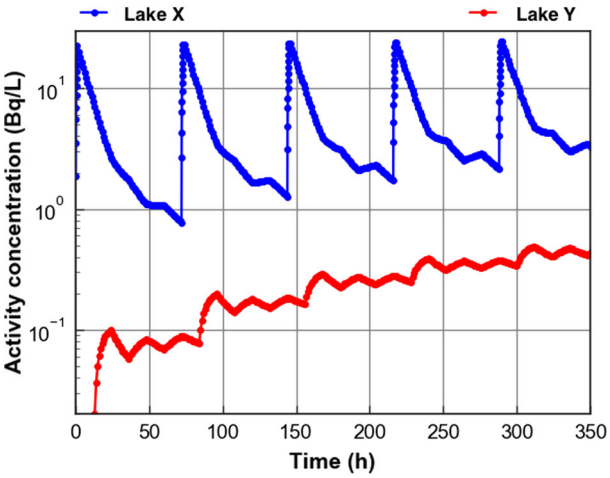

Fig. 5 Total activity concentration (Bq/L) in lake X and lake Y over the first 15 days of operation

lower than in lake X. Furthermore, since the rate of activity concentration entering the lake is considerably higher than the rate of activity concentration leaving the basin, an overall increase in the activity concentration can be observed over time, with ^{92m}Nb, ^{91m}Nb and ^{8}Zr being the radionuclides contributing the most in the long term due to their high Q values and relatively long half life. Ultimately, Fig. 5 compares the total activity concentration in lake X with the same quantitiy in lake Y.

Estimating the uncertainty of the proposed model is a challenging issue, being itself an error-prone process. A major source of uncertainty is likely to be the assumption that the activity concentration of the radionuclides is uniform throughout the lakes, at any time. Furthermore, the presented model does not account for deposition of radionuclides in the sediments. In fact, the evaluation of radionuclide fluxes from water to sediment, mainly by deposition of suspended matter, is particularly important for accurately predicting the temporal changes in the contamination levels of water. The radionuclide sedimentation is a complex process depending on a great number of factors, among which velocity and turbulence of water. Moreover, it depends on the processes of interaction of radionuclides with suspended matter that vary according to the chemical and physical characteristics of the aquatic environment and of the suspended matter [8]. As a consequence, further studies which take radionuclide sedimentation into account need to be undertaken. The best method of assessing the uncertainty associated with the proposed model will be by comparison of model predictions with empirical data.

6 Conclusions

This study presents a mathematical mixing model describing the migration of radioactive contaminant in the course of time between two water reservoirs, chosen as small lakes according to the definition of IAEA [5].

This model has been applied to a hypothetical scenario of an accelerator-driven fusion neutron source used for production of radioisotopes, taking as reference example ^{99}Mo [2], describing the timing of the potential liquid radioactive discharge on a daily basis. It proved

to be a valuable tool for improving the understanding of radioactive contaminant transport, making it possible to predict the concentration of radioactive material also in a basin connected to the one where the discharge takes place.

Hence, this model is feasible for environmental screening in complex contaminant dispersion situations, such as the scenario considered in this study, to plan the necessary actions to reduce the impact of radioactive releases on the environment and on human health during the operation of a plant.

Activity concentration data obtained from the present analysis were used to assess the effective dose to the representative person due to ingestion of water withdrawn from the two lakes of the scenario considered. The results of this analysis are presented in [4].

The mathematical model can be adapted to any systems of small water reservoirs, to evaluate the concentration of contaminant during time due to liquid radioactive discharges in the environment. Further work will be performed to model potential long term radiological impacts of the operation of a facility over the entire plant lifetime.

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